DECOMPOSING PERFORMANCE

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Decomposing Performance*

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Abstract

We present a new methodology for decomposing the (risk-adjusted) performance in empirical finance. Our technique offers the same straightforward economic intuition and all the statistical benefits of the portfolio sorts approach, in particular robustness to cross-sectional correlation, and in addition resolves the major drawbacks of portfolio sorts. Most importantly, our regression-based methodology handles multiple dimensions and continuous firm, fund, or investor characteristics. Moreover, the technique allows for relying on standard Wald-tests as an alternative to the popular Gibbons, Ross, and Shanken (1989) test. We illustrate our methodology with an asset pricing application and a long-horizon event study.

Keywords: Performance evaluation, Alpha decomposition, Portfolio sorts, Fama-French model

JEL classification: C21, G14, D1

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1. Introduction

Originally introduced by Jaffe (1974) and Mandelker (1974), the portfolio sorts methodology got promoted tremendously through the seminal work of Fama and French (1993, 1996). Nowadays, the portfolio sorts approach constitutes one of the most widely applied techniques for analyzing the cross-section of stock returns and for evaluating the (risk-adjusted) performance of firms, mutual funds, hedge funds, and private as well as institutional investors.\(^1\) The portfolio sorts methodology comprises a two-step procedure. The first step consists of calculating period-by-period average excess returns for a group of individual subjects.\(^2\) In the second step, these period-by-period averages are regressed on a set of market factors (e.g., the three Fama-French factors). The intercept term of this second-step time-series regression has a clear economic interpretation: it measures the subject group’s risk-adjusted performance.

By aggregating the returns of an entire cross-section into a single portfolio, the portfolio sorts methodology eliminates the problem of cross-sectional dependence between the individual subjects’ returns (Lyon, Barber, and Tsai, 1999). However, this statistical robustness comes at a cost. Cochrane (2011, p. 1061), for instance, observes that while it is customary to “sort assets into portfolios based on a characteristic […] we cannot do this with 27 variables”. He refers to this as the “multidimensional challenge” of the portfolio sorts approach. It emerges from the fact that an analysis based on portfolio sorts has to be limited to very few subject characteristics for the number of sub-groups not becoming too large.

In this research, we propose a new methodology for measuring and decomposing the (risk-adjusted) performance of multifactor models. Our approach relies on estimating a linear regression model on the individual subject level and to draw statistical inference based on Driscoll and Kraay (1998) standard errors that are robust to general forms of cross-sectional as well as temporal dependence. The model

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\(^1\) The portfolio sorts approach has been applied in many different areas of empirical finance, such as for example in asset pricing research (e.g., Novy-Marx, 2013; Ball, Gerakos, Linnaehmaa, and Nikolaev, 2015; Fama and French, 2015), in research on the performance of private investors (e.g., Barber and Odean, 2000, 2001; Ivkovic, Sialm, and Weisbenner, 2008; Seasholes and Zhu, 2010; Korniotis and Kumar, 2013), in research on insider trading (e.g., Jaffe, 1974; Jeng, Metrick, and Zeckhauser, 2003), and in studies analyzing the performance of mutual funds and hedge funds (e.g., Kacperczyk, Sialm, and Zheng, 2008; Fung, Hsieh, Naik, and Ramadorai, 2008).

\(^2\) For brevity and readability, we henceforth refer to “subjects” as a short form for “firms, mutual funds, hedge funds, or private (or institutional) investors”, thereby highlighting the wide area of applicability of the portfolio sorts approach.
 specification is such that the individual subjects’ monthly excess returns are regressed on a set of market factors (e.g., the three Fama-French factors), a series of subject characteristics, and all interaction terms between the market factors and the subject characteristics. As a result, our technique easily handles multiple (binary or continuously distributed) subject characteristics, thereby offering a straightforward to implement solution to the “multidimensional challenge”.

We prove theoretically and confirm empirically that the proposed regression approach can be specified such that it exactly reproduces the results of single and multiple portfolio sorts. Hence, the proposed method shares all the positive statistical properties and the straightforward economic interpretation of the portfolio sorts approach. Since the portfolio sorts methodology in the literature sometimes is referred to as the “calendar time portfolio approach” (Kothari and Warner, 2008), we name our methodology the “Generalized Calendar Time regression model” or, in short, the GCT-regression model.

Previous research by Dahlquist, Engström, and Söderlind (2000), Ivkovich and Weisbenner (2005), and Korniotis and Kumar (2011), among others, attempted to cope with the “multidimensional challenge” of the portfolio sorts approach by estimating cross-sectional regressions on the subject level. Like the portfolio sorts approach, this alternative technique (which we refer to as the CrossReg methodology) constitutes a two-step procedure. The two steps in the CrossReg methodology are the same as in the portfolio sorts approach, but their order is reversed. Hence, the first step consists of estimating a Fama and French (1993) type time-series regression for each individual subject. The second step then decomposes the risk-adjusted performance of the subjects by regressing the intercept terms (i.e., the Fama-French alphas) from the first step on a set of subject-specific explanatory variables. By reversing the order of the two steps, the CrossReg approach allows for the inclusion of multivariate and continuous subject characteristics and, hence, offers a potential remedy for the “multidimensional challenge” of the portfolio sorts methodology.

The major drawback of the CrossReg approach, however, is that by changing the order of the steps the CrossReg methodology sacrifices much of the statistical robustness of the portfolio sorts approach. In fact, Driscoll and Kraay (1998) point out that it is impossible to obtain cross-sectional dependence consistent standard errors for a linear regression on a single cross-section without relying on very strong
assumptions about the form of the cross-sectional correlation. The second-step regression of the Cross-Reg approach therefore (implicitly) assumes that the subjects and the error terms of the regression are independent. This assumption, however, is problematic for two reasons. On the one hand, cross-sectional correlation is a common phenomenon in microeconometric datasets since market trends, social learning, herd behavior, neighborhood effects, and macroeconomic shocks all can lead to contemporaneously correlated actions (and returns) of firms, funds, and investors. On the other hand, Driscoll and Kraay (1998) show that the erroneous ignorance of cross-sectional correlation in the estimation of linear regression models can lead to severely downward-biased standard error estimates. It is therefore not surprising that the Monte Carlo simulations of Gow, Ormazabal, and Taylor (2010) reveal that the Cross-Reg approach “produces substantially overstated t-statistics and rejects a true null hypothesis more than 80% of the time at the 1% level.” As a consequence, the CrossReg approach is at risk of producing unreliable statistical results.

The Fama and MacBeth (1973) procedure, to which we henceforth refer to as the FM-approach, is another regression-based technique routinely applied for analyzing the cross-section of stock returns. The FM-approach shares a number of important properties with the GCT-regression model. Petersen (2009) and Gow, Ormazabal, and Taylor (2010), among others, show that statistical inference based on the FM-procedure is also robust with respect to cross-sectional dependence in the data. Moreover, the FM-approach allows for the inclusion of multivariate and continuous subject characteristics as does the GCT-regression model. Apart from these similarities, however, the GCT-regression model fundamentally differs from the FM-approach in at least two important dimensions.

First, the FM-procedure, as it is usually applied in empirical finance, is generally limited to the analysis of (excess) returns and does not allow to investigate and decompose risk-adjusted performance. After all, it is impossible to estimate cross-sectional regressions with variables that are constant in the cross-section. For a given point in time, risk factors like the market excess return (which are required

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3 For example, see Welch (2000), Hirshleifer and Teoh (2003), Feng and Seasholes (2004), or Dorn, Huberman, and Sengmueller (2008). Besides, there are also technical reasons for spatial dependence in microeconometric datasets. For instance, the monthly portfolio return of any investor, mutual fund, or hedge fund is a linear combination of the underlying assets’ returns. As a consequence, the number of independent observations in a dataset on the monthly portfolio returns of (private) investors, mutual funds, or hedge funds is limited by the number of underlying assets and not the number of investors or funds in the sample (Seasholes and Zhu, 2010).
for estimating the risk-adjusted performance) do not vary across subjects. Therefore, they cannot be included in the FM-approach in a straightforward way. By contrast, the GCT-regression model is suitable for analyzing both (excess) returns as well as risk-adjusted performance: Structured as a pooled linear regression on the subject level, the GCT-regression model easily handles both market level variables (that do not vary across subjects but differ over time) and subject level variables (which vary in the cross-section and may or may not change over time). Second and most important, the GCT-regression model has the same intuitive economic interpretation as the portfolio sorts methodology. This, however, does not apply to the FM-approach. While the results from FM-regressions are often interpreted in a similar way as those from portfolio sorts, coefficient estimates from a FM-regression can only be considered “conceptually similar to a multifactor model alpha obtained from portfolio sorts” (Ball, Gerakos, Linnainmaa, and Nikolaev, 2015, p. 228). To the best of our knowledge, our paper therefore is the first to directly address and resolve the “multidimensional challenge” of the portfolio sorts approach. We are not aware of any prior research on the cross-section of stock returns, on (long-term) performance evaluation, or on panel data estimation procedures providing a comparable solution to this important issue.

The portfolio sorts approach has not only been criticized for its limitation to just a few subject characteristics. For instance, Fama and French (2008) and Patton and Timmermann (2010) criticize that in the portfolio sorts approach it is difficult to comprehensively assess the statistical results. In fact, statistical inference is often exclusively based on a comparison of the top and bottom sub-groups for simplicity. When working with continuous subject characteristics this issue gets exacerbated: the lack of a natural grouping criterion necessitates the formation of somewhat arbitrary (e.g., quintile or decile) subject groups which may have consequences for the empirical results. As it is capable to handle multivariate and continuous subject characteristics, the GCT-regression model easily overcomes this shortcoming of the portfolio sorts approach. Moreover, since the GCT-regression model allows for reproducing the results from multiple portfolio-sorts by estimating a single linear regression on the individual subject level, a standard Wald test can be applied for testing whether the “alphas” of a series of portfolio sorts are jointly equal to zero. Such a Wald test constitutes an easy-to-implement alternative to the popular “GRS-test” proposed by Gibbons, Ross, and Shanken (1989).
In the context of long-horizon event studies, Loughran and Ritter (2000) criticize that (for an unbalanced panel) the *portfolio sorts* approach underweights observations from periods with large cross-sections and overweights observations from periods with small cross-sections. Specifically, they argue that “tests that weight firms equally should have more power than tests that weight each time period equally” (Loughran and Ritter, 2000, p. 363). Our regression-based extension to the *portfolio sorts* approach allows for a natural remedy of the critique of Loughran and Ritter (2000). It is straightforward to estimate the *GCT-regression* model with ordinary least squares (OLS) where all observations are equally weighted. In contrast, Fama and French (2008), among others, caution that in asset pricing applications the plethora of small- and microcap stocks can be influential for the results when observations on micro- and megacap stocks are equally-weighted. This, however, does not pose a problem to the *GCT-regression* model either as it can easily be estimated with weighted least squares (WLS) where observation weights are set equal to, say, the value weights of the stocks.

In the empirical part of the paper, we illustrate the application of the *GCT-regression* model in an asset pricing setup and a long-horizon event study, respectively. We start our empirical analysis by replicating some of the key results in Ball, Gerakos, Linnainmaa, and Nikolaev (2015), a study analyzing the cross-section of stock returns. We then show how to exactly reproduce the results from the *portfolio sorts* approach, as employed in their paper, with the *GCT-regression* model in case of a single portfolio, for multiple portfolio sorts (e.g., for ten decile portfolios), and for the differences between two portfolios. Next, we empirically illustrate the flexibility of the *GCT-regression* model. Specifically, we show that the *GCT-regression* model is not confined to the analysis of portfolio sorts but can also handle multivariate and continuous subject characteristics. This allows us, for example, to run a horse race between alternative predictor variables.

Long-horizon event studies represent yet another important area of application for the *portfolio sorts* approach (Kothari and Warner, 2008). In our second empirical application, we therefore illustrate the *GCT-regression* model in a long-horizon event study. To this end, we replicate parts of Barber and

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4 Typical applications of the *portfolio sorts* methodology in long-term event studies include research on the performance of private investors (e.g., Barber and Odean, 2000, 2001), IPOs (Ritter and Welch, 2002; Cao, Jiang, and Ritter, 2015), M&As (Andrade, Mitchell, and Stafford, 2001; Moeller, Schlingemann, and Stulz, 2004), and mutual and hedge funds (e.g., Kacperczyk, Siaim, and Zheng, 2008; Fung, Hsieh, Naik, and Ramadorai, 2008).
Odean’s (2000) study analyzing the impact of portfolio turnover on the performance of private investors with both the portfolio sorts approach and the GCT-regression model. Moreover, to illustrate the flexibility of the GCT-regression model we also add Barber and Odean’s (2002) online trading variable as a second subject characteristic to the analysis.

The remainder of the paper is organized as follows. Section 2 presents the GCT-regression model and proofs mathematically that the GCT-regression model can be specified such that it exactly reproduces the coefficient estimates and standard errors of the portfolio sorts approach. Section 3 empirically illustrates the application of the GCT-regression model in a typical asset pricing setup. Section 4 demonstrates the application of the GCT-regression model in the context of long-horizon event studies. Section 5 concludes.

2. The GCT-regression model

In this section, we describe the model setup, discuss the properties of the GCT-regression model, and show how to interpret its coefficient estimates. Moreover, we show how to implement a standard Wald test as an alternative to the popular Gibbons, Ross, and Shanken (1989) “GRS-test” for testing whether the “alphas” of multiple portfolio sorts are jointly equal to zero.\(^5\) Finally, we demonstrate the flexibility of the GCT-regression model and contrast the (statistical) properties of the GCT-regression model with those of the portfolio sorts approach.

2.1 The model setup

We propose the following subject-level regression model for analyzing the cross-section of stock returns as well as for evaluating the long-term performance of firms, funds, and investors:

\[
y_{ht} = (z_{ht} \otimes x_t) \theta + v_{ht}
\]

\(^5\) A detailed comparison of the properties of the proposed Wald test on the GCT-regression model with the properties of the well-known Gibbons, Ross, and Shanken (1989) GRS-test is left for future research.
Here, \( z_{ht} \) is a \((1 \times M)\)-dimensional vector of subject characteristics, \( x_t \) is a \((1 \times (K + 1))\)-dimensional vector of market-level variables, and \((z_{ht} \otimes x_t)\) denotes the Kronecker product of vector \( z_{ht} \) with vector \( x_t \). Overall, regression (1) comprises a total of \( M \times (K + 1) \) explanatory variables whose coefficients are stored in vector \( \theta \).

While the subject characteristics in vector \( z_{ht} \) may vary across both the time dimension and the cross-section, the elements in vector \( x_t \) only vary over time but not across subjects. The variables in vector \( x_t \) therefore typically comprise a constant and, possibly, a set of factor variables like the market excess return \((R_{m,t} - R_{f,t})\), the Fama and French (1993) size and value factors, or the Carhart (1997) momentum factor. We propose to estimate regression model (1) with Driscoll and Kraay (1998) standard errors that are robust to heteroscedasticity, autocorrelation, and very general forms of cross-sectional dependence to obtain the model we refer to as the “Generalized Calendar Time regression model” or, in short, the GCT-regression model.\(^6\)

### 2.2 Performance evaluation of single subject groups

In this section, we demonstrate that the GCT-regression model in (1) is able to exactly reproduce the results of the portfolio sorts approach. The portfolio sorts approach comprises two steps. In the first step, one computes for each period the portfolio (excess) return \( y_t \) for a group of individual subjects. \( y_t \) is obtained as the weighted sum of the period \( t \) (excess) returns \( y_{ht} \) of all \( N_t \) subjects \( h \):

\[
y_t = \sum_{h=1}^{N_t} w_{ht} y_{ht}
\]

Here, \( w_{ht} \) denotes the beginning of period \( t \) portfolio weight of subject \( h \). The second step of the portfolio sorts methodology then evaluates the (risk-adjusted) performance of the subject group by aid of a linear \( K \)-factor time-series regression with \( y_t \) from (2) as the dependent variable:

\[
y_t = \beta_0 + \beta_1 x_{1t} + \ldots + \beta_K x_{Kt} + \epsilon_t
\]

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\(^6\) A Stata ado-file implementing our methodology, a tutorial, and a sample dataset can be obtained from the authors upon request.
In most applications, the factor variables \( x_{kt} \) \((k = 1, ..., K)\) are specified such that Equation 3 represents a Jensen (1968), Fama and French (1993), or Carhart (1997) type time-series regression model. To evaluate whether the risk-adjusted performance of the subject group is abnormal, the coefficient estimate for the intercept term (\( \hat{\beta}_0 \)) and its statistical significance are considered.

To reproduce the results of the portfolio sorts methodology in (3) with the GCT-regression model (1), we specify vector \( z_{ht} \) in (1) as \( z_{ht} = [1] \) and set vector \( x_t = [1 \ x_{1t} \ ... \ x_{Kt}] \). In this most basic case, the GCT-regression model in (1) is specified as the following subject-level regression model:

\[
y_{ht} = (z_{ht} \otimes x_t) \theta + v_{ht} = ([1] \otimes [1 \ x_{1t} \ ... \ x_{Kt}])\theta + v_{ht}
\]

\[
= \theta_0 + \theta_1 x_{1t} + ... + \theta_K x_{Kt} + v_{ht}
\] (4)

For ease of mathematical tractability, but without loss of generality, we limit our formal analysis to the case of a balanced panel with \( N \) subjects, \( T \) time periods, and portfolio weights \( w_{ht} \) being specified as \( w_{ht} = 1/N \) (i.e., all subjects are equally weighted). Under these assumptions, the following result holds true:

**Proposition 1 (Single subject group)**

- **Part A – Coefficient estimates.** Estimating the linear regression model in (4) with pooled OLS yields identical coefficient estimates as estimating time-series regression (3) with OLS, i.e., \( \hat{\beta}_k = \hat{\theta}_k \) \((\forall k = 0, 1, ..., K)\).

