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Price dynamics in electricity spot markets

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Particularities of electricity prices (1)

Extremely large price movements (spikes):
- Caused by small changes in demand and supply
- Electricity is not storable, load and generation are balanced on a knife’s edge

Negative prices:
- Occur because of wind infeed, on the other hand limited load change flexibility
- More frequently observed in night hours and on weekends
Particularities of electricity prices (2)

Electricity prices show characteristic seasonal patterns:

The price variation can be decomposed in a deterministic and a stochastic part:

- The deterministic part is derived as a “seasonality shape” (Blöchlinger, 2008). We determine the weights of the daily prices relative to the year as well as the hourly prices relative to the day.
- The stochastic component is modeled as variation around the hourly price-forward curve (HPFC). This takes information from the markets on future prices into account.
Model for electricity prices: Idea

- We define a band around the hourly price-forward curve
- Within the band: Deviations of prices from the HPFC are lognormally distributed → normal ("Gaussian") regime
- Below lower or above upper limit: Prices are exponentially distributed → spike regimes
- The HPFC is derived from market prices of traded futures with the procedure by Benth et al. (2007): The deviation from the seasonal component is modeled by a spline with maximum smoothness
**Model for electricity prices: Definitions**

- $S_t$ market spot price for hour $t$
- $f_t$ forward price for hour $t$ derived from HPFC
- $f^L_t$ lower limit of normal regime
- $f^U_t$ upper limit of normal regime

The limits of the normal regime are derived as

$$
 f^L_t := f_t \cdot e^{-\alpha^L_{\delta}} \quad \quad f^U_t := f_t \cdot e^{\alpha^U_{\delta}}
$$

where the parameters $\alpha^L_{\delta}, \alpha^U_{\delta}, \delta \in \{1, 2\}$, must be estimated.

We distinguish between Monday to Saturday ($\delta = 1$) and Sunday ($\delta = 2$).
Model for electricity prices: Time blocks

The price volatility or the probability and magnitude of spikes may be different
- in summer and winter,
- at weekdays (Mo-Fr), Saturday and Sunday,
- at different times of the day.

Times of the day are distinguished by the blocks:

<table>
<thead>
<tr>
<th>season</th>
<th>night</th>
<th>morning</th>
<th>high noon</th>
<th>afternoon</th>
<th>rush hour</th>
<th>evening</th>
</tr>
</thead>
<tbody>
<tr>
<td>summer</td>
<td>1–6</td>
<td>7–10</td>
<td>11–14</td>
<td>15–18</td>
<td>–</td>
<td>19–24</td>
</tr>
<tr>
<td>winter</td>
<td>1–6</td>
<td>7–10</td>
<td>11–14</td>
<td>15–16</td>
<td>17–20</td>
<td>21–24</td>
</tr>
</tbody>
</table>

→ $H = 33$ hourly blocks for which parameters must be estimated (normal regime).

For the spike regimes we distinguish only between days and seasons
→ $D = 6$ parameter sets

We define two functions

$$h(t) : t \rightarrow \{1, \ldots, H\} \quad d(t) : t \rightarrow \{1, \ldots, D\}$$

that assign to the hour $t$ the index $h$ or $d$, respectively.
Model for electricity prices

\[
S_t := \begin{cases} 
  f_t^L - \text{spike}_t^- & \text{if system in lower spike regime} \\
  f_t \cdot \exp(r_t) & \text{if system is in normal regime} \\
  f_t^U + \text{spike}_t^+ & \text{if system is in upper spike regime}
\end{cases}
\]

with

\[
\text{spike}_t^- \sim \text{Exp}(\lambda_d^-), \quad \text{spike}_t^+ \sim \text{Exp}(\lambda_d^+), \quad d = 1, \ldots, D
\]

\[
r_t = a_h + \sum_{i=1}^{6} b_{i,h} \cdot \hat{r}_{t-i} + b_{24,h} \cdot \hat{r}_{t-24} + \varepsilon_t, \quad \varepsilon_t \sim \text{N}(0, \sigma_h^2), \quad h = 1, \ldots, H
\]

\[
\hat{r}_{t-i} = \begin{cases} 
  \ln S_{t-i} - \ln f_{t-i} & \text{if system is in normal regime at time } t-i, \\
  \mathbb{E}(r_{t-i}) & \text{otherwise.}
\end{cases}
\]

