

# A DYNAMIC NORTH-SOUTH MODEL OF DEMAND-INDUCED PRODUCT CYCLES\*

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## Abstract

This paper presents a dynamic North-South general-equilibrium model where per capita incomes shape demand patterns across regions. Innovation takes place in a rich North while firms in a poor South imitate products manufactured in North. Allowing a role for per capita incomes in determining demand delivers a complete international product cycle as described by Vernon (1966), where the different stages of the product cycle are not only determined by supply-side factors but also by the distribution of income between North and South. We analyze how changes in the gap between North and South due to changes in Southern labor productivity, population size in South and inequality across regions affect the international product cycle. In line with presented stylized facts, we predict a negative correlation between adoption time and per capita incomes.

**JEL classification:** F1, O3

**Keywords:** international product cycles, inequality, international trade

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# 1 Introduction

Recent years have seen an unprecedented expansion of international trade. This expansion has gone together with the entry of important emerging economies like China into the world market. This raises the following question: What are the global implications for innovations and long-run productivity growth? At first sight, there are two opposing forces at play. On the one hand, the rise of emerging economies triggers quicker imitation of Northern innovations, and on the other hand, when per capita incomes play a role for demand, growing incomes of consumers raise market demand for innovative products.

To analyze this question we take up the idea of the international product cycle, first proposed by Akamatsu (1961) and Vernon (1966). In particular, Vernon (1966) hypothesized that new goods would be introduced in countries with high per capita incomes (catering to the needs of a rich market), after some time demand for these goods emerges in poorer countries abroad as incomes grow and exports start. Later on, goods are imitated by less advanced countries, which have a relative cost advantage, such that the production moves there. Completing the cycle, goods that were once exported by rich countries are eventually imported by them. In a follow-up paper, Vernon (1979) explicitly emphasized the role of the demand side in shaping the typical product cycle:

In the early part of the post-war period, the US economy was the repository of a storehouse of innovations not yet exploited abroad, innovations that responded to the labour-scarce high-income conditions of the US market. As the years went on, other countries eventually achieved the income levels and acquired the relative labour costs that had prevailed earlier in the United States. As these countries tracked the terrain already traversed by the US economy, they developed an increasing demand for the products that had previously been generated in response to US needs. That circumstance provided the consequences characteristically associated with the product cycle sequence ... (Vernon 1979, p. 260).

To model the idea that per capita income plays a role for demand, we modify Grossman and Helpman's (1991) formalization of the product cycle by replacing constant-elasticity-of-substitution (CES) utility with non-homothetic utility. This setup provides a demand-based dynamic model which is able to generate the three stages of the product cycle as Vernon (1966) described: (1) a product is exclusively produced and consumed in North, (2) a product is produced in North and exported to South and (3) a product is imitated and exported from South to North.

What is the contribution of non-homothetic utility to the theory of international product cycles? Homothetic, separable CES utility cannot deliver a complete product cycle without introducing additional parameters (e.g. fixed export costs), as each consumer always buys all goods, irrespective of her income. Hence, supply-based approaches cannot capture the fact that countries with lower per capita incomes consume products later in the cycle (i.e. the first stage mentioned above is missing). This is inconsistent with the stylized fact that product

adoption strongly correlates with the level of per capita income (see Section 4).<sup>1</sup>

We analyze our question of interest in a dynamic general-equilibrium model of two regions, a wealthy North and a poor South. Households have identical non-homothetic utility functions defined over differentiated products such that consumption patterns differ across regions. In particular, households in North can afford to consume more and newer products than households in South. Monopolistic firms in North innovate new products (horizontal innovations) whereas competitive firms in South randomly target Northern products for imitation. Trading products across regions is costless (see Section 2.4.2 for a discussion of this simplifying assumption). In steady state, products follow the following cycle: A new product is developed and introduced in North. Only after a certain time have households in South become rich enough to afford a "new" product that is produced in North. This demand lag increases in the degree of inequality across regions and decreases, *ceteris paribus*, with the innovation rate.<sup>2</sup> Southern firms must invest resources in order to reverse engineer the production process of a randomly chosen product. Once they have invested the necessary resources, they enter into price competition with the innovating firm in North. Because they have a cost advantage due to lower wages, they can underbid the Northern innovator and capture the whole market. Hence, South becomes an exporter of that product. In this model, the average time span a product is being manufactured in North is determined endogenously. This mechanism describes, in the aggregate, a product cycle as described by Vernon (1966).

Attempts to formalize Vernon's product cycle date back to Krugman (1979). In his model, an advanced North introduces new products at a constant exogenous rate and a less advanced South copies those goods, also at a constant exogenous rate. Higher per capita income in North depends on quasi rents from the Northern monopoly in new goods, i.e. North must continually innovate to maintain its relative and absolute position. Subsequently, Grossman and Helpman (1991) endogenized innovation and imitation rates. In their model, long-run growth is faster the larger the resource base of South and the more productive its resources in learning the production process. The reason is that profits during the monopoly phase are higher when a smaller number of Northern firms compete for resources in the manufacturing sector, which outweighs the effect of a higher risk-adjusted interest rate since profits accrue on average for a shorter period of time. Both models, as well as recent work by Acemoglu, Gancia and Zilibotti (2012), focus on supply-side aspects of the product cycle theory, i.e. how the diffusion of technology and the determination of relative wages depend on technology parameters. In all these approaches, demand patterns in North and South are identical because agents have

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<sup>1</sup>Melitz (2003), for example, introduces fixed export costs, which incur when entering a foreign market, such that products can be ranked by technology (i.e. prices, which reflect productivity). In fact, fixed export costs are necessary (and sufficient) for the existence of an equilibrium with partitioning of firms by export status in which trade liberalization induces reallocation and distributional changes among firms. In our model, products can be ranked by preferences (i.e. reservation prices). More generally, Murata (2009) shows in a static general equilibrium model that products can be ranked by their relative importance of desirability and technological feasibility.

<sup>2</sup>The assumption of random imitation allows for the possibility that some products skip the export stage. However, our model implies that a product follows, on average, the typical product cycle. We discuss this in Section 2.5.

homothetic utility.

There is only a small strand of literature dealing with demand-side explanations of the product cycle. Flam and Helpman (1987) and Stokey (1991) focus on vertical innovations with quality differences between North and South. Different from our approach, Flam and Helpman (1987) rely on exogenous technical progress, whereas Stokey (1991) presents a static Ricardian trade model of different demand structures (similar examples include Markusen 1986, Falkinger 1990, Matsuyama 2000, Behrens and Murata 2012, and Li 2011). More closely related to our paper is Kugler and Zweimüller (2005), who propose a dynamic North-South model with non-homothetic utility. However, the exogenous determination of interest rates makes theirs a partial-equilibrium model. Furthermore, the focus of their analysis is on the cross-sectional composition of aggregate demand rather than on product cycles. Our paper, in contrast, studies endogenous imitation with horizontal innovations in a dynamic general-equilibrium "new" trade model. To our knowledge, we are the first to provide a tractable dynamic general-equilibrium in a two-country setting with non-homothetic utility that is able to deliver a complete product cycle in line with stylized facts.

An extensive strand of product cycle literature deals with intellectual property rights (IPR). For instance, Helpman (1993), Glass and Saggi (2002), and Lai (1998) find that stronger IPR reduce the rate of innovation when technology is transferred through imitation. Our model is consistent with these results. Although we are not interested in the effects of IPR on the product cycle and do not address this issue, we can still think of ways how IPR could enter the model. Tighter IPRs might be implemented by higher costs of imitation which imply a longer lifetime of the Northern product which in turn is related to a lower innovation rate.

The remainder of the paper is structured as follows. In Section 2, we introduce the model and solve for the steady state and transitional dynamics. Comparative statics results of changes in Southern productivity, relative country sizes and changes in inequality across regions are discussed in Section 3. To illustrate the different stages of the product cycle we look at the case study of the countertop microwave oven (and five other typical consumer durables in the twentieth century) in Section 4. Section 5 discusses heterogeneity on the demand and supply side. Eventually, Section 6 concludes.

## 2 Model

### 2.1 Distribution and endowments

The economy consists of two regions  $i \in \{N, S\}$ , an industrialized North ( $N$ ) and a less developed South ( $S$ ). The population size of the economy is  $L$ ; a fraction  $\beta$  lives in South and a fraction  $(1 - \beta)$  in North. We assume that each household regardless of its residence inelastically supplies one unit of labor on the local labor market. This implies that aggregate labor supply in South is given by  $\beta L$ , and by  $(1 - \beta)L$  in North. Furthermore, suppose that each household holds domestic and foreign assets. Hence, income inequality is endogenously determined and originates from differences in labor and capital incomes across countries.

With non-homothetic utility, the difference between gross domestic and gross national product, or generally, current account imbalances play a non-trivial role. Since we want to discuss a reduction of the income gap between North and South (see Section 3.3), we allow for a non-zero trade balance. To this end, we allow for a transfer system, e.g. foreign aid, between North and South. The transfer system assumes that each household in North pays a lump-sum transfer  $T_N(t) \geq 0$  and each household in South receives  $T_S(t)$ . The transfer system must run a balanced budget in each period such that  $(1 - \beta)LT_N(t) = \beta LT_S(t)$ , and transfers grow at the same rate as incomes. We will take  $T_S(t)$  as the exogenous variable so that through the balanced budget condition  $T_N(t)$  is endogenously determined.

## 2.2 Preferences

We specify the objective function of the household's static maximization problem as follows. There is a continuum of differentiated products in the economy indexed by  $j \in [0, \infty)$ , where only a subset  $N(t)$  is available on the market at a given point in time  $t$ . We assume differentiated products to be indivisible, and model consumption as a binary decision. Hence, households consume either 1 unit of product  $j$  at time  $t$ , or they don't consume that product at all. Note that for the sake of readability, we drop the region index  $i$  where no confusion arises. All households have the same preferences, with a non-homothetic instantaneous utility function given by

$$u\left(\{c(j, t)\}_{j=0}^{N(t)}\right) = \int_0^{N(t)} c(j, t) dj \quad (1)$$

where  $c(j, t)$  is an indicator function that takes the value one if product  $j$  is consumed, and zero otherwise. The indicator function  $c(j, t)$  will be specific to the income group, i.e. the region. The specification of the utility function contrasts with the constant-elasticity-of-substitution (CES) form as follows. For a given set of products, households can only choose consumption along the extensive margin with 0-1 preferences, i.e. choose how many different products they want to purchase. Instead, with CES utility they can only choose consumption along the intensive margin, i.e. how many units of each product they want to buy since households will always buy all available products.<sup>3</sup> Furthermore, note that the utility function in (1) is symmetric, i.e. no product is intrinsically better or worse than any other product. In other words, there is no explicit consumption hierarchy. This allows us to order products in ascending order from old to new, such that product  $j$  was developed before product  $j'$ , where  $j' > j$ .