- **Part B – Standard errors.** For a given lag length choice \( H \), Driscoll and Kraay (1998) standard errors for coefficient estimates \( \hat{\theta}_k \) in (4) are identical to Newey and West (1987) standard errors for coefficient estimates \( \hat{\beta}_k \) \((\forall k = 0, 1, ..., K)\) in time-series regression (3), i.e., \( \text{SE}(\hat{\beta}_k) = \text{SE}(\hat{\theta}_k) \) \((\forall k = 0, 1, ..., K)\).

Proof: See Appendix A.1.

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7 In our empirical analysis, however, we consider unbalanced panels with time-varying cross-sections and alternative weighting schemes when constructing the portfolios (e.g., value weighted portfolios). In Sections 3 and 4, we demonstrate that our theoretical results also hold in this more general setup. However, in this case the GCT-regression model needs to be estimated with weighted least squares (WLS) rather than with OLS.
Part A of Proposition 1 is an application of a well-known property from portfolio theory which says that the portfolio beta is equal to the weighted sum of the individual asset betas. Part B of Proposition 1 is intuitive since the Driscoll and Kraay (1998, p. 552) “covariance matrix estimator is precisely the standard Newey and West (1987) heteroskedasticity and serial correlation consistent covariance matrix estimator, applied to the sequence of cross-sectional averages” of the moment conditions.

2.3 Portfolio sorts

When applying the portfolio sorts approach, the analysis usually is not restricted to a single subject group as in the case of Section 2.2. Rather, it is common to sort subjects into a series of (five, ten, or more) portfolios with predefined properties. In many cases, the portfolios are constructed on the basis of a single characteristic such as, for example, firm size or individual investors’ portfolio turnover. Alternatively, researchers form double (or even higher dimensional) sorts where portfolios are constructed based on multiple subject characteristics such as portfolios formed on size and book-to-market ratio. For each portfolio, the researcher then independently evaluates the risk-adjusted performance by aid of a Jensen (1968), Fama and French (1993), or Carhart (1997) type time-series regression along the lines described in Section 2.2 above.

Formally, the first step of the portfolio sorts approach in this more general setup thus involves sorting the subjects into characteristic-based portfolios \( p \) for which the average month \( t \) excess return, \( y_{pt} \), can be obtained as follows:

\[
y_{pt} = \sum_{h=1}^{N_t} w_{ht}^{(p)} z_{ht}^{(p)} y_{ht}
\]  

As before, \( y_{ht} \) denotes the month \( t \) excess return of subject \( h \) and \( w_{ht}^{(p)} \) refers to the subject’s beginning-of-the-period weight in portfolio \( p \). \( z_{ht}^{(p)} \) is a dummy variable which is equal to one if subject \( h \) belongs to portfolio \( p \) (with \( p = 1, \ldots, P \)) and zero otherwise. For each portfolio \( p \), the weights sum up to 1 (i.e., \( \sum_{h=1}^{N_t} w_{ht}^{(p)} z_{ht}^{(p)} = 1 \)), and the period \( t \) cross-section comprises \( N_t = \sum_{p=1}^{P} \sum_{h=1}^{N_t} z_{ht}^{(p)} \) subjects. The second step of the portfolio sorts approach then evaluates the risk-adjusted performance \( (\beta_{p,0}) \)
of portfolio \( p \) by aid of a linear \( K \)-factor time-series regression with \( y_{pt} \) from Equation (5) as the dependent variable:

\[
y_{pt} = \beta_{p,0} + \beta_{p,1} x_{1t} + \ldots + \beta_{p,K} x_{Kt} + \epsilon_{pt}
\]  

(6)

If the coefficient estimate for \( \beta_{p,0} \) is positive (negative) and statistically significantly different from zero, then subjects with the characteristics of portfolio \( p \) generate, on average, an abnormally good (poor) risk-adjusted performance.

It is possible to specify the GCT-regression model in (1) such that it exactly reproduces the results of multiple portfolio sorts by estimating a single panel regression on the subject level. To this end, we specify vectors \( z_{ht} \) and \( x_{t} \) in equation (1) as \( z_{ht} = [z_{ht}^{(1)} z_{ht}^{(2)} \ldots z_{ht}^{(P)}] \) and \( x_{t} = [1 x_{1t} \ldots x_{Kt}] \) to obtain the following regression model:

\[
y_{ht} = \left( \left[ z_{ht}^{(1)} z_{ht}^{(2)} \ldots z_{ht}^{(P)} \right] \otimes \left[ 1 x_{1t} \ldots x_{Kt} \right] \right) \theta + v_{ht}
\]

\[
= \theta_{1,0} z_{ht}^{(1)} + \theta_{1,1} x_{1t} z_{ht}^{(1)} + \ldots + \theta_{1,k} x_{kt} z_{ht}^{(1)}
+ \theta_{2,0} z_{ht}^{(2)} + \theta_{2,1} x_{1t} z_{ht}^{(2)} + \ldots + \theta_{2,k} x_{kt} z_{ht}^{(2)}
+ \ldots
+ \theta_{P,0} z_{ht}^{(P)} + \theta_{P,1} x_{1t} z_{ht}^{(P)} + \ldots + \theta_{P,k} x_{kt} z_{ht}^{(P)} + v_{ht}
\]  

(7)

Under the same assumptions as for Proposition 1, and provided that portfolios \( p = 1, \ldots, P \) are constant over time (i.e., \( z_{ht}^{(p)} = z_{ht}^{(p)} \) for all \( t \)), the following result holds true:

**Proposition 2 (Portfolio sorts)**

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8 For Proposition 2 to hold in the general case of an unbalanced panel with time-varying portfolios, the GCT-regression model in (7) has to be estimated with weighted least squares (WLS). Details on the weighting scheme reproducing the results of the portfolio sorts approach in the case of value weighted portfolios are provided in Section 3.3 where we discuss the application of the GCT-regression model in a typical asset pricing setup. In the context of a long-horizon event study, observation weights \( w_{ht}^{(p)} \) need to be set to \( w_{ht}^{(p)} = N_{pt}^{-1} \) in order to match the results of the portfolio sorts approach. See Section 4.3 for details.
Part A – Coefficient estimates. OLS coefficient estimates for $\beta_{p,k}$ in time-series regression (6) are identical to pooled OLS coefficient estimates for $\theta_{p,k}$ in subject-level panel regression (7), i.e.,

$$\hat{\beta}_{p,k} = \hat{\theta}_{p,k} \quad (\forall k = 0, 1, ..., K \text{ and } p = 1, ..., P).$$

Part B – Standard errors. For a given lag length choice H, Newey and West (1987) standard errors for coefficient estimates $\hat{\beta}_{p,k}$ in time-series regression (6) are identical to Driscoll and Kraay (1998) standard errors for coefficient estimates $\hat{\theta}_{p,k}$ in subject-level panel regression (7), i.e.,

$$\text{SE}(\hat{\beta}_{p,k}) = \text{SE}(\hat{\theta}_{p,k}) \quad (\forall k = 0, 1, ..., K \text{ and } p = 1, ..., P).$$

Proof: See Appendix A.2.

According to Proposition 2, the coefficient estimates of the GCT-regression model in (7) have a straightforward economic interpretation. As in the portfolio sorts regression (6), the coefficient estimates $\hat{\theta}_{p,0}$ ($p = 1, ..., P$) from subject-level regression (7) measure the risk-adjusted performance of portfolio $p$ whereas coefficient estimates $\hat{\theta}_{p,k}$ (with $k = 1, ..., K$) represent portfolio $p$’s exposure versus factor $k$.

Moreover, note that the GCT-regression model in (7) reproduces the results of a set of $P$ (independent) time-series regressions (6) by aid of a single linear regression on the subject level. As a result, a standard Wald test can be applied in order to test whether the risk-adjusted performance of the $P$ portfolios is jointly equal to zero:

$$H_0: \beta_{1,0} = \beta_{2,0} = \ldots = \beta_{P,0} = 0 \quad \text{vs.} \quad H_1: \beta_{p,0} \neq 0 \text{ for at least one } p \text{ in } 1, ..., P \quad (8)$$

The multiple hypothesis test in (8) offers a straightforward alternative to the widely applied Gibbons, Ross, and Shanken (1989) or “GRS” test, a finite-sample $F$-test commonly used to test the joint significance of the “alphas” across a set of (e.g., decile) portfolios. Estimating GCT-regression model (7) with Driscoll and Kraay (1998) standard errors ensures that the Wald test in (8) allows for valid statistical inference if the error terms ($v_{ht}$) of the regression are heteroscedastic, autocorrelated, and cross-sectionally dependent.
2.4 Performance difference between two portfolios or subject groups

Another important application of the portfolio sorts approach is the analysis of performance differences between two portfolios or subject groups. For instance, when analyzing the results of portfolio sorts, researchers routinely test whether the risk-adjusted performance of the top (e.g., decile or quintile) group’s portfolio is statistically significantly different from the performance of the bottom group’s portfolio. Similarly, in long-horizon event studies, it is common to investigate whether the risk-adjusted performance of, say, young firms (“IPOs”) is different from that of more mature firms (Ritter, 1991) or whether women outperform men on a risk-adjusted basis (Barber and Odean, 2001).

Formally, the first step of the portfolio sorts approach in this type of application involves the computation of the average month $t$ excess return for both portfolios (or subject groups) $p$ as follows:

$$y_{pt} = \sum_{h=1}^{N_t} w_{ht}^{(p)} z_{ht}^{(p)} y_{ht}$$

(9)

As before, $y_{ht}$ denotes the month $t$ excess return of subject $h$ and $z_{ht}^{(p)}$ is a dummy variable which is equal to one if subject $h$ belongs to group $p$ (with $p = high, low$) and zero otherwise. $w_{ht}^{(p)}$ refers to the subject’s beginning-of-the-period weight in portfolio $p$. Thereby, the weights of both portfolios sum up to 1 (i.e., $\sum_{h=1}^{N_t} w_{ht}^{(p)} z_{ht}^{(p)} = 1$), and the cross-section in this case comprises a total of $N_t = \sum_{h=1}^{N_t} z_{ht}^{(low)} + \sum_{h=1}^{N_t} z_{ht}^{(high)}$ subjects in period $t$. The period $t$ return difference between the two portfolios can then be obtained as

$$\Delta y_t = y_{high,t} - y_{low,t}$$

(10)

The second step of the portfolio sorts approach then evaluates the risk-adjusted performance of zero investment portfolio (10) based on a $K$-factor time-series regression as follows:

$$\Delta y_t = \beta_0 + \beta_1 x_{1t} + \ldots + \beta_K x_{Kt} + \epsilon_t$$

(11a)

For simplicity and without loss of generality, we label the portfolios or subject groups as “high” and “low”. However, $p$ could also refer to “IPO firms” and “mature firms” or to “women” and “men”, for example.
If the coefficient estimate for $\beta_{\Delta 0}$ is positive (negative) and significantly different from zero, portfolio "high" is considered to outperform (underperform) portfolio "low".

As shown in Section 2.2 above, we can evaluate the risk-adjusted performance of portfolio $p$ by using $y_{pt}$ from (9) as the dependent variable in the second-step time-series regression. Consequently, the performance of portfolio "low" is evaluated based on the following time-series regression:

$$y_{low,t} = \beta_{low,0} + \beta_{low,1}x_{1t} + \ldots + \beta_{low,K}x_{Kt} + \epsilon_{low,t} \quad (11b)$$

It is possible to specify the $GCT$-regression model in (1) such that it reproduces all the coefficient estimates and standard errors of both time-series regressions (11a) and (11b), respectively. For this purpose, we set $z_{ht} = \begin{bmatrix} 1 & z_{ht}^{(high)} \end{bmatrix}$ and $x_t = \begin{bmatrix} 1 & x_{1t} & \ldots & x_{Kt} \end{bmatrix}$ to specify the $GCT$-regression model as follows:

$$y_{ht} = (z_{ht} \otimes x_t) \theta + \nu_{ht}$$

$$= \theta_{low,0} + \theta_{low,1} x_{1t} + \ldots + \theta_{low,K} x_{Kt}$$

$$+ \theta_{\Delta 0} z_{ht}^{(high)} + \theta_{\Delta 1} x_{1t} z_{ht}^{(high)} + \ldots + \theta_{\Delta K} x_{Kt} z_{ht}^{(high)} + \nu_{ht} \quad (12)$$

Under the same assumptions as for Proposition 1 and provided that portfolios “low” and “high” are both constant over time, the following result holds true:\(^{10}\)

**Proposition 3 (Performance difference between two groups)**

**Part A – Coefficient estimates.**

- **OLS coefficient estimates for $\beta_{\Delta k}$ in time-series regression (11a) are identical to pooled OLS coefficient estimates for $\theta_{\Delta k}$ in subject-level panel regression (12), i.e., $\hat{\beta}_{\Delta k} = \hat{\theta}_{\Delta k}$ ($\forall k = 0, 1, \ldots, K$).**

\(^{10}\) In the general case of an unbalanced panel or time-varying portfolios (or subject groups), the regression model has to be estimated by weighted least squares (WLS) rather than with OLS. See Sections 3.4 and 4.4 for details.
- For subject group \( p = \text{“low”} \), OLS coefficient estimates for \( \beta_{\text{low},k} \) in regression (11b) are identical to pooled OLS coefficient estimates for \( \theta_{\text{low},k} \) in subject-level panel regression (12), i.e., \( \hat{\beta}_{\text{low},k} = \hat{\theta}_{\text{low},k} \) (\( \forall k = 0, 1, ..., K \)).

**Part B – Standard errors.**

- For a given lag length choice \( H \), Newey and West (1987) standard errors for coefficient estimates \( \hat{\beta}_{\Delta k} \) of time-series regression (11a) are identical to Driscoll and Kraay (1998) standard errors for coefficient estimates \( \hat{\theta}_{\Delta k} \) in subject-level panel regression (12), i.e., \( \text{SE}(\hat{\beta}_{\Delta k}) = \text{SE}(\hat{\theta}_{\Delta k}) \) (\( \forall k = 0, 1, ..., K \)).

- For portfolio \( p = \text{“low”} \) and a given lag length choice \( H \), Newey and West (1987) standard errors for coefficient estimates \( \hat{\beta}_{\text{low},k} \) of time-series regression (11b) are identical to Driscoll and Kraay (1998) standard errors for coefficient estimates \( \hat{\theta}_{\text{low},k} \) in subject-level panel regression (12), i.e., \( \text{SE}(\hat{\beta}_{\text{low},k}) = \text{SE}(\hat{\theta}_{\text{low},k}) \) (\( \forall k = 0, 1, ..., K \)).

Proof: See Appendix A.3.

2.5 Continuous and multivariate subject characteristics

The *portfolio sorts* approach is generally limited to the analysis of one, two, or at most a few subject characteristics (Cochrane, 2011). With the *portfolio sorts* approach it is therefore challenging to perform robustness checks or to test for competing hypotheses. Moreover, it is difficult to comprehensively assess the functional relationship across, say, ten decile portfolios. Researchers therefore routinely focus on a comparison of the top and bottom groups for simplicity (Patton and Timmermann, 2010).

The *GCT-regression* model (1) has no such limitations. In fact, it is straightforward to specify vector \( \mathbf{z}_{ht} \) with multivariate and continuous subject characteristics. Notwithstanding its flexibility, the *GCT-regression* model also facilitates a clear-cut economic interpretation of the regression coefficients. To substantiate the economic interpretation of the model, let us assume for simplicity, but without loss
of generality, that vector $x_t$ is specified as $x_t = [1 \ (R_{mt} - R_{ft})]$ whereas vector $z_{ht}$ is defined as $z_{ht} = [1 \ z_{ht}]$. In this case, the GCT-regression model (1) can be rewritten as follows:\(^{11}\)

$$y_{ht} = (z_{ht} \otimes x_t) \theta + v_{ht}$$

$$= ([1 \ z_{ht}] \otimes [1 \ (R_{mt} - R_{ft})]) \theta + v_{ht}$$

$$= (\theta_{a,1} + \theta_{a,2}z_{ht}) + (\theta_{\beta,1} + \theta_{\beta,2} \times z_{ht}) \times (R_{mt} - R_{ft}) + v_{ht}$$

$$= \alpha_{ht} + \beta_{ht} \times (R_{mt} - R_{ft}) + v_{ht}$$

(13)

where $\alpha_{ht} = \theta_{a,1} + \theta_{a,2}z_{ht}$ and $\beta_{ht} = \theta_{\beta,1} + \theta_{\beta,2}z_{ht}$. The last row in (13) reveals that the GCT-regression model actually decomposes the risk-adjusted performance and the exposure to the factor variables in vector $x_t$ with respect to the various subject characteristics comprised in vector $z_{ht}$. Thereby, the relationship between the risk-adjusted performance and the subject characteristics is assumed to be linear. In accordance with Cochrane’s claim, the GCT-regression model represents a regression-based alternative to portfolio sorts which “are really the same thing as nonparametric cross-sectional regressions, using nonoverlapping histogram weights” (Cochrane, 2011, p. 1061).

