Modeling negative electricity prices

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Outlook

• We discuss the particularities of electricity prices
• We propose a novel price modeling approach
• We test the new model against other classical time series models
Motivation

• Modeling electricity prices is particularly challenging among commodities, given their particularities:

  – **Extremely large price movements (spikes)** occur because of the *inelastic demand* and *the lack of efficient storing capacities*

  – **Negative prices** occur given *the limited storing capacities* and *the limited load change flexibility* (since Sept. 2008 negative prices are allowed at EEX)

• Literature on modeling spikes and negative prices: Keles et al. (2011), Fanone et al. (2012)

• Most of the existing models for electricity do not account for negative prices so far. However, this is important for production planning or for realistic stress testing techniques at EEX.
Negative prices at EEX

- Between September 2008 and December 2011, historical spot market data show a total amount of about 100 hours with negative prices. Mostly, they occur in the night and morning hours (23:00 to 08:00)

Figure 1: *Occurrence of negative prices Sept. 2008-Dec. 2011 on different hours*
Data

- In our analysis we will refer to EEX Phelix hourly electricity prices between 01.01.2008–01.01.2012
- Within a time span of 24 hours, prices increase and decrease with a distinct hourly pattern. The price behavior clearly follows the electricity demand
- Prices start to increase at around 5 a.m. when people wake up and start working, and they decrease after 8 p.m. when work and household activities are over
- The intra-day behavior shows obvious differences between the different seasons. In winter there are two price peaks during a day, one at noon and one around 7 p.m., whereas in summer there is only one peak at noon
- One can also observe that intra-day patterns differ between weekdays. The weekend patterns in particular diverge distinctively from the workday patterns
Modeling assumptions

• Electricity prices have one *deterministic* (seasonality) and one *stochastic* component (ex: power plant outages)

• In our modeling approaches we capture both patterns:
  
  – we derive hourly price forward curves (HPFC) which capture the typical seasonality pattern (estimation procedure following Fleten & Lemming (2003) and Blöchlinger (2007)).
   
  – we consider that spot prices fluctuate around the HPFC, due to uncertain parameters (power plant outages, fluctuant renewable electricity generation, etc.)

• **Regime switching model**: electricity prices jump into another price level “spike regime” and remain there for some hours. Afterwards prices jump back to the “base regime”
Derivation of the seasonality shape

- We identify and forecast the daily and yearly seasonality structure of electricity prices:

\[ f_{2y_d} = \frac{S^{\text{day}}(d)}{\sum_{k \in \text{year}(d)} S^{\text{day}}(k) \frac{1}{K(d)}} \]  \hspace{1cm} (1)

\[ f_{2y_d} = a_0 + \sum_{i=1}^{6} b_i D_{di} + \sum_{i=1}^{12} c_i M_{di} + \sum_{i=1}^{3} d_i \text{CDD}_{di} + \sum_{i=1}^{3} e_i \text{HDD}_{di} + \varepsilon \]  \hspace{1cm} (2)

- \text{CDD}_{di}: Cooling degree days for 3 different German cities
- \text{HDD}_{di}: Heating degree days for 3 different German cities

\[ f_{2d_t} = \frac{S^{\text{hour}}(t)}{\sum_{k \in \text{day}(t)} S^{\text{hour}}(k) \frac{1}{24}} \]  \hspace{1cm} (3)

\[ f_{2d_t} = a_o^c + \sum_{i=1}^{23} b_i^c H_{t,i} + \varepsilon_t \text{ for all } t \epsilon c. \]  \hspace{1cm} (4)

- where \( H_i = 0, \ldots, 23 \) represents Dummy variables for the hours of one day.

- The shape \( s_{wt} \) of the HPFC can be calculated by \( s_{wt} = f_{2y_t} \cdot f_{2d_t} \).
Derivation of HPFC

- Let $f_t$ be the price of the forward contract with delivery in a certain hour of the day $t$, and let $F(T_1, T_2)$ be the price of forward contract with delivery in the interval $[T_1, T_2]$. This latter contract is a portfolio of basic forward contracts $f_t$ which under the arbitrage condition becomes:

$$F(T_1, T_2) = \sum_{t=T_1}^{T_2} \frac{1}{\sum_{t=T_1}^{T_2} \exp(-rt)} \sum_{t=T_1}^{T_2} \exp(-rt)f_t$$

(5)

Since no real prices are observed, but bid/ask spread, we replace the equality condition with the following:

$$F(T_1, T_2)_{bid} \leq \frac{1}{\sum_{t=T_1}^{T_2} \exp(-rt)} \sum_{t=T_1}^{T_2} \exp(-rt)f_t \leq F(T_1, T_2)_{ask}$$

(6)

- A realistic price forward curve should capture information about the hourly seasonality pattern of electricity prices. Thus, the objective function accordingly to Fleten & Lemming (2003) is to minimize:

$$\min \left[ \sum_{t=1}^{T} (f_t - s_t)^2 + k_{smooth} \sum_{t=1}^{T-1} (f_{t-1} - 2f_t + f_{t+1})^2 \right]$$

(7)
Regime switching model

- Data used: hourly spot and HPFC data for EEX Phelix 01.01.2008–01.01.2012

- We employ a *regime-switching approach* to simulate the transition of prices among regimes. We consider three different regimes: “base regime”, upper or lower “spike regimes”

- As *input* for our model, the HPFC is generated for the next trading day at EEX (day-ahead HPFC). We consider that the spot price fluctuates around the HPFC, due to uncertain parameters such as power plant outages, fluctuant renewable electricity generation

