The credit model risk of interest rate derivatives and regulatory implications

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Abstract

In this paper we investigate the role of model risk in the context of counterparty risk of derivatives. For this purpose, a simple call option on zero bonds is analyzed. The role of model specification and parameter estimation risk is illustrated. Some implications with respect to the regulatory recognition of internal models are discussed. It is argued that under no circumstances should „model risk" (i.e. the risks associated with the specification, estimation and implementation of specific models) be transferred to regulatory agencies. This could have dramatic and adverse implications for the safety of the financial system.

Introduction: Modeling credit risk of derivatives

Counterparty risk plays an important role in the pricing of over-the-counter derivatives. Respective models were developed by Johnson and Stulz (1987), Hull and White (1995), Jarrow and Turnbull (1995), Klein (1996), Ammann (1999). Also, the new Basle capital proposal released in 1999 puts a lot of emphasis on the modeling of credit risk. While the 1988 capital standard took a very rough and undifferentiated view on the credit quality of counterparties (sovereign and non-sovereign debtors, banks in OECD countries and elsewhere, no differentiation for corporate debtors), external and internal ratings are now suggested to assess counterparty quality. In a last step, capital requirements can be based even on banks` quantitative credit risk and portfolio models. This approach does not only apply to traditional credit portfolios, but also to OTC derivatives which are subject to counterparty risk. Here we argue that the model risk associated with credit risk models is substantial and must be taken into account in regulatory efforts to regulate credit risk along the lines described before[1]. The approval of credit risk models by regulatory authorities could well result in a delegation of the model risk from banks to regulators and thus create adverse moral hazard effects.

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[1] Important paper addressing the role of model risk in the context of value-at-risk are Beder (1995) and Jackson/ Maude/ Perraudin (1997).

In this paper, we demonstrate the role of model risk in the context of interest rate derivatives. Model risk has, at least, three different although related dimensions:

- Specification risk: The choice of the model, the underlying assumptions and its architecture
- Estimation risk: Specification and statistical estimation of the input parameters, calibration of the model inputs to observed prices
- Implementation risk: Choice of estimation methods, numerical procedures, simulation techniques

In this paper, we illustrate model risk in the context of counterparty risk of derivative securities. While modeling credit risk causes some difficulties even if simple bonds and loans are likely to default, the task becomes definitively more challenging when derivatives on bonds or other corporate securities are subject to counterparty risk themselves. The added complexity comes from the fact that the exposure of the contract is not known in advance. Given that the time horizon of interest coincides with the maturity of the contract, the exposure of a deterministic contract, such as a simple bond or loan, is known in advance to be the face value of the debt. In the case of derivatives, the exposure is stochastic. For model risk, this means that for deterministic contracts, the credit risk of a contract does not depend directly on the market component and can therefore be modeled separately. (an indirect dependence may occur. For example, credit risk may change with interest rate movements).

With stochastic exposures, on the other hand, the relationship between market and credit component is immediate because the market component changes the credit risk exposure of the contract and thus directly the amount of credit risk. For example, an out-of-the-money, OTC call option on a Treasury Bond carries a small credit risk exposure. If interest rates fall, the option may move into the money and counterparty exposure rises for the party that owns the option. Does the counterparty risk rise as well? This depends on the effect of interest rate movements on the credit risk of the counterparty. If lower interest rates reduces the risk of the counterparty defaulting on the obligation, the counterpary risk of the option does not increase proportionally with the exposure and may even decrease. If, on the other hand, lower interest rates affects the ability of the counterparty to honor the contingent claim adversely, then decreasing rates might greatly increase the credit risk for the owner of the option.

The dependency between market and credit risk inherent in derivatives with counterparty risk exacerbates the model risk situation. An explicit model of these dependencies may be subject to severe specification and estimation risk. No explicit model – for example because market and credit risk management is not integrated – implies a model nevertheless, namely that market and credit risk are independent of each other. Thus, model risk is present in either situation.

