Life-Cycle Unemployment, Retirement, and Parametric Pension Reform*

WALTER H. FISHER AND CHRISTIAN KEUSCHNIGG

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Abstract

This paper investigates the consequences of pension reform for life-cycle unemployment and retirement. We find that (i) improving actuarial fairness in pension assessment not only boosts old age participation but also reduces unemployment among prime age workers and raises welfare; (ii) strengthening the tax benefit link boosts life-cycle labor supply on all margins and welfare; (iii) excluding unemployment benefits from the pension assessment base reduces unemployment, encourages later retirement and boosts efficiency; and (iv) extending the calculation period favors employment of young workers, might possibly lead to more unemployment among older ones, encourages postponed retirement and most likely yields positive welfare gains.

Keywords: Pensions, tax benefit link, retirement, unemployment.


Fisher: Institute for Advanced Studies, Stumpergasse 56, A-1060 Vienna, Austria. Ph: +43-159991-253; Fax: -555; Email: fisher@ihs.ac.at.

Keuschnigg: University of St. Gallen (FGN-HSG), CEPR, CESifo and Netspar, Varnbühlstrasse 19, CH-9000 St. Gallen, Switzerland. Ph: +41-71-224-2311; Fax: -2874; Email: christian.keuschnigg@unisg.ch.

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1 Introduction

The effects of the public pension system on economic performance is an issue that attracts ongoing interest from policy makers and scholars. Beyond issues of fiscal sustainability, one of the key concerns is whether labor market outcomes are adversely affected by the existing design of Pay-as-you-go (PAYG) systems. A large literature has studied these issues, see Feldstein and Liebman (2002), Bovenberg (2003), Lindbeck and Persson (2003) and Fenge and Pestieau (2005) for a few important reviews. Part of the recent policy debate, particularly in the U.S., has focussed on the choice between increased capital funding (e.g. Kotlikoff (1997), Feldstein (2005a,b), and Feldstein and Samwick (2002)) versus parametric reform of existing pay-as-you-go (PAYG) systems (e.g. Diamond (2004), Diamond and Orszag (2005)). Apart from its impact on national savings, the potential labor market implications of public pensions have played an important role in this debate.

The crucial question is the extent to which the contributions to an earnings linked pension system are actually perceived as a tax. The effective tax rate can amount to roughly half of the statutory contribution rate, as recent calculations for Germany by Fenge and Werding (2004) have shown. Beginning with Feldstein and Samwick (1992), the literature has calculated a much higher tax component for young workers far from retirement, while the effective tax is, in contrast, much lower for workers nearing retirement. Disney (2004) provided recent computations of the effective tax rates implied by PAYG contributions and econometric estimates of the employment effects. The results are consistent with usual findings of the empirical literature on intensive labor supply, namely that male employment is not particularly responsive to tax incentives, while female activity rates are highly adversely affected by the effective contribution tax.

According to the influential studies of Gruber and Wise (1999, 2004), a serious problem associated with PAYG systems is that they impose significant disincentives for labor market participation of older workers.\footnote{Gruber and Wise (1999) calculate the implicit tax rates of working another year for a number of} Gruber and Wise (2005) provide calculations for the relationship between later retirement and the additional benefits that lead to
actuarial fairness. Börsch-Supan (2000, 2003) provides evidence on the participation decisions of older German workers. Scarpetta (1996) finds empirical evidence supporting this phenomenon in a cross-country study. A major factor behind the trend toward early retirement is that existing PAYG systems distort the labor supply decision on the extensive margin and thereby encourage early retirement. Blöndal and Scarpetta (1999) suggest that early retirement provisions in many countries have led to a dramatic decrease in the labor force participation among older workers. The fact that benefits are not adjusted in an actuarially fair manner is a key reason for this large distortion. Theoretical work on social security and retirement decisions is inspired by the seminal contributions of Feldstein (1974), Sheshinski (1978) and Diamond and Mirrlees (1978). More recent theoretical work on the (optimal) design of pension systems in the presence of a retirement decision is found, for example, in Cremer and Pestieau (2003) and Cremer et al. (2004). Another strand of the literature has studied the effects of the pension system on aggregate unemployment (see Demmel and Keuschnigg (2000), Corneo and Marquardt (2000), and countries. For Germany the implicit tax of working between ages 60-61 is roughly 35%, while in France it is close to 70%. They find a much lower, approximately 0%, implicit tax in the U.S., which is due to the fact that not only is the replacement rate much lower in U.S., but also because of actuarial adjustment between ages 62 and 65 and smaller payroll tax rates. In their later paper using microestimation Gruber and Wise (2004) consider, among other issues, a reform package that incorporates early retirement at 60, normal retirement at 65, a 60% replacement rate, and actuarial adjustment. They show that this simulated reform has mixed effects across countries depending on the specific benchmark provisions of national programs. For the U.S. this reform increases the incentives to retire early due both to the lower eligibility age and the higher replacement rate, while for other countries, such as Italy, France, and Germany, these measures represent a substantial increase in the incentives to stay in the workforce.

In the case of Germany Gruber and Wise (2005) estimate that an actuarially fair reduction in benefits increases the retirement age by about 3 years. Moreover, they calculate a significant fiscal effect on the order of 1.2% of GDP.

See Fenge and Pestieau (2005) for a review. Breyer and Hupfeld (2010) point out the distributional effects of pension reform towards more actuarial fairness. Cremer et. al. (2004) focus on redistribution towards the ill. The redistributional implications of retirement incentives are, nevertheless, not the focus of this paper.
Cigno (2008) and the empirical analysis of Adam (2007)). It is fair to say, we believe, that neither the theoretical nor empirical literature on pensions and unemployment focusses to a great extent on the differential impact of young versus old workers and the interaction with respect to the retirement decision.

The novel contribution of this paper is an analysis of how a PAYG pension system affects life-cycle unemployment among young and old workers and how this interacts with labor market participation of older workers. Building on Fisher and Keuschnigg (2010), which abstracts from unemployment, we set up a model with endogenous retirement and life-cycle unemployment resulting from job search subject to labor market frictions that is combined with a general formulation of a PAYG pension system. We believe that the novel focus on life-cycle unemployment and retirement is important in the face of high unemployment rates in OECD countries and the large share of social security contributions in the overall labor tax burden.

More specifically, the analysis should be interesting for at least three reasons: (i) it sheds some light on the popular claim that increased labor market participation of older workers might adversely affect unemployment among younger workers. The argument is that raising the retirement age boosts the number of older workers who might crowd out younger employees and thereby add to prime age unemployment. Jousten et al. (2010) do not find empirical support for this mechanism. Our theoretical results show that raising the retirement age is likely to improve the fiscal stance, leading to a lower labor tax burden, more job search and, in turn, lower prime age unemployment; (ii) Our analysis highlights some features of real world PAYG systems that have been rather neglected in both theoretical and empirical work. Some countries allow periods of unemployment to

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4 In Cigno (2008) a Beveridge-type system in which contributions are unrelated to benefits discourages overall labor supply, while a Bismarckian system has ambiguous effects depending on whether the system is actuarially fair and whether agents are credit constrained.

5 An exception is the recent empirical work of Gruber et al. (2010), who find, among other results, that the implicit retirement tax faced by older workers is slightly positively related to the unemployment rate of the young.
create pension entitlements by adding a fraction of the last earnings prior to unemployment to the pension assessment base, presumably to prevent old age poverty. We find that this feature undermines search incentives and boosts unemployment; (iii) These results also connect to the literature on unemployment insurance savings accounts (UISA) as a novel policy tool to fight unemployment (see Stiglitz and Yun (2005), Brown et al., (2008), Feldstein and Altman (2007), and Bovenberg, Hansen and Sorensen (2008), among others). UISAs lead to a reduction in pension income whenever an individual is unemployed and withdraws benefits from her account. The econometric results in Reyes et al. (2010) provide strong support to the idea that UISAs can improve work incentives and reduce unemployment. Including the replacement income of unemployment insurance in the pension assessment base is exactly the opposite to UISAs and is likely to boost unemployment. The quantitative results in Keuschnigg et al. (2010) indeed show a substantial impact on the unemployment rate.

The present paper sets up a model of endogenous retirement and life-cycle unemployment in a frictional labor market. We first derive effective tax rates on all three margins of life-cycle labor supply, which is in itself a novel contribution to the literature, and show how they depend on retirement incentives and other features of an earnings linked PAYG pension system. We then study the real effects and the welfare consequences of several policy scenarios that avoid intergenerational redistribution and isolate the efficiency gains and losses of pension reform (see Keuschnigg (1994) on the concept of intergenerational neutrality). More specifically, we investigate four scenarios that play a prominent role in many countries: (i) increasing actuarial fairness of the pension system towards retirement behavior; (ii) strengthening the tax benefit link by a harmonization of the system; (iii) reforming the pension assessment base by excluding periods of unemployment; and (iv) extending the calculation period.

