Medium-term planning for thermal electricity production

Florentina Paraschiv

(joint work with Raimund Kovacevic, University of Vienna)

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Outlook

• We aim at a simplified model for **mid-term planning for thermal electricity production** that can be used for repetitive calculation

• **Optimization model:**
  - Costs: fuel, fixed and variable operating costs
  - Different fuels are bought at the spot market and stored to produce electricity
  - We allow for trading at CO2 spot market (emission certificates)
  - Production is sold at the spot market
  - **Maximization of the asset value** (cash + value of stored fuels) at the end of the planning horizon
Production

- Consider **time periods** \( t \in 0, 1, \ldots, T \) with length \( \Delta_t \).
- We model **thermal generators** \( i \) which may use different **fuels** \( j \) to produce energy \( x_{t,i,j} \) and are characterized by **efficiencies** \( \eta_{i,j} \) and **maximum power** \( \beta_i \), in particular
- We consider \( \Delta_t \) as **weeks**. If \( \Delta_t \) smaller, integer decisions related to switching, ramping, minimum power production constraints etc. become relevant
- **A cost model** for the generators:
  - **Fuel costs** (spot markets) are given by \( P_{t+1,j}(\omega) \cdot x_{t,i,j} / \eta_{i,j} \).
  - **Variable operating costs** are estimated by \( \gamma_i \cdot \sum_{j=1}^{J} x_{t,i,j} / (\beta_i \Delta_t) \)
  - In addition we consider **fixed operating costs** \( \kappa_i \) per time unit.
Storage

- We model storage $s_t$, cumulated CO$_2$-emissions $e_t$, cumulated CO$_2$-certificates $a_t$ and a **cash position** $w_t$.

- With $f_{t,j}$ denoting the amount of fuel $j$ bought at time $t$ storage develops as

  \[ s_{0,j} = s_j^0 \]  

  \[ s_{t,j} = s_{t-1,j} - \sum_{i=1}^{I} \frac{x_{t-1,i,j}}{\eta_{i,j}} + f_{t,j} \ \forall t > 0, j \]  

  \[ 0 \leq s_{t,j} \leq \bar{s}_j \ \forall t, j, \]  

and production is restricted by

  \[ \sum_{i=1}^{I} \frac{x_{t,i,j}}{\eta_{i,j}} \leq s_{t,j} \ \forall t, j. \]
CO₂-accounting

• If $\varepsilon_{ij}$ denotes the CO₂-emissions (t per MWh) of fuel $j$ if burned by generator $i$, the amount $e_t$ of CO₂ emitted is

$$e_0 = e^0.$$  

(5)

$$e_t = e_{t-1} + \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\varepsilon_{ij}}{\eta_{i,j}} \cdot x_{t-1,i,j} \quad \forall t > 0.$$  

• At each time it is possible to buy ($c_t \geq 0$) or sell ($c_t < 0$) certificates at the market for CO₂ allowances at prices $P^c_t$. Hence the accumulated amount of pollution covered by certificates is

$$a_0 = a^0$$

$$a_t = a_{t-1} + c_t \quad \forall t > 0.$$
Cash accounting

- The cash position starts with \( w_0 = w^0 - \sum_{j=1}^{J} P_{0,j} f_{0,j} \).

and develops by

\[
\begin{align*}
  w_t &= (1 + \rho_l) w_{t-1}^+ - (1 + \rho_b) w_{t-1}^- \\
  &+ P_t^x \cdot \sum_{i=1}^{I} \sum_{j=1}^{J} x_{t-1,i,j} \\
  &- \sum_{j=1}^{J} P_{t,j}^f \sum_{i=1}^{I} f_{t,j} \\
  &- P_t^c c_t \\
  &- \sum_{j=1}^{J} \zeta_j \left( s_{t,j} + s_{t-1,j} \right) \\
  &- \sum_{i=1}^{I} \frac{\gamma_i}{\beta_i} \cdot \sum_{j=1}^{J} x_{t-1,i,j} - \kappa_i \cdot \Delta_{t-1} \\
  &\quad \quad 0 < t < T
\end{align*}
\]

- At time \( T \) no fuel is bought anymore, but a penalty has to be paid if certificates are not sufficient: \((\theta + P_T^c)(e_T - a_T)^+\)

Medium-term planning for thermal electricity production – p.6
Optimization problem: Objective

- The producer aims at the asset value (excluding the value of generating units) at the end of the planning horizon

\[ v_T = w_T + \sum_{j=1}^{J} s_{T,j} \cdot P_{T,j}^f. \]  \hspace{1cm} (6)

- All prices are stochastic processes. Decisions at time \( t \) have to be taken with information available at time \( t \). Hence the decision variables are also stochastic. The equations and inequalities have to be understood as “holds almost surely”.