Households maximize (1) with respect to the budget constraint  $E(t) = \int_0^{N(t)} p(j, t) c(j, t) dj$ , where  $E(t)$  denote expenditures at time  $t$ . Expenditures  $E(t)$  are given in the static problem but are determined endogenously in the intertemporal problem presented in Section 2.5 below. The second stage of two-stage budgeting applies here because

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<sup>3</sup>Generally, if income per capita increases, single-product firms will both increase quantities and prices if you hold the number of firms and products constant, when non-homothetic utility are assumed. In the homothetic CES case, only quantities increase and prices/markups stay constant. With 0-1 preferences, only prices increase and quantities stay constant. For intermediate cases, e.g. quadratic preferences giving rise to linear demand, both quantities and prices increase. See Appendix A.1 for a comparison of different non-homothetic utility functions.

preferences are separable. We get the following first order conditions:

$$c(j, t) = \begin{cases} 1 & p(j, t) \leq z(j, t) \\ 0 & p(j, t) > z(j, t) \end{cases} \quad (2)$$

where  $z(j, t) \equiv 1/\lambda(t)$  denotes the willingness to pay and  $\lambda(t)$  the Lagrange multiplier. Figure 1 below shows the individual demand curve (2) for product  $j$ . The Lagrange multiplier  $\lambda(t)$  can be interpreted as marginal utility of wealth. Households purchase one unit of a product if the price of that product does not exceed their willingness to pay. Since the utility function is symmetric over all products, the willingness to pay is identical for all products  $j$ . However, the willingness to pay depends on  $\lambda(t)$ , i.e. on the shadow price of expenditures. Hence, consumption patterns differ across regions since by our distributional assumptions expenditures differ across regions. Wealthy households in North, with a lower equilibrium value of  $\lambda$ , consume a larger set of products than poor households in South. Lemma 1 below makes this statement precise. It shows how the willingness to pay  $z(t)$ , equal to the inverse of the Lagrangian multiplier, depends on static expenditures and the distribution of prices.

**Lemma 1.** *Because of symmetry let goods' prices be ordered such that  $p(j', t) \geq p(j, t)$  for  $j' > j$ . We define  $P(N_i, t) = \int_0^{N_i} p(j, t) dj$  where  $N_i \leq N(t)$ . For given expenditures  $E_i$ , the willingness to pay  $z(j, t) = z(t) = 1/\lambda(t)$  is the same for all goods at any given point in time, and is given as follows*

$$z(t) = \begin{cases} p(N_i, t) & E_i(t) = P(N_i, t) < P(N(t), t) \\ p(N(t), t) & E_i(t) = P(N(t), t) \\ \infty & E_i(t) > P(N(t), t). \end{cases}$$

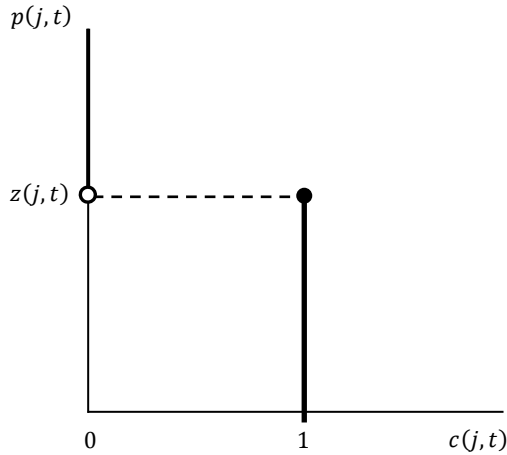


Figure 1: Individual demand

## 2.3 Technology and trade integration

### 2.3.1 Innovation technology in North

New products are designed and developed in high-income countries. Each firm in North is a single-product firm, which has access to the same innovation technology. The creation of a new product requires  $F^N(t) = F^N/N(t)$  units of labor, once this set-up cost has been incurred, the firm has access to a linear technology that requires  $b^N(t) = b^N/N(t)$  units of labor to produce one unit of output, with  $F^N, b^N > 0$  being positive constants. Innovations constitute an important spillover because they imply technical progress. We assume that the knowledge stock of the economy equals the number of known designs  $N(t)$ . The labor input coefficients are inversely related to the stock of knowledge. New products are protected by infinite patents but face a positive probability of being copied by a Southern firm (i.e. patent infringement). We assume that firms in North cannot license technology to Southern firms, or set up manufacturing plants in South (i.e. engage in foreign direct investment).

### 2.3.2 Imitation technology in South and transportation costs

As in Grossman and Helpman (1991), we assume that each new product, which has been developed in North at time  $t$ , faces the same positive probability of being imitated by a Southern firm at some time  $\tilde{t} > t$ . At the time the product is developed, date  $\tilde{t}$  is unknown. In other words,  $\tilde{t}$  is a random variable that represents the age of a product at the time of imitation. A Southern firm selects at random one of the existing products in North, which has not yet been copied, for imitation. We assume that firms in South benefit in reverse engineering and production from the total stock of knowledge (i.e. there are international knowledge spillovers).<sup>4</sup> Imitation of a selected product requires  $F^S(t) = F^S/N(t)$  units of labor, with  $F^S > 0$ . Investing  $F^S(t)$  allows a Southern firm to learn the production process of the randomly chosen product with probability one. Hence, there is complete certainty for a Southern imitator that reverse engineering succeeds. Subsequent production of the copied good requires  $b^S(t) = b^S/N(t)$  units of labor per unit. Finally, we assume that product markets are fully integrated and trade costs are zero.

## 2.4 Equilibrium

Depending on parameter values, two decentralized equilibria can emerge: (i) households in South are too poor to afford any Northern products or (ii) they can afford at least some Northern products. In case (i) no trade equilibrium exists. Hence, we focus on the interesting case (ii), and assume in the following that households in South can afford some Northern products. In proving the existence of the equilibrium, we will derive the necessary assumption on parameters. Let us denote the set of all products available in the economy as  $N(t) =$

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<sup>4</sup>The assumption of international knowledge spillovers is not essential for the model. We may also assume other forms of spillovers. In Appendix A.2, we explore the case of different knowledge spillovers where imitation in South is easier with respect to imported Northern products relative to non-imported Northern products (learning-by-importing).

$N^N(t) + N^S(t)$ , where  $N^N(t)$  denotes the subset of products that have not yet been imitated by South, and  $N^S(t)$  the subset of products that have been copied by South.

### 2.4.1 World demand

In the equilibrium we consider, households in North consume all products available in the market  $N_N(t) = N(t)$ , whereas households in South consume only a subset of all products  $N_S(t) \subset N(t)$ , which includes all products manufactured in South and some but not all Northern products. World demand for product  $j$  can be derived by horizontally aggregating individual demand (2) across regions. It is determined by:

$$C(j, t) = \begin{cases} 0, & p(j, t) > z_N(t) \\ (1 - \beta)L, & z_S(t) < p(j, t) \leq z_N(t) \\ L, & p(j, t) \leq z_S(t) \end{cases} \quad (3)$$

where  $z_i(t)$ , with  $i \in \{N, S\}$ , denotes the willingness to pay of households in North and South, respectively. From Lemma 1 we know that the willingness to pay is the same for all products  $j$ , hence aggregate demand is the same for all products. World demand (3) is depicted in Figure 2 below.

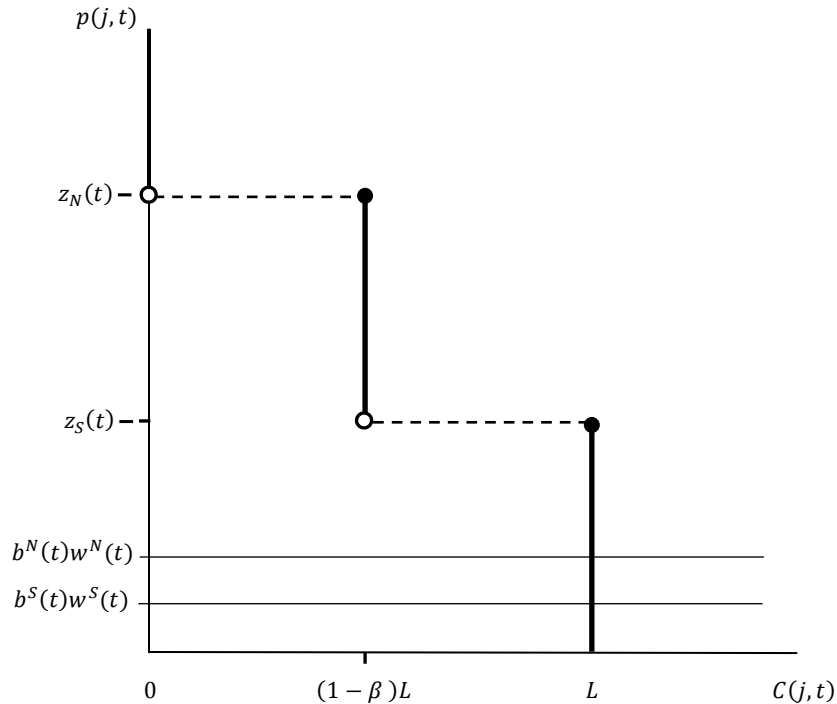


Figure 2: World demand

If the price of a product exceeds the willingness to pay of Northern households, there is no demand for that product. With a price between the willingness to pay of Southern and Northern households only the latter purchase the product. If the price falls short of the



willingness to pay of households in South everyone purchases it. Figure 2 is drawn under the assumption that the willingness to pay of Southern households exceeds marginal costs  $b^N(t)w^N(t)$ , which holds true in the equilibrium of interest.

#### 2.4.2 Aggregate supply

Let us first consider the problem of a monopolistic firm  $j$  located in North. Firm  $j$  maximizes operating profits

$$\pi^N(j, t) = [p(j, t) - w^N(t)b^N(t)] C(j, t) \quad (4)$$

subject to aggregate demand (3) by choosing a price  $p(j, t)$  such that marginal revenue equals marginal cost. From Figure 2 and the discussion in the previous section it follows that there are two candidates for the price that maximizes profits (4). Firm  $j$  either sets a high price equal to the willingness to pay of Northern households  $z_N(t)$  and sells exclusively to domestic households, or it sets a low price equal to the willingness to pay of Southern households  $z_S(t)$  and serves both markets.

Firms cannot price discriminate across regions. As there are no trade costs, arbitrageurs would take advantage of any price differential between North and South.<sup>5</sup> Thus, exporters set the same price in both regions. This implies that in equilibrium not all Northern firms export. To see this, suppose that at every point in time all Northern firms would set prices equal to the willingness to pay of Southern households and sell to everyone. In that case, households in North would not exhaust their budgets. According to Lemma 1, the shadow price of their (lifetime) income would become zero. That would imply an infinitely large willingness to pay for an additional product. Consequently, Northern firms have an incentive to deviate from selling to everyone and sell exclusively in North. Hence, a situation where all Northern firms serve all households cannot be an equilibrium.

