PUBLIC POLICY FOR EFFICIENT EDUCATION

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(March 2001; revised January 2002)

ABSTRACT

We study the role of public policy in promoting efficiency in human capital accumulation. Agents accumulate human capital by allocating time to home study and school attendance. The return to time spent in school is subject to congestion. The individual also faces an aggregate externality in skill accumulation. We find that a tuition fee combined with personal stipends can correct the resulting distortions by partly shifting educational effort from schools and universities to noninstitutional forms of learning, such as home study. The dynamic effects of education policy as well as their welfare implications are also calculated in the paper.

1. INTRODUCTION

Manpower quality has long been recognized as one of the most important sources of economic growth.¹ Education and training are also believed to yield important social returns.² Thus, promoting human capital formation has long been a prime public policy concern. In fact, most governments have devised a large array of programs to improve

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* We gratefully acknowledge financial support by the Austrian National Bank (OeNB) under Jubilaeumsfondsprojekt No. 7394. Earlier versions of this paper were presented at the 1998 European Econometric Society Meetings in Berlin, Germany, and the economic theory seminars of Louisiana State University, CERGE/Charles University, Czech Republic, the University of Bern, and CentER at Tilburg University. We appreciate the comments of Dean Jolliffe, Thomas Renström and the other seminar participants. We are particularly grateful for constructive suggestions by two anonymous referees.


² A strain in the endogenous growth literature assumes that spillovers are so strong that they result in constant returns to aggregate human capital accumulation. See Lucas (1988) and Chamley (1993).
educational attainment. Most, if not all, countries have a system of public schools, with primary and secondary education virtually free. Not only are numerous grants, subsidies and loans available to encourage higher education, but students are also offered a wide variety of institutions. Access to schools and public universities is free in continental Europe and in many other countries as well. Could these policies possibly neglect or even crowd out private inputs by students, parents or firms outside the formal educational institutions? Should governments, apart from distributing personal subsidies, provide free access to higher educational institutions, as in continental Europe? Or should they charge tuition fees, as is commonly the case in the USA? Will this policy induce students to work harder at home and thereby help them to attain the same or even higher educational standards with shorter duration of school enrollment? In this paper we consider human capital formation, both within and outside the formal institutions, and in the presence of spillovers in learning and potential crowding of public educational infrastructure. We then discuss optimal policies that provide agents with the appropriate incentives for educational effort at home and in schools.

The effect of education policy on human capital formation has been the subject of intensive empirical and theoretical research. A large part of the literature emphasized the effects of schooling on the distribution of income among heterogeneous agents, dealing with issues such as educational signaling and screening, the role of stipends or vouchers to overcome credit constraints or missing markets, or the trade-off between disparate local versus uniform national funding of schools. This paper, in contrast, asks a somewhat different question that is not directly related to distributional concerns: to what extent do agents rely on formal educational institutions when self-study and learning at home are alternative means of acquiring skills? To address this question, we present a simple model of homogeneous agents in which self-study and school attendance are substitutable inputs in skill formation. This additional decision margin, which has previously been neglected in the literature, is the novel feature of our approach. Home study activity, which includes learning within families and firms, may be viewed as a non-institutional

3 Spence (1973) is the seminal contribution in this branch of the literature.
4 Benabou (2001), for example, studies the role of progressive education finance, while Fernández and Rogerson (2001a) investigate various voucher policies. Fernández and Rogerson (2001b) show that credit constraints can be an important determinant of social stratification in a model of educational choice.
5 See Benabou (1996) and Fernández and Rogerson (1998), among others. These and the previous citations exemplify a very extensive literature on this subject.
source of skill formation. Heckman (1999), for one, considers this type of learning to be as important a source of skill acquisition as formal schooling that uses public infrastructure, such as schools and universities. This extension allows us to analyze how governments may prevent the possible congestion of educational infrastructure by creating incentives for non-institutional learning outside of school facilities.

There is growing empirical support for our main assumptions regarding home learning, school attendance and congestion of classrooms. The effect of class size has been addressed in a large empirical literature studying the ‘educational production function’, which investigates the statistical relationship between educational attainment and class size, teachers’ qualifications and salaries, family and neighborhood characteristics, among other controls. Krueger (1999) finds significant and positive effects of smaller classes on educational achievement, with this effect larger for black, economically disadvantaged and inner city children. Teacher characteristics, in contrast, have notably weak explanatory power. Hanushek (1992), in contrast, found no significant effects from smaller classes. Based on an analytical model of educational production, Lazear (2001) offered a resolution to the empirical puzzle regarding the effect of class size: according to the envelope theorem, variations of class size must have zero first-order effects on achievement if it is already at its optimal level and can have significant effects only if it is not optimally chosen beforehand.

Lazear’s (2001) analysis is based on the public goods character of classroom education that is characterized by congestion externalities. Students who are ill-prepared or have short attention spans tend to interrupt classes and prevent other students from benefiting from classroom education. While the school administrators prefer, in general, to raise the number of students per class in order to economize on school infrastructure and teachers’ salaries, increasingly ‘damaging’ congestion effects eventually make larger classes very costly in terms of educational attainment. Congestion results from the fact that the probability of classroom interruptions increases with a higher number of students, with the consequence that the effectiveness of lectures declines very rapidly. Students who have a shorter attention span, who are ill disciplined or ill prepared, are particularly damaging to classroom education, making congestion effects even more severe. Since this is particularly true for

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6 See also the review of Hanushek (1999). Hoxby (2000), exploiting jumps in class sizes between US school districts, found small and insignificant effects of smaller classes on educational achievement.
students with unfavorable family background and for very young children, kindergarten classes or classes in economically disadvantaged city neighborhoods tend to be smaller. Lazear (2001) however, did not, discuss the implications of this problem for the aggregate level of school infrastructure. It is nevertheless clear that smaller classes require higher spending on teachers and infrastructure in order to accommodate overall school enrollment. In the light of Lazear’s model and his supporting empirical evidence, we conclude that the congestion of educational infrastructure is an important consideration in deciding the level of education expenditures. Note, however, that there may also exist positive spillovers among students, such as the role model provided by good students, which run counter to the negative congestion effects. This type of school-based interaction has been posited by Benabou (1993) in his model of occupational and residential choice. For this reason, it is important to allow for a variable degree of ‘net’ congestion, as we do in our formal model.

Another central assumption of our model—backed also by empirical evidence—is that students can shift their effort between school attendance and home study. At the university level, this trade-off is quite evident as the recent emergence of Internet-based courses or the existence of the ‘Fernuniversitaet’ in Germany demonstrates. These institutions do not offer classroom education. Students meet only for test and grading purposes but otherwise prepare at home. Even at the primary and secondary school level, a significant trade-off exists between the intensity of home study and the services appropriated by school attendance. Lindahl (2001) points to the importance of summer learning when

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7 Recent authors who have examined the macroeconomic implications of congestion of government infrastructure include Fisher and Turnovsky (1998) and Eicher and Turnovsky (2000). Fisher and Turnovsky (1998) focus on the impact of congested public capital on private investment, while Eicher and Turnovsky (2000) examine the effects of congestion on the balanced growth rate and the rate of transitional dynamics.

8 Since in the Benabou (1993) framework pupils (endogenously) segregate into high and low skill categories, this specification of pupil interaction—which Benabou (1993) also terms local human capital spillovers—is an appropriate one for the problem he analyzes. Nevertheless, we believe that aggregate congestion externalities are more relevant to our fiscal policy model than local peer group effects.

9 Policies to encourage peer-group learning at the college and university level include requiring students to live in campus dormitories and discouraging automobile use. We are indebted to an anonymous referee for pointing this out.