The importance of scenario (i) is evident from the findings of the overall Gruber-Wise research program on pensions and retirement behavior. We find that more actuarial fairness by making pensions conditional on retirement choice not only encourages post-
poned retirement but also stimulates employment among young workers. Our model thus excludes crowding out of younger workers by older ones, a result that is in line with the empirical findings of Jousten et al. (2010). However, there is an ambiguous effect on search incentives and employment of older workers. Despite of this ambiguity, the net welfare gain is clearly positive. Relating to (ii), the empirical results in Adam (2007) confirm that in countries with a strong tax benefit link, a higher pension income, holding statutory tax rates constant, reduces unemployment. This effect is explained in our model because the experiment implies a lower effective tax rate and thereby stimulates search. However, the relation breaks down in countries with a Beveridge-type system with flat pensions unrelated to previous earnings. We model the strengthening of the tax benefit link (also called harmonization of the system) by shifting the composition of total retirement benefits from flat to earnings linked pensions. In particular, we find that harmonization boosts work incentives on all three margins of life-cycle labor supply, reduces unemployment (consistent with the empirical results of Adam (2007)), and unambiguously raises welfare. Regarding (iii), a reform that eliminates periods of unemployment from the pension assessment base strengthens search incentives, boosts employment over the entire life-cycle, encourages late retirement, and promises unambiguous efficiency gains. The empirical importance of our results is clearly backed up by the evidence in Reyes et al. (2010). Finally, extending the calculation period for pension assessment as in scenario (iv) favors young workers and encourages postponed retirement while employment rates of older workers close to retirement respond ambiguously. Welfare rises if job search among old workers is inelastic or if it is not distorted in the first place.

The paper proceeds as follows: Section 2 sets up the model. Section 3 derives the comparative statics behavior. Section 4 analyzes the consequences of four specific reform scenarios, and Section 5 concludes. Some technical computations are moved to an appendix.
2 The Model

To explore the implications of pension reform on life-cycle unemployment and old age participation, we use a highly stylized 2-period model with three overlapping generations. In period one, an “initial old” generation 0, living in its second period of life and consuming $C^o$, coexists with a young generation 1 living in its first period, consuming $C_1$. When generation 1 grows old in period 2, it consumes $C_2$ and coexists with a final “future” generation which, in turn, lives in its first period of life, consumes $C^f$ and dies thereafter. Normalizing the size of each cohort to one, aggregate consumption is $C^o + C_1$ in the first period and $C_2 + C^f$ in the second period. The world ends at the end of period 2. We focus on labor supply of generation 1 in both life-cycle periods and exclude any labor market decisions of other generations, i.e. we assume that the initial old generation 0 is fully retired and labor supply of the future generation 2 is exogenous. The production side is deliberately kept simple. Assuming a Ricardian technology, labor earns a fixed wage $w$, equal to the output of an employed worker. A unit of savings and investment generates output $R > 1$ next period. The labor market is subject to search frictions such that a given effort in job search results in employment only with a probability less than one, and ends in unemployment otherwise. All agents are risk-neutral.

2.1 Households

Life-cycle labor market behavior of generation 1 consists of job search in both periods and a retirement decision in the second period of life. To focus on labor supply, we abstract from savings and intertemporal consumption choice and assume that present and future consumption are perfect substitutes. With the interest factor $R$ equal to the rate of time preference, households are concerned with the present value, but not with the timing of consumption. They spend effort $l_s$ on job search in periods $s = 1, 2$ and choose a “retirement date” $x$ in the second period equal to the fraction of time $x$ of actively remaining in the workforce, where $1 - x$ represents the time in retirement. Job search and continued la-
bhor market participation lead, respectively, to increasing and convex effort costs $\zeta (l_s)$ and $\phi (x)$, so that $\zeta' > 0$, $\zeta'' > 0$, $\phi' > 0$ and $\phi'' > 0$. As a further simplification, we assume preferences to be linearly separable between consumption and effort, thereby eliminating income and wealth effects on labor supply.\(^6\) Thus, the specification of preferences is

$$V = C_1 - \zeta (l_1) + [C_2 - x\zeta (l_2) - \phi (x)] / R. \tag{1}$$

Given a frictional labor market, households supply a variable search effort $0 < l_s < 1$ and incur an effort cost $\zeta (l_s)$ to obtain suitable employment. Market frictions imply that this effort results in employment with probability $l_s$ and in unemployment with probability $u_s = 1 - l_s$. We do not further specify labor market frictions. With independent risks and large numbers, the ex ante probability $l_s$ is equal to the ex post employment rate. The (un-)employment rate is, thus, exclusively determined by the incentives for job search. If employed, the worker produces output $w$ per capita equal the gross wage rate. If unemployed, she derives utility from home production. To avoid complicated interactions between unemployment insurance and the pension system, we normalize unemployment benefits to zero. This a harmless restriction given our assumption of risk-neutrality. Wage earnings of an employed worker are subject to the statutory contribution rate $t$. By choosing search effort, households determine their individual employment probability and expected wage income $(1 - t)wl_s$. The budget identities equal

$$C_1 = (1 - t)wl_1 - A, \quad C_2 = x \cdot (1 - t)wl_2 + (1 - x) \cdot P + RA, \tag{2}$$

where $A$ is (indeterminate) savings and $P$ denotes pension earnings. In the second period, expected wage earnings accrue only while the agent is active, i.e., during $x$, and are replaced by pension income during retirement, $(1 - x)$. Similarly, the search effort cost in (1) is incurred only during the active part of the second period.

A central part of our analysis concerns the relationship determining the size of pension benefits

$$P = m (x) z + p_0. \tag{3}$$

\(^6\)However, job search will depend on future pensions if there is a tax benefit link.
Public pensions have three key features: (i) an assessment base $z$ that equals past earnings; (ii) a conversion factor $m(x)$ that depends on the old-age participation decision $x$ and is key in determining retirement incentives; and (iii) “flat”, basic pension payments $p_0$ that are independent of the individual’s earnings history. The assessment base is a weighted average of lifetime earnings

$$z \equiv \delta \cdot w [l_1 + \mu (1 - l_1)] R^P + x w [l_2 + \mu (1 - l_2)]. \quad (4)$$

The weight $\delta$ on first period earnings allows us to investigate the consequences of lengthening the calculation period for pension assessment. For example, if $\delta = 0$, only the most recent earnings count in the assessment base. Frequently, pensions are based only on a limited number of years with the best earnings. Since earnings typically rise with age, older workers prior to retirement often have the highest earnings. If $\delta = 1$, the calculation period extends over the entire earnings history. In many cases, PAYG systems pay a notional interest on accumulated earnings points in the assessment base, which introduces the factor $R^P$ in (4). The notional interest rate reflects the implicit rate of return of the PAYG system, which equal to the growth rate of wage earnings, and, thus, falls short of the market interest factor, i.e. $R^P < R$. If $\mu > 0$, periods of unemployment create future pension claims by crediting a fraction $\mu$ of the last earnings (prior to unemployment) to the pension assessment base, a feature instituted by several countries, such as Austria and Switzerland. On the other hand, if $\mu = 0$, households acquire pension entitlements only when employed and making contributions. An analysis of this rule is important since it is diametrically opposed to the concept of an unemployment insurance savings account (UISA) that countries such as Chile have implemented. UISA reduces pension rights whenever an individual is jobless and claims unemployment benefits, and thereby makes unemployment individually more costly. It is, therefore, often considered an important policy measure to improve labor market performance.

The conversion factor incorporates important institutional features of PAYG systems. Depending on its specification, it determines incentives for early or postponed retirement
and thereby influences the old age participation rate. We specify

\[ m(x) = m_0 + \frac{\alpha}{1 - x}. \tag{5} \]

A Beverage-type system is one in which households receive only flat pensions \( p_0 \), independent of earnings, so that \( m = 0 \). A Bismarckian system is one with a fixed earnings-benefit link, \( m = m_0 > 0 \), with \( p_0 = 0 \) and \( R^P = 1 \). A “Gruber-Wise” pension scheme features an actuarial adjustment of pension earnings that is conditional on the retirement date and reflects the length of the remaining retirement period, \( m'(x) > 0 \), requiring \( \alpha > 0 \) in (5). Postponing retirement (higher participation rate \( x \)) thus raises pension benefits in (3).