In an equilibrium, where some Northern firms serve all households in both regions and others serve exclusively the domestic region, firms must be indifferent between selling only to Northern households and selling to all households at any point in time. Hence, the following arbitrage condition must hold

$$[z_N(t) - w^N(t)b^N(t)] (1 - \beta) L = [z_S(t) - w^N(t)b^N(t)] L. \quad (5)$$

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<sup>5</sup>The threat of arbitrage opportunities imposes a price setting restriction on firms. If there are no trade costs the price setting restriction is always binding. However, in the presence of iceberg trade costs the price setting restriction might not be binding. In particular, if the difference in per capita incomes between North and South were sufficiently low, all newly invented products would be exported to South right away. Hence, the introduction of iceberg trade costs contracts the set of all possible combinations of parameter values for which the equilibrium discussed in the text exists, however, that set does not become empty. For a discussion on the characteristics of the equilibrium emerging with iceberg trade costs in the case of static trade models with non-homothetic preferences see e.g. Foellmi, Hopenstrick and Zweimüller (2017) and Fajgelbaum, Grossman and Helpman (2011). Clearly, pricing to market is relevant, as e.g. Atkeson and Burstein (2008) demonstrate. With segmented markets and/or positive trade costs, firms would price discriminate across markets and set a higher price in North. In equilibrium, more firms would export to South compared to a situation where no price discrimination is possible. In the model, allowing for price discrimination would affect the arbitrage condition (5) such that the right hand side would be larger. While the *RS-curve* is unaffected, the *NA-curve* would shift to the left in Figure 4. In equilibrium, the share of products consumed by South  $m$  would be higher and - via the *RS-curve*, the labor market clearing in South - the growth rate  $g$  would be lower.

In the aggregate, a fraction  $n$  of firms sells in both North and South whereas  $(1 - n)$  firms sell only in North. Due to symmetric subutility, however, the behavior of an individual firm is indeterminate. Because we are free to order the different goods, we may think of the following firm behavior at the micro level that generates the described outcome at the macro level: After developing a new product each firm starts marketing its product solely in North and after a certain period of time has elapsed, i.e. the time it takes for incomes in South to have grown sufficiently, begins exporting. In that case, there are at any point in time new products that are sold exclusively in the domestic market and older products that are exported as well. Section 5 discusses extensions where the product cycle at the firm level is determinate. We argue that, while the model would become substantially more complex, the basic structure and intuition of the baseline model is preserved.

The Northern firm, which develops product  $j$  at time  $t$ , faces a positive probability that its product will be copied by a Southern firm. After a product has been imitated, the Southern firm maximizes operating profits

$$\pi^S(j, t) = [p(j, t) - w^S(t)b^S(t)] C(j, t)$$

where  $C(j, t) = L$  is given by (3). After the firm in South has copied the Northern product  $j$ , it enters into price competition with the Northern firm currently producing  $j$  (i.e. the innovating firm). This forces the Southern firm to set a limit price equal to the marginal costs of the competing firm in North. Hence, optimal prices of Southern products are equal to  $w^N(t)b^N(t)$ .<sup>6</sup>

### 2.4.3 Labor markets

Labor is immobile across regions but regional labor markets are assumed to be perfect. In particular, in North labor is completely mobile between production and R&D, and in South between production and reverse engineering. Labor market clearing in North demands that

$$(1 - \beta) L = g(t)F^N + b^N L [n(t) - m(t)] + (1 - \beta) b^N L [1 - n(t)] \quad (6)$$

where we defined  $g(t) \equiv \dot{N}(t)/N(t)$ , and the share of goods consumed and produced in South as  $n(t) \equiv N_S(t)/N(t)$  and  $m(t) \equiv N^S(t)/N(t)$ , respectively. The first term in (6) on the right-hand side denotes labor demand from the R&D sector, the second term labor demand from the production of older Northern products consumed by all households in both regions, and the third term labor demand from the production of newer Northern products exclusively

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<sup>6</sup>Grossman and Helpman (1991), henceforth GH, have to distinguish two cases: the wide-gap case and the narrow-gap case. In the wide-gap case the Southern monopoly price falls short of Northern marginal costs. This is the case when the gap between Southern and Northern wages is large. If the gap between wages is small, the Southern monopoly price would be larger than Northern marginal costs. A Southern firm that charges its monopoly price would be undercut by its Northern rival. Hence, in this narrow-gap case Southern firms charge prices marginally below the marginal cost of Northern firms. Our case is similar to GH's narrow-gap case. A situation where Southern firms set the monopoly price cannot occur in a trade equilibrium here since  $z^S(t) > w^N(t)b^N(t) > w^S(t)b^S(t)$ , as otherwise no firm in North would export to South.

consumed by Northern households. Similarly, labor market clearing in South requires

$$\beta L = g^S(t)m(t)F^S + m(t)b^S L \quad (7)$$

where we defined  $g^S(t) \equiv \dot{N}^S(t)/N^S(t)$ . The right-hand side in (7) denotes labor demand from reverse engineering, and production of imitated products which are consumed by all households in both regions.

#### 2.4.4 Capital markets

We assume that international capital markets are perfect, hence, interest rates equalize across regions. The expected present discounted value of profits of product  $j$  that was introduced at time  $t$  is determined by equation (8) below, given the instantaneous rate of imitation  $\mu(t) \equiv \dot{N}^S(t)/N^N(t)$ . We make the standard assumption of free entry into product development in North. Hence, the expected value  $v^N(j, t)$  of product  $j$  must equal R&D costs  $w^N(t)F^N(t)$ ,

$$v^N(j, t) = \int_t^\infty \exp\left(-\int_t^s (r(\tau) + \mu(\tau)) d\tau\right) \pi^N(j, s) ds = w^N(t)F^N(t). \quad (8)$$

Note that profits are discounted using the risk-adjusted interest rate  $r(\tau) + \mu(\tau)$ , where  $r(\tau)$  is the risk-free interest rate and  $\mu(\tau)$  the risk premium. Since we assume capital markets to be perfect, households can diversify away the idiosyncratic risk of a Northern firm being copied by holding a portfolio of shares in all Northern firms. Free entry also prevails in the reverse engineering sector in South, which is not an uncertain activity, so that their present discounted value of profits  $v^S(j, t)$  must equal the imitation cost  $w^S(t)F^S(t)$ ,

$$v^S(j, t) = \int_t^\infty \exp\left(-\int_t^s r(\tau) d\tau\right) \pi^S(j, s) ds = w^S(t)F^S(t). \quad (9)$$

#### 2.4.5 Asset holdings and balance of payments

The balance of payments in present value terms is determined by

$$\begin{aligned} 0 &= \int_0^\infty \{[(1 - \beta)LN^S(t)w^N(t)b^N(t) - \beta L(N_S(t) - N^S(t))z_S(t)] \\ &+ \beta LT_S(t)\} \exp\left(-\int_0^t r(s)ds\right) dt \end{aligned} \quad (10)$$

where the first term in brackets on the right-hand side denotes the trade balance and the second term net transfer payments. We assume that net foreign assets (portfolio investments) are zero.<sup>7</sup> Note that if  $T_S(t) > 0$  for all  $t$ , South runs a (permanent) trade deficit, i.e. the value of its exports falls short of the value of its imports.

<sup>7</sup>Because of equal interest rates, consumption growth is identical across regions in steady state. Hence, net foreign assets will remain zero forever. If net foreign assets are non-zero,  $T_S$  is to be interpreted as sum of transfer and interest payments. For a formal derivation of the balance of payments see Appendix A.3.

## 2.5 Steady state

The economy is in a steady state if Northern firms introduce new products at a constant rate  $g$  and Southern firms imitate at a constant rate  $\mu$ . In steady state, shares of resources devoted to R&D and production are constant, and the fraction of Northern products that have not yet been imitated is constant. Furthermore, prices of Northern and Southern products and therefore, profits of Northern firms are constant. Let us choose the marginal costs of production of Northern firms as the numeraire, and set  $w^N(t)b^N(t) = 1$  for all  $t$ .

First, we turn to the first-order conditions of the household's (intertemporal) maximization problem. The representative household maximizes utility over an infinite horizon with the following objective function

$$U(0) = \int_0^{\infty} e^{-\rho t} \log u(t) dt$$

where  $\rho > 0$  denotes the time preference rate and  $u(t)$  is the consumption aggregator of the different goods, whose value follows from maximizing (1) with respect to the static budget constraint  $E(t) = \int_0^{N(t)} p(j,t) c(j,t) dj$ . Applying the first stage of two-stage budgeting is appropriate because expenditures and the number of goods consumed are proportional in steady state. Households maximize their lifetime utility subject to their lifetime budget constraint

$$\int_{t=0}^{\infty} e^{-R(t)} E(t) dt \leq a(0) + \sum_{t=0}^{\infty} e^{-R(t)} [w(t) + T(t)] dt$$

where  $R(t) = \int_{s=0}^t r(s) ds$  denotes the cumulative interest rate,  $a(0)$  initial wealth, and  $w(t)$  the market-clearing wage rate. Along the optimal path, the growth rate of expenditures is characterized by the standard Euler equation

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \quad (11)$$

The Euler equation (11) implies equal growth rates of consumption in North and South. We guess and will verify in equilibrium that the growth rate of expenditures equals the growth rate of products

$$g = r - \rho. \quad (12)$$

Households' budget constraints in steady state are given in Appendix A.4.

Now, consider the equilibrium in the labor markets. The resource constraint in South (7) becomes

$$\beta L = gmF^S + mb^S L. \quad (13)$$

A higher fraction of products that have been imitated and produced  $m$  implies that less resources are available for imitation, hence  $g$  is lower ceteris paribus. The resource constraint of North (6) can be written as follows in the steady state

$$(1 - \beta) L = gF^N + Lb^N (n - m) + (1 - \beta) Lb^N (1 - n) \quad (14)$$

where  $n$  denotes the "consumption gap" between South and North. Note that a higher share of South in total production  $m$  releases resources from the production sector in North that can be reallocated to the R&D sector, *ceteris paribus*. This allows North to introduce new products at a higher rate  $g$ . Furthermore, a higher consumption share of South  $n$  induces a reallocation of resources from the R&D sector to the production sector in North to satisfy the additional demand for existing Northern products by South, thereby depressing innovation in North, *ceteris paribus*.

Next, a fixed inter-sectoral allocation of labor implies that prices of Northern products must be constant in steady state. We denote the price of a new product that is sold exclusively to Northern households as  $z_N$ . Since all firms face the same demand curve and have the same cost structure,  $z_N$  is identical for all new products  $j \in (N_S(t), N(t)]$ . From the arbitrage condition (5) follows that prices for all old Northern goods  $j \in (N^S(t), N_S(t)]$ , which are sold to all households, are also constant and determined by  $z_S = \beta + (1 - \beta) z_N$ . Moreover, this implies that profits are constant over time. Prices of Southern products  $w^N(t)b^N(t)$  are equal to 1 due to our choice of numeraire. This is consistent with the steady state, else demand for Southern labor would change over time.