10 In the contrasting framework of Epple and Romano (1996), households are restricted to choose either public or private schools. In their model, the quality of public schools depends on the interactions of different income classes.
schools are out of session and therefore only family background matters. In fact, taking account of home learning in the summer period turns out to be important in identifying significant class size effects. The study by Betts (1997) is particularly relevant for our arguments. He finds that the amount of homework assigned significantly improves student test scores and is even more important for educational attainment than traditional policies such as increasing spending per student. It seems, furthermore, that additional homework is a potentially cost-effective way to raise educational performance, since it is the amount of homework assigned rather than the amount graded that has the larger influence on achievement. Therefore, a policy of increasing homework standards need not overburden teachers and promises to be a cost-effective way for improving school quality. In fact, Betts (1997, p. 27) finds that the correlation between class size and homework assigned is positive and equal to 0.096 (or 0.062 if homework is also graded and corrected). This finding is very much consistent with Lazear’s (2001) argument that well-prepared students allow for a larger optimal class size. Well-prepared students are better motivated, more attentive in class, and also provide a role model to fellow students. In this way, additional home study mitigates the congestion problem in schools. We take account of this mechanism in our formal model by making educational attainment dependent on both home study and school attendance that is subject to congestion. In shifting total study time towards home study, agents reduce congestion associated with a given level of educational infrastructure. We then proceed to determine the optimal level of infrastructure as well.

We believe that our model relates well to current debates on higher education policies, especially in continental Europe. These countries have traditionally offered free access not only to primary and secondary schools but also to universities and other institutions of higher education. There is some evidence, however, that the supply of public educational infrastructure has not kept pace with the rise in the number of students in recent decades. Universities, in particular, are severely under-equipped to serve this increase in demand. This has led to overcrowded classrooms, a rising number of students per professor, an increasingly restricted access to libraries and computer equipment and, finally, an increase in the average length of time required to complete a degree.11 Such indicators of

11 For the data on the increase in the duration of study in the Austrian case, see the Report on Higher Education by the Austrian Federal Ministry of Science (1999). This increase has taken place across almost all fields of study and been particularly severe in business administration, medicine, law, electrical engineering and pharmacy. Using data from previous years of the Report, we also find evidence of rising student-to-faculty ratios.
congestion have prompted a number of governments to consider the imposition of tuition fees as a means both of shortening the average study length and of discouraging the attendance of marginal students. For example, the Austrian government formally introduced tuition fees starting in 2001 that are partly compensated by student stipends. A number of German federal states have proposed charging fees to students who exceed the average study length. We interpret such measures as an attempt to reduce the congestion of educational infrastructure. If, in addition, tuition fees are combined with personal stipends, students are then encouraged to shift to other non-institutional forms of learning, rather than to completely sacrifice the opportunity of advanced training. Our model implies that governments, for reasons of efficiency, should encourage education in general, but should also charge tuition fees to avoid the congestion of school infrastructure. We thus provide a rationale for granting educational subsidies that is independent of distributional considerations, which tend to dominate public discussions of this issue.

These arguments are formalized in a model that captures some of the key factors that influence human capital formation. These include individual effort, both in terms of home study and school attendance, aggregate educational spillovers and the supply of public educational infrastructure. Among the policy instruments, we consider a personal subsidy (or stipend), which does not discriminate between the allocation of time spent at home and in schools, plus a tuition fee which taxes the time spent in schools or universities. We also calculate the optimal level of educational infrastructure in this framework. Section 2 describes the model and develops the decentralized equilibrium. Section 3 then compares the market allocation with the social optimum. Here we show that the optimal policy can be to subsidize overall educational effort and, simultaneously, to tax school attendance, i.e. the time spent in using the publicly provided educational infrastructure. Section 4 derives the log-linearized form of the model, with the mathematical details contained in an appendix, and discusses the comparative dynamic effects of public infrastructure and subsidy/tuition policies. Section 5 briefly concludes.

2. DECENTRALIZED EQUILIBRIUM

2.1 The model

In this section we will describe a representative agent model in which skills, or human capital, are accumulated through effort and educational
infrastructure. Unlike Lucas (1988) and Chamely (1993), human capital formation is bounded, reflecting diminishing returns to education. Unbounded accumulation is prevented by the fact that old agents lose part of their skills in the process of aging and, eventually, all skills with death. In a state of demographic and economic equilibrium, the education of young agents and the death of old agents just balances to give a finite constant stock of aggregate skills. One way to incorporate bounded human capital accumulation in an aggregate model—short of explicitly modeling and aggregating the life-cycle education decisions of overlapping generations—is to assume depreciation of human capital and diminishing returns to education in an infinitely lived representative agent model. We believe that the representative agent framework is useful for the purpose of this paper, since the emphasis is on efficiency aspects of government education policy rather than on questions of intergenerational distribution.

The process of individual skill accumulation in our model depends on own and aggregate effort, as well as on the services of schools. To acquire skills, individuals typically spend some time attending schools. Expenditures on education include salaries for teachers as well as outlays for maintaining and possibly expanding the number of schools and their facilities. These services encourage individual learning, i.e. the accumulation of human capital. We consider schools in this framework to be ‘impure’ public goods. Schools are also to a certain extent non-rival, since it is not automatically the case that an additional student ‘in class’ reduces one-for-one the educational services received by another student. The government in our model provides an aggregate level of educational infrastructure, $K$. We will assume a complementarity between effort and the level of schools, so that individuals become more effective in improving their skills if the level of public expenditures on schools is high. To keep the dynamics simple, we will model $K$ as a flow variable. Like other forms of government expenditure, an increase in

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12 Glomm and Ravikuma (1992) employ an overlapping generations model to analyze the effect on growth and income inequality of private versus public education regimes.

13 Barro and Sala-i-Martin (1995) view congestion as a pervasive problem for most publicly provided goods. We follow the definition of Oakland (1972), who cites congestion externalities as leading to ‘impurity’ in public goods. Of course, the existence of tuition fees implies that schools are also excludable goods whether or not they are publicly or privately provided.

14 A more complete, although more complicated, model would specify $K$ as a stock that depreciates at a certain rate. Such an approach has been recently adopted by Fisher and Turnovsky (1998).
expenditures on schools will divert resources from private consumption. We also consider the existence of educational spillovers in making individual achievement dependent on the aggregate level of human capital. We shall then specify individual human capital accumulation in the following way:

\[
\dot{h} = G(e, K^0)H^\gamma - \delta h \quad K^0 = i(i/I)^\gamma K \quad 0 \leq \sigma \leq 1, 0 < \gamma < 1
\]

(2.1)

where \( h \) is the individual, \( H \) the aggregate stock of human capital, and \( G(e, K^0) \) the component of human capital accumulation that depends on individual effort and the supply of public school infrastructure.\(^{15}\) The agent allocates his time endowment (normalized to one) to total educational effort or to work, \( l \). We further assume that the time devoted to education can be broken down into a home study component \( e \) and a component representing the time spent in schools, \( i \), so that \( l = 1 - e - i \). As indicated above, we term the time spent in acquiring skills away from school as ‘home study’, but this should be thought of as any type of educational activity independent of public educational facilities.

In addition to devoting time to home study, the student gains skills by using the services, \( K^0 \), of schools. We assume in (2.1) that the services of schools depend on the individual, \( i \), and the aggregate amount of time \( I \) spent in schools as well as on the supply of schools, \( K \). In our formulation of \( K^0 \), the influence of the aggregate time spent in school depends on the parameter \( \sigma \). If \( \sigma = 0 \), then schools are strictly non-rival public goods that can be consumed simultaneously by all students. In this case each student appropriates \( iK \) in public school services if he spends \( i \) units of time attending classes. If, on the other hand, \( 0 < \sigma \leq 1 \), then the services of public schools are subject to congestion externalities. Here, the same amount of time spent in ‘crowded’ schools yields only \( i[(i/I)^\gamma K] \) in services, because congestion deteriorates the quality of services, with a larger value of \( \sigma \) corresponding to a higher degree of congestion. For the special case in which \( \sigma = 1 \), the effective capacity of schools equals \( (i/I)K \). This case represents proportional congestion, since school capacity remains constant only if the aggregate

\(^{15}\) Lower case variables will refer to individual quantities, while upper case variables will denote aggregate levels. Unless indicated, we will suppress a variable’s functional dependence on time. A dot over a variable indicates a time derivative.
level of schools, \( K \), increases in direct proportion to aggregate attendance, \( I \). Our interpretation is similar to that used by Fisher and Turnovsky (1998) in their model of the macroeconomic implications of congestion externalities in public capital.\(^{16}\)

Along with the congestion externalities arising from school attendance, we specify in (2.1) that individual human capital accumulation depends positively on the level of aggregate skills, \( H \), where \( \gamma \) parametrizes the extent of this spillover. In other words, we consider, as do Cooper R. and John (1988), that it is easier to acquire skills if others also have them.\(^{17}\) For simplicity, and with no loss in generality, we impose linear homogeneity on \( G \). Since, as indicated, this is a model of bounded human capital accumulation, we specify in (2.1) that skills deteriorate at the constant rate \( \delta \).