Transition matrix:

\[
\Pi_h = \begin{pmatrix} 
  1 - \pi_{h}^{LG} & \pi_{h}^{LG} & \pi_{h}^{GU} & 0 \\
  \pi_{h}^{GL} & 1 - \pi_{h}^{GL} - \pi_{h}^{GU} & \pi_{h}^{GU} & 0 \\
  0 & \pi_{h}^{UG} & 1 - \pi_{h}^{UG} & 0 \\
  \pi_{h}^{UG} & 0 & 0 & 1 - \pi_{h}^{UG}
\end{pmatrix}.
\]
**Estimation: Assigning observations to regimes**

The allocation of observations to regimes depends on the choice of $\alpha_{\delta}^L$ and $\alpha_{\delta}^U$, $\delta \in \{1, 2\}$.

Estimation of all model parameters in two steps:

1. A log-likelihood function $\ln \mathcal{L}(\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U)$ is maximized with respect to the four parameters.

2. Estimation of parameters for the three regimes based on current values of $\alpha_1^L, \alpha_2^L, \alpha_1^U$ and $\alpha_2^U$.

Observations given at hourly time points $t = 1, \ldots, T$ are assigned to sets

\[
\mathcal{D}^G(h') := \{t = 1, \ldots, T \mid h(t) = h' \land f_t^L \leq S_t \leq f_t^U \} \quad h' = 1, \ldots, H,
\]

\[
\mathcal{D}^L(d') := \{t = 1, \ldots, T \mid d(t) = d' \land S_t < f_t^L \} \quad d' = 1, \ldots, D
\]

\[
\mathcal{D}^U(d') := \{t = 1, \ldots, T \mid d(t) = d' \land S_t > f_t^U \} \quad d' = 1, \ldots, D
\]

Parameters of the exponential distributions of the spike magnitudes:

\[
\hat{\lambda}_{d'}^- = \frac{\# \text{elements in } \mathcal{D}^L(d')}{\sum_{t \in \mathcal{D}^L(d')} (f_t^L - S_t)}, \quad \hat{\lambda}_{d'}^+ = \frac{\# \text{elements in } \mathcal{D}^U(d')}{\sum_{t \in \mathcal{D}^U(d')} (S_t - f_t^U)}.
\]
Estimation: Parameters for the normal regime

Residuals of the autoregressive process for the blocks in the normal regime:

\[ e_t = \begin{cases} 
  r_t - a_{h(t)} - \sum_{i=1}^{6} b_{i,h(t)} \cdot \hat{r}_{t-i} - b_{24,h(t)} \cdot \hat{r}_{t-24}, & t \in \mathcal{D}^G(h(t)) \\
  0, & \text{otherwise}
\end{cases} \]

Missing values (because a spike occurred at this time) are replaced by their expectations:

\[ \hat{r}_\tau = \begin{cases} 
  \ln S_\tau - \ln f_\tau, & \tau \in \mathcal{D}^G(h(\tau)) \\
  a_{h(\tau)} + \sum_{j=1}^{6} b_{j,h(\tau)} \cdot \hat{r}_{\tau-j} + b_{24,h(\tau)} \cdot \hat{r}_{\tau-24}, & \text{otherwise}
\end{cases} \]

Parameters are estimated by minimizing the sum of squared errors:

\[ \min \sum_{t=1}^{T} e_t^2 \]

Unbiased estimator for the volatilities of block \( h' = 1, \ldots, H \):

\[ \hat{\sigma}_{h'}^2 = \frac{\sum_{t \in \mathcal{D}^G(h')} e_t^2}{\# \text{elements in } \mathcal{D}^G(h') - 8} \]
Estimation: Log-likelihood function

\[ \ln L(\alpha^L_1, \alpha^L_2, \alpha^U_1, \alpha^U_2) = \sum_{h'=1}^{H} \sum_{t \in D^G(h')} \ln \phi(e_t | 0, \hat{\sigma}_{h'}) \]

\[ + \sum_{d'=1}^{D} \sum_{t \in D^L(d')} \ln \varphi(f^L_t - S_t | \hat{\lambda}^-_{d'}) + \sum_{d'=1}^{D} \sum_{t \in D^U(d')} \ln \varphi(S_t - f^U_t | \hat{\lambda}^+_{d'}) \]