To calculate optimal choices, we substitute the budget identities (2) into preferences (1), leading to the following two-period problem:

\[
V = \max_{l_1, x} (1 - t) w l_1 - \zeta (l_1) + V_2 / R, \tag{6}
\]

\[
s.t. : \quad V_2 = x \cdot (1 - t) w l_2 + (1 - x) \cdot P - x \cdot \zeta (l_2) - \phi (x). \]

Noting the dependence of pension benefits \( P \) on the retirement date and previous earnings as specified in (3)–(5), we obtain the following necessary conditions:

\[
(a) : \quad \zeta' (l_1) = (1 - \tau_1) w, \quad \tau_1 \equiv t - \delta \cdot (1 - \mu) (1 - x) m R^P / R, \\
(b) : \quad \zeta' (l_2) = (1 - \tau_2) w, \quad \tau_2 \equiv t - (1 - \mu) (1 - x) m, \tag{7}
\]

\[
(c) : \quad \phi' (x) = (1 - \tau_R) w l_2 - \zeta (l_2), \quad \tau_R \equiv t + \frac{P}{w l_2} - \frac{(1 - x) P'}{w l_2},
\]

where postponing retirement raises benefits over the remaining retirement period by

\[ P' = \partial P / \partial x = m' z + m w [l_2 + (1 - l_2) \mu]. \]

We define effective tax rates \( \tau_1, \tau_2, \) and \( \tau_R \) to summarize the joint impact of social security on labor market behavior. In the absence of government, these rates would be zero. Agents then equate the marginal disutility cost with the marginal expected return along each dimension of labor supply. More search effort boosts the employment probability \( l_s \) by 1 at the margin and raises gross expected earnings by \( w \). Job search is
optimal when the marginal return is equal to marginal effort cost, \((1 - \tau_s) w = \zeta'(l_s)\). Postponing retirement by an instant of time raises life-time earnings by \(wl_2\) and life-time utility by \(wl_2 - \zeta_2\), after taking into account the effort cost spent on actually obtaining employment. Continued participation adds an extra utility cost \(\phi'\), due, for example, to increasing health problems. Again, the optimal retirement date is found when the marginal return to continued participation is equal to the marginal disutility of postponed retirement.

The design of the PAYG system determines the size of effective tax rates. For example, the effective contribution tax rates (often also called “implicit” tax rates) \(\tau_1\) and \(\tau_2\) are less than the statutory contribution rate \(t\) if pension benefits are linked to past earnings due to the tax benefit link. More intensive search leads, then, not only to greater expected current wage income, but also to greater benefits during the retirement period of \(1 - x\). Consequently, the PAYG contribution rate is not a pure tax since households receive part of it back as a retirement benefit. Moreover, assuming that pension savings earn less than the market return, \(R^P < R\), the effective contribution tax on the young exceeds the effective tax on older workers, \(\tau_1 > \tau_2\). If the pension calculation period includes only the most recent earnings prior to retirement (\(\delta = 0\)), higher earnings of young workers no longer raises future pension benefits, which implies that statutory contribution rate is a “full” tax, i.e. \(\tau_1 = t\). Extending the calculation period by raising the weight \(\delta\) of first period earnings in the assessment base reduces the effective tax rate \(\tau_1\). The system thereby shifts the effective tax burden from young to old workers. Furthermore, the system inflates the effective contribution tax and undermines incentives for job search if it allows periods of unemployment to create pension claims by crediting a fraction \(\mu\) of “past earnings” \(w\) to the assessment base.\(^7\) Finally, the participation tax \(\tau_R\) on continued work during old age deserves special attention. When retirement is postponed, the individual continues to pay the statutory contribution rate \(t\), but incurs foregone pension benefits as an opportunity cost that adds the pension replacement rate \(P/(wl_2)\) to the effective tax

\(^7\)In reality, this fraction is often equal to the replacement rate for unemployment benefits which we have normalized to zero to reduce complexity.
rate $\tau_R$. However, the system can substantially reduce the participation tax if it rewards postponed retirement by a pension supplement $P' \equiv \partial P/\partial x > 0$ over the remaining retirement period.

We may further shed light on the labor market incentives of a PAYG pension system by showing that an ideal capital funded system would be fully neutral and reduce all effective tax rates to zero. An ideal funded pillar would strictly limit benefits to actual mandatory retirement savings, i.e. to contributions actually paid, $\mu = p_0 = 0$ and $\delta = 1$. It would pay the market rate of return on accumulated contribution capital, $R^D = R$. It would also convert contribution capital into benefits in an actuarially fair way such that the individual account is balanced at each date, which implies a conversion factor equal to $m = t/(1 - x)$ with $m_0 = 0$. In this situation effective tax rates on job search are zero, $\tau_s = 0$. The funded system would not harm incentives for job search. Evaluating (3)–(4), we obtain $P = tz/(1 - x)$ where $z = w_1 R + x w_2$ so that $P' = [w_2 + z/(1 - x)] t/(1 - x)$. Turning to (7), we find a zero participation tax, $\tau_R = 0$, which establishes full labor market neutrality. Another way to see the neutrality of the fully funded system is to evaluate indirect utility in (6). Since (6) reduces to $V = (wl_1 - \zeta_1) + (xwl_2 - x\zeta_2 - \phi)/R$ in this case, all pension parameters drop-out and the funded system does not affect life-time wealth and utility.

**Proposition 1** A pure capital funded system is neutral towards life-cycle unemployment and retirement behavior.

### 2.2 General Equilibrium and Welfare

The central feature of a PAYG system is that working generations pay contributions that are spent on pension income of retired persons. Hence, the system implements an inter-generational transfer from young to old generations. In our 2-period, three generations model, the initial old generation 0 is fully retired and collects pensions $P^o$, paid from the contributions of the young generation 1. Accordingly, we exclude a retirement decision
of the initial old generation in order to focus on the life-cycle choices of generation 1. In the same spirit, we add a “future” (and final) generation in period 2 that lives only for one period. We posit that job search of generation 2 is exogenous. Even simpler, and without loss of generality, we set its employment rate to unity, so that consumption is \( C^f = (1 - t^f) w \). Hence, the future generation contributes a fixed transfer \( t^f w \) to generation 1 to finance part of its pension income. Given that only a fraction \( 1 - x \) of generation 1 is retired and collects benefits, the periodic PAYG budget restriction would require that payouts be equal to contributions in each period

\[
P^o = twl_1, \quad (1 - x) P = twxl_2 + t^f w.
\]

In the following, we wish to analyze intergenerationally neutral reforms that avoid redistribution across generations. Hence, welfare of the initial old and the future generation must be kept constant, requiring \( P^o \) and \( t^f \) constant. In this case, any policy change necessarily leads to a periodic deficit or surplus. Sustainability of the PAYG system requires intertemporal budget balance, i.e. contributions and payouts must balance in present value, or satisfy

\[
tw (l_1 R + xl_2) + t^f w = RP^o + (1 - x) P \quad \text{in terms of end of period value}
\]

\[
(1 - x) P - t^f w - twxl_2 = RS, \quad S \equiv twl_1 - P^o.
\]

Intertemporal solvency thus requires that the second period deficit on the left-hand side must not exceed today’s surplus together with accumulated interest.

In this simple model, and with historically accumulated assets equal to zero, output in period 1 is \( Y_1 = wl_1 \). It is spent on consumption of the young and old generations and on savings/investment equal to \( I = A + S \). The GDP identity \( Y_1 = C^o + C_1 + I \) is identically satisfied when substituting \( C^o = P^o \) and using the budget constraints in (2) and (9). In period 2, GDP consists of the return on invested capital and on output of the still active workers of generation 1 and the future generation, \( Y_2 = IR + w (xl_2 + 1) \). Using \( C^f = (1 - t^f) w \) and substituting the budgets in (2) and (9), we again obtain the GDP identity of period 2, \( Y_2 = C_2 + C^f \).
We wish to evaluate pension reform by calculating welfare changes. Recall that the old generation in period 1 simply consumes PAYG pensions and, thus, enjoys welfare $V^o = C^o/R = P^o/R$. The welfare of the current generation corresponds to (6). In turn, the future generation in period 2 consumes after-tax wage earnings and dies thereafter, leading to welfare $V^f = C^f = (1 - t^f)w$. We adopt the Calvo and Obstfeld (1988) approach and define social welfare as $\Omega = V^oR + V^r + V^f/R$ which is implicitly restricted by the intertemporal budget constraint in (9). Substituting all welfare terms and imposing (9) yields the following welfare criterion:

$$\Omega = w\ell_1 - \zeta (\ell_1) + [w(x\ell_2 + 1) - x\zeta (\ell_2) - \phi (x)]/R.$$ (10)

3 Comparative Statics and Welfare

We explore the labor market consequences of pension reform by considering policy experiments involving changes in the parameters that define the pension assessment base, determine retirement incentives and change the composition between earnings linked and flat pensions. Given that effective tax rates and life-cycle search are functions of the retirement date and the contribution rate, we treat $x$ and $t$ as the two equilibrating variables. The comparative static analysis derives changes in variables relative to initial equilibrium values. As discussed below, we assume that the initial equilibrium is characterized by $R^P = \delta = 1$ and $\mu = 0$.