- Our objective is a mixture of expectation and \( AV@R \) with a mixing factor \( 0 \leq \lambda \leq 1 \)

\[
\max_{x, f, c, (s, w, v, a, e)} (1 - \lambda) \cdot E[v_T] + \lambda \cdot AV@R_\alpha(v_T)
\]

s.t. all constraints

\[ x, f, c \prec \Sigma \]
\[ s, w, v, a, e \prec \Sigma \].
Modeling the risk factors

- We look at daily European commodity prices:
  - Gas prices: Gaspool (GPL), April 2007-December 2011
  - Crude oil prices: Brent Crude oil, May 2003-December 2011
  - EUA: April 2008-December 2011
  - Coal: North West Europe (NWE) steam coal marker, December 2005-May 2012
  - Electricity prices: EEX Phelix, September 2008-December 2011

- We employ a common model for simulating commodity prices: gas, oil, coal and emissions allowances (EUA)
  - Similar patterns among commodity prices: leptokurtic distribution, negatively skewed returns, non-stationary variation are described by Geometric Brownian Motion with Jump Process (GBMPJ)/Merton model

- Spot electricity prices behave considerably different from other commodities and need a separate modeling approach: Regime Switching Model
Regime switching model for electricity prices

\[ MCP_t := \begin{cases} 
  f_t^L - Spike_t^- & \text{with } p_t^- \\
  f_t \cdot \exp(r_t) & \text{with } 1 - p_t^- - p_t^+ \\
  f_t^U + Spike_t^+ & \text{with } p_t^+ 
\end{cases} \]

with

\[ Spike_t^+ \sim \text{Exp}(1/\lambda_t^+) \]
\[ Spike_t^- \sim \text{Exp}(1/\lambda_t^-) \]
\[ r_t \sim N(0, \sigma_t^2) \]
\[ f_t^L = f_t \ast \exp(\alpha_L \ast \sigma_t) \]
\[ f_t^U = f_t \ast \exp(\alpha_U \ast \sigma_t) \]
## Energy prices: Results

<table>
<thead>
<tr>
<th>Sample</th>
<th>Parameter estimation</th>
<th>α</th>
<th>σ</th>
<th>λ</th>
<th>μ</th>
<th>δ</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude oil (monthly)</td>
<td>01.05.2003-01.12.2011</td>
<td>0.325</td>
<td>0.259</td>
<td>80.373</td>
<td>-0.0017</td>
<td>0.027</td>
<td>-5314.05</td>
</tr>
<tr>
<td></td>
<td>01.05.2003-01.12.2010</td>
<td>0.283</td>
<td>0.271</td>
<td>68.981</td>
<td>-0.0013</td>
<td>0.028</td>
<td>-4705.27</td>
</tr>
<tr>
<td>Heating oil (monthly)</td>
<td>01.05.2003-01.12.2011</td>
<td>0.218</td>
<td>0.245</td>
<td>99.953</td>
<td>-0.0005</td>
<td>0.028</td>
<td>-5405.37</td>
</tr>
<tr>
<td></td>
<td>01.05.2003-01.12.2010</td>
<td>0.158</td>
<td>0.253</td>
<td>103.751</td>
<td>0.0000</td>
<td>0.028</td>
<td>-4781.89</td>
</tr>
<tr>
<td>EUA (monthly)</td>
<td>01.04.2008-01.12.2011</td>
<td>0.178</td>
<td>0.254</td>
<td>81.165</td>
<td>-0.0051</td>
<td>0.036</td>
<td>-2152.87</td>
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<tr>
<td></td>
<td>01.04.2008-01.12.2010</td>
<td>0.327</td>
<td>0.268</td>
<td>78.921</td>
<td>-0.0057</td>
<td>0.036</td>
<td>-1595.28</td>
</tr>
<tr>
<td>Gas (monthly)</td>
<td>01.04.2007-01.12.2011</td>
<td>0.321</td>
<td>0.379</td>
<td>99.790</td>
<td>-0.0006</td>
<td>0.068</td>
<td>-2015.13</td>
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<tr>
<td></td>
<td>01.04.2008-01.12.2010</td>
<td>0.316</td>
<td>0.423</td>
<td>105.479</td>
<td>0.0003</td>
<td>0.071</td>
<td>-1514.65</td>
</tr>
<tr>
<td>Coal (weekly)</td>
<td>09.12.2005-01.12.2011</td>
<td>0.308</td>
<td>0.170</td>
<td>21.749</td>
<td>-0.0082</td>
<td>0.053</td>
<td>-552.264</td>
</tr>
<tr>
<td></td>
<td>09.12.2005-01.12.2010</td>
<td>0.437</td>
<td>0.172</td>
<td>25.860</td>
<td>-0.0098</td>
<td>0.052</td>
<td>-450.484</td>
</tr>
</tbody>
</table>