Consider the *average* life cycle in steady state of some product  $j$ , which is introduced at time  $t$ . In expectation, the product is imitated at  $\tilde{t} = t + 1/\mu$ , since the imitation rate  $\mu$  is constant in steady state. Between  $t$  and  $\tilde{t}$  production takes place in North. Since products are symmetric, all firms in North are indifferent between selling to Northern consumers or the whole world population. However, we are free to order the products in the way that the newest products are sold in North only and then exported to South.<sup>8</sup> At the time of introduction product  $j$  is sold at price  $z_N$  exclusively to Northern households. At time  $t + \Delta$ , where  $N(t) = N_S(t + \Delta) = N_S(t) \exp(g\Delta)$ , the Northern firm producing good  $j$  lowers the price to  $\beta + (1 - \beta) z_N$  and exports it to South. The average time length  $\Delta$ , until when a product is exported, is endogenously determined in the model. The average demand lag equals  $\Delta = -\log(n)/g > 0$ , decreasing in the consumption share  $n$  and the innovation rate  $g$ . In the equilibrium of interest,  $\tilde{t} > t + \Delta$  or  $1/\mu > -\log(n)/g$ .<sup>9</sup> Northern products are exported to South for some time before they are copied by a Southern firm. After imitation, due to lower production costs in South, Southern firms can set a price marginally below 1, the marginal costs of Northern firms. Hence, the Northern firm stops producing product  $j$  and the product is now exported to North. Of course, this discussion is only relevant for the *average* product. By the random nature of imitation there will be some products that skip the export stage (i.e. those products become "prematurely old"). The average life cycle of some product  $j$  in terms of sales volume is depicted in Figure 3 below.

From the definition of the imitation rate  $\mu = \dot{N}^S(t)/N^N(t)$ , we can express the production

<sup>8</sup>If there is asymmetry between goods, varieties with the highest utility/cost ratio are invented first and the length of the first stage of the product cycle is deterministic, we come back to this in Section 5.1.

<sup>9</sup>This condition follows from the fact that  $n > m$  must hold true otherwise the South would import no goods, which would violate the balance of payments with  $T \geq 0$  as required by Assumption 1. Since  $-\log(n) < -\log(m)$ , it is sufficient to show that  $1/\mu \geq -\log(m)/g$ . Using (15), this is equivalent to  $g/\mu \geq -\log(1 + g/\mu)$ , which always holds.

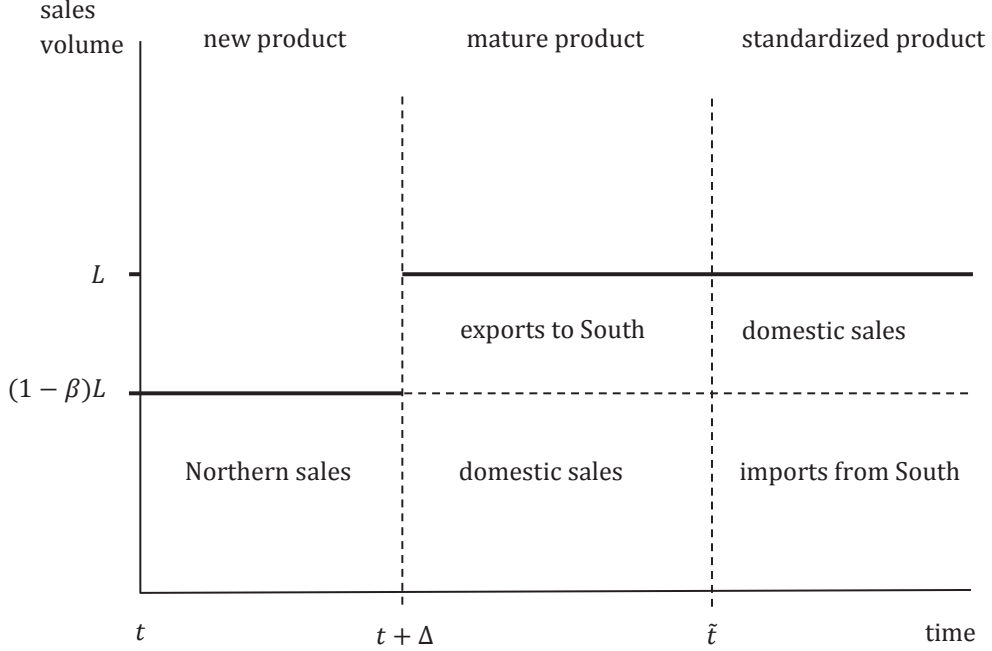


Figure 3: Average life cycle (in terms of sales volume)

share of South in the total number of differentiated products as

$$m = \frac{\mu}{g + \mu} \quad (15)$$

which must be constant in steady state. Next, the zero-profit condition (8) together with the arbitrage condition (5) in North implies that in steady state the value of a firm is equal to the expected present discounted value of its future profits

$$\frac{[z_N - 1](1 - \beta)L}{r + \mu} = \frac{F^N}{b^N}. \quad (16)$$

Similarly, in South the zero-profit condition (9) yields

$$\frac{[1 - \omega^S b^S]L}{r} = \omega^S F^S \quad (17)$$

where  $\omega^S \equiv w^S(t)/N(t)$ . From equation (17) follows that  $\omega^S$  must be constant in steady state. Hence, wages in South grow at rate  $g$ . Last, in steady state, the balance of payments (10) becomes

$$(n - m)[\beta + (1 - \beta)z_N]\beta = m(1 - \beta) + \beta T \quad (18)$$

where  $T \equiv T_S(t)/N(t)$ . Note that due to Walras' law the balance of payments is implied by the budget constraints, the zero-profit conditions and the resource constraints. Further, instantaneous expenditures equal  $z_N N(t)$  and  $z_S m N(t)$  in North and South, respectively. This

is consistent with our initial guess that expenditures grow at rate  $g$ .

Equations (12) - (18) in unknowns  $g$ ,  $\mu$ ,  $n$ ,  $m$ ,  $r$ ,  $z_N$ , and  $\omega^S$  fully characterize the steady state. We can reduce this system to 2 equations in 2 unknowns  $m$  and  $g$ . The first equation, the *RS-curve*, describes a steady state relationship between  $g$  and  $m$  that is consistent with labor market clearing in South:

$$m = \frac{\beta L}{g F^S + b^S L}. \quad (19)$$

The second equation, the *NA-curve*, describes a steady state relationship between  $g$  and  $m$  that is consistent with labor market clearing in North, the balance of payments, free entry in North, and the no arbitrage condition

$$\left(1 + \rho \frac{F^N}{b^N L} + \frac{g}{1-m} \frac{F^N}{b^N L}\right) \left((1-\beta) \left(\frac{1}{b^N} - 1 + m\right) - g \frac{F^N}{b^N L}\right) = m(1-\beta) + \beta T. \quad (20)$$

Intuitively, the *NA-curve* is upward sloping because a higher growth rate is only compatible with North's resource constraint (14) when  $m$  is higher, i.e., less products are produced in North. Further, a higher  $g$  implies higher interest rates which mean higher  $z_N$  and  $z_S$ , see (16) and (6). The increase in the export price  $z_S$ , means that for given  $m$ , the share of products consumed in South  $n$  decreases for a given  $m$ . This terms-of-trade effect further relaxes the resource constraint (14).

To guarantee that the *NA-curve* defined by (20) has a positive  $x$ -axis intercept in the  $(m, g)$  space we make the following assumption.

**Assumption 1.**  $\left(1 + \rho \frac{F^N}{b^N L}\right) (1-\beta) \left(\frac{1}{b^N} - 1\right) \geq \beta T \geq 0$ .

**Proposition 1.** *Given Assumption 1 holds, a steady state equilibrium with positive growth rate  $g$  and a constant share of imitated products  $m$  exists.*

*Proof.* The *RS-curve* (19) is downward sloping in the  $(m, g)$ -space. To determine the shape of the *NA-curve* we rewrite (20) as  $NA(m, g) = 0$ . The left hand side of this equation is a quadratic function in  $g$  with inverted U-shape. If Assumption 1 holds,  $NA(m, g)$  has a negative and a positive solution for  $g$ . Thus,  $NA_g(m, g) < 0$  at the relevant solution. Further, differentiation shows that  $NA_m(m, g) = \frac{g}{(1-m)^2} \frac{F^N}{b^N L} \left((1-\beta) \left(\frac{1}{b^N} - 1 + m\right) - g \frac{F^N}{b^N L}\right) + \left(\rho \frac{F^N}{b^N L} + \frac{g}{1-m} \frac{F^N}{b^N L}\right) (1-\beta) > 0$ . Hence, the *NA-curve* has a positive slope and positive intercept with the  $x$ -axis and a negative intercept with the  $y$ -axis in the  $(m, g)$ -space. Figure 4 below depicts the graphical solution of the steady state.<sup>10</sup>  $\square$

## 2.6 Transitional dynamics

The transitional dynamics are easy to characterize. The full derivation of the transitional dynamics, including a phase plane illustrating the dynamics, is given in Appendix A.5. If we

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<sup>10</sup>Note that the  $y$ -axis intercept  $m|_{g=0}$  implied by  $NA|_{g=0}$  is given by  $m = -\frac{\left(1 + \rho \frac{F^N}{b^N L}\right) (1-\beta) \left(\frac{1}{b^N} - 1\right) - \beta T}{\rho (1-\beta) \frac{F^N}{b^N L}} < 0$  due to Assumption 1.

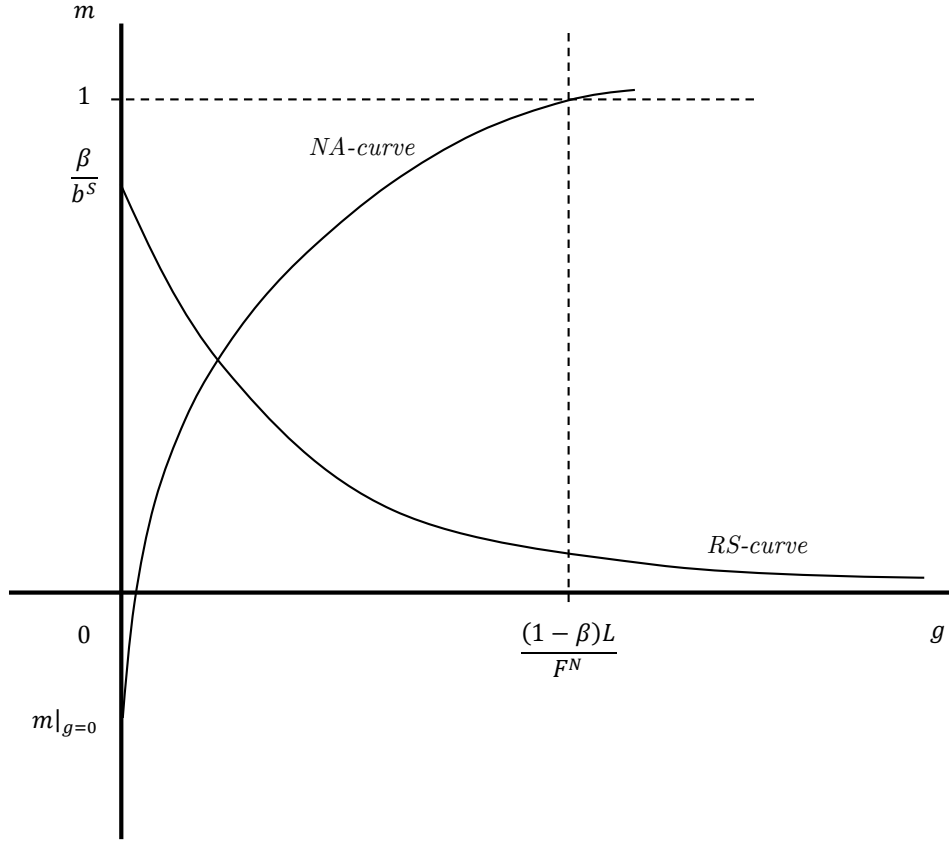


Figure 4: Steady state

replace  $g$  with  $g^S$  on the x-axis in Figure 4, the *RS-curve* now determined by equation (7), representing the Southern full employment condition only, must hold also outside the steady state. Hence, along a transition path,  $m$  and  $g^S$  move along the *RS-curve*. The *NA-curve* (20), instead, is a steady state condition. Appendix A.5 demonstrates that the steady state is saddle-path stable. When the number of varieties in South is below its steady state value,  $m(0) < m$ , then  $\dot{m}/m = g^S - g > 0$ , i.e. the growth rate of imitation is higher than the growth rate of innovation during the transition process. Thus,  $m$  converges monotonically to its steady state value.