3.1 Retirement Choice

Throughout the rest of the paper, we work in terms relative changes, i.e., $\hat{x} \equiv dx/x$ for variables and $\hat{\tau} \equiv d\tau/(1 - \tau)$ for implicit tax rates. We first show how the retirement date and the statutory contribution rate affect the earnings-benefit link and the effective tax rates on job search. A shift in pension policy, together with changes in the two
endogenous variables, affects the conversion factor in (5) according to

$$dm = m'x \cdot \hat{x} + d\hat{m}, \quad d\hat{m} \equiv dm_0 + \frac{d\alpha}{1 - x}. \quad (11)$$

When the government grants higher pensions, it raises the conversion factor, $d\hat{m} > 0$. To provide incentives for postponed retirement, it sets $\alpha > 0$ to make benefits sensitive to the chosen retirement date. The conversion factor then rises automatically when people postpone retirement. For later use, note that $(1 - x) m' = m - m_0$.

The effective tax rates on job search, defined in (7), after substituting for (11), change by

$$\hat{\tau}_1 = \frac{1}{1 - \tau_1} \left\{ \frac{m_0 x}{R} \cdot \hat{x} \cdot dt + \frac{1 - x}{R} [m \cdot (d\mu - d\delta) - d\hat{m}] \right\}, \quad (12)$$

$$\hat{\tau}_2 = \frac{1}{1 - \tau_2} \left\{ m_0 x \cdot \hat{x} \cdot dt + (1 - x) [m \cdot d\mu - d\hat{m}] \right\}.$$

A higher statutory contribution rate $t$ naturally raises effective tax rates. Less obviously, effective taxes also rise if workers choose a longer a working life, i.e., $\hat{x} > 0$. This is because workers must not only wait longer until more earnings today are rewarded with higher pension benefits under an operative tax benefit link, but also because the benefit is available for a shorter remaining retirement period. Crediting imputed earnings during periods of unemployment, $d\mu > 0$, raises the effective tax rate and discourages search. In contrast, strengthening the tax-benefit link, as parameterized by $d\hat{m}$, reduces effective tax rates and stimulates job search. We will use the parameter $\delta$ to capture the implications of lengthening the earnings calculation period. We note that a greater weight of first period earnings reduces the effective tax rate on young workers. The logarithmic derivatives of the optimality conditions in (7a)–(7b) reveal how employment rates respond to shifts in the implicit tax rates

$$\hat{l}_1 = -\sigma_1 \cdot \hat{\tau}_1, \quad \hat{l}_2 = -\sigma_2 \cdot \hat{\tau}_2, \quad (13)$$

where the elasticities are defined as $\sigma_s \equiv \zeta_s'/(l_s \zeta_s'')$. Changes in employment and unemployment rates are unemployment negatively related, $\hat{u}_t = -\hat{l}_s \cdot l_s/(1 - l_s)$. 

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The individual retirement choice determines the participation rate in the cross-section of the older population. Using \( \zeta' (l_2) = (1 - \tau_2) w l_2 \) from (7b) gives

\[
\eta \phi' \cdot \dot{x} = -w l_2 \cdot dt - (t - \tau_2) w l_2 \cdot \dot{l}_2 - d [P - (1 - x) P'] , \quad \eta \equiv x \phi'' / \phi'.
\]  

To obtain a response of participation in terms of policy variables, we must derive the term \( d [P - (1 - x) P'] \). After some tedious (though straightforward) algebra in appendix A, the retirement condition emerges as

\[
\nabla_X \cdot \dot{x} = -\nabla_T \cdot dt + (1 - x) \nabla_M \cdot d \bar{m} - z \cdot dm_0 - dp_0 + \nabla_{\mu} \cdot d\mu - \nabla_\delta \cdot d\delta,
\]

where, in general, we define all \( \nabla \)-coefficients to be positive:

\[
\begin{align*}
\nabla_X &\equiv \phi' \eta + (wl_2 + \nabla_M) m_0 x, & \nabla_M &\equiv wl_2 - m_0 \Psi_1 - m_0 \Psi_2 > 0, \\
\nabla_T &\equiv wl_2 - m_0 R \Psi_1 - m_0 \Psi_2 > 0, & \Psi_1 &\equiv \frac{\sigma_{1 \mu} w l_1}{(1 - \tau_1) R}, & \Psi_2 &\equiv \frac{\sigma_{2 \mu} w l_2}{1 - \tau_2}, \\
\nabla_\mu &\equiv (w - \nabla_M) (1 - x) m - m_0 Z_\mu, & \nabla_\delta &\equiv m_0 [wl_1 + (1 - x) m \Psi_1], \\
Z_\mu &\equiv w (1 - l_1) + wx (1 - l_2).
\end{align*}
\]

The reduced-form coefficients \( \Psi_1 \) and \( \Psi_2 \) in (15) reflect the influence of job search on the participation decision. Although the statutory tax rate \( t \) is an endogenous variable, it is illustrative to consider the implications of a rise in \( t \) on the retirement decision \( \dot{x} \). The direct effect is, of course, a decline in after-tax earnings. This leads people to retire earlier, since collecting a pension is marginally more attractive compared to working. On the other hand, this effect is offset by the fact that a higher tax rate erodes the incentives for job search which lowers earnings and thereby reduces pensions by a factor of \( m_0 \) per wage unit. This, in turn, raises the gains to old-age participation and leads agents to retire later. If the direct effect dominates, then a higher contribution rate results in early retirement, i.e. \( \nabla_T > 0 \), which implies a negative relationship between the two endogenous variables in (15). Note that \( \nabla_T > 0 \) necessarily implies \( \nabla_M > 0 \). We postpone the discussion of the impact of other policy parameters on retirement to section 4 where we investigate several alternative policy reforms.

\footnote{To solve for (15), we must also substitute for the responses of employment, \( \dot{l}_s \), both directly and through the term \( d [P - (1 - x) P'] \). For further details see appendix A.}
3.2 Intertemporal Solvency

The second equilibrium condition is the intertemporal budget constraint in (9). To exclude intergenerational redistribution, we hold $P^o$ and $t^f$ fixed and obtain the differential

$$\left( l_1 R + x l_2 \right) w \cdot dt = - \left( t + \frac{P}{w l_2} \right) w x l_2 \cdot \dot{x} - t w l_1 R \cdot \dot{l}_1 - t w x l_2 \cdot \dot{l}_2 + (1 - x) \cdot dP,$$

(16)

where the coefficient $w l_1 R + w x l_2$ on $dt$ represents a person’s life-time contribution base, expressed in period 2 values. Sustainability requires a higher contribution rate if employment in periods 1 and 2 declines, if households retire earlier, and if pension benefits to the retired fraction $1 - x$ of the old generation become more generous. To derive the reduced-form response of the contribution rate, we substitute for the employment response $\dot{l}_s$ as stated in (13) and the benefit changes $dP$ induced by the pension rule in (3). Detailed derivations in appendix B show that the solvency condition requires an adjustment of the contribution rate equals

$$B_T \cdot dt = -B_X \cdot \dot{x} + (1 - x) \left( dp_0 + B_M \cdot d\tilde{m} \right) + (1 - x) m \left( B_\mu \cdot d\mu + B_\delta \cdot d\delta \right),$$

(17)

where coefficients are defined as follows (note $B_T > B_M$):

$$B_T \equiv \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) w l_1 R + \left( 1 - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) w x l_2,$$

$$B_X \equiv \tau_R w x l_2 - w l_1 \frac{\tau_1 \sigma_1}{1 - \tau_1} m_0 x - w x l_2 \frac{\tau_2 \sigma_2}{1 - \tau_2} m_0 x,$$

$$B_M \equiv \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) w l_1 + \left( 1 - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) w x l_2,$$

$$B_\mu \equiv Z_\mu + w l_1 \frac{\tau_1 \sigma_1}{1 - \tau_1} + w x l_2 \frac{\tau_2 \sigma_2}{1 - \tau_2}, \quad B_\delta \equiv \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) w l_1.$$

Like the retirement condition, the PAYG solvency condition implies a negative relationship between retirement and contribution rates: extending working life boosts revenues which supports solvency with a lower contribution rate.