- Since the prices on the spot market vary in general for each hour of the day, *probabilities for upwards or downwards spikes* are derived for each hour of each week day (168 parameters)

- Furthermore, we assume that *upwards or downwards spikes are exponentially distributed* and we determine the expected spike size for each hour of each week day
Model for electricity prices

\[ MCP_t := \begin{cases} 
  f_t^L - Spike_t^- & \text{with } p_t^- \\
  f_t \cdot \exp(r_t) & \text{with } 1 - p_t^- - p_t^+ \\
  f_t^U + Spike_t^+ & \text{with } p_t^+ 
\end{cases} \]

with

\[ \begin{align*}
  Spice_t^+ & \sim \text{Exp}(1/\lambda_t^+) \\
  Spike_t^- & \sim \text{Exp}(1/\lambda_t^-) \\
  r_t & \sim N(0, \sigma_t^2) \\
  f_t^L &= f_t \cdot \exp(\alpha_L \cdot \sigma_t) \\
  f_t^U &= f_t \cdot \exp(\alpha_U \cdot \sigma_t)
\end{align*} \]

where:

- \( MCP_t \): Market clearing price (or spot price) for hour \( t \)
- \( f_t \): 1st day HPFC, forward price for hour \( t \)
- \( t \): Week hour (1,...,168)
Results

- The model parameters are estimated using maximum likelihood.

- To calculate the goodness of fit we generate 1000 scenarios *in sample* (estimation sample September 2008-December 2011) as well as *out of sample* (based on the parameters estimated between September 2008-December 2010).

- We compute quality factors like $R^2$ and **mean average percentage error (MAPE)** to compare the simulated and historical price paths for different estimation samples.

<table>
<thead>
<tr>
<th>Estimation sample</th>
<th>$R^2$</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.09.2008-01.12.2011</td>
<td>0.5596</td>
<td>0.168</td>
</tr>
<tr>
<td>01.09.2008-01.12.2010</td>
<td>0.5727</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Table 1: *Statistics over 1000 spot prices scenarios versus historical prices*
Results in sample

Figure 2: 1000 EEX Phelix spot prices in sample scenarios quantiles starting in 01.09.2008 on a horizon of 1 month
Robustness check

• We test the performance of our model versus classical time series models applied for electricity

• We derive the stochastic component of electricity prices by deducing from the hourly observed prices the level shape $lsw_t$ (multiplication of the index shape $sw_t$ with average yearly electricity prices)

\[
StochP_t = P_t - lsw_t
\]  

(8)

• We performed an Engle’s ARCH test in the stochastic component and we found significant evidence in support of ARCH and GARCH effects

• The following time series models were estimated for the stochastic component of electricity prices:
  
  – $ARMA(1, 1)$
  
  – $ARMA(5, 1)$
  
  – $GARCH(1, 1)$ with Gaussian innovations
  
  – $GARCH(1, 1)$ with $t$ innovations
Autocorrelation function before and after deseasonalization

Sample ACF of spot electricity prices before deseasonalization

Sample ACF of spot electricity prices after deseasonalization
Robustness check

- We look at statistics over 1000 scenarios simulated spot prices and we compute the MAPE and $R^2$ for the different models

<table>
<thead>
<tr>
<th>Sample</th>
<th>Quality factor</th>
<th>$ARMA(1, 1)$</th>
<th>$GARCH(1, 1)$ G</th>
<th>$GARCH(1, 1)$ t</th>
<th>RS model</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.09.2008-01.12.2011</td>
<td>MAPE</td>
<td>0.200</td>
<td>0.253</td>
<td>0.248</td>
<td>0.168</td>
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<td></td>
<td>$R^2$</td>
<td>0.213</td>
<td>0.199</td>
<td>0.201</td>
<td>0.560</td>
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<tr>
<td>01.09.2008-01.12.2010</td>
<td>MAPE</td>
<td>0.205</td>
<td>0.244</td>
<td>0.240</td>
<td>0.157</td>
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<tr>
<td></td>
<td>$R^2$</td>
<td>0.346</td>
<td>0.338</td>
<td>0.339</td>
<td>0.573</td>
</tr>
</tbody>
</table>

Table 2: Statistics over 1000 spot prices scenarios versus historical prices
GARCH(1,1) model performance

Horizon: 1 month, starting in 01.09.2008
EUR/MWh

- GARCH(1,1) with Gaussian innov simulated
- Observed spot
Conclusion

- From a graphical comparison of simulated and historical prices it can be concluded that the RS model reflects:
  - The daily, weekly and annual cycles are realistically described by the simulated prices
  - The model generates important properties like: single peaks or jump groups
  - The mean-reverting property is captured very well by the model

- The statistics show that the model performance is not sample dependent

- ARMA and GARCH processes are less suitable for the simulation of electricity prices, although it can handle the heteroscedasticity
  - ARMA and GARCH processes deliver price paths which are more volatile than the historical ones
  - The error terms are generally higher than RS model and the $R^2$ is significantly lower
Critical reflection

• The modeling approach for the deterministic components considers not only a long-term trend, but it takes into account changes in the price level due to structural changes, since the seasonality shape index is aligned to annual average prices. This is a value added to the existing literature (see Keles, 2011)

• The advantage of looking at spot prices instead of taking log-prices is that on average they lead to better parameter estimates (see Janczura and Weron, 2010a)

• Our novel approach offers the possibility to forecast spot prices on long term, if we simulate them around a long-term HPFC

• The above described models simulate different electricity prices characteristics: trend, seasonal cycles, jumps and stochastic volatility. However, to capture changes of the power plant structure other models are necessary, e.g. a fundamental model
References