### 3 Changing gap between North and South

How will the product cycle and therefore the international division of production and long-run innovation incentives change when the gap between North and South changes? In this section, we consider the cases of a rise in labor productivity in South, a larger Southern population, and an increased net wealth position of South in turn, and explore their steady state effects. A special focus lies on the implications on the length of the three product cycle stages, which are discussed at the end of each subsection. Simulated comparative statics results are relegated to



Appendix A.6.<sup>11</sup> In Section 3.4, we further highlight the difference between non-homothetic and homothetic utility by comparing the results from our model to those of Grossman and Helpman (1991).

### 3.1 Increase in Southern labor productivity

In recent years, relative productivity of the global South has risen. Jorgenson and Vu (2011) mention that "while labor has become nearly twice as productive over the last 20 years worldwide - it has risen even more so in the developing countries, with Asia in the lead." According to McMillan and Rodrik (2011) labor productivity growth in Asian countries between 1990-2005 was on average 3.87 percent, compared to 1.46 percent in high income countries. This raises the natural question of how changes in labor productivity in South affect the product cycle.

**Proposition 2.** *An increase in Southern productivity, i.e. a decrease in  $b^S$  or  $F^S$ , results in a higher growth rate  $g$ , Southern imitation share  $m$  and imitation rate  $\mu$ . Hence, the average time span a product is being manufactured in North  $1/\mu$  becomes shorter. While the terms of trade move in favor of North ( $z_S$  increases), two opposing effects move relative wages and the consumption share. Higher Southern productivity tends to increase Southern relative wages while the higher growth rate  $g$  tends to decrease them. A higher imitation rate expands the Southern consumption share whereas the higher growth rate lowers it.*

*Proof.* A decrease in  $b^S$  shifts the  $RS$ -curve upwards, whereas a decrease in  $F^S$  rotates the  $RS$ -curve upwards, both leaving the  $NA$ -curve unaffected. Hence, both a decrease in  $b^S$  and  $F^S$  lead to a higher growth rate  $g$  and Southern imitation and consumption share  $m$ . The imitation rate increases, as  $\mu$  depends positively on  $g$  and  $m$ . According to the Northern zero profit condition (16)  $z_N$  and  $z_S$  both increase. Using the Southern zero profit condition (17) we see that  $\omega^S$  increases with higher productivity in South but decreases in  $g$ . This implies that relative wages  $w^N(t)/w^S(t)$  decrease due to the direct productivity effect and increase because of a higher growth rate. Using the Northern resource constraint, we see that a higher  $g$  reduces  $n$  while the higher  $m$  raises  $n$ .  $\square$

Intuitively, a reduction in  $F^S$  or  $b^S$  triggers more imitation because it is cheaper to produce imitated goods. Thus, the imitation rate  $\mu$  and the share of products manufactured in South  $m$  rise, ceteris paribus. On the one hand, the higher risk-adjusted interest rate  $r + \mu$  lowers the present discounted value of profits earned from innovation. On the other hand, as the set  $m$  of cheap products produced in South expands the (real) income of households in both regions increases, which translates into a higher willingness to pay for both ( $z_S$  and  $z_N$  increase). The higher willingness to pay of Southern households implies that they can afford to buy more new products manufactured in North ( $n$  rises). Second, the higher willingness to pay of households

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<sup>11</sup>Note that the wage rate of North relative to South is determined by  $w^N(t)/w^S(t) = (\omega^S b^N)^{-1}$ , where  $\omega^S = (b^S + (\rho + g)F^S/L)^{-1}$  is pinned down by (17). Using the zero-profit condition (16) and the arbitrage condition in North, we get an expression for the terms of trade of North,  $z_S = 1 + [\rho + g/(1 - m)]F^N/b^N L$ .

in North makes the innovation of new products more attractive. The positive effect on the present discounted value of profits through higher prices  $z_N$  dominates the negative effect of a higher risk-premium  $\mu$  such that the innovation rate  $g$  rises.<sup>12</sup> Still, the imitation rate  $\mu$  goes up more than the innovation rate, hence  $m$  rises.

Looking at relative wages, the increase in Southern labor productivity directly increases the wage rate  $\omega^S$ , holding  $g$  constant. Moreover, there is also an indirect effect through the increase in  $g$ , which leads to an increase in the interest rate  $r$ , and therefore to a decrease in the present discounted value of profits earned from copying Northern products. The indirect effect induces less firms to enter the market in South, depressing labor demand, and hence the wage rate  $\omega^S$ . In simulations, the first effect dominates such that the Northern relative wage rate  $w^N(t)/w^S(t)$  falls.

### The effect on the product life-cycle

The time length  $\Delta$  where products are exclusively sold in North becomes shorter due to two reasons: households in South are relatively richer ( $n$  rises) and the overall growth rate  $g$  is higher. Since the imitation rate  $\mu$  increases, the average time span a product is being manufactured in North  $1/\mu$  becomes shorter as well. The third stage during which North imports a product clearly increases. The time period during which North exports a product ( $1/\mu - \Delta$ ) decreases according to our simulations.

## 3.2 Increase in Southern population

The entry of China into the world market in the 1980s can be seen as a rapid expansion of the Southern population. A fortiori, developing countries show higher population growth rates than developed countries. The World Bank (2014) reports a population growth rate for low and middle income countries of approx. 1.3 percent p.a. and for high income countries of approx. 0.6 percent p.a. for the period of 2002-2013. Those events and developments raise the question what are the consequences of a larger Southern population on the product cycle.

An increase in Southern population has very similar effects as the productivity changes discussed above. However, comparative statics are much more complex as not only the *RS-curve* shifts upwards but also the *NA-curve* shifts to the left. While the effects on  $m$  are clearly positive, our simulations show that for a wide range of parameter values, the innovation rate increases with higher Southern population. Only for an extremely unproductive South the negative effect on  $g$  starts to dominate for large values of  $\beta$ .

**Proposition 3.** *A higher Southern population, while leaving Northern population  $(1 - \beta)L$  constant, increases the imitation rate. The effect on the innovation share is ambiguous.*

<sup>12</sup>Alternatively, this can be seen from the labor market clearing condition in North. A marginal increase in  $n$  leads to an increase in labor demand in North's production sector by  $b^N \beta L$ , whereas a marginal increase in  $m$  leads to a decrease in labor demand of  $b^N L$ . Since we assume  $\beta < 1$ , it is straightforward to see that the latter effect outweighs the former, leaving more resources available in North to be reallocated to the R&D sector.

*Proof.* Leaving  $(1 - \beta)L = c$  constant, implies that we replace  $L = c / (1 - \beta)$  in all equations. We can now analyze the effects of higher Southern population (an increase in  $\beta$ ) without changing population in the North. A higher Southern population share leads to an upward shift of the *RS-curve*. For any given  $m \in (0, 1)$ , the *NA-curve* shifts to the left. This can be easily shown by total differentiation of  $NA(m, g; \beta) = 0$ . In Proposition 1, we showed that  $NA_g(m, g) < 0$  and  $NA_m(m, g) > 0$  at the relevant solution. We get  $NA_\beta(m, g) = -\left(\rho \frac{F^N}{b^N c} + \frac{g}{1-m} \frac{F^N}{b^N c}\right) (1 - \beta) \left(\frac{1}{b^N} - 1 + m - g \frac{F^N}{b^N c}\right) - \frac{1}{1-\beta} T < 0$ .  $\square$

Higher population in South increases demand for existing products and resources in South for imitating Northern products and for producing imitated goods. Thus, the imitation rate  $\mu$  and the share of products manufactured in South  $m$  rise, *ceteris paribus*. On the one hand, this releases resources in North for innovation. On the other hand, the higher risk-adjusted interest rate  $r + \mu$  lowers the present discounted value of profits earned from innovation. As the set  $m$  of cheap products produced in South expands, the (real) income of households increases, which translates into a higher consumption share in South ( $n$  rises). The higher willingness to pay of households in North makes the innovation of new products more attractive. Our simulations show that the positive effect on the present discounted value of profits through higher prices  $z_N$  dominates the negative effect of a higher risk-premium  $\mu$  such that the innovation rate  $g$  rises. According to our simulations with  $T = 0$ ,  $g$  starts to decline for a higher  $\beta$  only if South is extremely unproductive. In the case of positive transfer from North to South ( $T > 0$ ), the negative effect on  $g$  starts to dominate already for much lower productivity levels (i.e. South is only relatively unproductive compared to North).

### The effect on the product life-cycle

Our simulations show that the effect on the product life-cycle are similar to the productivity case discussed above. Both the time length  $\Delta$  where products are exclusively sold in North and the average time span a product is being manufactured in North  $1/\mu$  become shorter. The third stage during which North imports a product therefore increases. The time period during which North exports a product  $(1/\mu - \Delta)$  decreases.

### 3.3 Changes in net foreign asset positions

Our model can be used to understand the impact of changes in net foreign asset positions, or more broadly macroeconomic imbalances, on the international division of production. Since the early 2000s the net foreign asset position of industrial countries as a whole, and the United States in particular, has deteriorated, whereas for developing countries it has improved (Lane and Milesi-Ferretti 2007). While the net foreign asset position is exogenous to our model we may ask what are the effects on the product cycle of net wealth transfers from North to South.

**Proposition 4.** *An increase in  $T$ , i.e. lowering world income inequality, leads to a new steady state where the growth rate  $g$  is lower and the share of imitated and consumed products,  $m$  and*

$n$ , are higher. Northern relative wages deteriorate. There are opposing effects on the terms of trade and on the three stages of the product life cycle.