3.3 Equilibrium and Welfare

The retirement and solvency conditions (15) and (17) form a simultaneous system in $\dot{x}$ and $dt$ where the determinant of the coefficient matrix is $\Gamma$. Inverting the matrix system
yields the equilibrium solution

\[
\begin{bmatrix}
\dot{x} \\
\frac{dt}{\hat{x}}
\end{bmatrix} = \frac{1}{\Gamma} \begin{bmatrix}
B_T & -\nabla_T \\
-B_X & \nabla_X
\end{bmatrix} \begin{bmatrix}
E_X \\
E_T
\end{bmatrix}, \quad \Gamma \equiv \nabla_X B_T - \nabla_T B_X,
\]

(18)

where the changes in pension policy are collected in the following terms:

\[
E_X = (1 - x) \nabla_M \cdot \tilde{m} - dp_0 - z \cdot dm_0 + \nabla_\mu \cdot d\mu - \nabla_\delta \cdot d\delta,
\]

\[
E_T = (1 - x) (dp_0 + B_M \cdot d\tilde{m}) + (1 - x) m (B_\mu \cdot d\mu + B_\delta \cdot d\delta).
\]

Figure 1 below illustrates the stable solution, where the retirement locus \( R \) and the government’s solvency locus \( G \) are negatively sloped, i.e., \( dt/\hat{x}_R = -\nabla_X/\nabla_T < 0 \) and \( dt/\hat{x}_G = -B_X/B_T < 0 \). Moreover, stability requires that the retirement locus is steeper than the solvency locus

\[
\nabla_X/\nabla_T > B_X/B_T \iff \Gamma > 0.
\]

To conduct welfare analysis, we compute the differential of (10). To avoid intergenerational redistribution and isolate the efficiency effects of policy changes, we hold constant the tax \( t^f\cdot w \) paid by the future generation and the pension benefit \( P^o \) received by the initial old generation so that their welfare levels, \( V^o \) and \( V^f \), are not affected. Calculating the differential of (10) and substituting the optimality conditions (7) yields

\[
d\Omega = \tau_1 \cdot \hat{l}_1 + \tau_2 \frac{wxl_2}{R} \cdot \hat{l}_2 + \tau_R \frac{wxl_2}{R} \cdot \hat{x}.
\]

(19)

Welfare changes are proportional to effective tax rates \( \tau_1, \tau_2, \text{ and } \tau_R \) that measure the distortions to life-cycle labor supply. The greater is the initial distortion, the larger is the welfare loss from further discouraging labor supply on that margin.

4 Pension Reform

In studying labor market and welfare consequences of pension reform, we consider four widely discussed policies: (i) greater actuarial fairness, (ii) strengthening the tax benefit link, (iii) reforming the assessment base, and (iv) extending the calculation period.
4.1 Greater Actuarial Fairness

A policy of greater actuarial fairness is one that rewards households for postponed retirement or, conversely, penalizes them for leaving the workforce early, in a way that is more consistent with intertemporal balance of lifetime benefits and contributions.\footnote{Recall the discussion on actuarial fairness at the end of section 2.1.} Specifically, this requires making the pension conversion factor more sensitive to a variation of the retirement date \( x \) by raising the parameter \( \alpha \). To prevent the system from becoming more generous, the flat component \( m_0 \) must be simultaneously reduced to keep the level of the conversion factor constant. By (11), the restriction \( d\bar{m} = 0 \) requires \( dm_0 = -d\alpha/(1-x) < 0 \). Setting all other changes of policy parameters in (18) to zero yields the equilibrium impact on retirement behavior and the required contribution rate

\[
\hat{x} = -\frac{zB_T}{\Gamma} \cdot dm_0 > 0, \quad dt = \frac{zB_X}{\Gamma} \cdot dm_0 < 0. \tag{20}
\]

The effect of greater actuarial fairness is to strengthen incentives for later retirement. In Figure 1, the retirement locus defined in (15) shifts to the right, while the solvency condition in (17) is not directly affected. More actuarial fairness boosts the retirement age and allows for a smaller contribution rate. Analytically, this is the case if the coefficients \( B_T \) and \( B_X \) are positive. The coefficient \( B_T \) measures the net effect of a higher contribution rate on the budget. A higher rate directly boosts revenue, but also discourages job search, which erodes the tax base. The net effect in each period is positive if \( \tau_s\sigma_s/(1 - \tau_s) < 1, \ s = 1,2 \), implying \( B_T > 0 \). The coefficient \( B_X \) measures the net effect of an increase in the retirement age on the life-time contribution base and on tax revenue. Again, the direct effect of a higher \( x \) raises revenue in proportion to the effective tax rate \( \tau_R \). When the conversion factor is not actuarially fair and contains a fixed component \( m_0 \), later retirement, according to (12), also raises effective tax rates on prime age workers and discourages job search. Higher unemployment shrinks the contribution base and erodes revenue. The net effect is positive as long as \( \tau_s\sigma_s/(1 - \tau_s) \) is small and the effective tax rate \( \tau_R \) is relatively large, i.e., as long as \( B_X > 0 \).
To determine the consequences for job search and welfare, we must calculate the change in effective tax rates $\tau_1$ and $\tau_2$ and the labor market response of prime age workers. Substituting (20) into (12),

$$\hat{\tau}_1 = \left( \frac{m_0x/R \cdot \hat{x} + dt}{(1 - \tau_1)} \right) \Gamma \cdot dm_0,$$

yields

$$\hat{\tau}_1 = \frac{B_X - B_T \cdot m_0x/R}{(1 - \tau_1) \Gamma} \cdot \frac{z \cdot dm_0}{(1 - \tau_1)}.$$  \hspace{1cm} (21)

Clearly, the changes in old age participation and the contribution rate in the numerator have opposite effects on $\hat{\tau}_1$. Postponing retirement raises the effective period 1 tax—by lengthening the time households have to wait for their pension—while a lower contribution rate reduces it. We can show that the influence of the contribution rate dominates that of the participation decision, as long as the search response in period 2 is not too strong. This implies that $\hat{\tau}_1 < 0$. Consequently, job search in period 1 intensifies which boosts employment, $\hat{l}_1 = -\sigma_1 \hat{\tau}_1 > 0$ and cuts the unemployment rate, $\hat{u}_1 < 0$.\(^{10}\)

![Figure 1: Incentives for Postponed Retirement](image)

Unlike the change in the effective first period tax, the sign of $\hat{\tau}_2$ can be positive or negative ($\hat{\tau}_2 \geq 0$), leading to an ambiguous response of search and unemployment among older workers. This ambiguity is, in fact, intuitive. The cost of deferring retirement benefits can outweigh the benefits of facing a lower PAYG contribution rate. While younger households discount the cost of delayed retirement by $R$, the cost for their older

\(^{10}\)Using $\tau_R \equiv t + \frac{m_0x + p_0 - (1 - x)mwl_2}{wl_2} \cdot \frac{\hat{x}}{\hat{\tau}_2}$, eventually leads to $B_X - B_T \cdot m_0x/R = \tau_2 wxl_2 + x p_0 + \left( 1 - \frac{\hat{\tau}_2}{\hat{\tau}_2} \right) wxl_2 \frac{\hat{\tau}_2}{\hat{x}} \cdot m_0x > 0$. A sufficient condition to guarantee $\hat{\tau}_1 < 0$ is $1 - \frac{\hat{\tau}_2}{\hat{\tau}_2} > 0$. 

counterparts is imminent and can discourage search in period 2. Raising the retirement age thus might benefit or hurt older workers but unambiguously stimulates employment among younger workers. Hence, increased old age participation on account of a higher retirement age does not crowd-out young workers in our framework.

We can show that moving towards greater actuarial fairness reduces labor market distortions and boosts welfare, which depends in (19) on the agent’s labor market responses over the life cycle. Even though the employment risk of older workers might rise if \( \hat{\tau}_2 > 0 \) and possibly lead to a welfare loss, we can calculate, by substituting \( \hat{l} = -\sigma_s \hat{\tau}_s \) into (19) and using (20)-(21) and the definition of \( B_X \), an overall welfare gain equal to

\[
d\Omega = \frac{B_X}{R} \cdot \hat{x} - \left[ \frac{\tau_1 \sigma_1}{1 - \tau_1} \omega l_1 + \frac{\tau_2 \sigma_2}{(1 - \tau_2) R} w x l_2 \right] \cdot dt > 0. \tag{22}
\]

Moving towards greater actuarial fairness in retirement incentives unambiguously boosts welfare since it mitigates the net distortions on life-cycle labor market behavior.