2.6 Time-series versus cross-section

By setting $x_t = [1]$ the GCT-regression model decomposes (excess) returns rather than risk-adjusted performance. In the context of asset pricing studies, an interesting application of return decompositions arises when splitting up a subject variable’s predictability of (excess) returns into a cross-sectional and a time-series component (Cochrane, 2011). To this end, we subdivide (a continuous and time-varying) subject characteristic $z_{ht}$ as follows:

$$z_{ht} = \tilde{z}_h + \tilde{z}_t + \tilde{z}_{ht}$$

(14)

\(^{11}\) Note that the specification of vectors $x_t$ and $z_{ht}$ can easily be extended to comprise multiple factor variables or subject characteristics without changing the interpretation of the regression coefficients of the GCT-regression model.
Thereby, $\bar{z}_h$ is subject $h$’s time-series average of characteristic $z_{ht}$, i.e., $\bar{z}_h = T_h^{-1} \sum_{t=1}^{T_h} z_{ht}$. Likewise, $\bar{z}_t$ refers to the time $t$ (value-weighted) cross-sectional average of the subject characteristic, i.e., $\bar{z}_t = \sum_{h=1}^{N_t} w_{ht} z_{ht}$. Finally, $\bar{z}_{ht}$ denotes subject $h$’s idiosyncratic component of characteristic $z_{ht}$ in period $t$. Based on expression (14), we can use the GCT-regression model for decomposing returns with respect to the time-series and cross-sectional component of subject characteristic $z_{ht}$ as follows:

$$y_{ht} = (z_{ht} \otimes x_t) \theta + \nu_{ht} = ([1 \quad \bar{z}_h \quad \bar{z}_t] \otimes [1]) \theta + \nu_{ht} = \theta_0 + \theta_1 \bar{z}_h + \theta_2 \bar{z}_t + \nu_{ht} \quad (15)$$

It is important to note that the Fama and MacBeth (1973) procedure (or FM-approach) is not suitable for carrying out such an analysis since it is impossible to estimate cross-sectional regressions with variables that are constant in the cross-section. For a given point in time, $\bar{z}_t$ does not vary across subjects. Therefore, variable $\bar{z}_t$ cannot be included in the FM-approach in a straightforward way and, hence, the FM-approach does not allow to distinguish between a subject characteristic’s cross-section and time-series predictability in a similar way.

3. Applying the GCT-regression model in an asset pricing setup

Novy-Marx (2013) shows that gross profit scaled by the book value of total assets (GPAT) is a better predictor for the cross-section of average stock returns than alternative measures which are based on bottom line net income, cash flows, or dividends. He argues that the GPAT measure’s good performance in predicting the cross-section of average stock returns is mainly due to its numerator, gross profit, being a cleaner measure of economic profitability than, say, net income. Ball, Gerakos, Linnainmaa, and Nikolaev (2015), henceforth abbreviated as BGLN, challenge the findings of Novy-Marx (2013). Their critique centers on the observation that Novy-Marx (2013) deflates net income by the book value of equity while deflating gross profit by the book value of total assets. BGLN demonstrate that the predictive power of net income and gross profit is comparable if the same deflator is used. Moreover, they find operating profitability, an alternative profitability measure which more closely matches current expenses with current revenues, to better predict the cross-section of average returns than both net income and gross profit.
In this section, we replicate some of the results in BGLN to empirically validate Propositions 1 to 3 derived in Section 2. Moreover, we show how to specify the *GCT-regression* model to reproduce the results of two-way portfolio sorts. Finally, we demonstrate the flexibility of the *GCT-regression* model by including multiple and continuously distributed subject characteristics in the analysis.

### 3.1 The portfolio sorts approach in BGLN

BGLN use the CRSP-Compustat merged database, covering a sample period from July 1963 through December 2013. In Section 6.2 of their paper, they apply the *portfolio sorts* approach to evaluate the performance of operating profitability in predicting the cross-section of stock returns. BGLN define operating profitability as gross profit minus selling, general, and administrative expenses (excluding R&D expenditures) deflated by the book value of total assets.

To implement the *portfolio sorts* approach, they first form 10 portfolios sorted on operating profitability using NYSE decile breakpoints. In each year, portfolios are set up at the end of June and are then held throughout the subsequent year. For each portfolio $p (p = 1, \ldots, 10)$, monthly value-weighted excess returns ($y_{pt}$) are then obtained as follows:

$$y_{pt} = \sum_{h=1}^{N_t} w_{ht}^{(p)} z_{ht}^{(p)} y_{ht}$$  \hspace{1cm} (16)

Here, $w_{ht}^{(p)}$ refers to the beginning-of-month $t$ portfolio weight of stock $h$ in decile portfolio $p$ and $y_{ht}$ denotes stock $h$’s month $t$ excess return. $z_{ht}^{(p)}$ is a dummy variable with value one if stock $h$ in month $t$ belongs to portfolio $p$, and zero otherwise. $N_t$ refers to the overall month $t$ number of stocks in the sample.

In the second step, BGLN then estimate a time-series regression with $y_{pt}$ from (16) as dependent variable and the three Fama and French (1993) factors as explanatory variables:

$$y_{pt} = \alpha + \beta_{RMRF} \left( R_{mt} - R_{ft} \right) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_t$$  \hspace{1cm} (17)
where \( (R_{mt} - R_{ft}) \) denotes the market excess return, \( SMB_t \) is the return of a zero-investment size portfolio, and \( HML_t \) refers to the return of a zero-investment book-to-market portfolio.

3.2 Empirical validation of Proposition 1

We now show how to reproduce the results from the portfolio sorts approach with the GCT-regression model. As a starting point, we focus on the case of a single portfolio \( p \), namely the decile 1 (low) portfolio with the lowest operating profitability. According to Proposition 1, we can reproduce the results from the portfolio sorts approach in (17) for portfolio 1 (low) by estimating the following subject-level regression with Driscoll and Kraay (1998) standard errors:

\[
y_{ht} = \theta_0 + \theta_1 (R_{mt} - R_{ft}) + \theta_2 HML_t + \theta_3 SMB_t + \nu_{ht}
\]  

(18)

The GCT-regression model in (18) only includes the set of observations with \( z_{ht}^{(1)} = 1 \), i.e., the set of observations with a non-zero weight in portfolio 1. Since the CRSP-Compustat merged dataset constitutes an unbalanced panel with time-varying cross-sections and because BGLN analyze value-weighted rather than equal weighted portfolios, we cannot rely on pooled OLS estimation when aiming to exactly reproduce the results of the portfolio sorts approach. We therefore estimate GCT-regression model (18) with weighted least squares (WLS) where observation weights are fixed such that they match the (implicit) weighting scheme of the portfolio sorts approach. Consequently, we set the weight of observation \( ht \) equal to the beginning-of-month \( t \) value weight of stock \( h \) in portfolio 1 (low):

\[
w_{ht} = \frac{ME_{ht,-1}}{\sum_{h=1}^{N_t} ME_{ht,-1}}
\]  

(19)

where \( ME_{ht,-1} \) refers to the (lagged) market value of equity of stock \( h \) and \( N_t \) is the number of month \( t \) stocks in portfolio 1 (low).

The results are reported in Table 1. The first row (“Replication (portfolio sorts approach”) reports our replication of the results for decile 1 (low) portfolio from Table 7 in BGLN (2015, p. 240) as obtained when estimating time-series regression (17). Our replication results closely match those of
BGLN: Regression coefficients are very similar and significance levels are identical. In the second row ("GCT-regression (WLS, Driscoll-Kraay SE)"), we report the results from estimating GCT-regression model (18). The results are identical to those from the portfolio sorts approach reported in the first row.\textsuperscript{12} This empirically validates our Proposition 1 from Section 2.2. The third row ("GCT-regression (WLS, White SE)") presents the results from estimating the GCT-regression model in (18) with White rather than Driscoll and Kraay (1998) standard errors. The reliance on White standard errors, which assume cross-sectional independence, results in $t$-statistics that are inflated by roughly 60\% compared to the cross-sectional dependence consistent Driscoll-Kraay standard errors. This demonstrates the importance of relying on standard error estimates that are robust to cross-sectional correlation. The last row ("GCT-regression (OLS, Driscoll-Kraay SE)") estimates the GCT-regression model in (18) with ordinary least squares (OLS). Thereby, all stocks are equally weighted such that microcaps receive the same weight as large caps. As a consequence, the SMB factor loading is more than twice as large as in the value-weighted regressions. Moreover, the change in weights also affects the risk-adjusted performance (or alpha) which is no longer statistically significantly different from zero. This confirms Fama and French’s (2008) concerns that the abundance of small- and microcap stocks can be influential for the results when observations on micro- and megacap stocks are equally-weighted.

3.3 Empirical validation of Proposition 2

According to Proposition 2 it is possible to reproduce the results of multiple portfolio sorts with the GCT-regression model by estimating a single linear regression on the subject level. To validate Proposition 2 empirically, we extend our replication of results from Table 7 in BGLN. Following BGLN, we apply the portfolio sorts approach outlined in Section 3.1 above for each of the ten decile portfolios sorted on operating profitability. The results are reported in Panel A of Table 2. Again, our replication closely matches the results reported in BGLN (2015, p. 240).

\textsuperscript{12} Note that the number of observations largely differs across the two rows with 594 monthly observations in the time-series regression of the first row and 449,790 observations in the subject-level GCT-regression.
In order to specify the *GCT-regression* model such that it replicates the results from multiple portfolio sorts, we make use of Equation (7). Correspondingly, we set vector $\mathbf{z}_{ht}$ as $\mathbf{z}_{ht} = \begin{bmatrix} z_{ht}^{(1)} & z_{ht}^{(2)} & \ldots & z_{ht}^{(10)} \end{bmatrix}$ and specify vector $\mathbf{x}_t$, which includes the market-level factor variables, as $\mathbf{x}_t = \begin{bmatrix} 1 & (R_{mt} - R_{ft}) & SMB_t & HML_t \end{bmatrix}$. Based on the argumentation in Section 3.2, we have to estimate the *GCT-regression* model with weighted least squares (WLS) in order to reproduce the results from the *portfolio sorts* approach. To match the weighting scheme of the value-weighted portfolio sorts, we therefore specify the observation weights for the *GCT-regression* model according to Equation (19) where $N_t$ now refers to the overall month $t$ number of stocks in the sample.\(^{13}\) Panel B of Table 2 displays the results from estimating the *GCT-regression* model in a two-dimensional matrix. Thereby, the portfolio dummy variables $z_{ht}^{(p)}$ (which together form vector $\mathbf{z}_{ht}$) define the columns whereas the factor variables in vector $\mathbf{x}_t$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and $t$-statistics) of the interaction term between subject variable $z_{ht}^{(p)}$ and factor variable $x_{kt}$. A comparison of the results in Panels A and B of Table 2 shows that both the coefficient estimates and $t$-statistics (based on Driscoll-Kraay standard errors) of the *GCT-regression* model are identical to those obtained by applying the *portfolio sorts* approach (independently) to the 10 portfolios sorted on operating profitability. This empirically confirms the theoretical result stated in Proposition 2 from Section 2.3.

### 3.4 Empirical validation of Proposition 3

To empirically validate Proposition 3, we replicate BGLN’s analysis in the bottom two rows of Table 7, thereby completing our replication of this table. In these two rows, BGLN apply the *portfolio sorts* approach for evaluating the performance difference between the decile 10 (high) portfolio of stocks with the highest operating profitability and the decile 1 (low) portfolio comprising the stocks with the lowest operating profitability. To replicate BGLN’s analysis, we first compute the month $t$ excess returns ($y_{pt}$) for the top and bottom-decile portfolios as outlined in Equation (16) above. We then calculate

\(^{13}\) Note that at a given point in time, $t$, stock $h$ belongs to exactly one single decile portfolio $p$. 

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the month $t$ return difference between the two groups as $\Delta y_t = y_{10,t} - y_{1,t}$ and evaluate the (risk-adjusted) performance difference between the two groups by estimating time-series regression (17) with $\Delta y_t$ as the dependent variable. Panel A of Table 3 reports the results which again closely match those published in BGLN (p. 240).

To reproduce the results of the portfolio sorts approach with the $GCT$-regression model, we make use of Equation (12). Specifically, we set vector $z_{ht}$ equal to $z_{ht} = \begin{bmatrix} 1 & z_{ht}^{(10)} \end{bmatrix}$ and specify vector $x_t$ as $x_t = \begin{bmatrix} 1 & (R_{mt} - R_{ft}) & SMB_t & HML_t \end{bmatrix}$. Moreover, based on our prior reasoning, we can only replicate the results of the portfolio sorts approach when estimating the $GCT$-regression model with weighted least squares (WLS). We therefore set the observation weights equal to the stocks’ value weight as outlined in Equation (19). However, in this application, we only consider stocks being allocated to either operating profitability decile 1 (low) or to decile portfolio 10 (high). As a consequence, $N_t$ in this case refers to the overall month $t$ number of stocks contained in the bottom and top decile portfolios. Panel B of Table 3 displays the results in a two-dimensional matrix. Thereby, the elements of subject characteristics’ vector $z_{ht}$ define the columns while the factor variables in vector $x_t$ identify the rows. All elements in the results matrix thus represent the coefficient estimates (and $t$-statistics based on Driscoll-Kraay standard errors) for the interactions between the variables in vector $z_{ht}$ and those of vector $x_t$. When comparing the results from the $GCT$-regression model in Panel B with those from the portfolio sorts approach in Panel A, we observe identical coefficient estimates and $t$-statistics. This empirically confirms Proposition 3.

### 3.5 Two-way sorts

The $GCT$-regression model allows to reproduce the results from two-way portfolio sorts by estimating a single regression. To illustrate this, we first replicate the results in Panels B and C of Table 8 in BGLN (2015, p. 241). BGLN use NYSE breakpoints to sort the stocks independently into quintiles based on operating profitability and market capitalization. Thereby, the portfolios are formed by the end of each June and then held for the subsequent year. BGLN perform the two-way sorts to investigate
whether the predictive power of operating profitability for the cross-section of average returns is a marketwide phenomenon or whether it is confined to certain size groups. Panel A of Table 4 reports our replication results based on the portfolio sorts approach applied to the two-way sorted portfolios. Again, the coefficient estimates and $t$-values closely match those of the original research.

Next, we turn to the $GCT$-regression model from Equation (1) and show how to replicate the results of all 25 two-way sorted portfolios by estimating a single linear regression on the firm-level. For this purpose, we specify vector $z_{ht}$ containing the subject characteristics as follows:

$$z_{ht} = \left[OA_{ht}^{(1)} \ldots OA_{ht}^{(5)} \otimes ME_{ht}^{(1)} \ldots ME_{ht}^{(5)} \right]$$

Here, $OA_{ht}^{(p)} (p = 1, \ldots, 5)$ refers to a dummy variable with value one if stock $h$ in month $t$ belongs to operating profitability portfolio $p$, and value zero otherwise. Similarly, $ME_{ht}^{(q)} (q = 1, \ldots, 5)$ is a dummy variable with value one if stock $h$ in month $t$ belongs to market capitalization portfolio $p$. As before, vector $x_t$ again includes a constant and the three Fama-French factors, i.e., $x_t = \left[1 \ (R_{mt} - R_{ft}) \ SMB_t \ HML_t \right]$. To match the stocks’ value-weights in the various two-way sorted portfolios, we again define observation weights according to Equation (19) and estimate the resulting $GCT$-regression model with weighted least squares (WLS).

Panel B of Table 4 reports the results for the regression coefficients measuring the portfolios’ risk-adjusted performance, i.e., the coefficient estimates (and $t$-statistics) for the variables contained in vector $z_{ht}$. The results are displayed in a two-dimensional matrix with the operating profitability quintile dummies, $OA_{ht}^{(p)}$, identifying the rows and the market capitalization quintile dummies, $ME_{ht}^{(q)}$, defining the columns. Element $(p, q)$ in the result matrix thus represents the coefficient estimate (and $t$-statistic) of the interaction term between operating profitability quantile dummy $p$ and market cap quintile dummy $q$. When comparing the results from estimating the $GCT$-regression model (Panel B) with the results of

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14 In fact, the regression coefficients of the subject characteristics contained in vector $z_{ht}$ are the interaction terms of the subject characteristics in vector $z_{ht}$ and the constant of vector $x_t$ comprising the factor variables.
the two-way sorted portfolios (Panel A), it is evident that they perfectly match. This provides further empirical confirmation of the theoretical result in Proposition 2.

3.6 Generalizing the portfolio sorts approach: Decomposing risk-adjusted performance

The GCT-regression model is by no means confined to the analysis of sorted portfolios but rather can easily handle continuous (and multiple) subject characteristics. In this subsection, we illustrate the flexibility of the GCT-regression model empirically. Specifically, building on our prior analysis investigating the predictive power of operating profitability with respect to the cross-section of average returns, we alter the specification of the GCT-regression model as follows:

\[
y_{ht} = (z_{ht} \otimes x_t) \theta + v_{ht}
\]

\[
= ([1 \quad OPAT_{ht}] \otimes [1 \quad (R_{mt} - R_{ft}) \quad SMB_t \quad HML_t]) \theta + v_{ht}
\]  

(21)

where subject characteristic \( OPAT_{ht} \) refers to the period \( t \) operating profitability of stock \( h \).