A simple specification of the production sector is to use a dynamic Ricardian model in which human capital is the sole input. Given a human capital stock \( h \) inherited from the individual’s previous educational decisions, effective labor supply is then \( lh = (1 - e - i)h \), which produces the following output level:

\[
y = (1 - e - i)h
\]

(2.2)

Effective labor earns its marginal product \( w = 1 \), i.e. the real wage is fixed at unity. Our framework then implies that if the individual decides to spend more time in acquiring skills, either in home study or in school, he sacrifices current wage income in exchange for a higher level of human capital in the future.

We assume that a continuum of identical agents with unit mass maximizes the discounted time-separable utility of consumption over an infinite horizon. Intertemporal preferences for each agent are given by

\[
U_0 = \int_0^\infty \exp(-\rho t) u(c)dt
\]

(2.3)

\(^{16}\) Observe that we exclude in (2.1) the case in which aggregate attendance spillovers are positive on net. In terms of the congestion parameter \( \sigma \), this corresponds to \(-1 < \sigma < 0\).

\(^{17}\) This formulation is akin to the models of Romer (1986, 1989) in which the aggregate capital stock embodies the stock of knowledge. Some suggestive empirical evidence for this has been provided by Glaeser (1994), who shows that the level of schooling has a powerful effect on the growth of schooling. Benabou (1996) offers a microfoundation of the aggregate spillovers in educational production that gives rise to a reduced form accumulation equation such as (2.1), although he does not introduce educational infrastructure.
where $\rho$ is the exogenous rate of time preference. A further simplifying specification restricts instantaneous preferences to the logarithmic case, $u(c) = \ln c$. In addition to the time constraint $l = 1 - e - i$, the individual’s actions are subject to the accumulation equations for assets and human capital:

\begin{align*}
\dot{a} &= ra + w[1 - (1 - \tau)e - (1 - \tau + z)i]h - \chi - c \quad (2.4a) \\
\dot{h} &= G(e, K^s)H^\beta - \delta h \quad (2.4b)
\end{align*}

where $a$ represents financial assets that yield a real return $r$, $c$ is consumption and $\chi$ are lump-sum taxes (the government budget constraint will be introduced below). In equation (2.4a), educational subsidy policy is introduced. We specify that the total time devoted to education, $e + i$, receives a subsidy, or stipend, $\tau$. This is a reasonable assumption, we believe, because educational stipends do not, in general, discriminate between the time spent in home study and the time spent in attending school. We assume, in addition, that the time spent using the educational infrastructure attracts a specific fee, denoted by $z$. For reference, equation (2.4b) repeats (2.1).  

2.2 Optimality conditions and market equilibrium

In solving the utility-maximizing problem, we attach the multipliers $\lambda$ and $\mu$, respectively, to the dynamic constraints $\dot{a}$ and $\dot{h}$. The optimizing choices satisfy the following first-order conditions:

\begin{align*}
c: \quad 1/c &= \lambda \quad (2.5a) \\
e: \quad (1 - \tau)wh &= (\mu/\lambda)G_e(e, K^s)H^\beta \quad (2.5b) \\
i: \quad (1 - \tau + z)wh &= (\mu/\lambda)G_i(e, K^s)H^\beta(1 + \sigma)(i/I)^\sigma K \quad (2.5c) \\
a: \quad \dot{\lambda}/\lambda &= \rho - r \quad (2.5d) \\
h: \quad \dot{\mu}/\mu &= \rho + \delta - (\lambda/\mu)w[1 - (1 - \tau)e - (1 - \tau + z)i] \quad (2.5e)
\end{align*}

18 We could also incorporate an explicit labor/leisure choice into the problem and specify that educational subsidies and infrastructure expenditure are financed using taxes on labor income. By making the admittedly strong assumption of lump-sum taxation, we focus on the allocating role of educational policies and keep the algebraic solution of the model as simple as possible.
According to (2.5a), the agent chooses a level of consumption so that its marginal utility equals its shadow value, $\lambda$. In deciding how much time to spend on private study or in school, agents compare the marginal forgone wage income today, net of subsidies and fees, with the present value $(\mu/\lambda)$ of future wage incomes that accrue because education raises the future stock of human capital. The existence of aggregate human capital spillovers and the externalities arising from the decision to accumulate human capital in school modifies the optimality conditions (2.5b), (2.5c) for $e$ and $i$. Observe that the return on home study, $G_e H^\gamma$, depends on the aggregate stock of human capital as well as on individual effort.

The return on the time spent in school depends on the term $H^\gamma(1 + \sigma)(i/I)^\sigma K$, which, in addition to $H^\gamma$, is a function of the ratio $i/I$, the supply of public schools $K$ and the congestion parameter $\sigma$. Under conditions of congestion, $\sigma > 0$, an individual student may not only exploit a given effective capacity per student, $(i/I)^\sigma K$, for a longer time span $i$, but may also attain for his own use a larger share of the overall effective capacity $(i/I)^\sigma K$. In other words, congestion leads individuals to believe that they may capture a larger fraction of the overall capacity by attending classes more intensively. The existence of aggregate human capital spillovers and the externalities arising from public school attendance must be taken into account in evaluating the overall benefits of time spent at home and in school and, as we shall show below, will determine socially optimal values for $\tau$ and $z$. The last two optimality conditions (2.5d), (2.5e) describe, respectively, the evolution of the shadow values of wealth and human capital if optimal paths of these two variables are chosen. In addition, the accumulation of financial assets and human capital must satisfy the usual transversality conditions.

To calculate the decentralized macroeconomic equilibrium, we use the assumption of a continuum of identical agents to derive that the aggregate times devoted to work, home study and time in school are given, respectively, by $L = l$, $E = e$ and $I = i$. Applying these relations to (2.1), we rewrite the aggregate human capital accumulation function as

$$G = G(e, ik) = eg(Ki/e)$$

where $K^s = iK$. Linear homogeneity allows us to write $G$ in the intensive form $eg(Ki/e)$, where $g$ is increasing and concave. Since there is no net trading of financial assets in equilibrium and, by assumption, no government debt, we can set $a = a = 0$ and $A = A = 0$. Using this
fact and the aggregate relationships, the economy-wide private sector budget constraint is given by
\[
C + T + ziH = [L + \tau(e + i)]H
\]  
(2.7)
where \(C, T, L\) and \(H\) correspond to aggregate consumption, lump-sum taxes, labor supply and human capital, respectively. The government pays for its expenditures on schools and finances its subsidy policies by levying lump-sum taxes and tuition fees. This implies that the aggregate government budget constraint equals
\[
K + \tau(e + i)H = T + ziH
\]  
(2.8)
Substituting the government budget constraint into its private sector counterpart and using the aggregate version of (2.2), we obtain the following market clearing condition:
\[
C + K = LH \equiv Y
\]  
(2.9)
where \(Y = (1 - e - i)H\) is aggregate output.\(^{19}\)
To derive the economy’s dynamics, we will first obtain an equilibrium restriction on the ratio of time spent in school to time spent in home study. Without loss of generality, we can simplify our analysis by assuming that the human capital accumulation function \(G\) has a Cobb–Douglas specification, so that \(\alpha = eG_e/G\) and \(1 - \alpha = iKG_K/G.\(^{20}\) Using the equilibrium version of (2.5b) and (2.5c), the ratio \(i/e\) is given by
\[
\frac{i}{e} = \frac{(1 + \sigma)(1 - \alpha)}{\alpha} \frac{1 - \tau}{1 - \tau + z}
\]  
(2.10)
Observe that the ratio \(i/e\) depends, in addition to the technology and congestion parameters, exclusively on \(\tau\) and \(z\). Rewriting (2.5b) and using our technological assumptions and the intensive form of \(G\), we obtain the ratio

\(^{19}\) Hereafter, we shall designate the equilibrium time spent in home study and in school, \(e\) and \(i\), with lower case letters, while all other variables will be denoted with upper case letters.