**Proposition 2** Introducing pension supplements and discounts for more actuarial fairness in pension assessment encourages postponed retirement, boosts job search and employment of younger workers, and raises welfare.

### 4.2 Strengthening the Tax Benefit Link

We next consider an alternative policy reform: strengthening the tax benefit link by a harmonization of the system. In reality, substantial parts of the population such as civil servants or farmers often are subject to separate rules that feature an incomplete tax benefit link. Arguably even more important is the fact that many countries feature minimum pensions that are not linked to past earnings. Some countries such as Switzerland also have maximum pensions so that contributions on earnings above a given upper income ceiling do not lead to higher pensions. In all these cases the tax benefit link is not operative, implying that contributions become a full labor tax. In our model, we capture the presence of flat pensions by the assessment rule \( P = m_0 z + p_0 \). Harmonization subjects a
larger population share to the common earnings linked system in which flat pensions \( p_0 \) are replaced by earnings linked benefits \( m_0 z \) such that the overall pension level remains constant for given behavior. To concentrate on the role of the tax benefit link, we set \( \alpha = 0 \) in (5) so that the conversion factor is not sensitive to the retirement choice \( x \). The scenario thus raises the conversion factor for a higher earnings linked pension and cuts the flat benefit by \( dp_0 = -zdm_0 = -z\tilde{m} \) to keep the overall benefit level fixed. Evaluating (18) yields an equilibrium change in the retirement date and the required contribution rate equal to\(^{11}\)

\[
\begin{align*}
\hat{x} &= \frac{1-x}{\Gamma} [\nabla_M B_T + (z - B_M) \nabla_T] \cdot dm_0 > 0, \\
\hat{dt} &= -\frac{1-x}{\Gamma} [\nabla_M B_X + (z - B_M) \nabla_X] \cdot dm_0 < 0.
\end{align*}
\]

As in the case of more actuarial fairness, harmonization results in more old-age participation that “funds” – through a longer working life and higher contributions – a lower PAYG contribution rate. The responses of \( \hat{x} \) and \( \hat{dt} \) follow from the fact that all coefficients in the square brackets including the term \( z - B_M \) are positive. Replacing a flat by an earnings linked benefit rewards continued work to a greater extent since the conversion factor \( m_0 \) rewards an increase in the assessment base due to postponed retirement with a higher pension over the remaining life-time, which is not the case with a flat benefit. Equation (15) shows that harmonization shifts the retirement locus \( R \) in Figure 1 (not drawn) to the right. According to (17), this reform also shifts down the fiscal locus \( G \), since a stronger tax benefit link reduces the tax character of contributions and, thereby, encourages job search and the augmentation of the contribution base. Indeed, the higher are the search elasticities \( \sigma_s \), the stronger is the growth in the assessment base and the more the solvency constraint can be relaxed to accommodate a lower value of the contribution rate \( dt \), which reinforces incentives to postpone retirement.

Turning to effective tax rates on job search, we can calculate from (12) that strengthening the tax benefit link directly reduces effective tax rates and stimulates employment.

---

\(^{11}\)Using \( B_M \) and \( z \) and evaluating at \( \mu = 0 \) and \( \delta = 1 \) gives \( z - B_M = \frac{\tau_1 \sigma_1}{1 - \tau_1} w l_1 + \frac{\tau_2 \sigma_2}{1 - \tau_2} w x l_2 > 0 \).
in both life-cycle periods

\[
\hat{\tau}_1 = \frac{1}{1 - \tau_1} \left[ \frac{m_0 x}{R} \cdot \hat{x} + dt - \frac{1 - x}{R} \cdot dm_0 \right] < 0, \tag{24}
\]

\[
\hat{\tau}_2 = \frac{1}{1 - \tau_2} [m_0 x \cdot \hat{x} + dt - (1 - x) \cdot dm_0] < 0.
\]

To see this most easily, consider an introduction of a small earnings linked pension in the absence of any (initial) tax benefit link \((m = m_0 = 0)\) so that all pensions are flat. Contributions are then a full tax, \(\tau_1 = \tau_2 = t\) and \(\tau_R = t + p_0 / (w_l)\). In this case, the influence of the retirement age \(x\) on effective tax rates disappears. Since the policy allows for a reduction of the contribution rate, \(dt < 0\), the introduction of a small tax benefit link clearly reduces effective tax rates on job search, and more so for older workers, \(\hat{\tau}_2 < \hat{\tau}_1 < 0\). Strengthening the tax benefit link thus boosts job search and reduces unemployment of both old and young workers. If the conversion factor is increased from an already positive level, the induced increase in the retirement age \(x\) starts to raise effective tax rates on job search and offsets the policy’s stimulating effect on employment. Although the computations are complicated, we nevertheless obtain reduced-form versions of (24), after substitution of (23), establishing that harmonization reduces both \(\hat{\tau}_1\) and \(\hat{\tau}_2\).\(^{12}\) In sum, harmonization stimulates all three margins of life-cycle employment and clearly improves efficiency, as is evident from (19), \(d\Omega = \tau_1 w_1 \hat{l}_1 + \tau_2 w_2 \hat{l}_2 + \tau_R w_2 \hat{x} > 0\).

**Proposition 3** Harmonization of the system and, hence, strengthening the tax benefit link boosts all margins of life-cycle employment and raises welfare.

### 4.3 Reforming the Assessment Base

Some countries, such as Austria and Switzerland, add a fraction of last earnings prior to unemployment towards the pension assessment base \(z\) so that periods of unemployment also create pension entitlements. To capture the consequences of this policy, we marginally raise the weight \(\mu\) of unemployment in (4), starting from a value of zero. For a given

\(^{12}\) A separate appendix with a detailed proof is available on request.
conversion factor, the larger assessment base, \( dz = Z_\mu d\mu \), translates into a higher pension, 
\( dP = mZ_\mu d\mu \), where 
\( Z_\mu \equiv w(1 - l_1) + wx(1 - l_2) \) if \( \delta = 1 \) initially. Presumably, the motivation is to provide better protection against old age poverty.\(^{13}\) For this reason, we do not impose another compensating policy and allow the pension size to increase. The policy is costly and must be financed with higher contributions. As a further simplification, we set \( m_0 = 0 \) so that the conversion factor is fully sensitive to a variation in the retirement date. Given that \( \mu \) is the only policy parameter to change, the shift terms in (18) are 
\( E_X = \nabla_\mu d\mu > 0 \) and 
\( E_T = (1 - x) mB_\mu d\mu > 0 \). Evaluating the solution (18) then yields 
\[
\hat{x} = \frac{w(1 - l_2) \cdot B_T - wl_2 \cdot B_\mu (1 - x) m \cdot d\mu}{\Gamma},
\]
\[
\hat{\tau}_1 = \frac{dt + (1 - x) m / R \cdot d\mu}{1 - \tau_1} > 0, \quad \hat{\tau}_2 = \frac{dt + (1 - x) m \cdot d\mu}{1 - \tau_2} > 0.
\]
Consequently, employment rates fall by \( \hat{l}_s = -\sigma_s \hat{\tau}_s \), leading to larger unemployment among young and old workers. Combining this with the shift towards early retirement,

\(^{13}\)We address here only the policy’s implications for economic efficiency. The present framework does not allow us to discuss intragenerational redistribution.
\( \dot{x} < 0 \), welfare declines along all margins of life-cycle labor supply

\[
d\Omega = \tau_1 w l_1 \cdot \dot{l}_1 + \tau_2 \frac{w x l_2}{R} \cdot \dot{l}_2 + \tau_R \frac{w x l_2}{R} \cdot \dot{x} < 0.
\]  

(27)

**Proposition 4** In an earnings linked system, crediting periods of unemployment towards the pension assessment base boosts unemployment and encourages early retirement. Welfare declines on all margins of labor supply.

Crediting unemployment periods towards pension assessment is likely intended to prevent old age poverty. However, this policy goal may be achieved using minimum pensions \( p_0 \) as an instrument. It is interesting to observe that the crediting of unemployment spells is just the opposite of unemployment insurance savings accounts (UISA). The balance of these accounts is reduced each time when individuals draw unemployment benefits. At the date of retirement, a smaller balance means a smaller pension.