The paper is structured as follows: In the next section we briefly address the current and prospective state of the regulatory treatment of credit risk of derivatives. The main focus of the paper is then on the model risk associated with the credit risk of derivatives which is then illustrated in the context of defaultable call options on zero coupon bonds. Three examples of model risk are examined. The final section summarized the regulatory implications of the paper.

The regulatory treatment of credit risk of derivatives

The credit risk of traditional financial instruments is straightforward to measure: the amount of risk corresponds to the principal amount of the instrument or the investment. For derivatives, the counterparty risk consists of the costs associated with a substitution (replacement) of the counterparty, which is expressed by the so called replacement cost of the contracts[2]. Replacement cost comprises two components: *current* replacement cost corresponds to the mark-to-market value of the contract; *potential* replacement cost accounts for changes due to expected volatility, interest rates, underlying asset price, time to maturity, and other factors. The Basle Committee of Banking Supervision proposes to estimate potential replacement cost by adding a certain percentage of the notional value of the contract, the so called add-on, to the current replacement cost. Of course, fixed add-ons are sometimes used in practice; however, more and more, banks use option pricing models and Monte Carlo simulation techniques trying to compute VaR-based values. So far, the Basle Committee did not recognize these models as a basis to compute the credit risk of derivatives. However, this is currently subject to controversial discussions after the Basle Committee has released a new proposal on capital adequacy in June 1999 according to which internal/ external ratings as well as credit risk models should be recognized under certain conditions (Basel Committee on Banking Supervision (1999))[3]. This proposal is quite surprising because the Basle Committee had typically a rather defensive view on recognizing banks` internal credit risk models. For instance, in a paper released in 1998, the Basle Committee discusses the relevance of data limitations and model validation problems in the context of credit risk (Basel Committee on Banking Supervision (1998)). The management of counterparty risk was subject to two other initiatives of the Basle Committee in 1999, namely the release of 17 principles for the management of credit risk, and a review on best practices for credit risk disclosure. These reports are, however, not specifically related to derivatives, or the role of credit model risk, but cover topics related to risk organization, disclosure, control, monitoring, and responsability.

The basic model

We analyze a contingent claim that entitles its holder to receive the amount of X in four years from a specific counterparty that may be subject to default. X can denote the promised payoff from a loan, bond, or a derivative contract. Default occurs if the value of the firm V falls below the total amount of liabilities D by the time of maturity of the option contract. In case of default, the claimholder receives instead of X only X times the asset-liability ratio of the firm. Thus, the actual payoff of the claim is given by

$$(1)\ldots \qquad X \max\left(\frac{V}{D},1\right),$$

where V denotes the value of the assets of the firm, D the value of the liabilities (free of credit risk). Consider the following special case: If the firm has only one claim outstanding and this claim is a zero bond, the payout at maturity of the claim becomes

[2] The homepage of the International Financial and Commodities Institute, IFCI, provides up-to-date information on derivatives and credit risk. See http://newrisk.ifci.ch/00013403.htm.

[3] See Jackson/ Perraudin (2000) for an extensive discussion of the regulatory treatment of credit risk.

(2)... $D\max(\frac{V}{D},1)=\max(V,D)$.

This is the seminal default model proposed by Merton (1974). We assume that the default-free value of the zero bond is driven by a mean-reverting interest rate process characterized by

(3)... $dr(t)=a(b-r(t))dt+\sigma_r dW(t)$

as proposed by Vasicek (1977), while the firm value is assumed to follow the diffusion process

(4)... $dV=V(r(t)dt+\sigma_V dZ(t))$

where $Z(t)$ and $W(t)$ are correlated Brownian motions under the risk-neutral probability measure.

In the subsequent examples, we investigate the modeling of credit risk in the context of three European call option written on the zero-coupon bond, with different exercise prices but all with the same maturity in two years. The maturity of the underlying zero bond is 2 years from option expiration. The underlying zero bond is free of credit risk. The correlation coefficient between interest rate and firm value is zero. The other parameters of the stochastic processes are displayed in Table 1. They imply a forward bond price (for delivery at maturity of the option in 2 years) of 89.64 and a credit yield spread of 27 bp on two-year bonds issued by the option counterparty.