Table 1: **ML Estimation results of the GMBJ model for oil, EUA, gas and coal spot prices. Standard errors are in paranthesis.**
Electricity prices: Out of sample results
Scenario trees

- The above introduced optimization problem (??) cannot be solved directly.
- It is common in stochastic programming to approximate the relevant stochastic processes by scenario trees (which represent the filtration of the process).
- To keep the error of this approximation small, the tree should be “close” to an original process which can be measured by an appropriate distance $d_l$
- It can be shown that, under certain conditions, it holds for the difference between the original and the approximated problem

$$|v(\mathbb{P}) - v(\tilde{\mathbb{P}})| \leq L \cdot d_l(\mathbb{P}, \tilde{\mathbb{P}}),$$

where $\mathbb{P}$ represents the original process and $\tilde{\mathbb{P}}$ its approximation.
- Thus, by keeping the distance $d_l$ between the processes as small as possible, we minimize the error on the left-hand-side of (??)
Minimizing the distance for distributions

- Consider a random variable $G$ that is either continuous or discrete (with a very large number of mass points)
- We want to approximate $G$ by a simpler random variable $\tilde{G}$
- The distance between $G$ and $\tilde{G}$ can be measured by the Wasserstein distance $d(G, \tilde{G})$
- It is known that the Wasserstein distance is related to a transportation problem
- The problem of minimizing the distance $d(G, \tilde{G})$ is solved by assigning data points to a few “clusters” (which represent the approximate distribution $\tilde{G}$)

\[
\mu = \begin{pmatrix} 2.37 \\ 1.83 \end{pmatrix} \\
\Sigma = \begin{pmatrix} 0.71 & 0.33 \\ 0.33 & 0.91 \end{pmatrix}
\]
The scenario generation problem

- In a multistage problem, decisions are taken at discrete time steps.
- The evolution of the data is described by a stochastic process:

![Graph showing stochastic process]

- This process is approximated by a *scenario tree* with corresponding path probabilities:

![Graph showing scenario tree]
Facility location applied to processes (1)

- Find a tree that is
  - small enough as a numerically tractable approximation
  - large enough to capture important features of the problem

- The scenario tree should be as close as possible to the observed stochastic process
- We apply the concept of the Wasserstein distance again to simulated paths to construct a multistage tree
- The picture on the left shows 1000 simulations of the oil price from January to March 2011
- Assume we generate a tree with two stages ($T = 2$); the right picture shows the facility location view of 1000 paths in $\mathbb{R}^2$
Facility location applied to processes (2)

- The facility location problem is now solved in a backward fashion, given a fixed number of nodes $n_1, \ldots, n_T$ per stage.
- We start at stage $T$ with $n_T$ clusters in $\mathbb{R}^T$, based on a multidimensional distance $d(\cdot)$.
- For the stages $t = T - 1, \ldots, 1$, we cluster $n_t$ points from the $n_{t+1}$ points found for the next stage, but using only data from stage 1 to $t$ (i.e., the facility location problem is solved in $\mathbb{R}^t$).
Facility location applied to processes (3)

