The impact of false-negative reads on the performance of RFID-based shelf inventory control policies

Christian Metzger\textsuperscript{a,c}, Frédéric Thiesse\textsuperscript{b,*}, Stanley Gershwin\textsuperscript{c}, Elgar Fleisch\textsuperscript{d}

\textsuperscript{a} ETH Zurich, Switzerland
\textsuperscript{b} Julius-Maximilian University of Wuerzburg, Josef-Stangl-Platz 2, 97070 Wuerzburg, Germany
\textsuperscript{c} MIT, USA
\textsuperscript{d} University of St. Gallen & ETH Zurich, Switzerland

\section{Introduction}

From an operational perspective, the core business of the retail industry is to partition bulk products into smaller lot sizes or individual items and then sell them to consumers at a convenient location. With intense competition in metropolitan areas retailers are obliged to provide high product availability at low operating costs to establish a profitable and sustainable position in the market and to achieve satisfaction and loyalty among customers. To manage the high complexity of current retail supply chains more efficiently, retailers introduced novel management concepts into their supply chains to obtain fine-grain information sharing among supply chain partners. However, despite the widespread deployment of information technology and industry initiatives such as ‘Efficient Consumer Response (ECR)’, inventory inaccuracies, stock-outs, shrinkage, invoice inaccuracies, and goods that become unsellable (e.g., perished or damaged goods) still result in significant unnecessary costs thereby deteriorating revenues.

Radio Frequency Identification (RFID) offers opportunities for further improvements of operational processes by providing automatic identification of pallets, cases, and items to increase supply chain visibility. One of the potential applications of RFID in retail is its use to improve in-store logistics processes such as shelf replenishment. Studies detailing out-of-stock rates for U.S. and European retailers identified that the majority of root causes for stock-outs are found in insufficient shelf replenishment practices. Stock-outs affect sales in numerous ways: consumers substitute one item for another, switch brands, delay the purchase, or buy the product at a different store. These consumer responses to stock-outs result in a profit reduction of up to 10% and dwindling consumer loyalty may have a long-term impact on market share. Gruen et al. estimate that 25% of all stock-out situations are caused by inefficiencies in the shelf replenishment process, that is, products are in the store, but not on the shelf. Stores’ product availabilities average only 91.7% while product availabilities at the manufacturer and distribution center usually range from 98% to 99%. A study on European retailers by Thonemann et al. found that service levels range from 90% in the worst case up to 98.7% in the best case. The retail executives involved in this survey ranked current in-store logistics processes as the most promising point for improvement in the retail supply chain.

*Corresponding author. Tel.: +49 931 3180242; fax: +49 931 3181768. E-mail address: frederic.thiesse@uni-wuerzburg.de (F. Thiesse).

Available online 16 February 2013

Keywords: RFID, Inventory, Retailing, Shelf replenishment, In-store logistics

Abstract

Effective retail in-store logistics are paramount to provide high product availability at minimal operating costs. Despite various efforts by retailers to lower out-of-stock rates on retail shelves, product availability remains insufficient thereby significantly degrading a store’s performance. Currently, retailers consider the introduction of Radio Frequency Identification (RFID) to improve the efficiency of replenishment processes in stores. However, the possibilities of RFID are ultimately limited by the physical characteristics of RF communications. Tag detuning and the absorption of radio waves by the tag’s environment may lead to so-called ‘false-negative’ reads, i.e., RFID tags in range being undetected by the reader device. Retailers ignoring the impact of false-negatives on the performance of RFID-based inventory control systems run the risk of overestimating the benefits to be expected from RFID. We develop an inventory control policy based on shelf stock information generated by RFID, which specifically accounts for inaccuracies associated with false-negative reads. The mathematical model is optimized for operating costs and compared to a basic periodic review strategy in a numerical study. The results indicate that the impact of false-negatives on cost remains modest for medium to high read rates. However, the system performance is sensitive to a number of exogenous parameters that must be considered when evaluating the practical use of RFID.
Currently, response-based in-store logistics are widely established in the industry [61]. Store clerks visually inspect inventory levels on retail shelves by regularly walking the aisles. They add products to picking lists for replenishment from the backroom if shelf inventory levels are low. Among the most important factors influencing the economical effectiveness of such a periodic review policy are labor cost that arises from the manual inspection of inventory levels and accuracy of data collection. A consequence of high labor cost is often a low observation frequency that may cause delays in the detection of low inventory levels. The introduction of RFID aims at automating the inventory monitoring process thereby reducing the cost for data collection while increasing the update rate.

However, although the automatic detection of goods using RFID seems trivial at first glance, technical constraints directly influence the readability of the RFID tags and lead to unexpected read events in the form of so-called ‘false-negative’ RFID tag reads [13,39,55]. A false-negative read describes the phenomenon of RFID tags in range being ‘invisible’ to the reader device. The reasons for this issue are manifold [26]. On the one hand, tag collisions (i.e., tags sending a signal to the reader device at the same time) may prevent tags from being read. Sophisticated anti-collision protocols such as ‘EPC Gen 2’ [23] developed in recent years allow for eliminating collision problems under most circumstances. On the other hand, tag detuning (i.e., shifts in resonance frequency caused by other tags or metal in vicinity) and the absorption of radio waves by the tag’s environment pose a source of low data quality. In contrast to tag collisions, the latter phenomenon is rooted in the physical principles of RF communications, which can only to a small extent be ‘engineered away’ by improvements in IC and antenna design. While read rates of up to 100% might be realistic in some areas, read rates as low as 30% are reported for bulk identification of several everyday consumer goods that contain metal or liquids [22] as well as for stacked textiles [20]. Though specialized on-metal tags exist that address the mentioned problems, the associated costs are prohibitively high and do not yet allow for mass applications in open-loop supply chains.