Table 1 displays the impact of credit risk upon the value-at-risk of the three call options; we call it *Credit-Value-at-Risk*. This CVaR denotes the VaR of the option in terms of a percentage of the option price today (theoretical option value with counterparty risk according to parameters) that is entirely due to counterparty risk. The range of values is between 9.4% (for the in-the-money option) and 17.1% (for the out-of-the-money option). VaR and CVaR are based on a time-horizon of one year and a confidence level of 99%. A CVaR of 11.3% of the option price therefore means that the owner of the option can be 99% confident not to lose more than 11.3% of today's value of the option due to a counterparty default within one year from today.

The pricing model used for this CVaR computation is based on the solution for the price of a bond option with counterparty risk as derived in Ammann (1999). It uses the firm-value default model in equation (1) and is compatible with the class of Gaussian interest-rate models, of which the Vasicek model used in this paper is a special case.

The CVaR is computed from a distribution of differences between the credit-risk-free value of the claim and the respective value including credit risk. A total of 100'000 joint simulations for the stochastic variables r and V are performed using the closed-form solutions by Ammann (1999) for prices of credit-risky (sometimes called vulnerable) options on zero-coupon bonds.

Table 1 – The Credit-Value-at-Risk (CVaR) for a two-year call option on a four-year zero coupon bond.

	Strike price of bond call option		
	80%	85%	90%
CvaR	*9.42%*	*11.31%*	*17.13%*
D/V	0.9	r(0)	ln(1.05)
Volatility of V	10%	a	0.1
Correlation r and V	0.0	b	ln(1.08)
		Volatility of r	0.02

The time horizon of the CVaR is 1 year, confidence is 99%.
CVaR include only counterparty risk and are relative to the counterparty risk-free value of the option.

Model Risk - Example 1: Estimation Risk

Although most financial models are subject to estimation risk, this risk is often critical in credit risk modeling because of limited availability of data in the area of credit risk and because credit events are rare events and thus make credit risk models especially prone to estimation risk.

Table 2 shows CVaR figures of credit-risky call options on zero-coupon bonds for various model parameters. The default parameters are the same as in Table 1. Parameter changes are on a ceteris paribus basis and are indicated in the first two columns. The table demonstrates that the specific contract greatly influences the CVaR. In our example, the relative CVaR substantially increases with shorter maturity of the option contract, all other factors being equal.

Table 2 – The Credit-Value-at-Risk (CVaR) for Call Options on Zero Coupon Bonds for various parameter estimates

Parameter	Value	1-year option on 4-year bond	*2-year option on 4-year bond*	3-year option on 4-year bond
σ_r	0.015	20.4%	*10.0%*	7.4%
	0.025	20.2%	*12.6%*	9.6%
a	0.15	21.9%	*11.1%*	7.8%
	0.05	18.0%	*11.9%*	9.0%
b	ln(1.12)	27.0%	*12.2%*	7.7%
	ln(1.04)	15.7%	*11.0%*	8.8%
D/V	0.95	32.5% (87bp)	*16.6% (58bp)*	11.8% (42bp)
	0.85	10.4% (7 bp)	*7.0% (11bp)*	5.6% (11bp)
σ_V	0.15	48.7% (128bp)	26.1% (106bp)	19.4% (87bp)
	0.08	10.3% (9bp)	6.1% (10bp)	4.6% (9bp)

Reference parameters as in Table 1. CVaR=11.3%.

Table 2 shows that the estimation of the interest-rate model affects the CVaR only to a limited extent in most cases. An exception is the mean reversion parameter b, which appears to have a strong effect on the counterparty risk of short-maturity options. As expected, the estimation of the credit risk parameters has a more pronounced effect on counterparty risk. The credit risk parameters also directly determine the term structure of credit spreads of bond yields. The yield spread implied by the credit risk model parameters and the respective maturity of the option contract are given in parentheses. Note that, in our example, the CVaR of the option increases with shorter contract maturity regardless of the shape of the term structure of yield spreads. Both V/D=0.85 and σ_V=0.08 imply slightly increasing yield spreads, all other parameters being equal, but nonetheless relative CVaR decrease with increasing contract maturity.