\( \phi \): density of the normal distribution with parameters \( \mu, \sigma \)

\( \varphi \): density of the exponential distribution with parameter \( \lambda \)

Determination of \( \alpha^L_1, \alpha^L_2, \alpha^U_1, \alpha^U_2 \):

- \( \ln L \) is maximized to find the optimal values for the regime limits
- In each step the other parameters are estimated as outlined above
- The optimal values for the limits are fixed and the log-likelihood function is maximized again with respect to the other parameters
- Standard errors are approximated from the outer products of the gradients
- Probabilities of transitions and of remaining in regimes are calculated from the number of corresponding observations.
Estimation results: Expected spike magnitudes

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Probabilities of transitions from normal to spike regimes
### Probabilities of transitions from spike to normal regimes & regime limits

<table>
<thead>
<tr>
<th>Sample period</th>
<th>01/01/2009 31/12/2010</th>
<th>01/01/2009 31/12/2011</th>
<th>01/01/2009 14/03/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>seas.</td>
<td>day</td>
<td>( \hat{\pi}_{h}^{LG} )</td>
<td>( \hat{\pi}_{h}^{UG} )</td>
</tr>
<tr>
<td>S</td>
<td>Mo-Fr</td>
<td>65.40</td>
<td>53.85</td>
</tr>
<tr>
<td>S</td>
<td>Sa</td>
<td>60.78</td>
<td>87.50</td>
</tr>
<tr>
<td>S</td>
<td>Su</td>
<td>70.24</td>
<td>95.83</td>
</tr>
<tr>
<td>W</td>
<td>Mo-Fr</td>
<td>65.22</td>
<td>80.36</td>
</tr>
<tr>
<td>W</td>
<td>Sa</td>
<td>73.08</td>
<td>33.33</td>
</tr>
<tr>
<td>W</td>
<td>Su</td>
<td>65.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day index</th>
<th>( \delta = 1 )</th>
<th>( \delta = 2 )</th>
<th>( \delta = 1 )</th>
<th>( \delta = 2 )</th>
<th>( \delta = 1 )</th>
<th>( \delta = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha}_{\delta}^{L} )</td>
<td>0.50</td>
<td>0.99</td>
<td>0.50</td>
<td>0.85</td>
<td>0.69</td>
<td>0.91</td>
</tr>
<tr>
<td>( \hat{\alpha}_{\delta}^{U} )</td>
<td>0.78</td>
<td>1.15</td>
<td>0.78</td>
<td>1.11</td>
<td>0.80</td>
<td>1.05</td>
</tr>
</tbody>
</table>
Risk premiums

- Risk premium = difference forward minus spot price
- Derived from simulation of 1000 scenarios with regime-switching model
- Higher magnitudes in winter than in summer
- Values are positive during the week and decrease or become negative for the weekend
- Premiums vary over time
Comparison with ARMA and GARCH

Time series models are frequently applied for electricity price simulations:

**ARMA(n, m)**

\[
X_t = c + \sum_{i=1}^{n} \alpha_i X_{t-i} + \sum_{j=1}^{m} \beta_j \varepsilon_{t-j} + \varepsilon_t
\]

**GARCH(p, q)**

\[
\sigma_t^2 = \phi_0 + \sum_{z=1}^{p} \phi_1 z \sigma_{t-z}^2 + \sum_{y=1}^{q} \phi_2 y \varepsilon_{t-y}^2
\]

- First electricity prices are deseasonalized
- ARMA(1, 1), ARMA(5, 1) and GARCH(1, 1) models are estimated for their stochastic components
- Model orders are defined by Akaike’s information criterion
- Engle’s ARCH test shows significant evidence for ARCH and GARCH effects
- Model parameters estimated with ML are not sample dependent, except ARMA(5, 1)
## Estimation results for ARMA and GARCH