Prior research on the value of RFID as a means to optimize shelf replenishment has usually not investigated the issue of false-negative reads. In fact, the respective models tend to consider RFID as a perfect identification technology which eliminates inaccuracy problems in the supply chain, but does not become a source of inaccuracies itself [10,54]. However, ignoring the impact of false-negatives on the performance of RFID-based inventory control systems may lead to an overestimation of the benefits achievable with RFID. The present study aims to fill this gap in the literature by an inventory control model that implements a threshold-based replenishment policy using shelf stock information generated by RFID. Through the mathematical analysis, we investigate how the introduction of RFID influences the current retail shelf replenishment process in terms of data accuracy, timeliness, and cost. Our objective is to go beyond previous research on RFID by accounting for measurement errors due to the physical limitations of RF communications (i.e., imperfect read rates). We examine the decision making under information uncertainty and evaluate the performance of optimal replenishment policies. The performance of an optimal shelf inventory management strategy based on RFID is compared to the traditional replenishment policy that relies on manual inspection of product availability.

The remainder of the paper is organized as follows. In Section 2, we provide a review of previous research on RFID in retail and in-store logistics in particular. Section 3 gives an overview of our general analytical framework. Next, we develop a model of the traditional replenishment process based on periodic review and a model of RFID-enabled continuous review. In Section 4, we compare the two approaches numerically and evaluate them with respect to cost and stock-out ratio. The paper closes with a summary and conclusions.

2. Related work

Since about the year 2000, RFID has gained significant attention in white papers and trade journals that discuss the technology’s potential impact on supply chain performance. Reports prepared by PwC Consulting [48], Accenture [4], A.T. Kearney [1], and GCI [30] were among the first publications that elaborated on potential benefits of introducing RFID into retail store logistics. Many of these early publications emerged from standardization consortia such as the Auto-ID Center – an industry-sponsored project at the Massachusetts Institute of Technology (1999–2003) – while others were commissioned by industry organizations such as the Grocery Manufacturers of America (GMA). Although these sources may serve as a valuable foundation for understanding the industry’s interest in RFID, the numerous estimates on actual business value and market potential should be interpreted with care. In fact, the vast majority of these publications suffer from a ‘credibility gap’ [18] because the provided estimates are hardly substantiated.

In recent years, the academic community has developed a number of solid models that enable a more realistic assessment of the value of RFID [46,50,54]. Lee and Özer [44] provide a comprehensive overview of various approaches from an operations management perspective. They credit RFID two distinct values—visibility of inventory movements and prevention of inventory record inaccuracies. Quantitative analyses on the potential improvements that will arise from the elimination of inventory inaccuracies were presented by Atali et al. [7], de Kok et al. [16], Fleisch and Tellkamp [25], Gaukler et al. [29], Heese [37], Kang and Gershwin [41], Körk and Shang [42], Rekik et al. [51], and Rekik et al. [52]. However, although these studies offer valuable insight into the potential of RFID to eliminate inaccuracies due to shrinkage, misplacements, and transaction errors, they usually do not provide a detailed model of the actual shelf replenishment processes in stores and the impact of RFID thereon.

A reason for the low number of publications on this topic may be that companies have tried to avoid retail store backroom inventory to establish so-called ‘one-touch replenishment’ policies. Cooper et al. [14] identify the elimination of backroom inventory as one of three major areas of logistics innovation in the UK grocery industry that started in the mid-1980s. Nevertheless, there are several reasons why retailers have still kept backroom inventory until today: more products can be stored per area in the backroom than on the sales floor, backroom inventory acts as an inventory buffer for uncertain or imperfect deliveries and long lead times, and there may not be enough shelf space to store all products on the sales floor for bulky products or products with high turn-over rates [61]. The downside of a replenishment strategy from the backroom is that the inventory handling costs significantly contribute to the total supply chain costs. For example, 38% of the total supply chain costs for non-perishable goods are inventory handling costs [60]. Furthermore, retailers have directed most of their efforts to the optimization of processes between the distribution center and the store by introducing automatic reordering systems, leaving retail store shelf replenishment with great potential for improvements in efficiency and cost.

Wong and McFarlane [61] provide a qualitative analysis of the main factors of suboptimal replenishment performance, e.g., delayed reviews or outdated pick lists. The authors describe the structure of the traditional process based on a push policy and a pull policy. They distinguish between the two policies based on whether the review occurs in the backroom or on the sales floor. Furthermore, they
discuss opportunities for improvement by using RFID to automate the monitoring of stock levels and product movements as well as the automatic compilation of pick lists.

Hardgrave et al. [36] report on Wal-Mart’s pilot project conducted from February to September 2005 that included 12 stores with varying store formats. In total 4554 different products were tagged at the case level to allow monitoring product movements between the backroom and the sales floor. In this trial, the use of RFID led to an average reduction of stock-outs by 16% over a control group consisting of 12 similar stores. The best improvement was observed for products with a daily demand of 6–15 units. The reduction in stock-outs for these products was recorded at 62%. The impact of RFID on stock-outs could be confirmed in a second study comparing the improvement in inventory record accuracy before and after implementing RFID-enabled adjustments to the inventory management system [35]. The authors report reductions in the percentage of stockouts ranged from 21% to 36%, depending on category. A similar study was also conducted by Bertolini et al. [8] in 30 retail stores. The results indicate that RFID not only improves accuracy and efficiency of logistics processes but also leads to additional sales through stockout reduction.