### 4.4 Extending the Calculation Period

Until recently, many countries have assessed pensions on the basis of the best years of earnings, rather than extending the calculation period over the entire working life. Typically, life-cycle earnings profiles tend to be rising and weakly hump-shaped, implying that the best years of earnings accrue late in the career. Extending the calculation period means including earlier life-cycle periods. In our framework, the parameter \( \delta \) would be low initially, or even zero, to reflect the fact that the first parts of earnings are only incompletely included in the assessment base since they typically do not belong to the best earning years. Extending the calculation period over the entire earnings history implies raising the weight \( \delta \) in (4) towards one, \( \delta \rightarrow 1 \). Obviously, with more years included, the assessment base is augmented so that pensions get larger. To prevent this, the conversion factor must be simultaneously scaled down.

To keep our calculations simple, we mimic the policy measure by considering the reverse experiment. We start from \( \delta = 1 \) initially and reduce it. We compensate the
reduction in the assessment base by raising the conversion factor \( m_0 \) such that the policy shift keeps pension size fixed for given, constant behavior. We can considerably simplify computations by starting from a situation in which \( m_0 = 0 \), so that the conversion factor \( m(x) \) is fully sensitive to retirement choice prior to the policy change. Given these assumptions, the experiment is \( d\delta < 0 < dm_0 = \tilde{d}\tilde{m} \). Without any behavioral response, the assessment base \( z = \delta w l_1 + wxl_2 \) shrinks by \( dz = wl_1 d\delta \). To keep pension benefits \( P = mz + p_0 \) constant, we raise, in turn, the conversion factor by

\[
z \cdot dm_0 = -mwl_1 \cdot d\delta, \quad \tilde{d}\tilde{m} = dm_0.
\] (28)

The scenario thus shifts the system in (18) by \( E_X = [(1-x) \nabla_M - z] dm_0 - \nabla_\delta d\delta \) and \( E_T = (1-x) B_M dm_0 + (1-x) mB_\delta d\delta \).\(^{14}\) Inserting the resulting \( \nabla \)- and \( B \)-coefficients from (15) and (17) and noting the policy restriction above, we obtain

\[
E_X = - [z - (1-x) wl_2] \cdot dm_0, \quad E_T = \left[ \frac{\tau_1 \sigma_1}{1-\tau_1} - \frac{\tau_2 \sigma_2}{1-\tau_2} \right] (1-x) wxl_2 \cdot dm_0.
\] (29)

Lacking evidence to the contrary, we assume less elastic search on the part of older workers, \( \sigma_1 \geq \sigma_2 \). Since \( \tau_1 > \tau_2 > 0 \), we find \( E_T > 0 \) and \( E_X < 0 \).\(^{15}\) In Figure 1, these changes would shift up the budget locus and move the retirement locus to the left, leading to earlier retirement and a higher contribution rate in the new equilibrium. Analytically, we derive from (18)

\[
\dot{x} = - \frac{[z - (1-x) wl_2] \cdot B_T + \left( \frac{\tau_1 \sigma_1}{1-\tau_1} - \frac{\tau_2 \sigma_2}{1-\tau_2} \right) wxl_2 \cdot (1-x) wl_2}{\Gamma} \cdot dm_0 < 0, \quad \text{and} \quad \frac{\phi' \eta \left( \frac{\tau_1 \sigma_1}{1-\tau_1} - \frac{\tau_2 \sigma_2}{1-\tau_2} \right) wxl_2 \cdot (1-x) wl_2 + [z - (1-x) wl_2] \cdot \tau_R wxl_2}{\Gamma} \cdot dm_0 > 0,
\] (30)

where \( B_T > 0 \). Intuitively, a shorter calculation period (recall that we consider the reverse experiment \( d\delta < 0 \)) shrinks the assessment base and calls for higher contributions when pension size is kept constant by a compensating increase in the conversion factor. This

\(^{14}\)With \( m_0 = 0 \), we have \( \nabla_\delta = 0 \), \( \nabla_M = wl_1 \), \( \nabla_X = \phi' \eta \), \( \nabla_T = wxl_2 \) and \( B_X = \tau_R wxl_2 \).

\(^{15}\)If life-cycle search is comparable between younger and older workers, i.e. \( l_1 \approx l_2 \), then \( z - (1-x) wl_2 = w (l_1 - l_2) + 2 w xl_2 > 0 \).
leaves lower net of tax earnings, compared with a constant pension, and thus induces people to retire earlier since continued work becomes less attractive relative to receiving benefits. As the graphical argument illustrates, these immediate adjustments reinforce each other and magnify the equilibrium responses. Clearly, the policy inflates the contribution tax and leads to early retirement. In retiring earlier, people willingly accept a decline in the conversion factor and of pension benefits in equilibrium.

To obtain the consequences for life-cycle unemployment, we evaluate the effective tax rates on job search in (12). Starting from a situation of $m_0 = 0$ eliminates the effect of a marginal change in the retirement age $x$ and leaves $d\tau_1 = dt - \frac{1 - x}{R} [md\delta + dm_0]$ and $d\tau_2 = dt - (1 - x) dm_0$. For young workers, the effective tax rate rises by

$$\hat{\tau}_1 = \frac{1}{1 - \tau_1} \left[ dt + \frac{1 - x}{wl_1 R} wxl_2 \cdot dm_0 \right] > 0.$$  

(31)

Clearly, the higher distortion causes more unemployment among young workers.

The effective tax on older workers, in contrast, changes ambiguously. The exogenous increase in the conversion factor reduces the effective tax rate since any marginal increase in earnings is rewarded with a larger pension increment. The higher contribution rate, in contrast, raises the effective tax. Hence, without further restrictions, the change in $\tau_2$ is ambiguous. Substituting for the solution (30), using $\Gamma$, and evaluating (12) yields

$$\hat{\tau}_2 = \frac{\left( \frac{\tau_1 \sigma_1}{1 - \tau_1} - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) wxl_2 (1 - x) wl_2 + \tau_R wxl_2 \cdot z - (1 - x) \phi' \eta B_T}{(1 - \tau_2) \Gamma} \cdot dm_0 \geq 0.$$  

(32)

If the retirement elasticity is small enough, $\eta \to 0$, the effective tax rises, $\hat{\tau}_2 > 0$. On the other hand, if $\eta$ is “large”, and $\tau_R = \tau_2 = 0$ (in the case of $p_0 = m_0 = 0$), then the effective tax rate on period 2 search could even fall, $\hat{\tau}_2 < 0$. In general, then, this policy change has an ambiguous impact on job search and unemployment among older workers. In any case, however, we expect the absolute magnitude of the change to be small.

Knowing behavioral responses, we can evaluate the welfare effects in (19). Our reverse experiment of shortening the calculation period, requiring a higher conversion factor for compensation, raises the distortions in retirement choice and young workers’ job search.
Welfare falls on both accounts. Given the ambiguous impact on the employment of older, still active workers, these losses may be reinforced or partly offset. Several cases lead us to believe that the net welfare effect remains negative, even if the effective tax on old workers declines. For example, if \( p_0 = m_0 = 0 \), then \( \tau_R = \tau_2 = 0 \), as mentioned above, results in \( d\Omega = \tau_1 \hat{w} l_1 \cdot \hat{l}_1 < 0 \). Furthermore, even if \( \tau_R \) and \( \tau_2 \) are positive, a possible increase in \( \hat{\tau}_2 \) would lead to a weak reduction of the employment rate of old workers if job search is very inelastic. With \( \sigma_2 \to 0 \), \( d\Omega \approx \tau_1 \hat{w} l_1 \cdot \hat{l}_1 + \tau_R \frac{w x l_2}{R} \cdot \hat{x} < 0 \). All things considered, our intuition is that this policy experiment likely reduces welfare.

Since this analysis relates to the reverse experiment, a *lengthening* of the calculation period yields the following effects:

**Proposition 5** Lengthening the pension calculation period together with a compensating reduction in the conversion factor encourages postponed retirement and strengthens job search and employment of young workers. Employment of older, still active workers may rise or fall. Overall welfare is likely to increase, subject to an ambiguous, and probably small welfare component, due an ambiguous change in the old age employment rate.