\(^{20}\) The Cobb–Douglas function implies \(G = eg(Ki/e)\) and \(g = (Ki/e)^{1-\alpha}\).
which we substitute into (2.5e) to calculate the dynamic equation for the shadow price. This, together with (2.4b), forms the dynamic system in $(\mu, H)$ that determines the evolution of intertemporal macroeconomic equilibrium:

\[
\dot{\mu} = (\rho + \delta - r^H)\mu \\
\dot{r}^H = \frac{\alpha g(Ki/e)H^{\gamma - 1}}{1 - \tau}[1 - (1 - \tau)e - (1 - \tau + z)i] \\
\dot{H} = eg(Ki/e)H^{\gamma} - \delta H
\]

where $r^H$ denotes the rate of return on human capital (so that the implicit return on financial assets is $r = r^H - \delta$). Consider the dynamics of the shadow value of new skills. If the return to human capital $r^H$ is high, the shadow value $\mu$ of new skills is also high. In response, agents increase their efforts at education. The shadow value then declines as skill levels improve and the return on human capital is driven down to the long-run rate equal to $\rho + \delta$.

To complete the description of intertemporal equilibrium, we next show how $e$ and $i$ depend on the dynamic variables and exogenous policy parameters. The time $e$ devoted to home study and the time $i$ spent in school are interdependent according to (2.10). Note further that the ratio $i/e$ does not depend on $\mu, H$ and $K$, but only on the subsidy and fee rates $\tau$ and $z$, the share parameter $\alpha$ and the congestion parameter $\sigma$. Nevertheless, both $e$ and $i$ can be expressed as a function of the dynamic variables $\mu$ and $H$. Substituting aggregate consumption $C = 1/\lambda$ into (2.5b), using the Cobb–Douglas specification of $g$, and replacing $C$ from the product market clearing condition (2.9) yields

\[
1 - \tau = \mu \alpha g\left(\frac{Ki}{e}\right)H^{\gamma} \left[1 - \left(1 + \frac{i}{e}\right)e - \frac{K}{H}\right]
\]

This equation solves for $e(\mu, H; \tau, z, K)$ and, since (2.10) fixes the ratio $i/e$, also determines the value of $g(Ki/e)$.21 In the appendix, we derive a log-linearized version of the dynamic equations (2.12). Using this system, discussed below in section 4, we can obtain analytical solutions

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21 Once $e(\mu, H; \tau, z, K)$ is obtained $i(\mu, H; \tau, z, K)$ is calculated using (2.10).
for $\mu$ and $H$ and can calculate the long-run comparative dynamic response of these variables with respect to government infrastructure expenditure and stipend/tuition policy.

The steady-state equilibrium occurs when $\dot{H} = \ddot{\mu} = 0$. It consists of the following relationships:

$$H_{\infty}^{1-\gamma} = \frac{e_{\infty} g(\cdot)}{\gamma} \quad (2.14a)$$

$$H_{\infty}^{1-\gamma} = \frac{\alpha g(\cdot)}{\rho + \delta} \left\{ \frac{1}{1 - \tau} - \left[ 1 + \frac{(1 + \sigma)(1 - \alpha)}{\alpha} \right] e_{\infty} \right\} \quad (2.14b)$$

where (2.10) has been used to obtain (2.14b) and the subscript $\infty$ denotes a steady-state value. We can now state the first proposition of the paper.

**Proposition 1 (Steady-state time allocations in decentralized equilibrium):**

The steady-state allocations of time spent in home study and time spent in school are given by

$$e_{\infty} = \frac{1}{1 - \tau} \frac{\alpha \delta}{\rho + \delta + \delta[\alpha + (1 + \sigma)(1 - \alpha)]} \quad (2.15a)$$

$$i_{\infty} = \frac{1}{1 - \tau + \sigma \frac{\delta(1 + \sigma)(1 - \alpha)}{\rho + \delta + \delta[\alpha + (1 + \sigma)(1 - \alpha)]}} \quad (2.15b)$$

The long-run expression for $e_{\infty}$ was found by equating the two conditions (2.14), while the long-run solution for $i_{\infty}$ was determined by substituting (2.15a) into the ratio (2.10). Observe that $e_{\infty}$ and $i_{\infty}$ depend on the parameters of the model, such as $\sigma$, and on the overall subsidy rate $\tau$, but not on the aggregate human capital spillover parameter $\gamma$ or the level of educational infrastructure $K$.22 The latter is a consequence of employing the Cobb–Douglas specification in the human capital accumulation function.23 Nevertheless, even if the time spent in skill accumulation does not depend on the level of public infrastructure, the stock of human capital, $H_{\infty}$, always does. With the $i/e$ ratio, and consequently $g(Ki/e)$, determined by (2.10), the long-run skill level $H_{\infty}$

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22 While the aggregate human capital spillover affects neither the level nor the division of time spent in home study or in school, it does affect their productivity.

23 In the case of non-unitary elasticity of substitution, the results would be more complicated. In general, the steady-state levels of $e$ and $i$ would then depend on $K$. 

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is then inferred from (2.14a), along with the shadow value \( \mu_\infty \) from (2.13). In examining the solutions for \( e_\infty \) and \( i_\infty \), we can also investigate the influence of the congestion parameter \( \sigma \). It is straightforward to show that \( e_\infty \) is smaller and \( i_\infty \) is larger if there are congestion externalities in school, \( 0 < \sigma \leq 1 \). The individual then strives to acquire human capital by attending school more intensively while reducing the time spent in home study.

To consider the steady-state influence of subsidy, tuition and infrastructure policy on the time spent at home and at school as well as on the stock of human capital and its shadow value, we next state the following proposition.24

**Proposition 2 (Impact of education subsidy and infrastructure policy):** The impact of education subsidies on the long-run time allocations to home and school effort is equal to

\[
\hat{e}_\infty = \frac{1 - \tau + z}{1 - \tau} \tilde{\tau} \quad (2.16a)
\]

\[
\hat{i}_\infty = \tilde{\tau} - \hat{\tilde{\tau}} \quad (2.16b)
\]

The effect of subsidy and infrastructure policy on steady-state human capital is

\[
\hat{H}_\infty = \frac{1}{1 - \gamma} \left[ (1 - \alpha)(\hat{\hat{K}} - \hat{z}) + \left( 1 + \frac{\alpha z}{1 - \tau} \right) \tilde{\tau} \right] \quad (2.17)
\]

The effect on the long-run shadow value of human capital equals

\[
\hat{\mu}_\infty = -\frac{1 - \alpha - (1 - \gamma)k}{(1 - \gamma)(1 - k)} \hat{\hat{K}} + \frac{1 - \alpha - (1 - \gamma)s_I}{(1 - \gamma)(1 - k)} \hat{z} - \frac{1 - (1 - \gamma)s - [(1 - \gamma)(s - s_I) - \alpha]z/(1 - \tau)}{(1 - \gamma)(1 - k)} \tilde{\tau} \quad (2.18)
\]

where \( s_I \equiv i/l \), \( s \equiv (1 - l)/l \) and \( k \equiv K/Y \).