Lee et al. [45] use simulation models to investigate the effects of the elimination of inventory inaccuracies, the redesign of the shelf replenishment process, and the exchange of inventory level information between a supplier and a retailer through the use of RFID. To draw conclusions for the redesign of the shelf replenishment process, the traditional process of inventory management based on periodic reviews is compared to continuous reviews accomplished by using RFID readers on shelves. The authors show that RFID allows for a better adaptation to actual demand thereby necessitating lower shelf stock levels. However, the significance of their findings is limited because they chose arbitrary inventory policies instead of optimized ones to examine the performance of the different replenishment strategies. In addition, they only consider stock levels and stock-outs as the main performance indicator instead of total cost. A further simulation study was presented by Abdulmalek et al. [3], who also observe RFID-induced stockout reductions but do not provide an analysis of cost and profitability metrics, too.

Other authors presented analytical models of RFID-based inventory control in stores. Szmerekovsky and Zhang [57] study the effect of RFID at the item level for a manufacturer and a retailer, relying on RFID in a vendor-managed inventory system. They compare a system of continuous review using RFID and a non-RFID system of periodic review. The authors determine the optimal inventory policies in a centralized system and establish conditions under which the RFID system is preferable to the system without RFID. The limitations of their single-period model include fixed shelf space, fixed review/replenishment intervals, and the fact that replenishment costs are not considered. Çakıcı et al. [12] analyze the incremental benefits of RFID technology over barcodes for managing pharmaceutical inventories. Based on a case study, the authors show that inventory managers can benefit from RFID by leveraging automatic counting and continuous review and by tracking shrinkage actively. Using a mathematical model, they show that the switch to continuous review achieves savings with regard to inventory holding, backorder, and ordering costs. The results indicate that the total cost savings of RFID combined with business process reengineering increases in all policy parameters except for cost per order under RFID. Gaukler [27] examines a retail store operation with backroom and shelf stock and the impact of RFID on profit improvements. The study confirms that the direct effect of more efficient and effective backroom-to-shelf replenishment contributes the majority of the total RFID benefit.

A simplification common to most models that incorporate RFID data into the decision making process is the disregard of RFID read rates. In fact, the majority of prior works implicitly assume perfect detection accuracy, which does not correspond to the reality of today’s RFID hardware components. There exist many sources for measurement errors during the operation of an RFID-based shelf inventory monitoring system, e.g., dysfunctional tags, incomplete labeling of products, tags that are accidentally removed or destroyed by employees or customers, physical shielding by metals or liquids in the product or its packaging, shielding by other tags and errors in the installation or the configuration of reader hardware and antennae, etc. Read rates will continue to pose challenges among all product types despite foreseeable technological advances (e.g., by new product packaging, process changes, improved antennae design). Even if the physical characteristics of a product, tag placement, and store environment do not influence read rates (e.g., in the case of textiles), RFID systems are still vulnerable to the problem of stacked tags, i.e., RFID labels shielding or detuning each other [26]. Imperfect read rates significantly distort inventory data thereby influencing the parameters of an optimal replenishment strategy. To allow for an accurate assessment of potential improvement of the shelf replenishment process, the fundamental limitations of RF communication hence have to be accounted for.

Kang [40] seems to be the first one to specifically address the issue of limited read rates of RFID tags in the context of inventory control systems. His study provides an approach that treats this inventory control problem as an imperfect state information problem where the measurement data represents stochastically uncertain observations of the stock quantities. Although imperfect state information dynamic programming provides a means to determine the optimal policy for a problem that is subject to measurement errors, it requires highly intensive computing as the dimension of the information vector containing all information available at a certain time grows excessively with time. The author provides an optimal policy for a control system with a planning horizon of only five days. The fast-growing state dimensions of the imperfect state information problem limits the feasibility of this approach to small and simplified problems.

Condea et al. [13] investigate the impact of suboptimal read rates in a simulation study of an inventory control policy based on RFID data with case-level tagging. They propose a heuristic extension to cope with the problem of ‘replenishment freezes’ and other downsides of RFID-based inventory control. The results indicate that RFID has the potential to improve cost efficiency and service levels depending on the achievable read rate. Similarly, Buyurgan et al. [10] investigate the impact of inventory decisions in the supply chain based on imperfect data generated by RFID systems. Again, the results reveal the relation between RFID system performance and the achievable benefits. However, both studies do not provide any deeper analytical insights beyond the numerical results generated from simulation runs.

Some further studies at least account for the suboptimal performance of RFID in an indirect way. De Kok et al. [16] make use of a parameter \( \pi \) that denotes the fraction of theft that can be eliminated through RFID. In their numerical evaluation, they consider different values for parameter \( \pi \) to illustrate the influence of RFID on balancing RFID expenses and theft reduction. Atali [7] includes three random variables that represent the amount of undetected sales, misplacements, and shrinkage. However, the impact of detection rates on replenishment efficiency is not explicitly investigated in these works.

### 3. Model definition

#### 3.1. Assumptions

In the following, we develop a single-product inventory model to analyze the impact of RFID-based shelf inventory monitoring on the
3.2. Notations

With the model's restrictions described above, the threshold-based policy is neither a proper \((r, Q)\) nor a proper \((s, S)\) policy. Therefore, the notations \((r, S, T)\) for the periodic review policy and \((r, S)\) for the continuous review policy are used where \(r\) is the threshold level, \(S\) the restock-up-to level and \(T\), the review period. A complete overview of the notations used in the following sections is given in Table 1.