5 Conclusion

It is well-known that a PAYG pension system can distort the labor market outcomes on both intensive and extensive margins of work. The goal of this paper is to develop a life-cycle framework of labor market search and retirement to study how pension reform affects labor market performance and social welfare. Within our simple OLG setting, we consider the following parametric policies: i) increasing actuarial fairness of the pension system towards retirement behavior; ii) strengthening the tax benefit link by a harmonization of the system; iii) reforming the pension assessment base by excluding periods of unemployment; and iv) an extension of the calculation period. Among these options, we find that harmonization boosts work incentives on all margins of life-cycle labor supply.
and unambiguously raises welfare. More actuarial fairness in making pensions conditional on retirement choice not only encourages postponed retirement but also stimulates employment among young workers. However, there is an ambiguous effect on search incentives and employment of older workers. Despite of this ambiguity, the net welfare gain is positive. A reform that eliminates periods of unemployment from the pension assessment base strengthens search incentives, boosts employment over the entire lifecycle, encourages late retirement, and promises unambiguous efficiency gains. Extending the calculation period for pension assessment similarly favors young workers and encourages postponed retirement while employment rates of older workers close to retirement respond ambiguously. Welfare rises if job search among old workers is inelastic or if it is not distorted in the first place.

**Appendix A: Retirement Condition**

Appendix A solves for the retirement condition (15). We start by calculating the term $d[P - (1 - x) P'] = dP + P' x \cdot \hat{x} - (1 - x) dP'$ given in (14). Considering the first component, we use (11) and get

$$dP = z (m' x \cdot \hat{x} + d\tilde{m}) + m \cdot dz + dp_0. \quad (A.1)$$

Given $P' = m' z + mw [l_2 + (1 - l_2) \mu]$ in (7), evaluated at $\mu = 0$, we calculate

$$dP' = z \cdot dm' + m' \cdot dz + wl_2 \cdot dm + mw l_2 \cdot \hat{l}_2 + mw(1 - l_2) \cdot d\mu,$$

$$dP' = \left[ z \frac{2m'}{1 - x} + wl_2 m' \right] x \cdot \hat{x} + \frac{z}{(1 - x)^2} \cdot d\alpha + wl_2 \cdot d\tilde{m} \quad (A.2)$$

$$: \quad + m' \cdot dz + mw l_2 \cdot \hat{l}_2 + mw(1 - l_2) \cdot d\mu,$$

where we use $dm = m' x \cdot \hat{x} + d\tilde{m}$ and $dm' = 2m' x \cdot \hat{x} / (1 - x) + d\alpha / (1 - x)^2$ to get the second equality. We substitute (A.1)–(A.2) into $d[P - (1 - x) P'] = dP + xP' \cdot \hat{x} - (1 - x) \cdot dP'$ and use the definition of $P'$ as well as $m - (1 - x)m' = m_0$ to obtain

$$: \quad d[P - (1 - x) P'] = xwl_2 m_0 \cdot \hat{x} + m_0 \cdot dz - (1 - x) mw l_2 \cdot \hat{l}_2 + dp_0 \quad (A.3)$$

$$: \quad - \frac{z}{1 - x} \cdot d\alpha + [z - (1 - x) wl_2] \cdot d\tilde{m} - (1 - x) mw(1 - l_2) \cdot d\mu.$$
As a last step, we need to substitute for \( dz \). Evaluating the derivative of \( z \) in (4) where \( \delta = 1 \) and \( \mu = 0 \), and using the definition \( Z_\mu \equiv w (1 - l_1) + xw (1 - l_2) \) yields
\[
dz = wx l_2 \cdot \hat{x} + w l_1 \cdot \left( d\delta + \hat{l}_1 \right) + wx l_2 \cdot \hat{l}_2 + Z_\mu \cdot d\mu. \tag{A.4}
\]
Substituting (A.4) into (A.3) leads to:
\[
d[P - (1 - x) P'] = 2m_0 wx l_2 \cdot \hat{x} + m_0 w l_1 \cdot \left( d\delta + \hat{l}_1 \right) \tag{A.5}
\]
\[+ [m_0x - (1 - x) m] w l_2 \cdot \hat{l}_2 + dp_0 \]
\[+ z \cdot dm_0 - (1 - x) w l_2 \cdot d\tilde{m} + [m_0 Z_\mu - (1 - x) mw (1 - l_2)] \cdot d\mu,
\]
where we use \([z - (1 - x) w l_2] \cdot d\tilde{m} - \frac{z}{1 - x} \cdot d\alpha = z \cdot dm_0 - (1 - x) w l_2 \cdot d\tilde{m} \). Combining (A.5) with (14) yields:
\[
(\eta \phi' + 2m_0 wx l_2) \cdot \hat{x} = -m_0 w l_1 \cdot \hat{l}_1 - m_0 wx l_2 \cdot \hat{l}_2 - w l_2 \cdot dt
\]
\[= -m_0 w l_1 \cdot d\delta - [m_0 Z_\mu - (1 - x) mw (1 - l_2)] \cdot d\mu \tag{A.6}
\]
\[+ (1 - x) w l_2 \cdot d\tilde{m} - z \cdot dm_0 - dp_0,
\]
where we employ the relationship \((t - \tau_2) = (1 - x) m \). The final step to derive the retirement locus (15) is to substitute the employment responses \( \hat{l}_s = -\sigma_s \hat{\tau}_s \) together with (12) into (A.6). This yields:
\[
[\eta \phi' + (2wl_2 - m_0 \Psi_1 - m_0 \Psi_2) m_0 x] \cdot \hat{x} = -(wl_2 - Rm_0 \Psi_1 - m_0 \Psi_2) \cdot dt
\]
\[= -dp_0 - z \cdot dm_0 + (wl_2 - m_0 \Psi_1 - m_0 \Psi_2) (1 - x) \cdot d\tilde{m} \tag{A.7}
\]
\[+ \{w (1 - l_2) + m_0 \Psi_1 + m_0 \Psi_2 \} (1 - x) m - m_0 Z_\mu \} \cdot d\mu
\]
\[= -m_0 [wl_1 + (1 - x) m \Psi_1] \cdot d\delta,
\]
where \( \Psi_1 \equiv \frac{\sigma_1 w l_1}{(1 - \tau_1) \rho} \) and \( \Psi_2 \equiv \frac{\sigma_2 w l_2}{1 - \tau_2} \). This equation is the retirement condition (15) stated in the main text after substituting for the \( \nabla \)-coefficients.

**Appendix B: Fiscal Balance**

To obtain the reduced form PAYG budget constraint, we substitute out the change in pensions and employment in (16). The change in benefits follows upon substituting (A.4)
into (A.1) and noting $P' = wlm + zm'$ in the coefficient of $\hat{x}$

$$dP = xP' \cdot \hat{x} + z \cdot d\bar{m} + dp_0 + mZ_\mu \cdot d\mu + mwl_1 \cdot \left( d\delta + \hat{l}_1 \right) + mwxl_2 \cdot \hat{l}_2. \quad \text{(B.1)}$$

Substituting (B.1) into (16), and recalling the definitions of the effective tax rates in (7), evaluated at $\mu = 0$ and $R' = \delta = 1$, yields

$$(wl_1R + wxl_2) \cdot dt = -\tau wxl_2 \cdot \hat{x} - \tau_1 wl_1 R \cdot \hat{l}_1 - \tau_2 wxl_2 \cdot \hat{l}_2 \quad \text{(B.2)}$$

When participation of the old and search induced employment of active workers change, the contribution rate must adjust in proportion to the effective tax rates on these margins, $\tau_R$, $\tau_1$, and $\tau_2$, where $wl_1R + wxl_2$ represents the life-time contribution base.

The final step to obtain the reduced form change in the contribution rate necessary to sustain intertemporal solvency is to substitute the employment responses $\hat{l}_s = -\sigma_s \hat{\tau}_s$ together with (12) into (B.2). After collecting terms, we obtain:

$$\begin{align*}
\text{[} & \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) wl_1 R + \left( 1 - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) wxl_2 \text{]} \cdot dt \\
= & - \tau wxl_2 - \left( \frac{\tau_1 \sigma_1}{1 - \tau_1} + \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) m_0 \cdot \hat{x} + (1 - x) dp_0 \quad \text{(B.3)} \\
\text{[} & \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) \left( 1 - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right) \text{]} \cdot dt \\
= & + \left[ Z_\mu + \frac{\tau_1 \sigma_1}{1 - \tau_1} - \frac{\tau_2 \sigma_2}{1 - \tau_2} \right] \cdot \hat{l}_1 \cdot (1 - x) m \cdot d\mu \\
\text{[} & + \left( 1 - \frac{\tau_1 \sigma_1}{1 - \tau_1} \right) \text{]} \cdot dt \\
= & + (1 - \tau_1 \sigma_1) \left( 1 - \tau_2 \sigma_2 \right) \cdot m \cdot d\delta.
\end{align*}$$

This equation is identical to (17) after substituting for the $B$-coefficients defined in the main text.

References