Model Risk - Example 1: Correlation between credit and market risk

In management processes and regulatory procedures, credit risk is often treated as a separate risk category apart from market risk. Depending on the correlation between market and credit risk, there are, however, diversification effects between the two risks. Instead of assuming that the two risks are perfectly correlated and are therefore additive, the use and regulatory recognition of credit risk models makes it possible to account for these diversification effects.

In the setting of our simple model, market risk is reflected in the interest rate, r, while credit risk comes from the specification of the firm value process V in relation to the nominal value of the debt D, which is given exogenously. As a matter of fact, the size of the relevant correlation coefficients is far from being easy to specify.

Consider the relevance of *the sign* of the coefficient first. Most likely, the firm value is negatively correlated with interest rates[4]. In this case, a *decreasing* interest rate implies increasing bond prices which, in principle, raises the debt obligation of the short side of the call and thus makes default more likely. However, this is party offset by the fact that the firm value of the counterparty increases (because of the negative correlation) which decreases the credit risk compared to a situation, where the correlation between firm value and interest rate is positive. In this case namely, the probability of default would coincide with high option payoffs (low interest rates, high debt value, low firm value). On the other hand, if interest rates are *rising*, the firm value increases on average, but the quality of the counterparty is not very relevant for the option holder in this situation because the option tends to be out-of-the money if interest rates are high.

Thus, *ceteris paribus*, the holder of a bond option is better off if the quality of the counterparty (i.e. the firm value) is negatively correlated with interest rates. This implies, that credit risk premia are *positively* related to the correlation coefficient (i.e. higher if the correlation coefficient is positive). The exact values are displayed in Table 2.

For regulatory purposes, diversification effects should be not overestimated and it would be more adequate to take high (positive) correlation coefficients as appropriate. In a stress scenario, it is very likely that interest rate changes (more generally, market breakdowns) are associated with high defaults of counterparties. Taking a correlation of 0.9 as a realistic figure for such a scenario, the table shows that the credit-value-at-risk would be underestimated by 2-3% compared to a scenario assuming uncorrelated risks. This figure appears rather low in absolute terms, so that the impact of the correlation coefficient would not seem a major source of model risk for the specific instrument analyzed here. However, should there be no agreement on the sign and the order of magnitude of the coefficient under stress (compare the

[4] This is, for example, the case for banks having “short” interest-sensitive liabilities and “long” assets (in terms of the average repricing period).

–0.9 and +0.9 figures) then the CVaR values differ by 200%. As a conclusion, correlation risk accounts for up to 50% of the credit risk of a simple bond option.

Table 2 - The Credit-Value-at-Risk (CVaR) for Call Options on Zero Coupon Bonds

Correlation coefficient between firm value and Interest rate	Strike price of bond call option		
	80%	85%	90%
0.9	12.33%	14.77%	19.28%
0.5	10.93%	12.63%	18.01%
0	*9.42%*	*11.31%*	*17.13%*
-0.5	8.00%	9.47%	14.88%
-0.9	6.11%	6.77%	10.71%

This table shows CVaR values for a call option on a zero-coupon bond, for several correlation coefficients between the firma value V and the short interest rate r, and several strike prices of the call option. CVaR are relative values and are compured with respect to the corresponding no-counterparty risk value of the option. The time horizon of the CVaR is 1 year. The call option expires in 2 years, the underlying bond matures in 4 years.

Model risk – Example 2: Stochastic liabilities of the counterparty firm.