3.3. Periodic review based on manual inspections

Threshold-based periodic review with regular review intervals is the most common shelf stock monitoring practice at retail stores [61]. Manual inspections are necessary because existing data sources (i.e., bar code based data on received goods and POS data) do not allow retailers to accurately distinguish between backroom and shelf inventory. At the beginning of each review interval, the on-shelf inventory positions are inspected and replenishments are triggered if the shelf stocks are equal to or below a predefined threshold. If replenishments are triggered, shelves are restocked up to the allocated shelf space after a constant replenishment time \(T_r\). The mathematically optimal time interval \(T_r\) for reviews depends on the system parameters and may vary. Except for the case of products with very high demand rates, the mathematical optimum is often rounded up or down to the next half or full day thus the implementation of the review period becomes practically feasible. The periodic review model will serve as reference model to which any potential process improvement will be compared to.

The periodic review model presented in this paper is adapted from the extensive analysis of periodic review inventory systems provided by Hadley and Whitin [34]. We adjusted the more general models to the single-cycle problem described above, i.e., each shelf is restocked up to the allocated shelf space. Since we are only concerned with on-shelf inventory management and consider backroom inventory management part of a separate effort to optimize store inventory, the on-shelf inventory optimization problem becomes independent of inventory holding costs (i.e., they apply in any case), but must account for the cost of allocated shelf space. Shelf-space cost comprises all costs associated with the allocation of shelf space for a specific product type. In contrast to holding cost, shelf-space cost does not depend on the product’s average inventory, but on the product’s maximum inventory position \(S\) [11]. The equivalent to purchasing cost in traditional inventory control is the inclusion of review cost and replenishment cost in our model.

The retailer optimizes total cost over the parameters \(r\), \(S\), and \(T_r\). The corresponding objective function is structured as follows:

\[
C(r, S, T_r) = \text{shelf-space cost + review cost + replenishment cost + penalty cost}
\]

As mentioned before, the initial shelf inventory level is always the same at the beginning of each replenishment cycle. It is hence possible to limit the analysis to a single-period system, which

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>Replenishment threshold</td>
</tr>
<tr>
<td>(S)</td>
<td>Order-up-to level</td>
</tr>
<tr>
<td>(T)</td>
<td>Planning horizon</td>
</tr>
<tr>
<td>(i)</td>
<td>Demand rate</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Service level</td>
</tr>
<tr>
<td>(\xi_k)</td>
<td>Cumulative of all items at state (k) that have become visible</td>
</tr>
<tr>
<td>(T_c)</td>
<td>Cycle time</td>
</tr>
<tr>
<td>(T_{r1})</td>
<td>Replenishment lead time</td>
</tr>
<tr>
<td>(N_r)</td>
<td>Number of replenishments</td>
</tr>
<tr>
<td>(n_r)</td>
<td>Number of review periods per replenishment cycle</td>
</tr>
<tr>
<td>(k)</td>
<td>Number of demands that have occurred</td>
</tr>
<tr>
<td>(y^-)</td>
<td>Unmet demand</td>
</tr>
<tr>
<td>(C(r, \ldots))</td>
<td>Long-run average cost function under periodic review</td>
</tr>
<tr>
<td>(C(r, \ldots)_p)</td>
<td>Long-run average cost function under imperfect RFID</td>
</tr>
<tr>
<td>(c_{\text{rep}})</td>
<td>Replenishment cost</td>
</tr>
<tr>
<td>(c_p)</td>
<td>Penalty cost of unmet demand</td>
</tr>
<tr>
<td>(c_{\text{tag}})</td>
<td>Tag cost</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>Detection rate</td>
</tr>
<tr>
<td>(A_0)</td>
<td>Initial difference between actual and recorded inventory</td>
</tr>
<tr>
<td>(y_0)</td>
<td>Recorded inventory level</td>
</tr>
</tbody>
</table>

Table 1

Notations.
The single-period cost function $C_T(r,S,T_r)$ is formulated as

$$
C_T(r,S,T_r) = c_r \cdot S \cdot \frac{E[T_r]}{T} + E[n_t] \cdot c_e + c_{rep} + c_p \cdot E[y^-] \tag{1}
$$

where $c_r$ is the shelf-space cost per item per planning horizon, $c_e$ the review cost of a store clerk visually inspecting on-shelf inventory positions, $c_{rep}$ the replenishment cost, $c_p$ the penalty for shortages, $E[y^-]$ the expected number of shortages per cycle, and $E[n_t]$ the expected number of reviews per cycle until cumulative demand equals or exceeds $S - r$. Replenishment is triggered when the demand in review period $j$ is equal to or greater than the threshold. Shortages occur if the demand in the $n$-th period plus lead time is greater than the on-hand inventory at the end of the $(n-1)$-th period.

Replenishment is triggered if, and only if, the inventory level $y$ at a review time is equal to or smaller than the threshold. The time between two replenishments is an integral multiple of the cycle length $T_c$ during planning horizon $T$. The single-period cost function $C_T(r,S,T_r)$ is given by

$$
C_T(r,S,T_r) = c_r \cdot S \cdot \frac{E[T_r]}{T} + E[n_t] \cdot c_e + c_{rep} + c_p \cdot E[y^-] \tag{1}
$$

where $p(x; T_r)$ be the probability that $x$ units will be demanded in one review period. A cycle will be exactly one review period if the demand in the first period is greater than $S - r$. The probability of this is $P(S - r; T_r)$, where $P(x; T_r)$ is the complementary cumulative distribution function (CCDF) of $p(x; T_r)$. A cycle will contain precisely $n$ ($n \geq 2$) review periods if after a demand of $S - r$ items in the first $n - 1$ periods after replenishment, $j$ or more items are demanded in period $n$. Thus, the probability that a cycle contains precisely $n$ review periods is

$$
\sum_{j=1}^{n} p^{n-1}(S-r-j; T_r) \cdot P(j; T_r)
$$

where $p^{n-1}(S-r-j; T_r)$ is the $(n-1)$-fold convolution of $p(S-r-j; T_r)$, $S$ the restock-up-to level, and $r$ the threshold. Hence the expected number of review periods in a replenishment cycle is

$$
\sum_{n=1}^{\infty} \sum_{j=1}^{n} p^{n-1}(S-r-j; T_r) \cdot P(j; T_r)
$$

Following the notation by Hadley and Whitin [34], $p^{n}(x; T_r)$ becomes $P(x; T_r)$ and $P(x; T_r)$ becomes $P(x; T_r)$ for the Poisson demand with demand rate $\lambda$. The expected number of review periods in a cycle is

$$
\sum_{n=1}^{\infty} \sum_{j=1}^{n} p(S-r-j; (n-1) \cdot \lambda; T_r) \cdot P(j; \lambda; T_r).
$$