*Proof.* An increase in  $T$  leads to an upward shift of the  $NA$ -curve since  $NA|_{g=0}$  is a positive function of  $T$ . Notice that  $g$  implied by the  $NA$ -curve as  $m \rightarrow 1$  is given by  $g = (1 - \beta)L/F^N$ , independent of  $T$ . As the  $RS$ -curve is unaffected by a change in  $T$ , the new steady state has a lower  $g$  and higher  $m$ . Using the Northern resource constraint, we see that a lower  $g$  together with a higher  $m$  increases  $n$ . Since  $\omega^S$  is a decreasing function of  $g$ , Northern relative wages  $w^N(t)/w^S(t) = (\omega^S b^N)^{-1}$  are lower. A lower  $g$  tends to decrease the terms of trade  $z_S$ , whereas a higher  $m$  tends to increase them.  $\square$

A higher transfer leads to higher incomes in South and lower incomes in North, *ceteris paribus*. Lower incomes in North depress the incentives to develop a new product, which decreases the innovation rate  $g$ . As Southern resources are fixed, the fraction of imitated products increase. At the same time, higher incomes in South translate into a higher willingness to pay for older products produced in North. This implies that profits of innovating firms in North from selling only to Northern households fall short of profits from selling to all households, creating a disequilibrium in North. This induces some Northern firms to start exporting. As Southern households consume more products, i.e.  $N_S(t)$  increases, their marginal willingness to pay, *ceteris paribus*, decreases until the equilibrium in North is restored. In the new equilibrium, households in South consume a higher fraction of all products  $n$ , and their (marginal) willingness to pay is lower. In our simulations, North's export prices  $z_S$  decrease, and as North's import prices are equal to one, the terms of trade move in favor of South.<sup>13</sup>

### Effect on product life-cycle

There are two opposing effects on the the first stage of the product cycle (the demand lag  $\Delta$ ). On the one hand, households in South are richer so that the Northern firm producing the latest product would like to export sooner (effect of higher  $n$ ). On the other hand, even though the level of income for Southern households is higher, their income grows at a lower rate. This induces the Northern producer of the latest product to export later (effect of lower  $g$ ). The simulations show that the first effect dominates so that the first stage, where new products are exclusively sold in North, becomes shorter. There are two opposing effects on the second stage of the product cycle. On the one hand, the imitation rate  $\mu$  decreases because of a lower growth rate  $g$ . On the other hand, the higher share of imitation increases  $\mu$ . In our simulations the effect of a lower growth rate dominates. Hence, the average time span a product is being manufactured in North  $1/\mu$  becomes longer so that the third stage during which North imports a product decreases. Moreover, the time period during which North exports a product  $(1/\mu - \Delta)$  becomes longer.

<sup>13</sup>Totally differentiating the Northern zero-profit condition (16) and the definition of the imitation rate (15) shows that  $dz_N > 0$  and  $d\mu < 0$  if and only if  $\beta/(\beta - b^S m) > m/(1 - m) > b^S L/F^S$ , where  $\beta/b^S > m$  and we used that along the  $RS$ -curve  $dm/dg < 0$ . Sufficient conditions are  $m < 0.5$  and  $b^S L/F^S < 1$ .

### 3.4 Comparison to CES utility case

What is the contribution of non-homothetic preferences to the theory of product cycles? To better understand the intuition of our model and the role of non-homothetic utility, we compare our results with Grossman and Helpman (1991), henceforth referred to as GH. Although our model and GH are not exactly nested, the models are similarly structured which allows to compare their mechanisms.

There are three main differences. First, homothetic separable CES utility (as assumed in GH) yield an incomplete product cycle only. There is no first stage where the product is exclusively produced and consumed in North. As the reservation price is infinite, a household always buys all goods, irrespective its income, and product adoption is uncorrelated with per capita income. This leads us to the second difference. With non-homothetic utility, income differences affect the average time span of all three product cycle stages. Hence, per capita incomes and net wealth positions affect the product cycle. With CES utility a transfer from rich North to poor South has no effects on innovation incentives and the imitation depends only on the aggregate efficiency units of South. Per capita incomes play no distinct role. Third, CES utility implies that prices are a constant mark-up over marginal costs, which pins down real wages. Hence, real wages do not change when exogenous non-productivity shocks occur, such as changes in population or wealth transfers. With non-homothetic utility prices and markups depend on the willingness to pay, so wages and prices can move differently.

The mechanism, through which imitation and innovation are affected, bears more similarities. Consider the experiment in Proposition 2 and 3. Both in GH and in our model, an increase in Southern productivity and country size raise the imitation and growth rate. With non-homothetic utility, a change in productivity has a stronger impact on imitation and growth than an increase in country size. To see this, consider a rise in Southern productivity but the aggregate amount of efficiency units in South is constant. In GH, such a change would not affect imitation and growth. In this setup, however, higher per capita incomes raise the willingness to pay of Southern consumers. Graphically, the *RS-curve* shifts up and the *NA-curve* to the left. Both the growth rate and the imitation rate rise, although the aggregate market size (measure in efficiency units) of the South is unchanged. Therefore, while the qualitative statements in Proposition 2 and 3 are the same in GH, our approach highlights the importance of per capita income effects.

Furthermore, the terms of trade move in opposite directions. As already mentioned, in GH the Northern terms of trade are connected to the change in relative wage rates (prices are a constant markup over marginal costs), and hence deteriorate. In this paper, the terms of trade depend on the willingness to pay of households. Terms of trade move in favour of North as higher Southern relative wages lead to a higher willingness to pay of Southern households for Northern products.

Our model is not the only one, however, to be able to explain why wealth transfers affect innovation. If we introduce trade costs into GH, a home market effect arises. If  $T$  rises, the Southern market gets larger. Since trade costs are non-zero, this raises aggregate income in

South and thus, imitation and growth rates raise via the standard intuition. An advantage of our approach is that our model is parsimonious in the sense that we do not need additional parameters to explain such effects.

To sum up, non-homothetic preferences account for different adoption times as a function of per capita income and hence complete the product cycle description. In particular, they generate a new first stage that is consistent with stylized facts discussed in Section 4 below. Furthermore, real wages react to changes in parameters under the assumption of non-homothetic utility, which alters the prediction on terms-of-trade effects.

## 4 Stylized facts

We discuss stylized facts about major consumer durables of the 20th century that highlight the features of the product cycle emphasized by the theory presented above. In particular, we focus on the product cycle of the countertop microwave oven, a typical household appliance of the 20th century. Our case study shows that the launch of the microwave oven across 16 European markets varies systematically with per capita income, which we use as a proxy for demand. In addition, we show that the pattern of introduction across countries is similar for other major consumer durables like the dishwasher, dryer, freezer, VCR, and washing machine. The list of countries and products can be found in Table 2 in Appendix A.7. The pattern found in the data is difficult to explain by theories emphasizing the supply side, without assuming a fix cost of exporting that differs across export markets (i.e. beachhead cost). Trade data shows that the United States, where the microwave was first introduced in 1967, started out as a net exporter of microwaves but became a net importer in the mid-1990s. Together with the increase in production in the UK, South Korea, Brazil and Russia in the 1980s and 1990s, it suggests that production of microwaves gradually shifted from North to South.

This section documents stylized facts about the product cycle. Obviously, it is not a test of the product cycle hypothesis, which is beyond the scope of this paper. For completeness, we briefly look at the empirical literature on the product cycle hypothesis, before discussing our case study in detail. Since Vernon proposed the theory of the product cycle in 1966 there have been numerous attempts to test the theory. Hirsch (1967) and Wells (1969) were among to first to find evidence for the product cycle theory for consumer durables and electronic products, respectively. Later, Hirsch (1975) and Mullor-Sebastian (1983) found that industrial product groups behave according to the product cycle theory. Perdakis and Kerr (1998) provide a more complete overview of the earlier empirical literature on the product cycle hypothesis. More recently, Feenstra and Rose (2000), and Xiang (2014) both find evidence for product cycles using U.S. import data.

### 4.1 Demand lags

In 1946, Percy Spencer, an American engineer, while working on radar technology for the U.S. defense company Raytheon Corporation accidentally discovered that microwaves are capable

of heating food almost instantly. The story goes that a candy bar in Spencer’s pocket melted during an experiment. Spencer realized the commercial potential of his discovery, especially for a high-income market like the US, and Raytheon Corp. filed for patents. In 1947, Raytheon produced the first commercial microwave oven named ”Radarange”, which was sold to businesses like restaurants. Twenty years later, in 1967, Amana, a division of Raytheon, introduced the first domestic countertop microwave oven, marking the beginning of the use of the microwave in American kitchens (see e.g. Osepchuk 1984).

In the second half of the 20th century, the microwave oven became a beloved household item in kitchens all over the world. We show evidence, which suggests that the pattern of introduction across 16 European countries depends on the level of demand as measured by per capita income. The data was kindly provided by Tellis, Stremersch and Yin (2003). Table 2 in Appendix A.7 shows the year of introduction defined by Tellis, Stremersch and Yin (2003) as the first year commercial sales for the microwave oven were registered and GDP per capita in the year the microwave was introduced in the United States. In 1967, the year the countertop microwave oven was first introduced in the US, GDP per capita was USD 19,522 in the US, whereas it was only USD 9,742 and USD 5,937 in Greece and Portugal, respectively, where the microwave was last introduced in our sample of countries. Spearman’s rank correlation coefficient, shown at the bottom of Table 2, suggests that on average the microwave oven was first introduced in markets with a high GDP per capita, and last introduced in markets with a low GDP per capita. The pattern of introduction is similar for the other consumer durables. Table 1 below shows for each consumer durable the result from regressing the introduction lag  $\Delta$  on (relative) GDP per capita and population size in country  $i$ , where all variables are in logs and relative to the US (see Appendix A.7 for details). Table 1 below further suggests that there is a negative relationship between the introduction lag of the microwave and (relative) GDP per capita, controlling for (relative) population sizes. Indeed, we find again a similar relationship for all other consumer durables, as well as for the average across all six consumer durables (i.e.  $\Delta_{mean}$  in the first column of Table 1).

Table 1: Correlation between (log) relative GDP per capita and (log) introduction lag  $\Delta$

	$\log(\Delta_{mean})$	$\log(\Delta_{dish})$	$\log(\Delta_{dryer})$	$\log(\Delta_{freeze})$	$\log(\Delta_{micro})$	$\log(\Delta_{vcr})$	$\log(\Delta_{wash})$
mean dep var	11.48	10.73	18.47	9.50	7.12	10.31	10.94
$\log(\text{rel GDPpc})$	-0.43 (-3.95)	-0.40 (-9.75)	-0.43 (-3.61)	-0.70 (-2.49)	-0.85 (-2.88)	-0.12 (-0.88)	-0.25 (-1.45)
$\log(\text{rel pop})$	-0.11 (-2.41)	-0.11 (-6.03)	-0.10 (-1.77)	0.10 (0.75)	-0.22 (-2.48)	-0.11 (-3.86)	-0.235 (-3.09)
adj. $R^2$	0.55	0.91	0.46	0.26	0.46	0.55	0.40
#obs	16	14	16	15	16	12	15

Notes: t-values in parentheses. Mean dependent variable is the lag in years relative to the United States.

## 4.2 Export and production patterns

A look at trade and production data suggests that production of the microwave oven gradually moved from the US to abroad.