The results from estimating the GCT-regression model in (21) with weighted least squares (WLS), and observation weights set equal to the month \( t \) value-weights of the stocks, are reported in Column 1 of Table 5. For brevity, only the estimates for the regression coefficients decomposing the risk-adjusted performance (i.e., the interaction terms of the subject characteristics in vector \( z_{ht} \) with the constant in vector \( x_t \)) are reported. Consistent with the previous results reported in Tables 2 and 4, the results show a positive and statistically significant relation between operating profitability and alpha (\( t \)-statistic of 5.27).

The remaining two columns in Table 5 present the results for variations in the composition of vector \( z_{ht} \). Specifically, we aim at verifying BGLN’s claim that operating profitability is a better predictor for the cross-section of average returns than gross profit scaled by total assets. In Column 2, we therefore re-estimate GCT-regression model (21) by replacing operating profitability in vector \( z_{ht} \) with gross profit scaled by total assets, \( GPAT_{ht} \), as the predictor variable. When compared to Column 1, the coefficient estimates, \( t \)-statistics, and R-squared are all smaller indicating that operating profitability indeed seems to better predict the cross-section of average stock returns than gross profit scaled by total assets.
In Column 3, finally, we take advantage of the flexibility of the GCT-regression model to include multiple (and continuously distributed) firm characteristics in the analysis. Specifically, we run a horse race between the two alternative profitability measures by specifying vector $z_{ht}$ as $z_{ht} = [1 \ OPAT_{ht} \ GPAT_{ht}]$. This allows us to directly test which of the two profit measures has better power for predicting the cross-section of average returns. Again, and consistent with BGLN’s results, we find operating profitability to outperform gross profit scaled by total assets.\footnote{Note that our analysis in Table 5 is similar in spirit to the Fama-MacBeth results reported in Table 5 of BGLN (2015). However, while the regression estimates from the Fama-MacBeth approach have “alpha-like interpretations” (BGLN, 2015, p. 234), the GCT-regression model analyses (and decomposes) the “alpha” directly. As explained in the introduction, Fama-MacBeth regressions do not allow for a decomposition of the risk-adjusted performance since it is impossible to estimate cross-sectional regressions with (factor) variables that are constant in the cross-section.}

\subsection*{3.7 Time-series versus cross-sectional predictability}

We next attempt to divide the predictive power of operating profitability with respect to predicting the cross-section of average returns versus predicting stock market returns over time. For this purpose, we rely on expression (15) and, hence, specify the GCT-regression model as follows:

$$y_{ht} = (z_{ht} \otimes x_{t}) \theta + v_{ht} = ([1 \ \overline{OPAT}_{h} \ \overline{OPAT}_{t}] \otimes x_{t}) \theta + v_{ht}$$ \hspace{1cm} (22)

Here, $\overline{OPAT}_{h}$ is firm $h$’s average operating profitability over time and $\overline{OPAT}_{t}$ refers to the time $t$ (value-weighted) cross-sectional average operating profitability of all $N_{t}$ firms in the sample.

The results from estimating the GCT-regression model in (22) with weighted least squares (WLS), and observation weights set equal to the month $t$ value-weights of the stocks, are reported in Columns 1 to 3 of Table 6. The results reveal that operating profitability is a statistically significant predictor for the cross-section of average stock returns while operating profitability has no power in predicting stock market returns over time. In Columns 4 to 6, variants of the GCT-regression model in (22) are estimated with $\overline{OPAT}_{h}$ and $\overline{OPAT}_{t}$ being replaced by variables $\overline{GPAT}_{h}$ and $\overline{GPAT}_{t}$, respectively. Thereby, $\overline{GPAT}_{h}$ is firm $h$’s time-series average of gross profit deflated by the book value of total assets. Likewise, $\overline{GPAT}_{t}$ refers to the period $t$ (value-weighted) average of gross profit deflated by the book value of total assets.
measured over all $N_t$ firms in the sample. Again, the results indicate that profitability is a statistically significant predictor for the cross-section of average stock returns but has no power in predicting stock market returns over time. Column 7 presents the results from estimating GCT-regression model (22) with vector $z_{ht}$ comprising all the profitability variables considered in this section, i.e., $z_{ht} = [1 \ OPA^h \ OPAT_t \ GPA^h \ GPA^h_t]$. By running a horse race between the various profitability measures we only find the coefficient estimate for variable $OPAT^h_t$ to be statistically significant. Consequently, operating profitability outperforms gross profit scaled by total assets in predicting the cross-section of average stock returns while both profitability measures have no power in predicting stock market returns over time.

In Panel B of Table 6 we repeat the analysis from Panel A. However, this time we specify vector $x_t$ as $x_t = [1 \ (R_{mt} - R_{ft}) \ SMB_t \ HML_t]$ in order to decompose risk-adjusted performance from a Fama-French three-factor model. The results are qualitatively similar to those from Panel A. Most importantly, operating profitability is a better predictor for the cross-section of average stock returns than gross profit scaled by total assets. Moreover, the higher operating profitability the better on average is a stock’s risk-adjusted performance.

4. Applying the GCT-regression model in long-term event studies

Long-term event studies constitute another important area of application of the portfolio sorts approach. Typical applications include research on private investors (e.g., Barber and Odean, 2000, 2001; Korniotis and Kumar, 2013), IPOs (Ritter and Welch, 2002; Cao, Jiang, and Ritter, 2015), and M&As (Andrade, Mitchell, and Stafford, 2001; Moeller, Schlingemann, and Stulz, 2004). In this section, we illustrate the benefits of applying the GCT-regression model in the context of long-horizon event studies. To this end, we replicate some results of Barber and Odean’s (2000) study analyzing the impact of portfolio turnover on the performance of individual investors.

\[\text{References}\]

16 For a detailed discussion of the portfolio sorts approach in long-term event studies, e.g., see Kothari and Warner (2008).
4.1 The portfolio sorts approach in Barber and Odean (2000)

Barber and Odean (2000), henceforth abbreviated as BO, investigate the common stock trading behavior of 66,465 individual investors at a large discount brokerage firm during the period 1991 through 1996. BO attempt to shed some light on two competing theories of trading activity. The first, proposed by Grossman and Stiglitz (1980), is based on a rational expectations framework and proposes that investors will only trade when the marginal benefit of a transaction is equal to or better than the marginal cost of trading. The second theory, developed by Odean (1998), Gervais and Odean (2001), and Caballé and Sákovics (2003), is based on investor overconfidence and predicts that investors trade to their detriment. Consistent with the overconfidence theory, BO find that accounting for transaction costs results in a negative relation between stock turnover and the risk-adjusted performance of individual investors. BO therefore conclude that private investors “pay a tremendous performance penalty for active trading” (p. 773).

In Panel D of their Table II, BO apply the portfolio sorts approach to evaluate the net (of transaction costs) performance of the average household’s stock portfolio. They first compute the net excess return of the average household’s common stock portfolio as

\[ y_{t,net} = \frac{1}{N_t} \sum_{h=1}^{N_t} y_{ht,net} \]  

(23)

where \( y_{ht,net} \) refers to household \( h \)’s net portfolio return (after accounting for transaction costs) in excess of the risk-free return \( (R_{ft}) \).\(^{17}\) In a second step, they then estimate the following two specifications of time-series regression (3):

- **CAPM:**  
  \[ y_{t,net} = \beta_0 + \beta_1 (R_{mt} - R_{ft}) + \epsilon_t \]  
  (24a)

- **Fama-French 3-factor:**  
  \[ y_{t,net} = \beta_0 + \beta_1 (R_{mt} - R_{ft}) + \beta_2 HML_t + \beta_3 SMB_t + \epsilon_t \]  
  (24b)

\(^{17}\) For details on how to compute \( y_{ht,net} \), see BO (2000, p. 782). Note that BO subtract the risk-free return from \( y_{t,net} \) rather than from the individual investor returns (\( y_{ht,net} \)). When applying the portfolio sorts approach, these two approaches are equivalent. In case of the GCT-regression model, however, we need to compute excess returns on the individual investor level (i.e., \( y_{ht,net} \)).
In both cases, $y_t^{net}$ from Equation (23) serves as the dependent variable and the market excess return ($R_{mt} - R_{ft}$) and, in case of time-series regression (24b), additionally the Fama and French (1993) size ($SMB_t$) and book-to-market ($HML_t$) factors constitute the explanatory variables.

4.2 Empirical validation of Proposition 1

According to Proposition 1 from Section 2.2, we can reproduce the results from the portfolio sorts approach in (24a) and (24b) by estimating the following subject-level regressions with Driscoll and Kraay (1998) standard errors:

**CAPM:**

$$ y_t^{net} = \theta_0 + \theta_1 (R_{mt} - R_{ft}) + \nu_{ht} \tag{25a} $$

**Fama-French 3-factor:**

$$ y_t^{net} = \theta_0 + \theta_1 (R_{mt} - R_{ft}) + \theta_2 HML_t + \theta_3 SMB_t + \nu_{ht} \tag{25b} $$

Note that the individual investor data analyzed by BO constitutes an unbalanced panel dataset with a time-varying number of $N_t$ investors holding common stocks in month $t$. Because the portfolio sorts approach weights time periods rather than observations equally, pooled OLS estimation of the GCT-regression models in (25a) and (25b) is inappropriate for exactly reproducing the results of the portfolio sorts approach. To match the results of the portfolio sorts approach in the general case of an unbalanced panel, the GCT-regression model has to be estimated with weighted least squares (WLS) where the observation weights match the (implicit) weighting scheme of the portfolio sorts, i.e., where observation weights are set equal to $w_{ht} = N_t^{-1}$.

Columns 1 and 4 in Table 7 report the results from estimating the portfolio sorts approach as outlined in Equations (24a) and (24b), respectively. The coefficient estimates and significance levels by and large are in line with those from Table II, Panel D, in BO (2000, p. 787). More importantly, however, the results from the portfolio sorts approach coincide with the results from estimating the GCT-

---

18 Compared to BO, our replication includes slightly less investors (66,458 vs 66,465). The only notable difference between the results reported in Column 4 and the original results in Panel D of Table II in BO (2000) is the coefficient on the HML factor which is significant at the 5% level in BO’s published article but is insignificant in our regression. The most likely explanation for this difference is that the monthly returns of the HML-factor as available from Kenneth French’s data library have changed over time.
regression models in (25a) and (25b) with weighted least squares (Columns 2 and 5). Consequently and as stated in Proposition 1, the GCT-regression model manages to reproduce the results of the portfolio sorts approach by aid of estimating a single linear regression on the individual household level.

Finally, in Columns 3 and 6, we report the results from estimating the GCT-regression models in (25a) and (25b) with pooled OLS rather than with WLS. This results in an equal weighting of the observations (i.e., household months) and thereby addresses Loughran and Ritter’s (2000, p. 363) argument according to which “tests that weight firms [here: households] equally should have more power than tests that weight each time period equally”. Columns 3 and 6 of Table 7, however, suggest that the alteration of the weighting scheme in this case has no major consequences for the statistical results: the coefficient estimates and significance levels remain qualitatively unchanged.

4.3 Empirical validation of Proposition 2

BO measure an individual households’ portfolio turnover as the average of its sales and purchase turnover.\(^{19}\) They then partition the sample investors into five quintiles where households with the lowest portfolio turnover are allocated to quintile portfolio “1 (Low)” while households with the highest portfolio turnover are allocated to quintile portfolio “5 (High)”. In their Table V, Panel A, BO report the average household’s stock turnover across the five quintile portfolios. We report our results from replicating BO’s analysis in Panel A of Table 8. All five figures coincide with those from BO’s original research at the reported two decimals.

Next, we follow BO in applying the portfolio sorts approach to measure the risk-adjusted net (of transaction costs) performance for each of the five quintile portfolios. For this purpose, we first compute the period \(t\) net excess return of portfolio \(p\) as follows:

\[
y_{pt}^{net} = \frac{1}{N_{pt}} \sum_{h=1}^{N_t} y_{ht}^{(p)} y_{ht}^{net}
\]

\(^{19}\) For a detailed description on how to compute the individual investors’ monthly sales and purchase turnover, see BO (2000, p. 781).
As before, $y_{ht}^{\text{net}}$ refers to household $h$’s net portfolio return in excess of the risk-free return. $z_{ht}^{(p)}$ is a dummy variable with a value of one if household $h$ in period $t$ belongs to turnover quintile $p$ (with $p = 1, \ldots, 5$), and zero otherwise. $N_{pt} = \sum_{h=1}^{N_t} z_{ht}^{(p)}$ is the period $t$ number of households $h$ that have been sorted into portfolio $p$ and $N_t = \sum_{p=1}^{5} N_{pt}$. For each of the five portfolios, we then estimate the CAPM time-series regression in (24a) with $y_{pt}^{\text{net}}$ as the dependent variable. The results are reported in Panel B of Table 8. They closely match those from Table V, Panel C, in BO’s original research.\(^{20}\)

According to Proposition 2 from Section 2.3 it is possible to specify the GCT-regression model such that it exactly reproduces the results from multiple portfolio sorts by aid of a single regression on the individual subject level. Making use of Equation (7), we set vector $z_{ht}$ equal to $z_{ht} = \left[ z_{ht}^{(1)} \ z_{ht}^{(2)} \ \ldots \ z_{ht}^{(5)} \right]$ and specify vector $x_t$ as $x_t = \left[ 1 \ (R_{mt} - R_{ft}) \right]$. To match the implicit weighting scheme underlying the quintile portfolios from the portfolio sorts approach, we therefore estimate the GCT-regression model with weighted least squares (WLS) where observation weights are fixed at $w_{ht}^{(p)} = N_{pt}^{-1}$.

Panel C of Table 8 displays the results in a two-dimensional matrix. The turnover quintile dummies $z_{ht}^{(p)}$ identify the columns of the result matrix and the elements from vector $x_t$ define the rows. Each element in the results matrix thus represents the coefficient estimate (and $p$-value) of the interaction between turnover dummy $z_{ht}^{(p)}$ and either the constant or market excess return from vector $x_t$. When comparing the results of the GCT-regression model with those of the respective portfolio sorts in Panel B, it is evident that both the coefficient estimates and $p$-values of the GCT-regression model exactly coincide with those from the portfolio sorts. This empirically confirms Proposition 2 from Section 2.3.

\(^{20}\) We also estimate the results for the Fama-French 3-factor regressions in (24b) but do not report them for brevity. However, compared to the CAPM case the results from estimating the Fama-French 3-factor regressions differ slightly more from BO’s original results. This supposedly is due to the fact that the Fama-French factor returns, in particular those of the HML factor, changed over time. See footnote 18 for details.
4.4 Empirical validation of Proposition 3

In the last column of Table V, Panel C, BO apply the portfolio sorts approach to evaluate the performance difference (after transaction costs) between the top and bottom quintile groups of individual investors. They compute the month $t$ return difference between the top and bottom turnover groups as

$$\Delta y^{\text{net}}_t = y^{\text{net}}_{2t} - y^{\text{net}}_{1t}.$$  

In the second step, they then estimate the time-series regression in (24a) with $\Delta y^{\text{net}}_t$ as the dependent variable. We report our results from replicating the respective analysis in Panel A of Table 9. Our replication of BO’s results is again in line with the originally published results.

To empirically validate Proposition 3 from Section 2.4, we make use of Equation (12) and specify the $GCT$-regression model such that it is capable to reproduce the results of BO’s portfolio sorts approach. Hence, we set vector $z_{ht}$ equal to

$$z_{ht} = \begin{bmatrix} 1 \\ z^{(Q5)}_{ht} \end{bmatrix}$$  

and vector $x_t$ as

$$x_t = \begin{bmatrix} 1 \\ (R_{mt} - R_{ft}) \end{bmatrix}.$$  

As before, we match the implicit weighting scheme of the portfolio sorts approach by estimating the $GCT$-regression model with weighted least squares (WLS) and fix observation weights at $w^{(p)}_{ht} = N_{pt}^{-1}$ (with $p = 1$ or 5). Panel B of Table 9 reports the results from estimating the $GCT$-regression model in the usual two-dimensional matrix. When comparing the results in Panels A and B of Table 9, we find the coefficient estimates and $p$-values of the $GCT$-regression model to exactly match those of the portfolio sorts approach. This empirically validates Proposition 3.

4.5 Continuous and multivariate subject characteristics

To empirically illustrate the flexibility of the $GCT$-regression model, we build on the analysis in Sections 4.2 and 4.3. We first use the $GCT$-regression model to investigate the impact of portfolio turnover on the net (of transaction costs) performance of individual households. Making use of the $GCT$-regression model’s capability to deal with continuous subject characteristics, we now specify the $GCT$-regression model as follows:

$$y^{\text{net}}_{ht} = (z_{ht} \otimes x_t) \theta + n_{ht} = \left( \begin{bmatrix} 1 \\ TO_h \end{bmatrix} \otimes \begin{bmatrix} 1 \\ (R_{mt} - R_{ft}) \end{bmatrix} \right) \theta + n_{ht}$$  

(27)
Here, subject characteristic $TO_h$ in vector $z_{ht}$ represents household $h$’s average monthly portfolio turnover as defined in BO (2000, p. 781). The results from estimating the GCT-regression model in (27) with pooled OLS are presented in the first column of Table 10. For brevity, the table only reports the coefficient estimates (and $p$-values) for the regression coefficients decomposing the “alpha” (i.e., the interaction terms between the subject characteristics in vector $z_{ht}$ and the constant in vector $x_t$). The results confirm that there is a negative and statistically significant relation between portfolio turnover and a household’s net (of transaction costs) performance.