With

$$
p(S-r-j; (n-1) \cdot \lambda; T_r) = e^{-(n-1) \cdot \lambda} \cdot \lambda^{S-r-j} \cdot (S-r-j)! / \lambda^{S-r-j},
$$

which is the probability that the demand in review period $(n-1)$ is equal to $S - r - j$, and

$$
P(j; \lambda; T_r) = \sum_{x=j}^{\infty} e^{-\lambda} \cdot \lambda^{x} / x!.
$$

which is the probability that the demand in review period $T_r$ is equal to or greater than $j$. The expected number of periods in a cycle becomes

$$
\sum_{n=1}^{\infty} \sum_{j=1}^{n} e^{-(n-1) \cdot \lambda} \cdot \lambda^{S-r-j} \cdot (S-r-j)! / \lambda^{S-r-j} \cdot \sum_{x=j}^{\infty} e^{-\lambda} \cdot \lambda^{x} / x!.
$$

Note that if the demand in the $n$-th review period is equal to or greater than $r$ and replenishment is triggered.
smaller than the actual inventory because measurement errors only occur when a tag is not detected. The opposite error, a tag is detected that does not exist, is not possible due to the operating principles of RFID. Therefore, the inventory system is likely to trigger replenishments early which results in more frequent replenishments than necessary. The expected time for replenishment depends on the progression of the difference between the actual inventory and the recorded inventory over time. Assuming that only one item is left after \( S - 1 \) customers have arrived, the one remaining item cannot be shielded by any other items. Since this tag must be readable, measurement errors in the inventory management system must have decreased to zero by this time. The measurement error decreases from an initial value of \( A_0 \) to zero over \( S - 1 \) steps according to a stochastic process unless replenishment occurs.

In order to limit the complexity of measurement errors’ progression over time, we assume that the maximum number of items that can be shielded by another item is restricted to one. This assumption reflects the often-found situation of stacked tags with one tag屏蔽ing or detuning another. An example of this issue can be found in apparel retail where sales items are usually stacked or hanging close to each other. Hence, if an item is removed it may or may not reveal another item that becomes visible to the RFID system. The number of items revealed in one cycle cannot exceed the number of measurement errors recorded at the beginning of that cycle. Although the latter assumption favors the mathematical tractability of the error propagation problem constitutes a loss of generality, it still suggests feasibility for a profound analysis of the system’s behavior and for meaningful results. The inventory management system has only knowledge about the inventory level of the recorded inventory that is subject to measurement errors and the initial value of the actual inventory. However, recorded inventory and actual inventory conform over time. For each event of demand, the actual inventory is reduced by one. If the item that is removed from the shelf has not revealed another item that now becomes visible to the system, the recorded inventory is also reduced by one and the difference between the two inventory counts remains the same. If, however, an item does become visible due to the fact that another item has been removed, the recorded inventory level remains unchanged, while the actual inventory is reduced by one. Consequently, the difference between the two inventory levels is reduced by one as well. The occurrence of an item becoming visible to the system when demand occurs is modeled as a stochastic process. This stochastic process depends on the state of the system and the difference between the two inventory counts.

The progression of the recorded inventory level \( y^*_{k} \) is described as a function of the actual inventory:

\[
y^*_{k} = S - A_0 - k + \xi_k,
\]

where \( S \) is the initial value of the actual inventory, \( A_0 \) the initial measurement error, \( k \) the number of customer arrivals at the shelf that have occurred since the beginning of a cycle, and \( \xi_k \) the cumulative of all items that have become visible since the beginning of a cycle after \( k \) customer arrivals. The number of customer arrivals \( k \) determines the state that the inventory system is in at a particular point in time. The system’s state contains all the information that is relevant for predicting the future of the system. The state is defined as the unique values for each of the state variables. \( \xi_k \) is given by

\[
\xi_k = \sum_{i=0}^{k} \epsilon_i, \quad \epsilon_i \in \{0,1\},
\]

where \( \epsilon_i \) is the event that at state \( i \) an item becomes visible (\( i \leq k \)). The states, at which items become visible during a period of \( S - 1 \) events of demand, show a binomial distribution. There are \( A_0 \) items that eventually become visible during the \( S - 1 \) events of demand, and therefore, there are \( S - 1 - A_0 \) events of demand at which an item is removed without another one becoming visible.

Replenishment occurs when \( y^*_{k} \) is equal to the threshold \( r \) which is equivalent to

\[
\xi_k = r - S - A_0 + k.
\]

Let \( P(\xi_k=n) \) be the event that in a sequence of \( k \) occurrences of demand, \( n \) items have become visible. Then

\[
P(\xi_k=n) = \frac{\left| \xi_k = n \right|}{\left| \Omega \right|},
\]

which is the number of results in \( \xi_k \) divided by the number of all possible results.

\[
\left| \Omega \right| = (S-1)^k = \frac{(S-1)!}{(S-1-k)!}.
\]

The sequence of \( n \) events of an item becoming visible over a period of \( k \) steps can be arranged in \( \binom{k}{n} \) possible ways. For each of these positions, there exist \( \binom{A_0}{n} \) \( \binom{S-1-A_0}{k-n} \) \( k \)-tuples.