Logarithmic derivatives of (2.15) were taken to obtain (2.16). Observe

---

24 A logarithmic derivative of a variable \( x \) is denoted \( \frac{dx}{x} \). The exceptions are \( \frac{d\tau}{1 - \tau + z} \) and \( \frac{dz}{1 - \tau + z} \), respectively. We also make the innocuous assumption that \( (1 - \tau + z) > 0 \).
that the long-run response of home study, \( \hat{e}_\infty \), depends solely, though more than proportionately, on the rate of change of the overall subsidy rate \( \hat{t} \). In contrast, the adjustment of school attendance, \( \hat{i}_\infty \), depends on the difference between the rates of change of the overall subsidy and the tuition fee, \( \hat{t} - \hat{z} \). Both \( \hat{e}_\infty \) and \( \hat{i}_\infty \), however, are independent of \( \hat{K} \).

To calculate the long-run impact of subsidy policy on \( \hat{H}_\infty \), we substituted \( \hat{e}_\infty \) and \( \hat{i}_\infty \) into the percentage change of \( g \),

\[
\hat{g} = (1 - \alpha)(\hat{K} + \hat{i}_\infty - \hat{e}_\infty) = (1 - \alpha)[\hat{K} - \hat{z} - z\tau/(1 - \tau)],
\]

and the resulting expression into the log-linearized version of (2.14a), which is equal to \( \hat{H}_\infty = (1 - \gamma)^{-1}(\hat{e}_\infty + \hat{g}) \). This yields (2.17). According to (2.17), improved educational infrastructure and more generous subsidies serve to expand human capital. Higher tuition fees would tend to lower \( \hat{H}_\infty \). The extent of the changes depends on the factor shares, the spillover parameter \( \gamma \), and the initial subsidy and tuition rates \( \tau \) and \( z \).

Next, substituting \( \hat{e}_\infty \), \( \hat{g} \) and \( \hat{H}_\infty \) into (A6) (derived in the appendix), we solve in (2.18) for the long-run comparative statics for the shadow value of human capital. Whether \( \hat{\mu}_\infty \) rises or falls depends, in general, on the resource cost of the policy change relative to its effect on the returns to educational activities. Consider the following illustrative cases.

In the absence of human capital spillovers, \( \gamma = 0 \), an increase in \( \hat{K} \) will lower (raise) \( \hat{\mu}_\infty \) if the output share of schooling infrastructure \( k \) is less (greater) than the infrastructure share in skill production, \( 1 - \alpha \). With positive spillovers, the same effect continues to hold for even higher values of \( k \). Similarly, if we set the initial subsidy and tax rates at zero, \( \tau = z = 0 \), assume there is no congestion, \( \sigma = 0 \), and substitute for \( s \) and \( s_I \), we can then show that increases in \( \hat{t} \) and decreases in \( \hat{z} \) reduce \( \hat{\mu}_\infty \).

3. SOCIAL OPTIMUM

3.1 Optimality conditions and equilibrium

Private agents take as given the number of schools and the educational efforts of other agents. Society at large, however, faces a trade-off between the resource cost of public infrastructure and the returns that

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25 The congestion parameter \( \sigma \) drops out of the steady-state expressions for \( \hat{e}_\infty \) and \( \hat{i}_\infty \). This is a consequence of having imposed a unitary elasticity of substitution in skill production.

26 Evaluating (2.15) in this case, we have \( e = \alpha\delta/(\rho + 2\delta) \) and \( i = e(1 - \alpha)/\alpha \). Equation (A1) then gives \( \hat{s} = \delta/(\rho + \delta) \) and \( s_I = (1 - \alpha)\hat{s} \).
result from changing $\tau$ and $z$, which include any externalities involved in
the process of acquiring skills. Weighing the per capita benefits against
the resource costs of marginally expanding the size of educational
infrastructure determines a welfare maximizing number of schools. This
trade-off and its welfare implications are addressed by a social planning
approach in which per capita intertemporal utility is maximized subject
to the aggregate resource constraints, i.e. the market clearing condition
for the output and the law of motion for human capital. In deciding on
behalf of the entire community, the planner chooses the effort of all
agents simultaneously. This takes into account the aggregate level of
externalities in school attendance. The planner internalizes these spill-
overs by setting $i = I$, which results in $K^s = iK$. By equating $h = H$,
the planner also takes into account how the aggregate level of human
capital contributes to skill accumulation. This planning problem can
therefore be stated as

$$\max \int_0^\infty \exp(-\rho t) \ln(C) dt$$

subject to

$$C + K = (1 - i - e) H$$
$$\dot{H} = G(e, iK) H^{\gamma} - \delta H$$  \hspace{1cm} (3.1)

The social optimum is characterized by the following maximum condi-
tions:

$$e: \quad \frac{H_{1-\gamma}}{C} = \mu G_e(e, iK)$$  \hspace{1cm} (3.2a)
$$i: \quad \frac{H_{1-\gamma}}{C} = \mu G_k(e, iK) K$$  \hspace{1cm} (3.2b)
$$K: \quad \frac{H_{-\gamma}}{C} = \mu G_k(e, iK)i$$  \hspace{1cm} (3.2c)
$$H: \quad \frac{\dot{\mu}}{\mu} = \rho + \delta - \gamma G(e, iK) H^{\gamma-1} - \frac{\theta}{\mu C} (1 - e - i)$$  \hspace{1cm} (3.2d)

Condition (3.2a) weighs the utility cost of forgone consumption against
the utility gain from increased skills due to the socially optimal time
spent in home study. Conditions (3.2b), (3.2c) apply the same criterion
to determine the optimal intensity of school attendance and educational infrastructure. Condition (3.2d) describes the socially optimal evolution of the shadow value of skills. Using the intensive form of $G$ described in footnote 20, we obtain from (3.2a)

$$\frac{1}{\mu C} = \alpha g\left(\frac{K_i}{e}\right) H^{\gamma - 1}$$

which is the socially optimal version of (2.11). Substituting (3.3) into (3.2d) yields

$$\frac{\mu}{\mu} = \rho + \delta - r^{H^*} \quad r^{H^*} = [ye + \alpha(1 - i - e)]g(K_i/e)H^{\gamma - 1}$$

where $r^{H^*}$ is the social return of human capital (henceforth, we will denote a socially optimal value by an asterisk). Next, combining (3.2a), (3.2b) yields the socially optimal ratio of time spent in school to time in home study:

$$\left(\frac{i}{e}\right)^* = \frac{1 - \alpha}{\alpha}$$

Equation (3.5) implies that $(i/e)^*$ should be set equal to the ratio of the factor shares of school services and home effort, $(1 - \alpha)/\alpha$. It is instructive to compare (3.5) to (2.10), the ratio in the decentralized equilibrium in which individual decisions are distorted by externalities, subsidies and fees. If there were no congestion, $\sigma = 0$, and the subsidy and tuition rates were set to zero, $\tau = z = 0$, then (2.10) would coincide with (3.5). Observe also that the optimal ratio (3.5), like its decentralized counterpart (2.10), is independent of $\gamma$, the spillover parameter for human capital accumulation. As we shall show below, it is the ‘wedge’ between private and social returns that implies a role for optimal government policy. Next, using (3.2b), (3.2c) or, alternatively, (3.2a), (3.2c) we obtain

$$K^* = Hi^* = \frac{1 - \alpha}{\alpha} He^*$$

which states that the fiscal policy authorities, given the historically accumulated stock of human capital, should supply the level of infrastructure given in (3.6) in order to accommodate optimal school attendance. We now state the following proposition.
Proposition 3 (Time allocation and infrastructure in the social optimum):
The socially optimal steady-state allocations of time spent in home study and in public schools are given by

\[ e^*_\infty = \frac{a\delta}{\rho + \delta + \delta(1 - \gamma)} \]  \hspace{1cm} (3.7a)  
\[ i^*_\infty = \frac{(1 - \alpha)\delta}{\rho + \delta + (1 - \gamma)\delta} \]  \hspace{1cm} (3.7b)  

The socially optimal share of infrastructure in output is equal to

\[ k^* = \frac{K^*}{Y^*} = \frac{(1 - \alpha)e^*_\infty}{\alpha - e^*_\infty} = \frac{(1 - \alpha)\delta}{\rho + \delta(1 - \gamma)} < 1 \]  \hspace{1cm} (3.8)  

To derive equations (3.7) of proposition 3, we note that the law of motion for skills, as in the decentralized case, equals \( H^{1-\gamma}_\infty = e^*_\infty g(\cdot)/\delta \) in the steady-state equilibrium. On the other hand, \( \mu = 0 \) in the social optimal requires \( H^{1-\gamma}_\infty = [g(\cdot)/(\rho + \delta)](\gamma e^*_\infty + \alpha - e^*_\infty) \), where we used (3.5) to eliminate \( i^*_\infty \). Equating the two relationships for \( H^{1-\gamma}_\infty \), we solve for \( e^*_\infty \) and then use (3.5) again to obtain \( i^*_\infty \).