We next consider an alternative household characteristic that has been shown to negatively impact on the net performance of private investors. Specifically, Barber and Odean (2002) find that investors switching from phone-based banking to online banking significantly underperform their phone banking peers after switching to online trading. They attribute this finding to overconfidence and excessive trading of online traders. In the second column of Table 10, we present the results from estimating GCT-regression model (27) with vector $z_{ht}$ being specified as $z_{ht} = [1 \ Online_{ht}]$ with variable $Online_{ht}$ referring to a dummy variable with a value of one if household $h$ traded stocks via online banking sometime before time-period $t$, and zero otherwise. Consistent with Barber and Odean (2002), we find the coefficient estimate for variable $Online_{ht}$ to be negative and statistically significant which confirms that online investors, on average, indeed underperform phone-based investors.

In Column 3 of Table 10, finally, we present the results from estimating a GCT-regression model including both the turnover and the online trading variables simultaneously. To this end, we define vector $z_{ht}$ as $z_{ht} = [1\ TO_h\ Online_{ht}]$. The results are in line with those from Columns 1 and 2 discussed before: The coefficient estimates for both variables $TO_h$ and $Online_{ht}$ are still negative and statistically significant at conventional levels. However, compared to Column 2 the coefficient estimate (and significance) of the $Online_{ht}$ dummy is substantially reduced indicating that a considerable portion of online investors’ underperformance results from their excessive trading, a result which is also in line with the findings of Barber and Odean (2002).
5. Conclusion

In this paper, we present a new methodology for decomposing the (risk-adjusted) performance with respect to multiple and continuously distributed subject characteristics. Our regression-based methodology, which we refer to as the \textit{GCT-regression} model, offers a straightforward to implement solution to the “multi-dimensional challenge” (Cochrane, 2011) of the widely applied \textit{portfolio sorts} approach. Statistical inference in the \textit{GCT-regression} model is based on Driscoll and Kraay (1998) standard errors that are heteroscedasticity consistent and robust to very general forms of temporal as well as cross-sectional dependence. The \textit{GCT-regression} model therefore shares all the statistical benefits of the \textit{portfolio sorts} approach and offers the same straightforward economic interpretation of the results. Moreover, the \textit{GCT-regression} model remedies a series of minor weaknesses of the \textit{portfolio sorts} approach. For example, it resolves the well-known problem of the \textit{portfolio sorts} approach underweighting (overweighting) observations from periods with large (small) cross-sections (Loughran and Ritter, 2000). Furthermore, the \textit{GCT-regression} model allows for using a standard Wald test as an alternative to the popular Gibbons, Ross, and Shanken (1989) “GRS-test” for testing whether the “alphas” of a series of portfolio sorts are jointly equal to zero.

We proof mathematically and confirm empirically that the \textit{GCT-regression} model can be specified and estimated such that it exactly reproduces the results of the \textit{portfolio sorts} approach. In the empirical part of the paper, we illustrate the \textit{GCT-regression} model by aid of two applications, one considering a typical asset pricing setup and the other implementing a long-horizon event study. After reproducing the results from the \textit{portfolio sorts} approach with the \textit{GCT-regression} model, we demonstrate the \textit{GCT-regression} model’s flexibility to include continuous and multiple subject characteristics in the analysis. This allows us, for example, to run a horse race between two or more competing hypothesis and to control for alternative subject characteristics.
References


This table reports the 3-factor model alpha along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. The portfolio sort is based on NYSE breakpoints at the end of each June and the portfolio is held for the subsequent year. The sample period goes from July 1963 through December 2013. Row (1) reports our replication of the results for decile portfolio 1 (low) as reported in Table 7 of BGLN (2015, p. 240) based on the portfolio sorts approach. Rows (2) to (4) present the results from estimating GCT-regression models. In Rows (2) and (3), the GCT-regression model is estimated with weighted least squares (WLS) where observation weights are set equal to the time t value-weight of the stocks. In Row (4), the regression is estimated with ordinary least squares (OLS). t-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference for the portfolio sorts approach in Row (1) and the GCT-regression model in Row (3) are based on heteroscedasticity-consistent White (1980) standard errors. The GCT-regression models in Rows (2) and (4) are estimated with Driscoll and Kraay (1998) standard errors with a lag-length of zero. These standard errors allow for valid statistical inference in the presence of cross-sectional dependence and heteroscedasticity. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 2: Portfolio sorts

<table>
<thead>
<tr>
<th>Decile Portfolio</th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10 (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.501***</td>
<td>-0.190**</td>
<td>-0.160**</td>
<td>-0.147**</td>
<td>0.039</td>
<td>0.015</td>
<td>-0.075</td>
<td>0.156**</td>
<td>0.084</td>
<td>0.272***</td>
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<tr>
<td></td>
<td>(-4.87)</td>
<td>(-2.27)</td>
<td>(-2.09)</td>
<td>(-2.03)</td>
<td>(0.56)</td>
<td>(0.20)</td>
<td>(-1.11)</td>
<td>(2.36)</td>
<td>(1.36)</td>
<td>(4.61)</td>
</tr>
<tr>
<td>( \beta_{MKT} )</td>
<td>1.178***</td>
<td>1.051***</td>
<td>0.984***</td>
<td>0.985***</td>
<td>0.958***</td>
<td>0.968***</td>
<td>1.005***</td>
<td>1.034***</td>
<td>0.971***</td>
<td>0.931***</td>
</tr>
<tr>
<td></td>
<td>(42.79)</td>
<td>(40.59)</td>
<td>(44.85)</td>
<td>(49.42)</td>
<td>(44.32)</td>
<td>(44.94)</td>
<td>(52.92)</td>
<td>(54.69)</td>
<td>(54.94)</td>
<td>(56.14)</td>
</tr>
<tr>
<td>( \beta_{SMB} )</td>
<td>0.523***</td>
<td>0.039</td>
<td>0.057*</td>
<td>0.065*</td>
<td>-0.049</td>
<td>-0.036</td>
<td>-0.073**</td>
<td>-0.025</td>
<td>-0.067**</td>
<td>-0.081***</td>
</tr>
<tr>
<td></td>
<td>(13.76)</td>
<td>(1.32)</td>
<td>(1.77)</td>
<td>(1.75)</td>
<td>(-1.39)</td>
<td>(-0.99)</td>
<td>(-2.19)</td>
<td>(-0.98)</td>
<td>(-2.45)</td>
<td>(-3.45)</td>
</tr>
<tr>
<td>( \beta_{HML} )</td>
<td>0.041</td>
<td>0.255***</td>
<td>0.353***</td>
<td>0.259***</td>
<td>0.188***</td>
<td>0.135***</td>
<td>0.150***</td>
<td>-0.007</td>
<td>-0.060*</td>
<td>-0.434***</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(6.57)</td>
<td>(9.23)</td>
<td>(6.52)</td>
<td>(5.71)</td>
<td>(3.37)</td>
<td>(4.66)</td>
<td>(-0.23)</td>
<td>(-1.94)</td>
<td>(-15.77)</td>
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<tr>
<td>( R^{2} )</td>
<td>0.862</td>
<td>0.850</td>
<td>0.860</td>
<td>0.873</td>
<td>0.866</td>
<td>0.859</td>
<td>0.884</td>
<td>0.912</td>
<td>0.901</td>
<td>0.918</td>
</tr>
<tr>
<td>( N )</td>
<td>594</td>
<td>594</td>
<td>594</td>
<td>594</td>
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Panel A: Replication of results in Table 7 of BGLN (2015) based on the portfolio sorts approach

Panel B: Replication of the results in Panel A based on the GCT-regression model (WLS estimation)

<table>
<thead>
<tr>
<th>( z_{t} ) (subject var.)</th>
<th>1 (Intercept)</th>
<th>( z_{t}^{(1)} )</th>
<th>( z_{t}^{(2)} )</th>
<th>( z_{t}^{(3)} )</th>
<th>( z_{t}^{(4)} )</th>
<th>( z_{t}^{(5)} )</th>
<th>( z_{t}^{(6)} )</th>
<th>( z_{t}^{(7)} )</th>
<th>( z_{t}^{(8)} )</th>
<th>( z_{t}^{(9)} )</th>
<th>( z_{t}^{(10)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector ( x_{t} ) (factor var.)</td>
<td>( \beta_{MKT} )</td>
<td>( \beta_{SMB} )</td>
<td>( \beta_{HML} )</td>
<td>( \alpha )</td>
<td>( \beta_{MKT} )</td>
<td>( \beta_{SMB} )</td>
<td>( \beta_{HML} )</td>
<td>( \alpha )</td>
<td>( \beta_{MKT} )</td>
<td>( \beta_{SMB} )</td>
<td>( \beta_{HML} )</td>
</tr>
<tr>
<td>( r_{t} - r_{f} )</td>
<td>1.178***</td>
<td>1.051***</td>
<td>0.984***</td>
<td>0.985***</td>
<td>0.958***</td>
<td>0.968***</td>
<td>1.005***</td>
<td>1.034***</td>
<td>0.971***</td>
<td>0.931***</td>
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<td></td>
<td>(42.79)</td>
<td>(40.59)</td>
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<td>(56.14)</td>
<td></td>
</tr>
<tr>
<td>SMB</td>
<td>0.523***</td>
<td>0.039</td>
<td>0.057*</td>
<td>0.065*</td>
<td>-0.049</td>
<td>-0.036</td>
<td>-0.073**</td>
<td>-0.025</td>
<td>-0.067**</td>
<td>-0.081***</td>
<td></td>
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<tr>
<td></td>
<td>(13.76)</td>
<td>(1.32)</td>
<td>(1.77)</td>
<td>(1.75)</td>
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<td>(-0.99)</td>
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<td>(-0.98)</td>
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<tr>
<td>HML</td>
<td>0.041</td>
<td>0.255***</td>
<td>0.353***</td>
<td>0.259***</td>
<td>0.188***</td>
<td>0.135***</td>
<td>0.150***</td>
<td>-0.007</td>
<td>-0.060*</td>
<td>-0.434***</td>
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<tr>
<td></td>
<td>(0.77)</td>
<td>(6.57)</td>
<td>(9.23)</td>
<td>(6.52)</td>
<td>(5.71)</td>
<td>(3.37)</td>
<td>(4.66)</td>
<td>(-0.23)</td>
<td>(-1.94)</td>
<td>(-15.77)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Replication of the results in Panel A based on the GCT-regression model (WLS estimation)

| \( R^{2} \)      | 0.230 |
| \( N \)          | 1,950,524 |
Table 2 – continued

This table reports value-weighted 3-factor model alphas along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A contains our replication of the results reported in Table 7 of BGLN (2015, p. 240). Panel B presents the results from estimating a single GCT-regression model with weighted least squares (WLS). Coefficient estimates and t-statistics (in parentheses) in Panel B (GCT-regression model) are for the product of the market-level variables (and constant) contained in the x-vector and the decile dummy variables for operating profitability in the z-vector. The dummy variables (Panel B) and decile portfolios (Panel A) are formed based on NYSE breakpoints at the end of each June and then remain unchanged throughout the subsequent year. The sample period is from July 1963 through December 2013. t-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference in the portfolio sorts approach is based on Newey and West (1987) standard errors. The GCT-regression models are estimated with Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for both Newey-West and Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 3: Performance difference between two groups

<table>
<thead>
<tr>
<th>Panel A: Portfolio sorts approach</th>
<th>Panel B: GCT-regression model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replication of BGLN (2015)</td>
<td>WLS estimation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Decile 1 (low)</th>
<th>High - low (deciles)</th>
<th>Vector $\mathbf{z}_t \rightarrow$</th>
<th>Vector $\mathbf{x}_t \downarrow$</th>
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<th>$\mu(010)$</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.501***</td>
<td>0.773***</td>
<td>I (Intercept)</td>
<td></td>
<td>-0.501***</td>
<td>0.773***</td>
</tr>
<tr>
<td></td>
<td>(-4.87)</td>
<td>(6.29)</td>
<td></td>
<td></td>
<td>(-4.87)</td>
<td>(6.29)</td>
</tr>
<tr>
<td>$\beta_{RMRF}$</td>
<td>1.178***</td>
<td>-0.247***</td>
<td>(R_m - R_f)</td>
<td></td>
<td>1.178***</td>
<td>-0.247***</td>
</tr>
<tr>
<td></td>
<td>(42.79)</td>
<td>(-7.15)</td>
<td></td>
<td></td>
<td>(42.79)</td>
<td>(-7.15)</td>
</tr>
<tr>
<td>$\beta_{SMB}$</td>
<td>0.523***</td>
<td>-0.604***</td>
<td>SMB</td>
<td></td>
<td>0.523***</td>
<td>-0.604***</td>
</tr>
<tr>
<td></td>
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<td>(-12.88)</td>
<td></td>
<td></td>
<td>(13.76)</td>
<td>(-12.88)</td>
</tr>
<tr>
<td>$\beta_{HML}$</td>
<td>0.041</td>
<td>-0.475***</td>
<td>HML</td>
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<td>0.041</td>
<td>-0.475***</td>
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<tr>
<td></td>
<td>(0.77)</td>
<td>(-7.43)</td>
<td></td>
<td></td>
<td>(0.77)</td>
<td>(-7.43)</td>
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</table>

| $R^2$          | 0.862          | 0.413                 | $R^2$                             | 0.207                            |
| $N$ Obs.       | 594            | 594                   | $N$ Obs.                          | 678,012                          |

This table reports value-weighted 3-factor model alphas along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for portfolios sorted by operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A contains our replication of the results reported in Table 7 of BGLN (2015, p. 240). Panel B presents the results from estimating GCT-regression models with weighted least squares (WLS). Coefficient estimates and $t$-statistics (in parentheses) for the GCT-regression models are for the product of the market-level variables (and a constant) contained in the $\mathbf{x}$-vector and a constant as well as a dummy variable with value one for stocks with top decile gross profitability in the $\mathbf{z}$-vector. The dummy variable in Panel B and the decile (difference) portfolios in Panel A are formed based on NYSE breakpoints at the end of each June and then remain unchanged throughout the subsequent year. The sample period is from July 1963 through December 2013. $t$-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference for the portfolio sorts approach is based on White (1980) standard errors. The GCT-regression model is estimated with Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for the Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 4: Two-way sorts

Panel A: Replication of results in Panels B and C of Table 8 in BGLN (2015) with portfolio sorts approach

<table>
<thead>
<tr>
<th>Operating profitability</th>
<th>Market capitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
</tr>
<tr>
<td>Q1</td>
<td>-0.45***</td>
</tr>
<tr>
<td></td>
<td>(-4.40)</td>
</tr>
<tr>
<td>Q2</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(-0.44)</td>
</tr>
<tr>
<td>Q3</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(1.08)</td>
</tr>
<tr>
<td>Q4</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(1.50)</td>
</tr>
<tr>
<td>Q5</td>
<td>0.30***</td>
</tr>
<tr>
<td></td>
<td>(3.93)</td>
</tr>
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</table>

Panel B: GCT-regression model (WLS estimation)

<table>
<thead>
<tr>
<th>Operating profitability</th>
<th>Market capitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{ht}^{(Q1)}$</td>
</tr>
<tr>
<td>OA_{ht}^{(Q1)}</td>
<td>-0.45***</td>
</tr>
<tr>
<td></td>
<td>(-4.40)</td>
</tr>
<tr>
<td>OA_{ht}^{(Q2)}</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(-0.44)</td>
</tr>
<tr>
<td>OA_{ht}^{(Q3)}</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(1.08)</td>
</tr>
<tr>
<td>OA_{ht}^{(Q4)}</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(1.50)</td>
</tr>
<tr>
<td>OA_{ht}^{(Q5)}</td>
<td>0.30***</td>
</tr>
<tr>
<td></td>
<td>(3.93)</td>
</tr>
</tbody>
</table>

This table reports value-weighted 3-factor model alphas and t-statistics (in parentheses) for portfolios sorted by market capitalization and operating profitability, defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets. Panel A contains our replication of the results reported in Panels B and C of Table 8 of BGLN (2015, p. 241). Panel B presents the results from estimating a single GCT-regression model with weighted least squares (WLS). Coefficient estimates and t-statistics (in parentheses) for the GCT-regression model are for the product of the market capitalization quintile dummy variables and quintile dummy variables for operating profitability. The dummy variables (Panel B) and two-way sorted portfolios (Panel A) are formed based on NYSE breakpoints at the end of each June and then remain unchanged throughout the subsequent year. The market capitalization and operating profitability sorts are independent of each other. The sample period is from July 1963 through December 2013. t-statistics from testing for significance against a value of zero are presented in parentheses. Statistical inference in the portfolio sorts approach in Panel A is based on White (1980) standard errors. The GCT-regression model in Panel B is estimated with Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for the Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 5: Continuous and multivariate subject characteristics