Therefore,

\[
P(\xi_k=n) = \binom{k}{n} \cdot \frac{\binom{A_0}{n} \cdot \binom{S-1-A_0}{k-n}}{(S-1)^k} = \frac{\binom{A_0}{n} \cdot \binom{S-1-A_0}{k-n}}{\binom{S-1}{k}}. \quad (5)
\]

Based on the probability distribution for \( \xi_k \), the expected time until replenishment is calculated as the cumulative for all states \( S \) and the time to go from the initial state \( S \) to state \( S - k \) multiplied by the probability that the recorded inventory at this state has just dropped down to the threshold level. Note that the recorded inventory level may remain unchanged despite the occurrence of demand because items may become visible. Therefore, it is important to only account for the time \( t_{rep} \) elapsed until the recorded inventory level drops down to \( r \) for the first time. This is the time after which the replenishment is triggered. The expected time until replenishment is given by

\[
E[t_{rep}] = \sum_{k=1}^{S} \sum_{r=S-A_0-k+\xi_k}^{S-A_0-(k-1)+\xi_{k-1}} p(S-A_0-k+\xi_k = r, S-A_0-(k-1)+\xi_{k-1} = r+1),
\]

where the first term is the time to go from \( S \) to \( S - k \), and the second term is the probability that the recorded inventory level is at \( r + 1 \) at state \( k - 1 \) (i.e., just one above the threshold level) and that the recorded inventory level drops down to \( r \) when the next demand occurs (i.e., at state \( k \)). This reduction of the recorded inventory level assumes that no item becomes visible with the latest event of demand. Applying Bayes’ theorem, the expected time until replenishment is reformulated as

\[
E[t_{rep}] = \sum_{k=1}^{S} \sum_{r=S-A_0-k+\xi_k}^{S-A_0-(k-1)+\xi_{k-1}} p(S-A_0-(k-1)+\xi_{k-1} = r+1) \times p(S-A_0-(k-1)+\xi_{k-1} = r+1)
\]

Computing the probability that the inventory level drops from \( r + 1 \) at state \( k - 1 \) to \( r \) at state \( k \) is simple. It is the probability that with the next demand no item becomes visible given that the system is at state \( k - 1 \). If the system is at state \( k - 1 \), there are \( S - 1 - (k - 1) \) states remaining until all shielded items have become visible. At state \( k - 1 \), \( \xi_{k-1} \) items out of a total of \( A_0 \) undetected items have already become visible which leaves \( A_0 - \xi_{k-1} \) items that have yet to become visible during the remaining \( S - 1 - (k - 1) \) steps. Conversely, during these \( S - 1 - (k - 1) \) steps, the number of times an item does not become visible is \( S - 1 - (k - 1) - (A_0 - \xi_{k-1}) \). Substituting \( \xi_{k-1} \) with
r+1−S+A0+(k−1) results in the number of times an item does not become visible which is equal to r. Exactly one of these occurrences is required, thus the recorded inventory level will drop from r+1 to r with the next demand. Consequently, the probability of this occurrence is
\[ p(c_k = 0) = \frac{r}{S−1−(k−1)} = \frac{r}{S−k}. \]

The probability that the inventory level is at r+1 at state k−1 is transformed as follows:
\[ p(S−A0−(k−1)+\xi_{k−1} = r+1) = p(\xi_{k−1} = r−S+A0+k) \]

The probability \( P(\xi_k=n) \) for \( \xi \) at state \( k \) was given above (Eq. (5)). At state \( k−1 \), \( P(\xi_{k−1} = n) \) transforms to
\[ P(\xi_k = n) = \frac{A0}{n} \frac{(S−1−A0)}{(k−n)} \frac{(S−1)}{(k−1)}. \]

where \( n=r−S+A0+k \). Hence, the expected time until replenishment is
\[ E[\tau_{rep}] = \sum_{k=1}^{S} \frac{r}{S−k} \frac{A0}{n} \frac{(S−1−A0)}{(k−n)} \frac{(S−1)}{(k−1)}. \]

4. Numerical evaluation

4.1. Optimal control parameters for periodic review

In this section, we illustrate the impact of read rate and technology cost on the performance of the RFID-based replenishment policy using a numerical example based on our practical experiences with RFID implementations at grocery and apparel retailers. The following set of parameter values characterizes products that are typically equipped with RFID transponders today, such as textiles, shoes, perfumery and drugstore products, CDs/DVDs, and others (i.e., medium- to high-value products with medium daily demand). As the evaluation baseline, we consider a product with a demand rate of \( \lambda=10 \) and a replenishment lead time of 30 min (0.02 days). The individual costs incurred in operating an inventory system significantly impact overall system performance and its optimal parameters. The shelf-space cost is assumed \$0.3 per item per day. Review cost and replenishment cost are labor costs that arise from store clerks visually inspecting and restocking shelves. The review cost is assumed \$2 per product type per shelf. The replenishment cost is assumed \$3 per product type per shelf. The penalty cost derives from the sales loss of each unit of unmet demand. The cost also accounts for lost profits on sales of other items (i.e., a customer may decide to buy some or all items on the shopping list at another store), future sales, and special procedures used to deal with customers that are confronted with an out-of-stock situation. The penalty cost is assumed \$55 per occurrence of unmet demand. The system operates with a planning horizon of 365 days, which suggests adequate approximations and is a meaningful time unit to the retail store manager.