Comparing (3.7) to (2.15) reveals how externalities cause the decentralized level of home effort and the time spent in school to depart from their socially optimal levels. For example, the decentralized length of time devoted to education (setting \( \tau = z = \sigma = 0 \)), whether at home or in school, will fall short of its socially optimal counterpart if the aggregate level of skills has a positive impact on human capital accumulation, i.e. if \( \gamma > 0 \). On the other hand, assuming \( \tau = z = \gamma = 0 \) but allowing for congestion, \( 0 < \sigma < 1 \), agents in the decentralized equilibrium spend too much in school relative to the social optimum. The optimal policies we will calculate below will eliminate these discrepancies.\(^{27}\)

To compute (3.8), the socially optimal output share of school infrastructure, we divide (3.6) by output \( Y = (1 - i - e)H \), and use (3.5). We find, after substituting for (3.7a), the expression for \( k^* \). As long as \( \gamma < \rho/\delta \), the optimal share of infrastructure is less than proportional to

\(^{27}\) As in the decentralized case, \( H^*_\infty \) can be determined: substitution of (3.5) and (3.7) into the stationary condition \( \dot{H} = 0 \) implicitly solves for \( H^*_\infty \). In turn, the shadow value of human capital \( \mu^*_\infty \) can be found by combining (3.3) with (3.1) and substituting for the socially optimal values we have calculated.
the share of school services in the human capital accumulation function, 
\( (1 - \alpha) \). Clearly, the socially optimal share of total resources devoted to 
school infrastructure is greater the larger is \( \gamma \), the aggregate human 
capital spillover parameter.\(^{28}\)

\[ 3.2 \textbf{Optimal tax/subsidy policy} \]

We will show how the private equilibrium derived in section 2.2 can 
replicate first-best equilibrium through the use of optimal government 
policy. We shall limit our attention to the replication of the steady-state 
equilibrium, though a time-varying government policy can be used to 
reproduce the first-best equilibrium along a dynamic path.

\textit{Proposition 4 (Optimal subsidy/tuition policy):} For the decentralized 
equilibrium to become socially optimal, the two economies must attain 
the same time allocation of educational activities for any given level of 
public infrastructure. The first step is to set the subsidy rates so that the 
private ratio \( i/e \) replicates the socially optimal ratio. Equating (2.10) with 
(3.5), we obtain the restriction

\[ (1 - \tau^* + z^*) = (1 + \sigma)(1 - \tau^*) \]  
\[ (3.9) \]

where \( 0 < \sigma \leq 1 \). To generate the same incentives for skill accumula-
tion, we equate the private and social rates of return to skills given in 
(2.12a) and (3.4), i.e. \( \tau^H = \rho^H \). Using (2.10) and the restriction on \( z^* \) 
in (3.9), we obtain the optimal overall subsidy:

\[ \tau^* = \frac{e[(1 - \alpha)\sigma + \gamma]}{\alpha + e[(1 - \alpha)\sigma + \gamma]} \]  
\[ (3.10) \]

The relationship (3.9) implies \( z^* = \sigma(1 - \tau^*) \), which is combined with 
(3.10) to derive the optimal tuition fee:

\(^{28}\) Note that the long-run shadow value in (2.18) will always decline due to an increase in 
\( K \), as long as the actual output share \( k \) is less than \( k^* \), or does not exceed \( k^* \) by too much. 
Substituting (3.8) into \( 1 - \alpha - (1 - \gamma)k \) yields \( (1 - \alpha)\rho/(\rho + (1 - \gamma)\delta) \), so that the 
coefficient on \( \bar{K} \) in (2.18) is negative.
Proposition 4 states that while a *tuition fee* should be charged for school attendance, \( z^* > 0 \), overall effort should receive a *subsidy*, \( \tau^* > 0 \). Observe, however, that if schools are strictly non-rival public goods, \( \sigma = 0 \), then no tuition fee should be charged, \( z^* = 0 \), while an overall subsidy should still be offered, \( \tau^* < 0 \), as long as there are aggregate human capital spillovers, \( \gamma > 0 \).\(^{29}\)

Consider, on the other hand, the case in which there is no aggregate human capital externality, \( \gamma = 0 \), but public education is subject to congestion, \( \sigma > 0 \). The optimal values of (3.10), (3.11) then become

\[
\begin{align*}
\tau^* &= \frac{(1 - \alpha)\sigma e}{\alpha + (1 - \alpha)\sigma e} \\
z^* &= \frac{\alpha \sigma}{\alpha + (1 - \alpha)\sigma e}
\end{align*}
\]  

(3.12)

This implies, as in the general case, that an overall subsidy to skill accumulation should be offered, while, at the same time, a tuition fee should be imposed. In the absence of spillovers from aggregate human capital, the trade-off between education and work is not distorted.\(^{30}\) Nevertheless, if agents fail to correctly evaluate the benefits of attending public schools due to congestion externalities, private decisions regarding the correct time allocation between the two educational activities will be distorted. As before, congestion \( (0 < \sigma \leq 1) \) calls for charging tuition fees and subsidizing overall effort. Charging a tuition fee to prevent congestion must therefore be accompanied by an overall subsidy in order to preserve the overall incentives for educational effort.

4. COMPARATIVE DYNAMICS OF EDUCATION REFORM

In this section we shall discuss the impact of educational policies on the dynamics of human capital investment. The basis of our analysis will be the log-linearized version of the system derived in (2.12). Since the derivation of this system involves some lengthy algebra, we relegate it to the appendix and simply state it here in matrix form:

\(^{29}\) In the model of Cooper S. (1998)—based on the work of Durlauf (1996) dealing with income inequality—voluntary educational subsidies (transfers) from rich neighborhoods to poor ones can obtain due to positive human capital spillovers.

\(^{30}\) According to Acemoglu and Angrist (1999), it is difficult to establish empirically the existence of aggregate human capital spillovers.
\[
\begin{bmatrix}
\dot{\mu}_t \\
\dot{H}_t
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_\mu & \varepsilon_H \\
\eta_\mu & \eta_H
\end{bmatrix}
\begin{bmatrix}
\dot{\mu}_t \\
\dot{H}_t
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_K & \varepsilon_t & \varepsilon_z \\
\eta_K & \eta_t & \eta_z
\end{bmatrix}
\begin{bmatrix}
\dot{K} \\
\dot{t} \\
\dot{z}
\end{bmatrix}
\] (4.1)

The \(\varepsilon\) and \(\eta\) elements of the coefficient matrices are defined in the appendix and the subscript \(t\) indicates the variables whose growth rates vary with time. For convenience, we use the following short-hand matrix notation to represent the dynamic system, \(X = AX + B\). The roots of the characteristic polynomial, \(\Psi(\omega) = |\omega I - A| = 0\), of (4.1) determine a pair of eigenvalues \(\zeta, \bar{\zeta}\). Since

\[\Psi(0) = \det A = \zeta \bar{\zeta} = \varepsilon_\mu \eta_H - \varepsilon_H \eta_\mu = -\eta_\mu (1 - \gamma)(\rho + \delta + \phi) < 0\] (4.2)

where \(\phi \equiv \delta[\alpha + (1 + \sigma)(1 - \alpha)] > 0\), there exists a positive and a negative root \(\zeta < 0 < \bar{\zeta}\) of \(\Psi(\omega) = 0\), which implies that the equilibrium \(X_\infty = -A^{-1}B\) of (4.1) is a saddlepoint. A particular solution of \(X\) is the steady-state equilibrium:

\[\begin{bmatrix}
\dot{\mu}_\infty \\
\dot{H}_\infty
\end{bmatrix} = \frac{-1}{\det A}
\begin{bmatrix}
\eta_H & -\varepsilon_H \\
-\eta_\mu & \varepsilon_\mu
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix}
\quad B = \begin{bmatrix}
\varepsilon_K \dot{K} + \varepsilon_t \dot{t} + \varepsilon_z \dot{z} \\
\eta_K \dot{K} + \eta_t \dot{t} + \eta_z \dot{z}
\end{bmatrix}
\] (4.3)

Equations (4.3) correspond to the steady-state solutions (2.17), (2.18) above.