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.323***</td>
<td>-0.257***</td>
<td>-0.359***</td>
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<tr>
<td></td>
<td>(-4.58)</td>
<td>(-3.73)</td>
<td>(-4.73)</td>
</tr>
<tr>
<td>OPAT</td>
<td>1.757***</td>
<td>1.248***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.27)</td>
<td>(2.93)</td>
<td></td>
</tr>
<tr>
<td>GPAT</td>
<td>0.788***</td>
<td>0.371*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.80)</td>
<td>(1.74)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.240</td>
<td>0.238</td>
<td>0.240</td>
</tr>
<tr>
<td>N Obs.</td>
<td>2,013,242</td>
<td>2,013,242</td>
<td>2,013,242</td>
</tr>
</tbody>
</table>

This table reports the coefficient estimates and t-statistics (in parentheses) of GCT-regression models with multivariate and continuous subject characteristics in vector z. The regression model in Column 1 is: \( y_{ht} = \left( [1 \ OPAT_{ht}] \otimes [1 \ (R_{mt} - R_{ft}) \ SMB_t \ HML_t] \right) \ \theta + v_{ht}. \) OPAT is operating profit (defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures)) deflated by the book value of total assets. In Column 2, OPAT is replaced by GPAT. GPAT is gross profit deflated by the book value of total assets. In Column 3, the z-vector includes both OPAT and GPAT and is defined as \( [1 \ OPAT_{ht} \ GPAT_{ht}] \). The table only shows the results for the coefficient estimates in the z-vector, i.e., the decomposition of the risk-adjusted performance (or 3-factor model alpha). The risk-adjusted performance of firm \( h \) thereby is obtained as \( \alpha_h = \hat{\theta}_{\text{const}} + \hat{\theta}_{\text{OPAT}} \times E(\text{OPAT}_h) + \hat{\theta}_{\text{GPAT}} \times E(\text{GPAT}). \) The results for the market (RMRF), SMB (small minus big), and HML (high minus low) factor exposure decompositions are not shown in the table. The sample period is from July 1963 through December 2013. All GCT-regressions are estimated with weighted least squares (WLS) with observation weights set proportional to monthly value weights. All t-statistics test for significance against a value of zero. Statistical inference is based on Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for the Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
### Table 6: Time-series and cross-section predictability of profitability measures

**Panel A: Decomposition of excess returns**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.162</td>
<td>1.166</td>
<td>1.05</td>
<td>0.333*</td>
<td>-0.536</td>
<td>-0.587</td>
<td>0.329</td>
</tr>
<tr>
<td>(0.77)</td>
<td>(0.37)</td>
<td>(0.34)</td>
<td>(1.71)</td>
<td>(-0.22)</td>
<td>(-0.24)</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>(\overline{OPAT}_h)</td>
<td>1.760***</td>
<td>1.777***</td>
<td>1.827***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.07)</td>
<td>(3.15)</td>
<td>(2.95)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\overline{OPAT}_t)</td>
<td></td>
<td>-3.311</td>
<td>-4.448</td>
<td>-16.26</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(-0.21)</td>
<td>(-0.28)</td>
<td>(0.72)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\overline{GPAT}_h)</td>
<td>0.467**</td>
<td>0.456**</td>
<td>-0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.13)</td>
<td>(2.14)</td>
<td>(-0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\overline{GPAT}_t)</td>
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<td>8.354</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.37)</td>
<td>(0.85)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| R-squared | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |

**Panel B: Decomposition of Fama-French three-factor model alpha**

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<th>(6)</th>
<th>(7)</th>
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<tbody>
<tr>
<td>Constant</td>
<td>-0.545***</td>
<td>0.366</td>
<td>0.014</td>
<td>-0.241***</td>
<td>0.421</td>
<td>0.379</td>
<td>0.052</td>
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<tr>
<td></td>
<td>(-6.04)</td>
<td>(1.16)</td>
<td>(0.04)</td>
<td>(-3.13)</td>
<td>(1.42)</td>
<td>(1.23)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>(\overline{OPAT}_h)</td>
<td>2.968***</td>
<td>2.979***</td>
<td>3.024***</td>
<td></td>
<td></td>
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<td></td>
<td>(6.57)</td>
<td>(6.61)</td>
<td>(5.49)</td>
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<tr>
<td>(\overline{OPAT}_t)</td>
<td></td>
<td>-1.654</td>
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<td></td>
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<td>(-1.62)</td>
<td>(-0.07)</td>
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<td>(\overline{GPAT}_h)</td>
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<td>0.758***</td>
<td>-0.031</td>
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<td>(3.98)</td>
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<tr>
<td>(\overline{GPAT}_t)</td>
<td>-1.016</td>
<td>-1.673**</td>
<td>-1.518</td>
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<td>(-1.27)</td>
<td>(-2.09)</td>
<td>(-1.51)</td>
<td></td>
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</tbody>
</table>

| R-squared | 0.240 | 0.228 | 0.240 | 0.238 | 0.228 | 0.238 | 0.241 |
Table 6 – continued

This table reports the coefficient estimates and t-statistics (in parentheses) of GCT-regression models with multivariate and continuous subject characteristics in vector \( z \). The GCT-regression models \( y_{ht} = (z_{ht} \otimes x_t) \theta + v_{ht} \) in Panel A rely on \( x_t = [1] \) and, hence, decompose excess returns. In Panel B, vector \( x_t \) is specified as \( x_t = [1 \ (R_{mt} - R_{ft}) \ SMB_t \ HML_t] \) such that the GCT-regression models decompose the risk-adjusted performance from a Fama-French three factor model. The table only displays the results for the coefficient estimates in the \( z \)-vector, i.e., the decomposition of the excess returns or the three-factor alpha. The results for the market (RMRF), SMB (small minus big), and HML (high minus low) factor exposure decompositions are not shown in Panel B of the table. \( \overline{OPAT}_h \) is firm \( h \)'s average operating profitability (defined as gross profit minus selling, general, and administrative expenses (excluding research and development expenditures) deflated by the book value of total assets) over time. \( \overline{OPAT}_t \) refers to the time \( t \) (value-weighted) cross-sectional average operating profitability of all \( N_t \) firms in the sample. \( \overline{GPAT}_h \) is firm \( h \)'s time-series average of gross profit deflated by the book value of total assets measured over all \( N_t \) firms in the sample. The sample period is from July 1963 through December 2013. All GCT-regressions are estimated with weighted least squares (WLS) with observation weights set proportional to monthly value weights. All t-statistics test for significance against a value of zero. Statistical inference is based on Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for the Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
### Table 7: Long-term event study – Single subject groups

<table>
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<tr>
<th>Method</th>
<th>Portfolio sorts approach</th>
<th>CAPM</th>
<th>GCT-regression model</th>
<th>Fama-French 3-factor</th>
<th>GCT-regression model</th>
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</thead>
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<tr>
<td></td>
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<td>(4)</td>
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</tr>
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<td></td>
<td></td>
<td>1.097</td>
<td>1.097</td>
<td>1.089</td>
<td>1.090*</td>
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<tr>
<td></td>
<td></td>
<td>(0.177)</td>
<td>(0.177)</td>
<td>(0.209)</td>
<td>(0.060)</td>
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<tr>
<td>SMB</td>
<td></td>
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<td>0.447***</td>
<td>0.447***</td>
<td>0.447***</td>
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<td>(0.000)</td>
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<td>(0.488)</td>
<td>(0.492)</td>
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<td>-0.181</td>
<td>-0.178</td>
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<td>(0.311)</td>
<td>(0.311)</td>
<td>(0.323)</td>
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<tr>
<td></td>
<td></td>
<td>-0.263**</td>
<td>-0.263**</td>
<td>-0.261**</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.029)</td>
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<tr>
<td>R-squared</td>
<td></td>
<td>0.824</td>
<td>0.089</td>
<td>0.087</td>
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<tr>
<td>N Investors</td>
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<td></td>
<td>66,458</td>
<td>66,458</td>
<td></td>
</tr>
<tr>
<td>N Obs.</td>
<td></td>
<td>72</td>
<td>3,770,842</td>
<td>3,770,842</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the market model alpha and RMRF (market) factor loading as well as the Fama-French 3-factor model alpha along with RMRF (market), SMB (small minus big), and HML (high minus low) factor loadings for the net return of a portfolio comprising all investors in the sample of BO (2000). The sample includes 66,465 individual investors at a large discount brokerage firm and covers the period 1991-1996. Columns (1) and (4) report our replication of the results reported in Panel D of Table II of BO (2000) based on the portfolio sorts approach and the market model (1) or the Fama-French 3-factor model (4), respectively. Columns (2) and (5) present the results from estimating GCT-regression models with weighted least squares (WLS) where observation weights are set equal to $w_{ht} = N_t^{-1}$. In Columns (3) and (6), the GCT-regression model is estimated with OLS resulting in equal weights. As in BO (2000), the null hypothesis for the coefficient estimate on the market excess return is $H_0: \beta=1$. All other t-statistics test for significance against a value of zero. Statistical inference for the portfolio sorts approach is based on Newey and West (1987) standard errors. GCT-regression models are estimated with Driscoll and Kraay (1998) standard errors. The lag-length choice for both Newey-West and Driscoll-Kraay standard errors is set to zero. p-values are presented in parentheses. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 8: Long-term event study – Portfolio sorts

Panel A: Mean monthly portfolio turnover (%)

<table>
<thead>
<tr>
<th>Quintile</th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replication of BO</td>
<td>0.19</td>
<td>1.24</td>
<td>2.89</td>
<td>5.98</td>
<td>21.49</td>
</tr>
</tbody>
</table>

Panel B: Portfolio Sorts Approach

<table>
<thead>
<tr>
<th>Quintile</th>
<th>1 (low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM intercept</td>
<td>0.078</td>
<td>-0.037</td>
<td>-0.144</td>
<td>-0.322</td>
<td>-0.705*</td>
</tr>
<tr>
<td>(0.394)</td>
<td>(0.743)</td>
<td>(0.420)</td>
<td>(0.206)</td>
<td>(0.052)</td>
<td></td>
</tr>
<tr>
<td>(R_{mt} - R_f)</td>
<td>0.982***</td>
<td>1.035***</td>
<td>1.090***</td>
<td>1.171***</td>
<td>1.293***</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
</tbody>
</table>

Panel C: GCT-regression Model

| Vector z_t (subject variables) → z_{ht}^{(1)} | z_{ht}^{(2)} | z_{ht}^{(3)} | z_{ht}^{(4)} | z_{ht}^{(5)} |
|----------|---------|---|---|---|----------|
| Intercept | 0.078 | -0.037 | -0.144 | -0.322 | -0.705* |
| (0.394) | (0.743) | (0.420) | (0.206) | (0.052) |
| (R_{mt} - R_f) | 0.982*** | 1.035*** | 1.090*** | 1.171*** | 1.293*** |
| (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |

Panel A of this table reports the average household’s stock turnover across five quintile portfolios sorted on the private households’ monthly portfolio turnover. Households with the lowest (highest) portfolio turnover are allocated to quintile portfolio “1 (low)” (quintile portfolio “5 (high)”). Panel B reports the coefficient estimates and p-values (in parentheses) from estimating a time-series regression of the quintile portfolios’ excess return on a constant and the market excess return (R_{mt} - R_f). Statistical inference for this portfolio sorts approach is based on Newey and West (1987) standard errors. Panel C reports the coefficient estimates and p-values (in parentheses) from estimating a GCT-regression model with Driscoll-Kraay (1998) standard errors. Coefficient estimates for the GCT-regression model are for the product of the subject-level variable of vector z_{ht} and the elements of vector x_t. The lag-length choice for both Newey-West as well as Driscoll-Kraay standard errors is zero. *** , **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 9: Long-term event study – Performance difference between two groups

**Panel A: Portfolio Sorts Approach**

<table>
<thead>
<tr>
<th></th>
<th>Quantile 1 (low turnover group)</th>
<th>Difference: Quintile 5 – Quintile 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replication, intercept</td>
<td>0.078</td>
<td>-0.784**</td>
</tr>
<tr>
<td></td>
<td>(0.394)</td>
<td>(0.010)</td>
</tr>
</tbody>
</table>

**Panel B: GCT-regression model**

|                        | 1                               | $z^{(Q5)}_{ht}$ |
|------------------------|                                 |                 |
| Vector $z_t$ (subject variables) → |                                 |                 |
| Vector $x_t$ (factor variables) ↓ |                                 | $z^{(Q5)}_{ht}$ |
| Intercept              | 0.078                           | -0.784**        |
|                        | (0.394)                         | (0.010)         |
| $(R_{mt} - R_{ft})$     | 0.982***                       | 0.310***        |
|                        | (0.000)                         | (0.001)         |

*R-squared* 0.075  
*N Obs.* 1,404,536

This table reports market model alphas along with RMRF (market) factor loadings for the quintile portfolio including investors with the lowest turnover and the portfolio difference between the quintile group of investors with the highest portfolio turnover and the quintile group of investors with the lowest portfolio turnover. Panel A contains our replication of the results reported in Panel C of Table V of BO (2000, p. 793) from the *portfolio sorts* approach. Panel B reports the results from a *GCT-regression* model. All *t*-statistics test for significance against a value of zero. Statistical inference for the *portfolio sorts* approach is based on Newey and West (1987) standard errors. The *GCT-regression* models are estimated with Driscoll and Kraay (1998) standard errors. The lag-length choice for both Newey-West and Driscoll-Kraay standard errors is zero. Coefficient estimates for the *GCT-regression* models are for the interaction of the subject-level variable of the $z$-vector and the market-level variable of the $x$-vector. *p*-values are presented in parentheses. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Table 10: Long-term event study – Continuous and multivariate subject characteristics

<table>
<thead>
<tr>
<th>Vector $z_t$ (subject variables)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0271</td>
<td>-0.164</td>
<td>-0.0211</td>
</tr>
<tr>
<td></td>
<td>(0.834)</td>
<td>(0.363)</td>
<td>(0.871)</td>
</tr>
<tr>
<td>TO$_h$</td>
<td>-3.267**</td>
<td>-3.192**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>Online$_h$</td>
<td>-0.381**</td>
<td>-0.248*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.092)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.088</td>
<td>0.087</td>
<td>0.088</td>
</tr>
<tr>
<td>$N$ Obs.</td>
<td>3,770,842</td>
<td>3,770,842</td>
<td>3,770,842</td>
</tr>
</tbody>
</table>

This table reports the coefficient estimates and $p$-values (in parentheses) of GCT-regression models with the individual households’ net returns (after transaction costs) as the dependent variable and with multivariate and continuous subject characteristics in vector $z$. The table only shows the results for the variables in the $z$-vector, i.e., the decomposition of the risk-adjusted performance (or market model alpha). The results for the market (RMRF) factor exposure decompositions are not shown in the table. Apart from a constant, vector $z$ contains the following variables: TO$_h$ in Column 1, Online$_h$ in Column 2, and both TO$_h$ and Online$_h$ in Column 3. All GCT-regressions are estimated with OLS. All $t$-statistics test for significance against a value of zero. Statistical inference is based on Driscoll and Kraay (1998) standard errors that are robust w.r.t. cross-sectional dependence and heteroscedasticity. The lag-length choice for the Driscoll-Kraay standard errors is fixed at zero, i.e., the regression disturbances are assumed to be uncorrelated over time. ***, **, and * indicate significance at the 1, 5, and 10 percent levels (two-tailed).
Appendix A: Proof of Propositions 1 to 3

In this appendix, we first set the ground by rewriting both the GCT-regression model and the portfolio sorts approach in matrix notation. We then perform a series of basic transformations that apply to all three propositions stated in Section 2. Finally, we proof Propositions 1 to 3 mathematically. For ease of mathematical tractability and as outlined in Section 2, we thereby restrict our formal analysis to the case of a balanced panel (⁴⁄ substec with ⁴⁄ regularly spaced observations), time-constant subject characteristics (i.e.,  \( z_{ht} \equiv z_h \)), and equally weighted portfolios (i.e.,  \( w_{ht} = 1/N \)). Under these simplifying assumptions, the GCT-regression model can reproduce the results of the portfolio sorts approach by aid of ordinary least squares (OLS) rather than weighted least squares (WLS) estimation.