The costs for all sets of \((r,S,T)\) were computed via complete enumeration in order to facilitate subsequent sensitivity analyses. The set of control parameters \( r, S, \) and \( T \) that minimize the expected inventory management costs constitutes the optimal \((r,S,T)\) replenishment policy. For the costs and parameters given above, the mathematically optimal policy for periodic review was found with \( r=13, S=17, \) and \( T=1.2 \). The operating cost for this policy is \$3614.48 with 0.1679 units short per cycle. Hence, the service level is 97.95%. Note that rounding the review cycle time to a full day \((T=1.0)\) makes its practical implementation more feasible. For a review cycle time \( T=1.0 \), the operating cost is \$3652.95 with a service level of 98.69%.

4.2. Optimal control parameters for RFID

In order to permit for comparable results between the inventory management systems, each system assumes the same initial costs. However, in contrast to periodic review, no labor costs arise...
for the visual inspection of shelf stocks. Instead, the system accounts for RFID infrastructure cost such as RFID antennae, reader, and data processing with an infrastructure cost $C_I$ of $12 per allocated shelf space per year. The RFID tag cost including the cost for item tagging represents a variable cost and contributes to the total cost with $0.2 per item sold. For the analysis of arbitrary cost for item tagging represents a variable cost and contributes to the total cost with $0.2 per item sold. For the analysis of arbitrary retail settings and a general understanding of the system dynamics, read rates ranging from 10% to 100% are taken into consideration. Optimal $(r,S)$ policies for RFID inventory management systems at different read rates are given in Table 2.

In comparison to optimal $(r,S,T_t)$ policies for periodic review, optimal $(r,S)$ policies for RFID inventory management systems show low replenishment threshold values $r$. Continuous monitoring combined with a short lead time for replenishments and a moderate demand rate allow for a low replenishment threshold because inventory levels that reach the threshold are detected instantly. Consequently, the higher the actual inventory level when replenishment is triggered, the lower the probability for stock-outs remains low despite a low threshold level. A low threshold reduces the total number of replenishments and the costs incurred thereof.

Additionally, Table 2 demonstrates that the service level increases as the read rate decreases. If the recorded inventory level reaches the threshold while the actual inventory still shows an inventory level higher than the threshold, an early triggering of replenishment occurs. Early triggering of replenishment is due to the difference in actual inventory and recorded inventory. With an increasing initial difference, the probability that the recorded inventory reaches the threshold and triggers replenishment before all shielding errors have been eliminated from the system increases as well. Consequently, the higher the actual inventory level when replenishment is triggered, the lower the probability for stock-outs. In addition, the earlier the recorded inventory reaches the threshold – in contrast to when the actual inventory level would have reached the threshold – the higher the number of additional replenishments. To some extent, the system tries to reduce the incidence of costs due to additional replenishment by increasing the allocated shelf space to carry higher inventory stocks.

For read rates of 30% and less, the behavior of the system differs from the pattern described above. With such a low initial recorded inventory level, it becomes more economical to reduce the threshold to zero and account for penalty costs due to units short rather than for additional replenishments. Because elimination of measurement errors from the system occurs with the removal of the last item, the recorded and actual inventory show the same inventory level right before shelf depletion. Consequently, an early triggering of the replenishment process cannot occur. However, with a replenishment threshold of zero, the system cannot meet any demand that occurs during the replenishment lead time. Therefore, the number of unmet demand is considerably higher than for better read rates.

### 4.3. Comparison of operating costs and service levels

Fig. 1 illustrates the change in minimal operating cost for RFID systems as the read rate changes. The minimal operating cost for a RFID inventory system depends on detection performance whereas the operating cost for periodic review systems do not. Therefore, the minimal cost for periodic review forms a horizontal line in Fig. 1 at operating cost of $3614.48 for a demand rate of $\lambda=10$. The results indicate that RFID system performance is not significantly affected by decreasing read rates. Plots are given for RFID systems operating with tag cost of $0.1$, $0.2$, and $0.4$, respectively. The figure reveals that in our example RFID inventory systems with tag costs of $0.1$ and $0.2$ operate at significantly lower cost than the periodic review system. At higher read rates, the improvement of an RFID system over periodic review is approximately 22% for $c_{\text{tag}}=0.1$ and 12% for $c_{\text{tag}}=0.2$; for tag cost of $0.4$, the RFID system performs worse. The introduction of RFID not only results in lower operating cost in the case of low tag costs but also in an increase in service level. The service level for the periodic review inventory management system is 97.95%, which compares to 99.8% or higher for a RFID-based continuous review system at read rates equal to or greater than 40%. Note that service levels are higher in the case of lower read rates due to early replenishments.

### 4.4. Sensitivity analysis

Our analyses in the previous subsections are limited to a base case that was defined by a number of model parameters assumed to be constant. However, the parameters cost and time may vary among retailers depending on store format, assortment, location, and geographic region, etc. For this reason, this section investigates the impact of parameter variations on total cost and service level. In order to identify the critical input factors that lead to a significant change of the system’s output if varied, the effects on the system output are examined for variations of all inputs with the exception of the read rate (the influence of $\varphi$ has already been examined before). In Fig. 2, the input parameters shelf-space cost $c_s$, shelf replenishment cost $c_{\text{rep}}$, penalty cost $c_p$, infrastructure cost $C_I$, tag cost $c_{\text{tag}}$, and $\lambda$ are varied to 25%, 50%, 200%, and 400% of their original values. The tornado charts show how the minimal operating costs change as a result of the variation of an input cost factor. The results indicate that total cost under periodic review is most sensitive to demand rate and shelf-space cost. This phenomenon can be attributed to the fact that an increase in demand