- We store the allocation of data points to clusters to reconstruct the scenario structure.
- The probabilities of the scenario paths are given by the probabilities of the clusters that correspond to the $n_T$ leaves (found in the first step, not shown).
Extension to nested distances

- Pflug/Pichler (2012) introduced and analyzed a generalization of the well known Wasserstein distance
- Kovacevic/Pichler (2012) propose an algorithm for improving the distance between the trees
- This *nested distance* $d_l$ takes the information from the filtration into account (rather than comparing only scenario paths)
- Based on this concept, the tree resulting from the first step is further improved by adjusting the probabilities and values
System specification

- The thermal system consists of:
  - Two combined cycle plants (gas/oil)
  - Three combustion turbines (gas/oil)
  - One steam turbine (coal)
- Premises:
  - We start with a small amount of small fuel
  - Cash position: 1 million EUR
  - Interest on cash: 2.5%; on debt: 12.5%
  - AV@R calculated at level $\alpha = 0.05$
  - Mixture parameter $\lambda$ is set to 0.5 in the standard case
- Implementation: AIMMS 3.12, solver GUROBI 4.6
GUI: input trees
GUI: input trees

Electricity prices

Fuel prices
GUI: input trees
Development of the asset value
Distribution of the asset value - end of the planning horizon

Asset value, $T=52$

Density

$N = 306 \quad \text{Bandwidth} = 1.566\times10^7$
Efficient frontier. Tradeoff expected end value vs. riskiness of the end value
Effect of increases in CO2 prices on the accumulated CO2 emissions
Effect of CO2 prices

- A (general) increase of 1% in CO2 prices decreases the final asset value by $\sim 1.66\%$
- The decrease in CO2-emissions is (on average) $\sim 0.035\%$
Effect of CO2 prices

Gas burned (MWh) over the planning horizon: normal CO2-price

N = 306, Bandwidth = 5.964e+05
**Effect of CO2 prices**

Gas burned (MWh) over the planning horizon: CO2-price +5%
Effect of CO2 prices

Gas burned (MWh) over the planning horizon: CO2-price +10%
Effect of CO2 prices

Gas burned (MWh) over the planning horizon: CO2-price +20%
Effect of CO2 prices

Gas burned (MWh) over the planning horizon: CO2-price +30%
Effect of CO2 prices

Gas burned (MWh) over the planning horizon: CO2-price +50%

N = 306  Bandwidth = 5.93e+05
Indifference pricing

- Given the thermal system as described above, consider in addition an electricity delivery contract: A fixed amount $E$ of electricity has to be delivered (produced) during all weeks (52) of the planning horizon at a fixed price $K$.
- Which price is the minimum price such that the producer is interested to sign the contract?
- Solve with indifference pricing:

\[
\min_{K,(\ldots)} K
\]

\[
s.t. \lambda \cdot E[v_T] + (1 - \lambda) \cdot AV@R_\alpha(v_T) \geq v^*
\]

- All constraints of the original problem, except

* It is possible to buy electricity $y_t$ at the spot market,

* $\sum_{i \in I, j \in J} x_{t,i,j} + y_t \geq E$

* The cash calculation has to be corrected: $P_t^x \cdot (\sum_{i=1}^I \sum_{j=1}^J x_{t-1,i,j} - E) + K \cdot E$. 

Medium-term planning for thermal electricity production – p.34
Indifference pricing
Conclusion

- We specified a flexible model for mid-term planning, such that iterative analysis – repeatedly using the optimization model can be done in reasonable time.

- We simulated the risk factors: oil, gas, coal and CO2 emissions by a GBMJ process and electricity prices by a spot-forward model.

- Simulated hourly/daily commodity prices were aggregated to weekly average price scenarios and reduced to stochastic trees suitable for multistage optimization.

- We show the sensitivity of the asset value and of CO2 emissions to increases in the prices of the CO2 allowances.

- We investigated the pricing of electricity delivery contracts with fixed amount and price in the framework of indifference pricing.