A.1 Matrix notation and proof of Proposition 1

A.1.1 Coefficient estimates and standard errors for the GCT-regression model

Applying matrix notation, we can write the GCT-regression model in (1) as

\[
\begin{bmatrix}
  y_{11} \\
  y_{12} \\
  \vdots \\
  y_{1T} \\
  y_{21} \\
  \vdots \\
  y_{NT}
\end{bmatrix} = \left( \begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_N
\end{bmatrix} \otimes \begin{bmatrix}
  x_1 \\
  \vdots \\
  x_T
\end{bmatrix} \right) \theta + \begin{bmatrix}
  v_{11} \\
  v_{12} \\
  \vdots \\
  v_{1T} \\
  v_{21} \\
  \vdots \\
  v_{NT}
\end{bmatrix}
\]

or, more briefly:

\[
\text{vec}(Y) = \begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_N
\end{bmatrix} = (Z \otimes X) \theta + \nu
\]

Here,  \( z_h \) is a  \((1 \times M)\)-vector of subject characteristics (which are assumed to remain constant over time),  \( x_t \) refers to a  \((1 \times (K + 1))\)-vector of market-level variables (which apart from the constant change over time but do not vary in the cross-section), and  \((Z \otimes X)\) denotes the Kronecker product of  \((N \times M)\)-dimensional matrix  \( Z = [z_1 \ldots z_N]' \) with  \((T \times (K + 1))\)-dimensional matrix  \( X = [x_1' \ldots x_T']' \). Estimating regression model (A2) with ordinary least squares (OLS) and applying the calculus rules for the Kronecker product yields the following coefficient estimates for  \( \theta \):

\[
\hat{\theta} = \left( (Z \otimes X)'(Z \otimes X) \right)^{-1}(Z \otimes X)'\text{vec}(Y)
\]
\[
= (Z'Z\otimes X'X)^{-1}(Z'X)\text{vec}(Y)
= ((Z'Z)^{-1}Z'\otimes (X'X)^{-1}X')\text{vec}(Y)
\]

Next, we use the following Lemma from linear algebra (e.g., see Sydsaeter, Strom, and Berck, 2000, p. 146):

**Lemma 1.** For any three matrices \( A \in \mathbb{R}^{r \times r}, B \in \mathbb{R}^{r \times s}, \text{ and } C \in \mathbb{R}^{s \times s} \) it holds true that \( \text{vec}(ABC) = (C' \otimes A)\text{vec}(B) \).

Applying Lemma 1 to expression (A3) above yields

\[
\tilde{\theta} = (X'X)^{-1}X'YZ(Z'Z)^{-1}
\]  

(A4)

Here, \( \tilde{\theta} \) refers to the \((K + 1) \times M\)-dimensional matrix of coefficient estimates \( \tilde{\theta}_{k,m} \) for the interaction of subject variable \( m \) (with \( m = 1, \ldots, M \)) from vector \( z_h \) and market-level (factor) variable \( k \) (with \( k = 0, \ldots, K \)) in vector \( x_t \).

We now turn to the Driscoll and Kraay (1998) covariance matrix estimator for the pooled OLS regression model in (A1). For \( H \) lags, it has the following structure:

\[
\bar{V}(\hat{\theta}) = ((Z\otimes X)'(Z\otimes X))^{-1}\tilde{S}_T((Z\otimes X)'(Z\otimes X))^{-1}
= ((Z'Z)^{-1}\otimes (X'X)^{-1})\tilde{S}_T ((Z'Z)^{-1}\otimes (X'X)^{-1})
\]  

(A5)

with

\[
\tilde{S}_T = \tilde{\Omega}_0 + \sum_{j=1}^{H} \omega_{j,H} (\tilde{\Omega}_j + \tilde{\Omega}_j') , \quad \tilde{\Omega}_j = \sum_{T=j+1}^{T} h_T(\hat{\theta}) h_T' - j(\hat{\theta}) ,
\]

\[
h_T(\hat{\theta}) = (Z\otimes x_T)' \hat{\nu} , \quad \text{and} \quad x_T = [1 \quad x_{1T} \ldots x_{kT}]
\]

Here, \( x_T = [1 \quad x_{1T} \ldots x_{kT}] \) is a \((K + 1)\)-dimensional row vector. The modified Bartlett weights \( \omega_{j,H} = 1 - j/(H + 1) \) ensure positive semi-definiteness of \( \tilde{S}_T \) and smooth the sample autocovariance function such that higher order lags receive less weight.

### A.1.2 Coefficient estimates and standard errors for the portfolio sorts approach

The *portfolio sorts* approach comprises two steps. First, one computes the month \( t \) average return for subject group \( p \) as outlined in Equation (5). In our case of a balanced panel, time-constant subject characteristics, and equally weighted portfolios (i.e., \( w_{ht} = N_p^{-1} = (\sum_{h=1}^{N} z_h^{(p)})^{-1} \)), we can rewrite Equation (5) as follows:
\[ y_{pt} = \frac{1}{N_{p}} \sum_{h=1}^{N} z_h^{(p)} y_{ht} = y_t' d_p (d_p' d_p)^{-1} \]  
(A6)

with \( y_t' = [y_{1t} \ y_{2t} \ \ldots \ y_{Nt}] \) and \( d_p' = [z_1^{(p)} \ z_2^{(p)} \ \ldots \ z_N^{(p)}] \).

In the second step of the procedure, \( y_{pt} \) from (A6) is regressed on a constant and the \( K \) factor variables as outlined in Equation (6) which yields OLS coefficient estimates as follows:

\[
\hat{\beta}_p = (X'X)^{-1} X'[y_{1t} \ y_{2t} \ \ldots \ y_{Nt}]d_p(d_p' d_p)^{-1} = (X'X)^{-1} X'[y_{1t} \ y_{2t} \ \ldots \ y_{Nt}]d_p(d_p' d_p)^{-1} 
\]

(A7)

The formula for computing the Newey and West (1987) covariance matrix with lag length \( H \) for the regression model in Equation (6) has the following structure:

\[
\tilde{V}(\hat{\beta}_p) = (X'X)^{-1} S_T (X'X)^{-1}
\]

with

\[
S_T = \sum_{t=1}^{T} \tilde{e}_{pt}^2 x'_t x_t + \sum_{j=1}^{H} \omega_{j,H} \sum_{t=j+1}^{T} \left( \tilde{e}_{pr} \tilde{e}_{pr-j}(x_t' x_{t-j} + x_{t-j}' x_t) \right) 
\]

(A8)

where \( x_t \) and \( \omega_{j,H} \) match their definitions in Equation (A5) above.

Restricting the sample to two portfolios (or, subject groups), arbitrarily denoted as \( p = \text{"high"} \) and \( p = \text{"low"} \), we can use matrix notation to rewrite Equation (10) computing the month \( t \) return difference as follows:

\[
\Delta y_t = y_{high,t} - y_{low,t} = y_t' Z(Z'Z)^{-1} e_2 = [y_{low,t} \ \Delta y_t]' [0 \ \ 1]
\]

(A9)

where matrix \( Z \) is specified as \( Z = [t \ \ d_{high}] \) and \( t \) is a \((N \times 1)\)-dimensional vector of ones. When regressing \( \Delta y_t \) from (A9) on a constant and the \( K \) factor variables according to Equation (11a), one obtains the following OLS coefficient estimates:

\[
\hat{\beta}_\Delta = (X'X)^{-1} X'[\Delta y_1 \ \Delta y_T]' = (X'X)^{-1} X' Y Z (Z'Z)^{-1} e_2
\]

(A10)
The Newey and West (1987) covariance matrix estimator for the coefficient estimates in (A10) has the same structure as the one displayed in Equation (A8), with \( \hat{\epsilon}_t \) replacing \( \hat{\epsilon}_{ht} \) in the formula.

A.1.3 Proof of Proposition 1

Proposition 1 states that the GCT-regression model can reproduce the results of the portfolio sorts approach for the case of a single subject group if vector \( z_h \equiv z_{ht} \) is specified as \( z_h = [1] \). In this case, matrix \( Z \) in Equation (A2) is given as \( Z = t \). As a result, the coefficient estimates of the GCT-regression model in this case are

\[
\hat{\theta} = (X'X)^{-1}X'Y_i(t')^{-1}
\]  

(A11)

When there is only a single subject group, then \( z_h^{(p)} \) for all subjects \( h \) is equal to 1, i.e., \( Z_p = t \). Consequently, the coefficient estimates for the portfolio sorts approach in Equation (A7) are equal to

\[
\hat{\beta} = (X'X)^{-1}X'Y_i(t')^{-1}
\]  

(A12)

As stated in Part A of Proposition 1, we thus have \( \hat{\theta} \equiv \hat{\beta} \). This completes the first part of the proof.

We next turn to the standard errors for the coefficient estimates. With \( y_t = N^{-1}\sum_{h=1}^{N}y_{ht} \) and \( \theta = \beta \), the (estimated) residual \( \hat{\epsilon}_t \) in Equation (3) is equal to

\[
\hat{\epsilon}_t = N^{-1}\sum_{h=1}^{N}\hat{\epsilon}_{ht} \equiv N^{-1}V_t
\]  

(A13)

where \( \hat{\epsilon}_{ht} \) is the (estimated) residual from pooled OLS regression (4). Replacing \( \hat{\epsilon}_t \) in (A8) by the corresponding term from (A13) yields

\[
N^2 \hat{S}_T = \sum_{t=1}^{T}2V_t^2x_t'x_t + \sum_{j=1}^{T}\sum_{j+1}^{T}V_tV_{t-j}(x_t'x_{t-j} + x_{t-j}'x_t)
\]  

(A14)

In case of the GCT-regression model, we plug in \( Z = t \) in Equation (A5). This gives

\[
\hat{V}(\hat{\theta}) = ((t')^{-1} \otimes (X'X)^{-1}) \hat{S}_T ((t')^{-1} \otimes (X'X)^{-1}) = (X'X)^{-1} \frac{\hat{S}_T}{N^2} (X'X)^{-1}
\]  

(A15)

Comparing \( \hat{V}(\hat{\theta}) \) in (A15) with \( \hat{V}(\hat{\beta}) \) from (A8) in case of a single subject group, we hence have to show that \( N^{-2}\hat{S}_T = \hat{S}_T \). Rewriting \( h_t(\hat{\theta}) \) in (A5), we obtain:
\[ h_t(\bar{\theta}) = (\otimes x_t)' \bar{\psi} = [x_t' \quad \cdots \quad x_t'] \bar{\psi} = x_t' V_t \quad (A16) \]

From (A16) it follows for \( \hat{\Omega}_j \) in (A5) that
\[
\hat{\Omega}_j = \sum_{t=j+1}^{T} h_t(\bar{\theta}) h_t'(\bar{\theta}) = \sum_{t=j+1}^{T} V_t V_t' x_t x_{t-j},
\]
and consequently
\[
\tilde{S}_T = \sum_{t=1}^{T} V_t^2 x_t x_t + \sum_{j=1}^{H} \omega_{jH} \sum_{t=j+1}^{T} \left( V_t V_t' x_t x_{t-j} + x_t' x_{t-j} \right) \equiv N^2 \tilde{S}_T \quad (A17)
\]
This completes the proof. □

A.2 Proof of Proposition 2

Proposition 2 states that the GCT-regression model can reproduce the results of the portfolio sorts approach for the case of multiple portfolio sorts if vector \( z_h \) is specified as
\[
z_h = \left[ z_h^{(1)} \quad z_h^{(2)} \quad \cdots \quad z_h^{(P)} \right].
\]
Using the definition of \( d_p \) in (A6), \((N \times P)\)-matrix \( Z \) in Equation (A2) is given as follows:
\[
Z = \begin{bmatrix} Z_1 \\ \vdots \\ Z_N \end{bmatrix} = \begin{bmatrix} Z_1^{(1)} & \cdots & Z_1^{(P)} \\ \vdots & \ddots & \vdots \\ Z_N^{(1)} & \cdots & Z_N^{(P)} \end{bmatrix} = [d_1 \quad \cdots \quad d_p] \quad (A18)
\]
The coefficient estimates for the GCT-regression model are derived in (A4). Specifying matrix \( Z \) according to Expression (A18) thus results in a \((K+1) \times P\)-dimensional matrix of coefficient estimates \( \bar{\theta} \). The \( p \)-th column of results matrix \( \bar{\theta} \) can be obtained as follows:
\[
\bar{\theta}_p = \bar{\theta} e_p = (X'X)^{-1}X'YZ(Z'Z)^{-1} e_p \quad (A19)
\]
where \( e_p \) is a \( P \)-dimensional vector of zeroes with a 1 on position \( p \). Proposition 2 claims that \( \bar{\theta}_p \equiv \hat{\theta}_p \) where \( \hat{\theta}_p \) refers to the portfolio \( p \) coefficient estimates from estimating the second-step time-series regression of the portfolio sorts approach. The respective coefficient estimates have been derived in Equation (A7). We thus have to proof that
\[
\bar{\theta}_p \equiv \hat{\theta}_p = (X'X)^{-1}X'Y d_p \quad (d'_p d_p)^{-1} \quad (A20)
\]

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To show that (A20) indeed constitutes an identity, we first note that \( \mathbf{d}_p = \mathbf{Z} \mathbf{e}_p \). Multiplying both sides in (A20) with \( \mathbf{d}'_p \) from the left yields
\[
d'_p \mathbf{Z} (\mathbf{Z}' \mathbf{Z})^{-1} \mathbf{e}_p = \mathbf{e}'_p \mathbf{Z}' (\mathbf{Z}' \mathbf{Z})^{-1} \mathbf{e}_p = 1
\]

\[\equiv d'_p d_p (d'_p d_p)^{-1} = 1\] 

(A21)

This shows that \( \mathbf{\theta}_p \equiv \mathbf{\hat{\beta}}_p \) indeed holds true and, hence, completes the first part of the proof of Proposition 2. \( \blacksquare \)

Next, we turn to the standard errors for the coefficient estimates. With \( y_{pt} = N_p^{-1} \sum_{t=1}^{T} z_{ht} y_{ht} \) and \( \mathbf{\hat{\theta}}_p = \mathbf{\hat{\beta}}_p \), the (estimated) residual \( \hat{\varepsilon}_{pt} \) for the portfolio sorts approach in Equation (6) is equal to
\[
\hat{\varepsilon}_{pt} = N_p^{-1} \sum_{h=1}^{H} \sum_{j=1}^{T} \alpha_{j,ht} \sum_{\tau=j+1}^{T} (V_{pt} V_{pt,\tau-j} (x'_{t-j} x_{t-j} + x'_{t-j} x_{t-j})) \equiv \hat{S}_{T}^{(p)}
\]

(A22)

where \( \hat{\varepsilon}_{ht} \) is the (estimated) residual from pooled OLS regression (7). Replacing \( \hat{\varepsilon}_{pt} \) in (A8) by the corresponding term from (A22) yields
\[
N_p^2 \overset{\circ}{S}_{T} = \sum_{\tau=1}^{T} V_{pt}^2 x'_{t} x_{t} + \sum_{j=1}^{H} \sum_{\tau=j+1}^{T} (V_{pt} V_{pt,\tau-j} (x'_{t-j} x_{t-j} + x'_{t-j} x_{t-j})) \equiv \hat{S}_{T}^{(p)}
\]

(A23)

As a consequence, we finally obtain the Newey and West (1987) standard errors in case of the portfolio sorts approach as follows:
\[
\mathbf{V} (\mathbf{\hat{\beta}}_p) = N_p^{-2} (X' X)^{-1} \hat{S}_{T}^{(p)} (X' X)^{-1}
\]

(A24)

We now consider the GCT-regression model with matrix \( \mathbf{Z} \) being defined according to Equation (A18). The \((P \times (K + 1))\)-dimensional column vector \( \mathbf{h}_t (\mathbf{\hat{\theta}}) \) from (A5) in this case is equal to
\[
\mathbf{h}_t (\mathbf{\hat{\theta}}) = (\mathbf{Z} \otimes \mathbf{x}'_t) \mathbf{\hat{\theta}} = \begin{bmatrix}
\sum_{h=1}^{N} z_{h}^{(1)} (1) \hat{\theta}_{ht} \\
x_{1t} \sum_{h=1}^{N} z_{h}^{(1)} \hat{\theta}_{ht} \\
\vdots \\
x_{Kt} \sum_{h=1}^{N} z_{h}^{(1)} \hat{\theta}_{ht}
\end{bmatrix} = \begin{bmatrix}
V_{1t} \\
x_{1t} V_{1t} \\
\vdots \\
x_{Kt} V_{1t}
\end{bmatrix} \equiv (I_P \otimes \mathbf{x}'_t) \mathbf{V}_t
\]

(A25)

With (A25) it follows for \( \mathbf{\hat{\Delta}}_j \) in Equation (A5) that
\[
\hat{\Omega}_j = \sum_{\tau=j+1}^{T} (I_p \otimes x'_{\tau}) V_{\tau} V'_{\tau-j} (I_p \otimes x_{\tau-j}) = \sum_{\tau=j+1}^{T} (V_{\tau} V'_{\tau-j}) \otimes (x'_{\tau} x_{\tau-j})
\]  

(A26)

As a result, matrix \( \bar{S}_T \) in (A5) can be written as follows:

\[
\bar{S}_T = \sum_{\tau=1}^{T} (V_{\tau} V'_{\tau}) \otimes (x'_{\tau} x_{\tau}) + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \left( (V_{\tau} V'_{\tau-j}) \otimes (x'_{\tau-j} x_{\tau-j}) + (V_{\tau-j} V'_{\tau}) \otimes (x'_{\tau-j} x_{\tau}) \right)
\]  

(A27)

We next define \( \bar{S}_T^{(p,q)} \) as follows

\[
\bar{S}_T^{(p,q)} \equiv \sum_{\tau=1}^{T} V_{p\tau} V_{q\tau} (x'_{\tau} x_{\tau}) + \sum_{j=1}^{H} \omega_{j,H} \sum_{\tau=j+1}^{T} \left( V_{p\tau} V_{q,\tau-j} (x'_{\tau-j} x_{\tau-j}) + V_{p,\tau-j} V_{q\tau} (x'_{\tau-j} x_{\tau}) \right)
\]  