The solution to (4.1) is \(X_t - X_\infty = D \exp(\zeta t)\), where \(D = [d_1, d_2]'\) is the eigenvector corresponding to the stable root \(\zeta < 0\). Solving \([A - \zeta I]D = 0\) yields the eigenvector up to a scalar multiple. From the second row, we obtain \(d_1 = [(\zeta - \eta_H)/\eta_\mu]d_2\), where \(d_2\) is determined by the initial condition on human capital. In this case, \(H_0 = 0\), which implies that \(d_2 = -\dot{H}_\infty\). The complete stable solution of (4.1) for time-invariant policy shocks then equals

\[\dot{H}_t = [1 - \exp(\zeta t)]\dot{H}_\infty \quad \dot{\mu}_t = \dot{\mu}_\infty + \frac{\eta_H - \zeta}{\eta_\mu} \dot{H}_\infty \exp(\zeta t)\] (4.4)

The stable saddlepath of the system is, in turn, given by
\[ \dot{\mu}_t - \dot{\mu}_\infty = \frac{\zeta - \eta_H}{\eta_\mu} (\dot{H}_t - \dot{H}_\infty) \]  

which implies that the forward-looking variable \( \dot{\mu}_t \) can jump at \( t = 0 \).

We can illustrate the dynamics of this system with two phase diagrams, figures 1(a) and 1(b). These depict the negatively sloped stable arm, denoted by \( \text{XX} \), and the \( \dot{H}_t = 0 \) and \( \dot{\mu}_t = 0 \) loci, whose slopes equal \( -\eta_H/\eta_\mu \) and \( -\varepsilon_H/\varepsilon_\mu < 0 \), respectively. The intersection of \( \dot{H}_t = 0 \) and \( \dot{\mu}_t = 0 \) at point A determines the steady-state values of \( \dot{H}_\infty \) and \( \dot{\mu}_\infty \). The distinction between the two diagrams is that figure 1(a) depicts the case in which \( \dot{H}_t = 0 \) is positively sloped, while figure 1(b) illustrates the case in which the slope of \( \dot{H}_t = 0 \) is negative. Since \( \eta_\mu > 0 \), figure 1(a) corresponds to \( \eta_H < 0 \), while figure 1(b) corresponds to \( \eta_H > 0 \). Rewriting the expression for \( \eta_H \) in (A9) as \( [k + \gamma/(1 - \gamma) - s] \delta (1 - \gamma)/s \), we can show that figure 1(a) corresponds to the case in which \( s > k + \gamma/(1 - \gamma) \), while figure 1(b) illustrates the case in which \( s < k + \gamma/(1 - \gamma) \). That is, the slope of \( \dot{H} = 0 \) is positive (negative) as the ratio of the time spent in educational activities to

![Figure 1](image-url)

\[ \frac{\zeta - \eta_H}{\eta_\mu} = \frac{d_1}{d_2}, \quad \frac{\varepsilon_H}{\varepsilon_\mu} < 0, \]

\[ \text{Figure 1.} \]

31 The stable arm \( \text{XX} \) is negatively sloped, since it is also the case that \( d_1 = [\varepsilon_H/(\zeta - \varepsilon_\mu)] d_2 \), where \( \varepsilon_H/(\zeta - \varepsilon_\mu) < 0 \).

32 Note that the slope of \( \dot{\mu}_t = 0 \) in figure 1(b) is greater in absolute value than that of \( \dot{H}_t = 0 \). The opposite case, i.e. the slope of \( \dot{\mu}_t = 0 \) being less in absolute value than that of \( \dot{H}_t = 0 \), can be ruled out by the saddlepoint property.
the time spent working, $\hat{s}$, is greater (less) than the share of educational infrastructure $k$ plus the term $\gamma/(1 - \gamma)$. The latter term is greater, the larger is the human capital spillover.

We can use the phase diagrams to illustrate the paths taken by human capital and its shadow value in response to a time-invariant shift in infrastructure, subsidy or tuition policy. To take one example, consider a permanent increase in $\hat{K}$, holding $\hat{t} = \hat{z} = 0$ and letting $\gamma = \alpha$. Using equations (2.17), (2.18), the long-run comparative statics for $H_\infty$ and $\mu_\infty$ and $d\hat{H}_\infty/d\hat{K} = -d\hat{\mu}_\infty/d\hat{K} = 1$. If we also assume that there is no congestion externality, $\sigma = 0$, and that $k = \delta(1 - \alpha)/(\rho + \delta)$, which is less, according to (3.8), than the corresponding social optimum $k^*$, then the $\hat{H}_t = 0$ locus is negatively sloped, since $\eta_H > 0$. Turning to figure 2 and using equations (4.4), (4.5), we can describe how the dynamic system for these parameter values adjusts to this shift in education policy. Because $\varepsilon_k < 0$, $\eta_k > 0$ in this case, an increase in $\hat{K}$ causes the $\hat{\mu}_t = 0$ locus to shift to the right while the $\hat{H}_t = 0$ locus shifts to the left. Their new intersection at point C illustrates that while human capital $\hat{H}_\infty$ increases, its shadow value $\hat{\mu}_\infty$ falls. In order to reach their new steady-state values at C, both $\hat{H}_t$ and $\hat{\mu}_t$ must start at point B and adjust down the new stable locus towards C. The shadow value, $\hat{\mu}_0$, must then jump instantaneously to point B when the policy change is implemented at $t = 0$. The rise in $\hat{\mu}_0$ reflects the initial drain on resources that a permanent increase in $\hat{K}$ imposes on the economy. Its subsequent fall reflects the productive impact of the rise in $\hat{K}$ on human capital accumulation.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Figure 2.}
\end{figure}
5. CONCLUSION

Acquiring human capital is naturally considered a form of investment. While public policies, such as tax credits and depreciation allowances, to promote physical investment are very widespread, government intervention in education in terms of subsidies, grants, loans, tuition and the direct provision of educational personnel and infrastructure has probably been even more pervasive in most countries, and surely so in terms of a share of GDP. In this paper we have developed a simple representative agent model to explore how the presence of non-institutional or home learning could change the rationale for public policy. In our framework individuals accumulate human capital by private self-study in non-institutional settings and by attending publicly provided schools. The economic returns to the individual, however, are affected by spillovers. We focused on two distortions in this paper. One was the presence of aggregate human capital spillovers while the other derives from congestion externalities in school attendance.

When deciding how much time to spend on home study and on school attendance, individuals disregard any external benefits from their contribution to aggregate human capital formation, but also ignore the external congestion costs they impose on other students in using educational infrastructure. A corrective subsidy/tuition policy can, in principle, elicit the socially optimal amount of time spent in home study and in school. We found that to eliminate an aggregate human capital externality, the total time spent in accumulating skills should receive a subsidy. At the same time, the government should impose a tuition fee to prevent congestion in schools which would impair scholastic achievement. In making school attendance more expensive, the fee shifts individual effort towards non-institutional forms of learning. Furthermore, even in the case in which there are no human capital spillovers, we determined on efficiency grounds that both a subsidy to overall activity and a fee on school attendance must be imposed to prevent school congestion from impairing skill accumulation. We believe that this policy prescription rationalizes, independently from distributional considerations, some recent continental European reforms such as the introduction of tuition fees combined with individual stipends.