Table 2

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>Optimal $(r,S)$</th>
<th>$C(r,s)$</th>
<th>$E[y]$</th>
<th>$\beta$</th>
<th>No. of add. repl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
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<td>0.0187</td>
<td>0.99809</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.9</td>
<td>(1,11)</td>
<td>3186.84</td>
<td>0.0169</td>
<td>0.99829</td>
<td>3.5430</td>
</tr>
<tr>
<td>0.8</td>
<td>(1,11)</td>
<td>3197.24</td>
<td>0.0152</td>
<td>0.99848</td>
<td>7.9698</td>
</tr>
<tr>
<td>0.7</td>
<td>(1,11)</td>
<td>3211.39</td>
<td>0.0135</td>
<td>0.99868</td>
<td>13.6581</td>
</tr>
<tr>
<td>0.6</td>
<td>(1,11)</td>
<td>3231.16</td>
<td>0.0115</td>
<td>0.99889</td>
<td>21.2370</td>
</tr>
<tr>
<td>0.5</td>
<td>(1,11)</td>
<td>3259.96</td>
<td>0.0097</td>
<td>0.99909</td>
<td>31.8366</td>
</tr>
<tr>
<td>0.4</td>
<td>(1,12)</td>
<td>3324.62</td>
<td>0.0071</td>
<td>0.99930</td>
<td>46.5561</td>
</tr>
<tr>
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<td>(0,11)</td>
<td>3362.54</td>
<td>0.0050</td>
<td>0.99950</td>
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<td>0.99970</td>
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<td>(0,11)</td>
<td>3362.54</td>
<td>0.0000</td>
<td>0.99970</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Fig. 1. Minimal operating costs for a periodic review system and RFID systems.
leads to more frequent reviews and replenishments. In addition, substantial amounts of shelf inventory are needed in order to prevent stock-outs between two reviews. The RFID-based policy is even more sensitive to demand since tagging costs must be incurred per item sold. Replenishment cost also exerts a stronger influence on total cost than under periodic review since the RFID-based policy tends to trigger more replenishments. In contrast, continuous review using RFID is less sensitive to an increase in shelf-space cost as it requires less shelf inventory in order to meet a specific demand rate. Not least, the performance of the RFID-based policy depends on tag cost, which does not apply to periodic review. All considered, we conclude that RFID is most suitable for slow-moving items, particularly if tag cost is small and the retailer is able to adapt shelf space flexibly to the optimal set of policy parameters.

5. Summary and conclusion

In this paper, we developed mathematical models for periodic review and RFID-enabled retail shelf inventory management. The latter model accounts for the occurrence of false-negative reads due to imperfect detection rates as well as technology costs accruing from the utilization of RFID. False-negatives may cause a discrepancy between actual and recorded inventory. We provided an extensive analysis of the error propagation of this discrepancy over time. Moreover, we identified minimal operating costs for periodic review and RFID inventory management systems for different demand rates, varying input cost factors, different read rates and tag costs. Our numerical results show that for low to medium demand rates and low tag cost, RFID systems may operate at lower costs than periodic review systems. In the investigated example, we have seen cost improvements of up to 25.7% and service levels increasing by up to 2%. The RFID system's performance in operating costs has been shown to be relatively robust to high to medium read rates. In contrast, low read rates often result in early triggering of the replenishment process thereby reducing the probability of out-of-stocks but leading to additional costs for more frequent replenishments. We also found that the introduction of RFID results in lower overall inventory levels due to continuous shelf stock monitoring and timely replenishments. In practice, a reduction in inventory levels leads to freed up capital that may be invested in other areas to generate additional revenues.

Models as the ones presented here may support retailers with evaluating the cost-efficiency of their current shelf replenishment policies and with deriving explicit results on potential benefits for introducing RFID to shelf inventory management systems. Our numerical example illustrated the dimension of potential benefits that may be achieved with an RFID-enabled continuous review strategy. In particular, the consideration of false-negative reads allows the retailer to come to a more realistic estimate of RFID system performance than other models that usually assume perfect detection rates. However, it should be noted that the input parameters to the control system vary from retailer to retailer (e.g., labor cost, shelf space allocation cost, etc.) and so vary the potential benefits. Retailers will have to conduct their own analysis with respect to input parameters specific to their business. The sensitivity analyses for the different models identify the input factors that should receive most attention during the data gathering process that precedes the actual system evaluation.

As for other studies of this kind, our research is not without limitations. First, our analysis was limited to the last mile of the supply chain, whereas the use of RFID in other parts of the chain was beyond our scope. While this limitation implies no loss of generalizability, it should be kept in mind that sub-optimal shelf availability may also be caused by other factors (e.g., improper demand forecasts, outdated store reordering practices), which must be taken into account. Second, we did not consider stochastic influences on replenishment lead times and the quality of manual activities. For instance, similar to the RFID-based process periodic reviews and replenishments might be imperfect, too. Third, we concentrated on stock-outs and did not consider consumer reactions to different shelf inventory levels. These and other psychological factors are beyond the scope of our study. Fourth, our research focus was set particularly on the impact of false-negative read events. In contrast, other sources of inaccuracies (e.g., misplacements, theft) were not considered in our analysis. We also did not consider the phenomenon of false-positive reads (i.e., reads of other tags that do not belong to the shelf inventory), which may have an impact on the accuracy of our RFID-based inventory control policy. Moreover, our model is limited to a single product and does not account for dependencies between the replenishment of different product types. As a consequence, it is not clear to what extent we may generalize from our study to multi-item inventory systems. Similarly, we did not consider substitution effects between product types and treated each case of unmet demand as a lost sale. These limitations should be regarded as opportunities for future research in this area.

References