The left-hand panel in Figure 5 below looks at U.S. import and export data for the same 16 European countries discussed above at the 5 digit SITC level from 1978-2006, which are provided by the Center for International Data at UC Davis (Feenstra 1996, 1997). We observe that the US starts out as a net exporter of microwave ovens at the beginning of the sample period in 1978 and ends up as a net importer at the end of 2006, switching in the mid-1990s. A possible interpretation for the decline in the ratio of exports to exports plus imports is that firms in the 16 European countries mastered the technology to produce microwave ovens, and due to lower production costs were able to compete with US firms in their home markets as well as in the US market. In other words, US firms became less competitive in their export markets and/or European firms became more competitive in the US market, such that US exports relative to U.S. imports decreased.<sup>14</sup> The export performance of the other products as well as on average across all products (see Appendix A.7) is similar, with the exception of the domestic deep freezer.

The right-hand side panel in Figure 5 shows data on the production of the microwave oven of various countries relative to the United States for the time period of 1982-2008, obtained from the Industrial Commodity Production Statistics Database (United Nations Statistics Division 2012).<sup>15</sup> We observe that the US production of the microwave is declining from the 1980s until 2008, first relative to the UK and South Korea, and later on relative to emerging countries like Brazil and Russia. Again, a possible explanation consistent with the product cycle theory is that the production of microwave oven moves from developed countries to developing countries as firms in these countries acquire the technology to produce microwave ovens and have lower production costs. The production pattern for the washing machine is very similar (see Appendix A.7). Data limitations prevent us from looking into the production patterns for other countries and for the rest of the consumer durables discussed above.

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<sup>14</sup>Note that during the first half of the 1980s until the Plaza Accord was signed in 1985, the USD strongly appreciated against all major currencies. This might have (temporarily) added to the decline in US net export of microwaves. However, the US export performance continues to deteriorate after 1985, although to a somewhat lesser extent.

<sup>15</sup>The data is collected through annual questionnaires sent to national statistical authorities. The data reported by the United Nations Commodity Statistics Yearbook reflect volume (and value) of production sold during the survey period, which is defined as the production carried out at some time, which has been sold (invoiced) during the reference period.



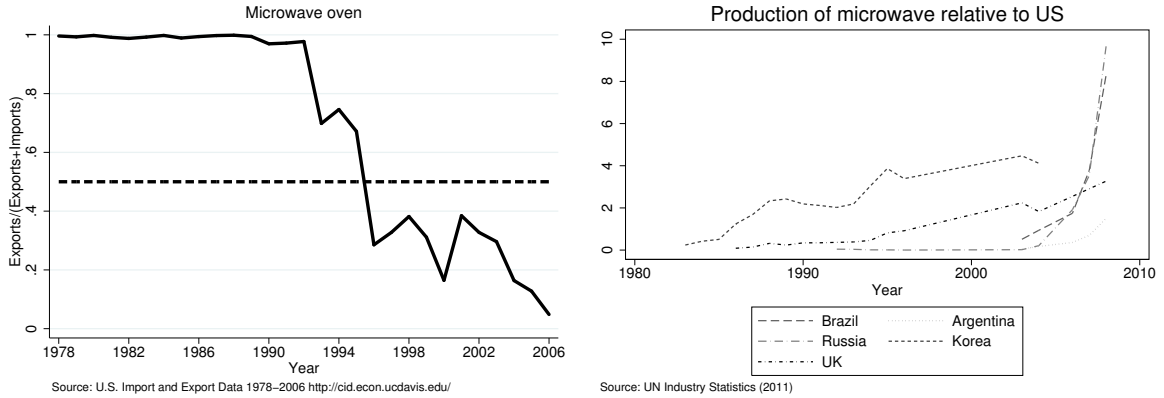


Figure 5: US export ratio for microwave ovens across 16 European countries (left-hand side), and production of microwave oven relative to US (right-hand side)

## 5 Discussion and extensions

Due to our assumption of symmetric preferences and identical cost structures the product cycle of product  $j$  (i.e. at the firm level) is indeterminate. In order to illustrate that the product cycle we impose in our baseline model emerges from more complex models, without changing the basic channels through which the income distribution operates, we discuss in what follows heterogeneity on the demand and supply side.

As it turns out, the slightest asymmetry (either on the demand side or the supply side) generates the (product) life cycle patterns we emphasize. Among the goods not yet invented, the next innovator chooses the product which is cheapest to produce (or most valued by consumers). This firm will first sell exclusively on the market of rich North. Later on, it will start to export before the product is finally imitated by producers in poor South. In this sense, among the many life cycle patterns that are possible in the symmetric case, the limit case of an asymmetric structure (when the asymmetry is about to vanish) is the most interesting one. In this latter case, the symmetric economy behaves - qualitatively, with respect to the life cycle patterns of a specific product - just like an asymmetric economy.

### 5.1 Heterogeneous demand-side

Following Foellmi and Zweimüller (2006), we assume that households have the following non-homothetic utility function

$$u(c(j, t)) = \int_0^{N(t)} j^{-\eta} c(j, t) dj$$

where the parameter  $\eta \in (0, 1)$  determines the “steepness” of the hierarchy, i.e. how fast marginal utility falls in the index  $j$ . One can view low-indexed products as satisfying more basic needs relative to higher-indexed products. It is straightforward to derive the willingness to pay for good  $j$ , which is given by  $z(j, t) \equiv j^{-\eta} [u(\cdot)\lambda(t)]^{-1}$ , and decreases in the index  $j$  (see

also Appendix A.1). In other words, households demand products (and therefore Northern firms develop products) along the hierarchy, starting with low-indexed products and gradually moving up the hierarchy ladder.<sup>16</sup> This implies that profit-maximizing prices for Northern products, and hence profits decrease in the index  $j$ , given all firms have the same cost structure.

We continue to assume that Southern households can afford to consume some products manufactured in North. Which Northern firms do not export and which firms do? First, suppose that no firm in North exports. In that case Southern households would not exhaust their budget constraints and their willingness to pay would become infinitely large. This implies that prices for the lowest-indexed products, which have not yet been imitated by South, become infinitely high. Hence, the firms producing the lowest-indexed products have an incentive to start exporting their products. Second, consider the case where all Northern firms export. In that case, Northern households would not exhaust their budget constraints, and their willingness to pay for an additional product would become infinitely high. This implies that new firms enter the market along the consumption hierarchy, manufacturing products that Southern households cannot afford, and that are therefore not exported.

We keep our assumptions about technology in North and South. However, instead of assuming that Southern firms target Northern products at random for imitation, we assume they always target the Northern product with the highest willingness to pay. In sum, this model would generate the following deterministic product cycle at the individual product-level in steady state. At some time  $t \geq 0$ , the Northern firm  $j$  introduces the lowest-indexed product that has not yet been invented. It starts selling its product to Northern households at the price  $z_N(j, t)$  since only they can afford to purchase new products that satisfy relatively non-essential needs. The price  $z_N(j, t)$  increases at rate  $\eta g$  until after  $\Delta$  periods, which is still determined by  $N(t) = N_S(t) \exp(g\Delta)$ , the Northern firm finds it attractive to lower the price to  $z_S(j, t)$  and starts exporting its product.<sup>17</sup> The price  $z_S(j, t)$  increases at rate  $\eta g$  until after  $\tilde{t} > t + \Delta$  periods a Southern firm copies the product and price competition drives the Northern firm out of the market. The price drops to the marginal cost of production of Northern firms, and stays constant from then on. Hence, such a model would eliminate the indeterminacy of the product cycle at the individual firm level. However, the analysis would be substantially more complicated without adding much additional insight.

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<sup>16</sup>Using hierarchic utility, Foellmi and Zweimüller (2006) study the impact of inequality on growth in a closed economy with two groups. In their model, there is no imitation.

<sup>17</sup>This follows from taking the time derivate of the willingness to pay for the most recently innovated product  $N(t)$ , which is given in steady state by  $\dot{z}_N(N(t), t)/z_N(N(t), t) = r - \rho - g$ . In a steady state where the allocation of resources in North is constant across sectors the price of the newest product must be constant, i.e.  $r = \rho + g$ . In steady state  $n = N_S(t)/N(t)$  must be constant too, so that the price of any product  $j$  evolves over time as follows  $\dot{z}_i(j, t)/z_i(j, t) = r - \rho - (1 - \eta)g$  for  $i \in \{N, S\}$ . Hence, using  $r = \rho + g$  yields  $\dot{z}_i(j, t)/z_i(j, t) = \eta g$ . Note that the firm selling the newest product must be indifferent in equilibrium whether to export or not, i.e.  $[z_N(N_S(t), t) - 1](1 - \beta) = [z_S(N_S(t), t) - 1]$ , where  $z_N(N_S(t), t) = n^{-\eta} z_N(N(t), t)$ .

## 5.2 Heterogenous supply-side

### Heterogenous firms

In the following we assume that subutility is CES (as in Grossman and Helpman, 1991) but firms have heterogenous technology à la Melitz (2003). More concretely, we assume that subutility is given by  $v(c) = c^\alpha$  and a firm  $j$  draws its productivity parameter  $b(j)$  from a common distribution. Thus, firms can be ranked by their technology or productivity  $b(j)$ . The solution to a monopolistic firm  $j$ 's profit maximizing problem is given by a fixed markup over marginal costs:  $p(j) = (w/b(j))/\alpha$ . Without fixed export costs, all firms will export due to the assumption of CES subutility. Introducing fixed trade costs implies that only the most productive firms are able to compete in foreign markets, which is consistent with empirical evidence showing that exporters are, on average, more productive (see e.g. Bernard et al. 2012). In that case, there is a cutoff productivity level  $b^*$  (determined by a zero profit condition) such that only firms with  $b(j) > b^*$  enter export markets. Note that in this model there is no relationship between the date of entry into the market (age) and an individual firm's productivity level, i.e. it is not the case that over time firms draw on average higher or lower productivity levels. However, if fixed entry costs decrease exogenously, new firms entering the market have on average lower productivity levels. Furthermore, an individual firm's productivity level is constant over time, i.e. there is no endogenous growth through intertemporal spillovers affecting (labor) productivity. This implies that an individual firm with productivity draw  $b(j) < b^*$  never starts exporting (unless  $b^*$  changes, e.g. due to an exogenous change in trade costs). Thus, firm  $j$  with  $b(j) < b^*$  entering the market at time  $t$  and firm  $k$  with  $b(j) = b(k) < b^*$  entering the market at time  $\tilde{t} > t$  behave the same. In other words, there is neither learning-by-doing (e.g. Irwin and Klenow 1994, Levitt, List and Syverson 2013) nor learning by exporting at the firm level (see Bernard et al. 2012).