APPENDIX

In deriving the log-linearized system (4.1), we start with the dynamic equation for human capital. Consider first the dynamics of working
hours, \( l = 1 - i - e \), and output, \( Y = lH \). Working hours evolve according to

\[
\dot{I}_t = -s_I i_t - s_E \dot{e}_t \\
\dot{I} = i/l \\
s_I \equiv e/l \\
s \equiv s_E + s_I = (1 - l)/l
\]

(A1)

where the circumflex indicates logarithmic (proportional) rates of change relative to the initial steady state. Note also that we have used the subscript \( t \) to distinguish those variables whose proportional rate of change is time variant from the policy variables that are not time variant. Taking the logarithmic derivative of (2.10) yields

\[
\dot{\bar{I}}_t = \dot{\bar{I}}_t - \dot{\bar{z}} - \frac{z}{1 - \tau} \hat{\tau}
\]

where \( \hat{\tau} \) and \( \bar{z} \) are equal to \( d\tau/(1 - \tau + z) \) and \( dz/(1 - \tau + z) \). Substituting this result and (A1) into the rate of growth of output gives us

\[
\dot{Y}_t = \dot{I}_t + \dot{H}_t = \dot{H}_t - \dot{s} \dot{e}_t + s_I \dot{z} + \frac{s_I z}{1 - \tau} \hat{\tau}
\]

(A2)

Turning to the demand side of the economy, income is devoted to consumption and public infrastructure, \( Y_t = C_t + K \), implying

\[
\dot{Y}_t = (1 - k) \dot{C}_t + k \dot{K} \\
k \equiv K/Y
\]

(A4)

We next compute the percentage change of the time spent in home study, \( e_t \). Substituting for \( C_t = 1/\lambda_t \), we calculate the rate of change of (2.11), \( (1 - \tau) \theta H_t^{-\gamma} = \mu_t \alpha g C_t \). This yields

\[
(1 - \gamma) \dot{H}_t - \frac{1 - \tau + z}{1 - \tau} \hat{\tau} = \dot{\mu}_t + \dot{g} + \dot{\lambda}_t
\]

(A5)

Substituting (A3), (A4) into (A5), we then obtain the following expression for \( \dot{e}_t \):

\[
\dot{e}_t = \frac{1 - k}{s} \left[ \frac{1}{1 - k} (1 - \gamma) \right] \dot{H}_t + \frac{1 - k}{s} (\dot{\mu}_t + \dot{g})
\]

\[
- \frac{k}{s} \dot{K} + \frac{1 - k}{s(1 - \tau)} \left[ (1 - \tau + z) + \frac{s_I z}{1 - k} \right] \hat{\tau} + \frac{s_I z}{s} \hat{\tau}
\]

(A6)
The log-linearized form of (2.12b) and the percentage change of \( g(K_i/e) \) are given by

\[
\begin{align*}
\dot{H}_t &= \delta[\dot{e}_t + \dot{g} - (1 - \gamma)H_t] \\
\dot{g} &= (1 - \alpha)\left(\dot{K} - \frac{z}{1 - \tau} \dot{t} - \dot{z}\right)
\end{align*}
\]

(A7a)

(A7b)

where we have used the steady-state restriction \( \dot{e}H_t = \delta H \) in (A7a) and (A2) in the expression for \( \dot{g} \). Substitution of (A6) and (A7b) into (A7a) then yields the dynamic equation (4.1) for human capital in log-linearized form:

\[
\dot{H}_t = \eta_H \dot{H}_t + \eta_\mu \dot{\mu}_t + \eta_k \dot{K} + \eta_\tau \dot{\tau} + \eta_z \dot{z}
\]

(A8)

where the \( \eta \) coefficients are defined as

\[
\begin{align*}
\eta_H &= \delta \left[\frac{1}{s} - (1 - \gamma)\left(1 + \frac{1 - k}{s}\right)\right] \\
\eta_\mu &= \delta \frac{1 - k}{s} > 0 \\
\eta_K &= \delta \left[(1 - \alpha)\left(1 + \frac{1 - k}{s}\right) - \frac{k}{s}\right] \\
\eta_z &= \delta \left[\frac{s_I}{s} - (1 - \alpha)\left(1 + \frac{1 - k}{s}\right)\right] \\
\eta_\tau &= \delta \frac{1}{1 - \tau} \left\{\frac{1 - k}{s} \left[(1 - \tau + z) + \frac{zs_I}{1 - k}\right] - (1 - \alpha)z\left(1 + \frac{1 - k}{s}\right)\right\}
\end{align*}
\]

(A9)

Turning to log-linearization of equation (2.12a), we obtain \( \dot{\mu}_t = -(\rho + \delta)r^H_t \), where \( r^H = \rho + \delta \) is the equilibrium interest rate. Using the definition of \( r^H \) in (2.12a), we denote the term in square brackets by \( x = [1 - (1 - \tau)e - (1 - \tau + z)i] \). Using the steady-state conditions \( H^1 = e^\sigma g()/\delta \) and \( x_{\infty} = (\rho + \delta)(1 - \tau)e_{\infty}/\alpha\delta \), and inserting (2.10), we obtain

\[
(\rho + \delta)x_t = -\delta \alpha \dot{e}_t + \delta \alpha \frac{1 - \tau + z}{1 - \tau} \dot{\tau} + \delta(1 - \sigma)(1 - \alpha)(\dot{\tau} - \dot{z} - \dot{i}_t)
\]

(A10)

Taking the differential of \( r^H = [\alpha g/(1 - \tau)]H^{n-1} \) \( x \) yields
\[ \dot{r}_t^H = \dot{g} - (1 - \gamma) \dot{H}_t + \frac{1 - \tau + z}{1 - \tau} \dot{\tau} + \dot{x}_t \]  
\hfill (A11)

Substituting for \( i_t \) from (A2) into (A10) and then the resulting expression into (A11), we find

\[ \dot{r}_t^H = \dot{g} - (1 - \gamma) \dot{H}_t + \frac{1 - \tau + z}{1 - \tau} \dot{\tau} + \frac{\phi}{\rho + \delta} \left( \frac{1 - \tau + z}{1 - \tau} \dot{\tau} - \dot{\varepsilon}_t \right) \]  
\hfill (A12)

where \( \phi \equiv [\alpha + (1 + \sigma)(1 - \alpha)] \). Finally, we substitute for \( \dot{g} \) and for \( \dot{\varepsilon}_t \) from (A7b) and (A6) into (A12) and then substitute the resulting expression into \( \dot{\mu}_t = - (\rho + \delta) \dot{r}_t^H \) to obtain the log-linearized equation (4.1) for the shadow value of human capital:

\[ \dot{\mu}_t = - (\rho + \delta) \dot{r}_t^H = \varepsilon_H \dot{H}_t + \varepsilon_{\mu} \dot{\mu}_t + \varepsilon_K \dot{K} + \varepsilon_{\tau} \dot{\tau} + \varepsilon_{\varepsilon} \dot{\varepsilon} \]  
\hfill (A13)

where the \( \varepsilon \) coefficients are given by

\[ \varepsilon_H = (\rho + \delta)(1 - \gamma) + \frac{\phi(1 - k)}{s} \left[ \frac{1}{1 - k} - (1 - \gamma) \right] > 0 \]

\[ \varepsilon_{\mu} = \frac{\phi(1 - k)}{s} > 0 \]

\[ \varepsilon_K = (1 - \alpha) \left[ \frac{\phi(1 - k)}{s} - (\rho + \delta) \right] - \frac{\phi k}{s} \]

\[ \varepsilon_{\tau} = - (\rho + \delta) \left[ 1 + \frac{\alpha z}{1 - \tau} \right] - \frac{\phi(1 - \tau + z)}{1 - \tau} \left( \frac{1 - k}{s} \right) \]

\[ - \frac{\phi(1 - k)z}{(1 - \tau)s} \left( 1 - \alpha - \frac{s_I}{1 - k} \right) \]

\[ \varepsilon_{\varepsilon} = \frac{\phi s_I}{s} - (1 - \alpha) \left[ \frac{\phi(1 - k)}{s} - (\rho + \delta) \right] \]  
\hfill (A14)

REFERENCES


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