(A28)

where \( V_{p\tau} \) is a scalar as defined in Expression (A22) above. Consequently, matrix \( \bar{S}_T \) is equal to

\[
\bar{S}_T = \begin{bmatrix}
\bar{S}_T^{(1,1)} & \ldots & \bar{S}_T^{(1,p)} \\
\vdots & \ddots & \vdots \\
\bar{S}_T^{(p,1)} & \ldots & \bar{S}_T^{(p,p)}
\end{bmatrix}
\]  

(A29)

Moreover, with matrix \( Z \) being defined according to (A18), \( Z'Z \) now is a \((P \times P)\)-dimensional diagonal matrix with element \((p,p)\) equal to \( N_p \) and all off-diagonal elements equal to zero. With \( \bar{S}_T \) structured according to Expression (A29), we can thus rewrite \( \bar{V}(\hat{\theta}) \) in (A5) as follows:

\[
\bar{V}(\hat{\theta}) = \begin{bmatrix}
N_1^{-2} (X'X)^{-1} \bar{S}_T^{(1,1)} (X'X)^{-1} & \ldots & N_1^{-1} N_p^{-1} (X'X)^{-1} \bar{S}_T^{(1,p)} (X'X)^{-1} \\
\vdots & \ddots & \vdots \\
N_1^{-1} N_p^{-1} (X'X)^{-1} \bar{S}_T^{(p,1)} (X'X)^{-1} & \ldots & N_p^{-2} (X'X)^{-1} \bar{S}_T^{(p,p)} (X'X)^{-1}
\end{bmatrix}
\]  

(A30)

The second part of Proposition 2 claims that \( \text{SE}(\hat{\theta}_{p,k}) = \text{SE}(\hat{\theta}_{p,k}) \) for \( k = 0, 1, \ldots, K \) and \( p = 1, \ldots, P \). To prove that this holds true, it is sufficient to show that \( N_p^{-2} (X'X)^{-1} \bar{S}_T^{(p,p)} (X'X)^{-1} \) in Expression (A30) is identical to \( \bar{V}(\hat{\theta}_p) \) in Equation (A24) for every \( p = 1, \ldots, P \). This in turn is equivalent to demonstrating that \( \bar{S}_T^{(p,p)} \) from (A23) coincides with \( \bar{S}_T^{(p,p)} \) in (A28). By comparing the two expressions we see that this in fact holds true, which completes the proof. \( \blacksquare \)
A.3 Proof of Proposition 3

Proposition 3 states that the GCT-regression model can reproduce the results of the portfolio sorts approach for the case of performance differences between two portfolios. In order to compare the performance of subject group “high” with that of subject group “low”, we have to specify vector $z_h$ as $z_h = [1 \ z_h^{(\text{high})}]$ such that the $(N \times 2)$-dimensional matrix $Z$ comprising the characteristics of all $N$ subjects is equal to $Z = [t \ d_{\text{high}}]$. This matches the definition of matrix $Z$ in Expression (A9) which is used for deriving the results of performance differences in case of the portfolio sorts approach. Estimating the GCT-regression model in (A2) with OLS yields the coefficient estimates, structured as a $((K + 1) \times 2)$-dimensional matrix, in Equation (A4).

The second column of $ar{\theta}$ in (A4) contains the coefficient estimates for the interaction terms between dummy variable $z_h^{(\text{high})}$ and the factor variables in vector $x_t$. In Equation (12), those coefficient estimates are named $\hat{\theta}_{\Delta k}$. To extract the coefficient estimates for $\hat{\theta}_{\Delta k}$ from $\bar{\theta}$, we post-multiply Expression (A4) with $e_2$. The resulting term coincides with the one in Equation (A10) for the portfolio sorts approach. This shows that $\hat{\beta}_{\Delta k} \equiv \hat{\theta}_{\Delta k}$ ($\forall k = 0, 1, \ldots, K$).

The first column of $ar{\theta}$ in (A4) comprises the coefficient estimates for subject group “low”. In Equation (12), the respective coefficient estimates are labeled as $\hat{\theta}_{\text{low},k}$ (with $k = 0, 1, \ldots, K$). The coefficient estimates for $\theta_{\text{low},k}$ are retrieved by post-multiplying (A4) with $e_1$. In case of the portfolio sorts approach, we obtain the coefficient estimates for subject group “low” ($\hat{\beta}_{\text{low},k}$) by repeating the analysis of (A9) and (A10) with $e_1$ replacing $e_2$. The resulting expressions for the portfolio sorts approach and the GCT-regression model again coincide. This demonstrates $\hat{\beta}_{\text{low},k} \equiv \hat{\theta}_{\text{low},k}$ ($\forall k = 0, 1, \ldots, K$) and, hence, completes the first part of the proof. □

For the second part of the proof, we note that due to $y_{\text{high},t} = y_{\text{low},t} + \Delta y_t$ the following corollary holds true:

Corollary 1. \[ \hat{\beta}_{\text{high},k} = \hat{\beta}_{\text{low},k} + \hat{\beta}_{\Delta k} = \hat{\theta}_{\text{low},k} + \hat{\theta}_{\Delta k} \quad \text{for all } k = 0, 1, \ldots, K. \] (A31)

Based on Corollary 1 and Proposition 2 and because $y_{pt} = N_p^{-1} \sum_{h=1}^N z_{h}^{P} y_{ht}$ (for $p = \text{“low”}$, “high”), residual $\hat{\varepsilon}_{\Delta t}$ in the second-step time-series regression (11a) of the portfolio sorts approach is equal to

\[ \hat{\varepsilon}_{\Delta t} = N_{\text{high}}^{-1} \sum_{h=1}^N z_{h}^{(\text{high})} \hat{\varphi}_{ht} - N_{\text{low}}^{-1} \sum_{h=1}^N z_{h}^{(\text{low})} \hat{\varphi}_{ht} \equiv N_{\text{high}}^{-1} V_{\text{high},t} - N_{\text{low}}^{-1} V_{\text{low},t} \] (A32)

where $\hat{\varphi}_{ht}$ is the (estimated) residual from pooled OLS regression (7). Replacing $\hat{\varepsilon}_{\Delta t}$ in the Newey and West (1987) covariance matrix estimator for the coefficient estimates in (A10) yields
\[
\hat{S}_T = \sum_{t=1}^{T} \left( \frac{V_{\text{high},t}}{N_{\text{high}}} - \frac{V_{\text{low},t}}{N_{\text{low}}} \right)^2 x'_t x_t
\]

\[
+ \sum_{j=1}^{H} \omega_j \sum_{\tau=1}^{T} \left( \frac{V_{\text{high},\tau}}{N_{\text{high}}} - \frac{V_{\text{low},\tau}}{N_{\text{low}}} \right) \left( x'_t x_{\tau-j} + x'_{t-j} x_{\tau} \right)
\]

We now turn to the GCT-regression model with matrix \( Z \) being defined as \( Z = [t \ d_{\text{high}}] \). The \((2 \times (K + 1))\)-dimensional column vector \( h_t(\tilde{\theta}) \) from (A5) in this case is equal to

\[
h_t(\tilde{\theta}) = (Z' \otimes x'_t) \tilde{v} = \begin{bmatrix}
\sum_{h=1}^{N} \tilde{\theta}_h x'_t x_h \\
x'_t \sum_{h=1}^{N} \tilde{\theta}_h \\
\vdots \\
\sum_{h=1}^{N} z'(h) \tilde{\theta}_h x'_t \\
x'_t \sum_{h=1}^{N} z(h) \tilde{\theta}_h \\
\vdots \\
x'_t \sum_{h=1}^{N} z'(h) \tilde{\theta}_h
\end{bmatrix} = \begin{bmatrix}
x'_t & x'_t & 0' \\
x'_t & x'_t & 0'
\end{bmatrix} V_t
\]

(A34)

From (A34) and \( V_{\text{high},\tau} + V_{\text{low},\tau} \equiv V_t \) it follows for \( \tilde{\Omega}_j \) in Equation (A5) that

\[
\tilde{\Omega}_j = \sum_{\tau=j+1}^{T} \begin{bmatrix}
V_t V_{\tau-j} x'_t x'_{\tau-j} & V_t V_{\text{high},\tau-j} x'_t x_{\tau-j} \\
V_t V_{\text{high},\tau-j} x'_t x_{\tau-j} & V_{\text{high},\tau} V_{\text{high},\tau-j} x'_t x_{\tau-j}
\end{bmatrix}
\]

(A35)

As a result, matrix \( \tilde{S}_T \) in (A5) can be written in block form as follows:

\[
\tilde{S}_T = \begin{bmatrix}
\tilde{S}_T^{(1,1)} & \tilde{S}_T^{(1,2)} \\
\tilde{S}_T^{(2,1)} & \tilde{S}_T^{(2,2)}
\end{bmatrix}
\]

(A36)

where

\[
\tilde{S}_T^{(1,1)} = \sum_{t=1}^{T} V_t^2 (x'_t x_t) + \sum_{j=1}^{H} \omega_j \sum_{\tau=j+1}^{T} \left( V_t V_{\tau-j} (x'_t x_{\tau-j} + x'_{t-j} x_{\tau}) \right)
\]

\[
\tilde{S}_T^{(1,2)} = \sum_{t=1}^{T} V_t V_{\text{high},t} x'_t x_t + \sum_{j=1}^{H} \omega_j \sum_{\tau=j+1}^{T} \left( V_t V_{\text{high},\tau-j} x'_t x_{\tau-j} + V_{\text{high},\tau} V_{\text{high},\tau-j} x'_t x_{\tau-j} \right)
\]

\[
\tilde{S}_T^{(2,1)} = \sum_{t=1}^{T} V_t V_{\text{high},t} x'_t x_t + \sum_{j=1}^{H} \omega_j \sum_{\tau=j+1}^{T} \left( V_{\text{high},t} V_{\text{high},\tau-j} x'_t x_{\tau-j} + V_{\text{high},\tau} V_{\text{high},\tau-j} x'_t x_{\tau-j} \right)
\]

\[
\tilde{S}_T^{(2,2)} = \sum_{t=1}^{T} V_{\text{high},t}^2 (x'_t x_t) + \sum_{j=1}^{H} \omega_j \sum_{\tau=j+1}^{T} \left( V_{\text{high},t} V_{\text{high},\tau-j} (x'_t x_{\tau-j} + x'_{t-j} x_{\tau}) \right)
\]
Next, we rewrite matrix \((Z'Z)^{-1} \otimes (X'X)^{-1}\) in (A5) as
\[
(Z'Z)^{-1} \otimes (X'X)^{-1} = \begin{bmatrix}
N_{\text{low}}^{-1}(X'X)^{-1} & -N_{\text{low}}^{-1}(X'X)^{-1} \\
-N_{\text{low}}^{-1}(X'X)^{-1} & (N_{\text{low}}^{-1} + N_{\text{high}}^{-1})(X'X)^{-1}
\end{bmatrix}
\]
(A37)
and insert (A37) into the Driscoll and Kraay (1998) covariance matrix estimator of (A5) to obtain
\[
\bar{\mathbf{V}}(\hat{\theta}) = ((Z'Z)^{-1} \otimes (X'X)^{-1}) \tilde{\mathbf{S}}_T ((Z'Z)^{-1} \otimes (X'X)^{-1}) = \begin{bmatrix}
\bar{\mathbf{V}}^{(1,1)} & \bar{\mathbf{V}}^{(1,2)} \\
\bar{\mathbf{V}}^{(2,1)} & \bar{\mathbf{V}}^{(2,2)}
\end{bmatrix}
\]
(A38)
with
\[
\bar{\mathbf{V}}^{(1,1)} = N_{\text{low}}^{-2}(X'X)^{-1} \left( \tilde{s}_T^{(1,1)} - \tilde{s}_T^{(1,2)} - \tilde{s}_T^{(2,1)} + \tilde{s}_T^{(2,2)} \right) (X'X)^{-1}
\]
\[
\bar{\mathbf{V}}^{(1,2)} = N_{\text{low}}^{-1}N_{\text{high}}^{-1}(X'X)^{-1} \left( \tilde{s}_T^{(1,2)} - \tilde{s}_T^{(2,2)} \right) (X'X)^{-1} - \bar{\mathbf{V}}^{(1,1)}
\]
\[
\bar{\mathbf{V}}^{(2,1)} = N_{\text{low}}^{-1}N_{\text{high}}^{-1}(X'X)^{-1} \left( \tilde{s}_T^{(2,1)} - \tilde{s}_T^{(2,2)} \right) (X'X)^{-1} - \bar{\mathbf{V}}^{(1,1)}
\]
\[
\bar{\mathbf{V}}^{(2,2)} = N_{\text{high}}^{-2}(X'X)^{-1} \tilde{\mathbf{S}}_T^2 (X'X)^{-1} - \bar{\mathbf{V}}^{(1,1)} - \bar{\mathbf{V}}^{(1,2)} - \bar{\mathbf{V}}^{(2,1)}
\]

According to the second part of Proposition 3, we have to show that \(\bar{\mathbf{V}}^{(2,2)}\) in (A38) coincides with the Newey-West covariance matrix estimator in (A10) with \(\tilde{\mathbf{S}}_T\) specified according to Expression (A33). Therefore, we simplify the “sandwich” expressions in (A38) as follows

\[
\tilde{s}_T^{(1,1)} - \tilde{s}_T^{(1,2)} - \tilde{s}_T^{(2,1)} + \tilde{s}_T^{(2,2)} = \sum_{t=1}^T V_{\text{low},t}(x'_t x_t) + \sum_{j=1}^H \omega_{j,t} \sum_{t=j+1}^T \left( V_{\text{low},t} V_{\text{low},t-j}(x'_t x_{t-j} + x'_{t-j} x_t) \right)
\]
\[
\tilde{s}_T^{(1,2)} - \tilde{s}_T^{(2,2)} = \sum_{t=1}^T V_{\text{high},t} V_{\text{high},t}(x'_t x_t) + \sum_{j=1}^H \omega_{j,t} \sum_{t=j+1}^T \left( V_{\text{high},t} V_{\text{low},t-j}(x'_t x_{t-j} + V_{\text{low},t-j} V_{\text{high},t} x'_{t-j} x_t) \right)
\]
\[
\tilde{s}_T^{(2,1)} - \tilde{s}_T^{(2,2)} = \sum_{t=1}^T V_{\text{low},t} V_{\text{high},t}(x'_t x_t) + \sum_{j=1}^H \omega_{j,t} \sum_{t=j+1}^T \left( V_{\text{high},t} V_{\text{low},t-j}(V_{\text{high},t} V_{\text{low},t-j} x'_t x_{t-j} + V_{\text{high},t-j} V_{\text{low},t} x'_{t-j} x_t) \right)
\]

and insert the resulting expressions into \(\bar{\mathbf{V}}^{(2,2)}\) from (A38). This finally yields
\[
\bar{\mathbf{V}}^{(2,2)} = (X'X)^{-1} \tilde{Q}_T^2 (X'X)^{-1}
\]
with
\[
\tilde{Q}_T^2 = \sum_{t=1}^T \left( N_{\text{high}}^{-1} V_{\text{high},t} - N_{\text{low}}^{-1} V_{\text{low},t} \right)^2 x'_t x_t
\]
(A39)
\[ Q^A_T + \sum_{j=1}^H \omega_{j,H} \sum_{\tau=j+1}^T \left( (N^{-1}_{high} V_{high,\tau} - N^{-1}_{low} V_{low,\tau})(N^{-1}_{high} V_{high,\tau-j} - N^{-1}_{low} V_{low,\tau-j})(x'_\tau x_{\tau-j} + x'_\tau x_{\tau-j}) \right) \]

Since \( Q^A_T \) in (A39) and \( S_T \) in (A33) are identical, this demonstrates that \( SE(\hat{\beta}_{\Delta,k}) = SE(\hat{\theta}_{\Delta,k}) \) for all \( k = 0, 1, ..., K \).

The last part of Proposition (3) claims that \( \tilde{\Psi}^{(1,1)} \) in (A38) coincides with the Newey-West covariance matrix estimator for the second-step time-series regression of the portfolio sorts approach applied to subject group “low”. The respective Newey-West covariance estimator has been derived in Expression (A24) above with \( p = \text{“low”} \). By replacing \( \tilde{S}_T^{(1,1)} - \tilde{S}_T^{(1,2)} - \tilde{S}_T^{(2,1)} + \tilde{S}_T^{(2,2)} \) with the corresponding term derived above, we finally obtain the following expression for \( \tilde{\Psi}^{(1,1)} \):

\[ \tilde{\Psi}^{(1,1)} = (X'X)^{-1} \tilde{Q}^{high}_T (X'X)^{-1} \]  

with

\[ \tilde{Q}^{high}_T = \sum_{\tau=1}^T N^{-2}_{low} V_{low,\tau} x'_\tau x_\tau + \sum_{j=1}^H \omega_{j,H} \sum_{\tau=j+1}^T \left( N^{-2}_{low} V_{low,\tau} V_{low,\tau-j}(x'_\tau x_{\tau-j} + x'_\tau x_{\tau-j}) \right) \]

Since \( \tilde{Q}^{high}_T \) in (A40) and \( S_T \) for subject group \( p = \text{“low”} \) in (A23) are identical, this demonstrates that \( SE(\hat{\beta}_{low,k}) = SE(\hat{\theta}_{low,k}) \) for all \( k = 0, 1, ..., K \). This completes the proof of Proposition 3. ■