In the previous example it was assumed that the firm value (V) is stochastic while the liabilities of the firm were assumed constant (D). Here, we assume that the liabilities (denoted by L) also follow a diffusion process which exhibits a certain correlation vis-a-vis the interest rate. Here, we assume for simplicity that the liabilities are uncorrelated with the firm value and interest rates, and that their volatility is 10%. In Table 3, the Credit-VaR results of Table 2 are replicated for this generalization. Not surprisingly, the figures increase for all choices of the correlation coefficient. The reason is the following: The risk of default is positively related to the volatility of the difference between firm value and liabilities which is given by

$$\sigma_{V-L} = \sqrt{\sigma_V^2 + \sigma_L^2 - 2\rho_{VL}\sigma_V\sigma_L}$$

and which is strictly larger than σ_V given our parameter specifications. Interestingly, the figures approximately double compared to the non-stochastic liability case. This demonstrates that credit-risk measures of derivatives are very sensitive with respect to the specification of the stochastic processes which determine the counterparties' default.

Table 3 – Extension of the credit risk model: Stochastic liabilities of the counterparty firm – The Credit-Value-at-Risk (CVaR) for Call Options on Zero Coupon Bonds

Correlation coefficient between firm value and Interest rate	Strike price of bond call option		
	80%	85%	90%
0.9	25.22%	30.73%	42.48%
0.5	22.78%	26.62%	39.49%
0	*19.60%*	*23.25%*	*35.73%*
-0.5	16.12%	19.17%	30.63%
-0.9	12.27%	13.72%	22.28%

Model risk – Example 3: Model class

So far, we have analyzed the model risk of counterparty risk only in one model class. We assumed that counterparty risk is adequately characterized by a firm value model, where the default probability and recovery rate are implicitly derived from the stochastic specification of the value of the assets and liabilities of the firm. Model risk, however, may also arise from the firm value model class not being adequate to model counterparty risk.

A popular alternative to firm value models are *intensity* or *reduced-form models*. They are based on a hazard process and a recovery rate, with the latter often being exogenous to the model. The hazard process models the probability of default, the recovery rate the associated losses in case of default.

In a single-asset context, the relative CVaR of a derivative security takes either a value of zero or one minus the recovery rate, depending whether the level of confidence is smaller or greater than the probability of default, respectively. Consider, for example, a loan portfolio consisting of one loan maturing in a year with an annual default probability of 0.8% and an estimated recovery in case of default of 50% of face value. At a 99% confidence level, the CVaR is zero for a one year horizon. Had the default probability been 1.2%, however, the relative CVaR would have amounted to approximately 50% of the face value of the loan (depending whether the current value of the loan is above or below par). Because of the binary nature of intensity models (the only two states are default and no-default), they cannot be meaningfully applied to compute VaR for a single position. In a portfolio context, this problem of simple intensity models used to compute quantile-based risk measures is mitigated, though not eliminated.

Finally, a risk measure such as VaR may not measure the relevant risk in some circumstances. If such a shortcoming of a risk measure manifests itself only in conjunction with a particular credit risk model or is exacerbated by a particular model, it can be viewed as another source of model risk. Consider again the loan with 0.8% probability of default within one year. If this one loan is replaced by two loans, each with half of the face value of the original loan and otherwise equal price and default characteristics, but independent occurrence of default, the CVaR actually increases from zero to approximately 25% of the loan amount because the probability of at least one loan defaulting is 1.59%. Contrary to common wisdom and because of the limitations of quantile-based risk measures in conjunction with hazard models, the act of diversification seemingly increased the risk of the portfolio.

Regulatory implications for the regulation of credit risk of derivatives

The modeling of credit risk is – not only in the context of derivative counterparty risk – more difficult and requires more assumptions than, for instance, market risk. The reasons are manifold:

- First, there is no consensus on a *"standard" model* or at least, a family of models, which is regarded to be appropriate to evaluate the credit risk of derivatives – as opposed to the valuation models for default-free stock options (Black/ Scholes, binomial) or interest rate derivatives (CIR, HJM models).
- Second, many of the models used by the banks were developed *in-house* and kept secret because they rely on bank-specific or client-specific information. As such they are not exposed to extensive academic discussion, refereeing by journals, and empirical testing to various data sets, which implies a substantial model risk.
- Third, the *availability of data* on bank specific credit risk is often very limited. Moreover, published data on credit risk, bond ratings, syndicated loan performance etc. are in many cases not representative for individual bank loans; see Carey (1998) and Altman/ Suggitt (2000) for respective evidence. Credit risk also also substantially differs between continents, countries, and sectors. This problem is somewhat limited for derivative counterparties because they are systematically rated by leading agencies.
- Fourth, backtesting credit risk models is sometimes problematic, because data are only available over a relatively short and possibly not *representative time period*. If back-testing is performed over a boom period, model risk is likely to be underestimated; see Carey (1998) for respective evidence on private placements.
- Fifth, in the case of OTC-derivatives, the credit risk is ultimately related to *market risk*: The risk of a counterparty default may be highest when the market moves in favor of our position (and thus, against the counterparty). But we know theoretically and empirically very little about the correlation between market and credit risk in stress scenarios.

With respect to the regulatory treatment of credit risk, the following aspects should be considered:

- Credit risk still accounts for the most substantial part of the regulatory capital. According to unpublished information of a major non-US central bank, credit risk represents approximately 90% of the overall regulatory capital of the banks in that country. Thus, regulators should keep in mind that inappropriate treatment of credit risk, including model risk or the credit risk related to derivatives, could substantially weaken the capital base and thus the systemic risk of the banking system.

- It can be observed that information disclosure has an impact on the behavior of firms. In the case of banks and other financial firms, the obligation to disclose information about the institutions' overall risk exposure (such as VaR, replacement values, major counterparties) tends to cause a „smoothing" of risk and earnings figures. This is, of course, more a hypothesis than an empirically verified fact. At least, one could observe during the emerging markets crises in fall 1998 that the VaR of the global market-risk of major Swiss banks remained virtually unchanged – or even slightly decreased – over the market breakdown. This implies that the exposures were drastically decreased in order to keep this figure stable. An analogous behavior would have drastic and adverse consequences in the credit risk area: If banks try to smooth CVaR over business or interest rate cycles, this could have extremely adverse effects on the overall supply of credit in the economy. Of course, not only smoothing, but also the length of the time horizon over which CVaR is calculated has an impact on the credit risk banks are willing to take.

- According to the Basle proposal on a new capital adequacy framework, regulators should recognize banks` internal credit risk models for determining the regulatory capital, under certain conditions. The results of this paper have shown that model risk is substantial even for very simple derivatives with counterparty risk. Given problems discussed in the previous paragraph, the regulatory recognition of internal models implies that part of the model risk, and thus part of the responsibility, is delegated to the regulatory authority. Once a model is accepted, it may be hard to create strong incentives for a bank to spend substantial resources to further improve the model (to a certain extent, this is done through the „multiplier“ which reflects the updated performance of the model). Of course, a more immediate question is who has the expertise outside the banking system to evaluate these models. There is a natural bias towards rather complex models that must be evaluated, because with simple models, the standard approach is likely to be more attractive given the lower implementation costs.

- Regulators should finally recognize that credit risk cannot be regulated separately from market risk. This is particularly true for OTC-derivatives. The counterparty risk is ultimately linked to market risk, as became clear in the numerical examples in this paper. Of course, the joint effect of market and credit risk must be derived from the same valuation or simulation model in order to avoid inconsistencies (or arbitrage opportunities). However, in many institutions, market and credit risk responsibilities are separated (even in the top management) which makes an integrated view difficult.

Conclusion

We have analyzed a simple derivative contract and found that the way in which credit risk is modeled has a substantial impact on the Credit-VaR. Because of the variety and complexity of the models used for quantifying credit risk and the resulting model risk, we argue that a regulatory system which is based on reviewing and approving internal credit risk models by regulatory agencies is very problematic. In our opinion, such a system might allow banks to transfer responsibility for model risk to regulators, an outcome that gives rise to behavioral risks and thus could adversely affect the banking system.

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