<table>
<thead>
<tr>
<th>Sample</th>
<th>ARMA(1,1)</th>
<th>ARMA(5,1)</th>
<th>GARCH(1,1)</th>
<th>GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/01/2009–31/12/2010</td>
<td>c</td>
<td>0.035</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>0.825*</td>
<td>0.763*, 0.051, -0.018, 0.009*, 0.020*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta_j$</td>
<td>0.168*</td>
<td>0.228</td>
<td>-2.167*</td>
</tr>
<tr>
<td></td>
<td>$\phi_0$</td>
<td></td>
<td></td>
<td>0.046*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{1z}$</td>
<td></td>
<td></td>
<td>0.900*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{2y}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01/01/2009–31/12/2011</td>
<td>c</td>
<td>-0.08</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>0.827*</td>
<td>1.765*, -0.903*, 0.161*, -0.062*, 0.028*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta_j$</td>
<td>0.079*</td>
<td>-0.877*</td>
<td>-2.135*</td>
</tr>
<tr>
<td></td>
<td>$\phi_0$</td>
<td></td>
<td></td>
<td>0.015*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{1z}$</td>
<td></td>
<td></td>
<td>0.948*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{2y}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01/01/2009–14/03/2013</td>
<td>c</td>
<td>0.058*</td>
<td>0.086*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>0.879*</td>
<td>0.409*, 0.394*, 0.026*, 0.021*, -0.031*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta_j$</td>
<td>0.0003</td>
<td>0.470*</td>
<td>1.657*</td>
</tr>
<tr>
<td></td>
<td>$\phi_0$</td>
<td></td>
<td></td>
<td>0.016*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{1z}$</td>
<td></td>
<td></td>
<td>0.892*</td>
</tr>
<tr>
<td></td>
<td>$\phi_{2y}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = significant at 1% level
Simulation of 1000 scenarios

Graphs showing price dynamics in electricity spot markets:
- **First week in March 2009**
- **EUR/MWh**
- **Spot price observed**
- **ARMA(1,1) modeled prices**
- **GARCH(1,1) modeled prices**
- **ARMA(5,1) modeled prices**
- **RS modeled prices**
Simulation results

- ARMA/GARCH: Stochastic component is simulated, then seasonality shape is added
- Regime-switching (RS) model: Prices are simulated directly (seasonality incorporated in HPFC)
- Goodness-of-fit is assessed by $R^2$ and the mean average percentage error ($MAPE$):

$$E(\text{MAPE}) = \frac{1}{N} \sum_{k=1}^{N} \frac{1}{T} \sum_{t=1}^{T} \frac{|S_{k,t}^{\text{sim}} - S_t|}{S_t}$$

where

- $S_{k,t}^{\text{sim}}$ is simulated price in path $k = 1, \ldots, N$ and
- $S_t$ is the real price at time $t = 1, \ldots, T$

<table>
<thead>
<tr>
<th>Sample</th>
<th>ARMA(1,1) MAPE</th>
<th>ARMA(5,1) MAPE</th>
<th>GARCH(1,1) MAPE</th>
<th>RS model MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/01/2009–</td>
<td>0.135</td>
<td>0.136</td>
<td>0.095</td>
<td>0.079</td>
</tr>
<tr>
<td>31/12/2010</td>
<td>0.490</td>
<td>0.493</td>
<td>0.490</td>
<td>0.607</td>
</tr>
<tr>
<td>01/01/2009–</td>
<td>0.144</td>
<td>0.143</td>
<td>0.096</td>
<td>0.084</td>
</tr>
<tr>
<td>31/12/2011</td>
<td>0.442</td>
<td>0.442</td>
<td>0.439</td>
<td>0.652</td>
</tr>
<tr>
<td>01/01/2009–</td>
<td>0.150</td>
<td>0.15</td>
<td>0.086</td>
<td>0.083</td>
</tr>
<tr>
<td>14/03/2013</td>
<td>0.414</td>
<td>0.414</td>
<td>0.409</td>
<td>0.617</td>
</tr>
</tbody>
</table>
Summary

- Classical time series models are based only on historical data, no information on future is used
- RS model is built on market view reflected by price forward curves, spot prices vary around HPFC
- A base and two spike regimes are distinguished:
  - Autoregressive process takes dependencies between consecutive hours into account (normal regime)
  - Negative prices and spike (clusters) can be reflected
  - Probabilities modeled by transition matrix
- Electricity prices show different volatilities and spike behavior depending on time of day, weekday or season
- Reflected by different parameter sets, estimated probabilities confirm this assumption
- Small estimated risk premiums, vary over time; consistent with literature
- Higher $R^2$ and smaller errors than time series models
- Potential applications: Simulation of spot prices for medium and long term planning of electricity production, risk management, valuation of power contracts etc.