### Learning-by-doing

Another way of introducing heterogeneity on the production side is learning-by-doing. In the following, we keep our assumptions from the basic model about preferences (Section 2.2) and technology in South (Section 2.3.2). However, we follow Matsuyama (2002) and assume that there is passive learning-by-doing (i.e. externality of the manufacturing process) in the production sector of North. In particular, we assume that producing one unit of output requires  $b^N(j, t) = b^N(Q(j, t))/N(t)$  units of labor, where  $b^N(\cdot)$  is a decreasing function of the discounted cumulative output determined by

$$Q(j, t) = \delta \int_{-\infty}^t C(j, s) \exp(\delta(s - t)) ds$$

where  $\delta > 0$  can be interpreted as both the speed of learning as well as the rate of depreciation of the learning experience. Again,  $C(j, t) \in \{0, (1 - \beta)L, L\}$  denotes market demand. Due to depreciation the cumulative learning experience  $Q(j, t)$  is bounded from above by  $C(j, t)$ ,

and can therefore not exceed  $L$ . We continue to assume that the creation of a new product requires  $F^N(t)$  units of labor. As in Section 5.1, we assume that Southern firms always target the Northern product with the highest willingness to pay.

Again, consider a situation where Southern households can afford to purchase some of the products made in North. Prices of Northern and Southern products are still determined as before. Our assumptions about technology imply that profits of Northern firms increase with production experience, *ceteris paribus*. In other words, firms which have been in the market for a longer time earn higher profits since their marginal costs are lower. In equilibrium, at any point in time some firms export and some sell exclusively to Northern households. Hence, there must be some threshold value  $Q(N_S(t), t)$ , implicitly defined by  $[z_N(N_S(t), t) - w^N(t)b^N(Q(N_S(t), t))/N(t)](1 - \beta) = [z_S(N_S(t), t) - w^N(t)b^N(Q(N_S(t), t))/N(t)]$ , at which a Northern firm is indifferent between exporting or not. Below this threshold value the profits from excluding Southern households exceed the profits from exporting, and vice versa. In other words, below the threshold value  $Q(N_S(t), t)$  the price effect dominates the market size effect, and vice versa.

Hence, this model would imply that products go through the following cycle in steady state. A new product introduced by a Northern firm is first sold at high prices  $z_N$  only in the domestic market since this firm has a relatively low productivity level at which the price effect dominates the market size effect. The Northern firm finds it optimal to lower the price to  $z_S$  and start exporting its product after  $\Delta$  periods (still determined as before) since incomes in South grow and the Northern firm becomes more productive due to learning-by-doing. At time  $\tilde{t} > t + \Delta$  the patent of the product expires, and it is imitated by a Southern firm. Price competition implies that the limit price drops to marginal costs of Northern firms, and the Northern firm exits the market. From then onwards the product is imported by North from South.

## 6 Conclusion

Vernon's (1966) celebrated product cycle theory hypothesizes that new products go through the following stages. In the first stage, new products are developed and introduced in high-income countries. Later in the cycle, incomes in the poorer countries have grown sufficiently such that demand for these products appears there. Thus, products that were only consumed in high-income countries before are now exported. In the third stage, production moves from high-income countries to low-income countries because they have learned the technology to produce these goods and are able to produce them at lower costs.

The paper contributes to the literature in building a dynamic general-equilibrium model that is able to generate the three stages of the product cycle described by Vernon (1966). In our model, a wealthy North develops new products, which a poor South randomly attempts to copy. The incentives to innovate and imitate are determined by the distribution of income across regions such that the demand side is an important determinant of the different stages of the product cycle. Aside from analyzing changes in Southern labor productivity and a larger

Southern population, we elaborate the effects of a redistribution of wealth between North and South such that inequality across regions decreases. We show that a decrease in inequality across regions leads to a decline in the innovation rate and hence a slowdown of imitation activity in South, for a given share of South in total production. Since Southern households are wealthier after the redistribution of income, they can afford to purchase a higher share of goods available in the world market - in particular more newer goods produced in North. Thus, firms in the North will export their products sooner. In other words, the first stage of the cycle becomes shorter. At the same time the average duration new products are manufactured in North increases because imitation activity in South has slowed down. Firms in South master the technology to copy a good later, so that on average it takes longer for the production to move to South because of the (comparative) cost advantage. Hence, the second stage of the product cycle where new goods are exported by North to South gets longer. Therefore, the third stage of the cycle where the products are imitated and exported to North becomes shorter.

Supply-based approaches cannot capture the fact that adoption time and per capita incomes are correlated, i.e. that poorer countries start consuming products later in the cycle. Incorporating non-homothetic utility into these types of models enables us to formalize the product cycle hypothesis and analyze the effects of the demand side on the product cycle. Our model is consistent with the stylized fact that product adoption strongly correlates with per capita income. We show that the microwave oven (and other common consumer durables) appear to have gone (or still go) through a "typical" product cycle. In particular, new products are not introduced simultaneously across countries and the lag in introduction depends negatively on relative GDP per capita, i.e. relative to the first country where a product is introduced. In other words, new products are introduced in affluent countries before they are introduced in less prosperous ones.

The relevance of the product cycle theory is essentially an empirical question. Our analysis delivers empirically testable predictions on the long-term movements in the terms of trade to exogenous changes in (relative) labor productivity, population size, and per capita income. At the same time, empirical tests help to put the product cycle theory into perspective. On the theoretical side as production processes are ever more segmented across countries, it might be interesting to explore the consequences for the product cycle of allowing R&D to take place in a different region than production (e.g. a product like the iPad, which is designed in the United States and assembled in China). We consider these topics promising for future research.

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## A Appendix

### A.1 Household problem

In this appendix, we discuss the household problem in a more general form. To keep the exposition short, we use the one-stage formulation of the household problem. Households maximize (logarithmic) intertemporal utility given by

$$U(0) = \int_0^{\infty} e^{-\rho t} \log u \left( \{c(j, t)\}_{j=0}^{N(t)} \right) dt$$



where  $u(\cdot)$  is instantaneous utility, and  $\rho > 0$  denotes the time preference rate.<sup>18</sup> Instantaneous utility is given by

$$u\left(\{c(j, t)\}_{j=0}^{N(t)}\right) = \int_0^{N(t)} j^{-\eta} v(c(j, t)) dj$$

where  $v(\cdot)$  denotes a concave subutility function with  $v'(c) > 0$  and  $v''(c) < 0$ , and  $j^{-\eta}$  is a weighting function with  $\eta \in (0, 1)$ . In our baseline case, subutility  $v(\cdot)$  is an indicator function  $c(j, t) \in \{0, 1\}$ . The household maximizes intertemporal utility above subject to non-negativity constraints  $c(j, t) \geq 0$ ,  $\forall j, t$ , and its intertemporal budget constraint  $\int_0^\infty e^{-R(t)} E(t) dt \leq a(0) + \int_0^\infty e^{-R(t)} [w(t) + T(t)] dt$ , where  $a(0) \geq 0$  denotes initial wealth,  $R(t) = \int_{s=0}^t r(s) ds$  the cumulative interest rate, and  $E(t) = \int_0^{N(t)} p(j, t) c(j, t) dj$  total consumption expenditures. We impose a no-Ponzi game condition of the following form  $\lim_{t \rightarrow \infty} e^{-R(t)} a(t) \geq 0$  on the intertemporal budget constraint. The first-order conditions including complementary slackness conditions to the household's optimization problem are given by

$$e^{-\rho t} \frac{\xi(j) v'(\cdot)}{u(\cdot)} - \Lambda e^{-R(t)} p(j, t) + \mu(j, t) = 0 \quad (21)$$

$$\mu(j, t) c(j, t) = 0, \quad \mu(j, t) \geq 0, \quad c(j, t) \geq 0 \quad (22)$$

$$\int_0^\infty e^{-R(t)} E(t) dt = a(0) + \int_0^\infty e^{-R(t)} [w(t) + T(t)] dt \quad (23)$$

$$\lim_{t \rightarrow \infty} e^{-R(t)} \lambda(t) a(t) = 0 \quad (24)$$

where  $\Lambda$  denotes the (present value) Lagrange multiplier on the intertemporal budget constraint, and  $\mu(j, t)$  the Lagrange multiplier on the non-negativity constraints. Note that the current value Lagrange multiplier on the budget constraint is given by  $\lambda(t) = \Lambda e^{-R(t) + \rho t}$ . Due to the transversality condition the intertemporal budget constraint will always be binding in optimum, i.e. preferences exhibit global non-satiation. However, preferences might exhibit local satiation (i.e. bliss points). We distinguish the following cases:

- (i) Non-negativity constraint is binding, i.e.  $c(j, t) = 0$ . This implies by (22) that  $\mu(j, t) > 0$ . Thus, the first-order condition (21) can be written as follows

$$v'(0) < \frac{p(j, t)}{z(j, t)}$$

where we defined the willingness to pay  $z(j, t) \equiv j^{-\eta} [\lambda(t) u(\cdot)]^{-1}$ .

- (ii) Non-negativity constraint is not binding, i.e.  $c(j, t) > 0$ . This implies by (22) that  $\mu(j, t) = 0$ . Therefore, the first-order condition (22) can be written as follows

$$v'(c(j, t)) = \frac{p(j, t)}{z(j, t)}$$

---

<sup>18</sup>Note that logarithmic intertemporal utility is a special case of constant-relative-risk-aversion (CRRA) utility of the following form  $u(\cdot)^{1-\sigma}/(1-\sigma)$ , where  $\sigma$  denoting the inverse of the intertemporal elasticity of substitution goes to 1.

Note that the willingness to pay  $z(j, t)$  decreases in  $j$  as long as  $\eta > 0$ , meaning that the willingness to pay is higher for necessities (low- $j$  varieties) than for luxuries (high- $j$  varieties). Furthermore,  $z(j, t)$  decreases in the marginal utility of wealth  $\lambda(t)$ , i.e. wealthy households with a low marginal utility of wealth have a higher willingness to pay.

A broad set of papers using non-homothetic utility typically assumes that the subutility  $v(\cdot)$  belongs to the HARA (hyperbolic absolute risk aversion) class (see Bertola, Foellmi and Zweimüller 2006). The utility function in this paper falls within that category as well. HARA utility bears the characteristic that income expansion paths and thus, Engel curves, are linear but need not go through the origin. The marginal utility of consumption takes the following form (we drop indices  $j$  and  $t$  for convenience)

$$v'(c) = \left( \frac{\zeta c}{\sigma} + \bar{c} \right)^{-\sigma} \quad (25)$$

where  $\zeta > 0$ ,  $\sigma$ , and  $\bar{c}$  are preference parameters. Grossman and Helpman (1991) use homothetic CES utility, which are a special case of (25) with  $\zeta = \sigma$  and  $\bar{c} = 0$  so that subutility is given by  $v'(c) = c^{-\sigma}$ . In their case,  $v'(0) \rightarrow \infty$  such that non-negativity constraints are never binding, i.e.  $v'(0) < p(j, t)/z(j, t)$  never holds. Although still small, the literature on non-homothetic consumer behavior in international trade has been growing in the recent past. Notable examples are Markusen (2013), Simonovska (2015), Melitz and Ottaviano (2008), and Behrens et al. (2014). Those authors all use variations of utility functions belonging to the HARA class. Behrens et al. (2014) use a utility function characterized by constant absolute risk aversion, which is again a special case in (25) by setting  $\bar{c} = -1$  and letting  $\sigma \rightarrow -\infty$ , this yields  $v'(c) = e^{-\zeta c}$  such that  $v'(0) = 1 < \infty$ , and non-negativity constraints might be binding. Markusen (2013) and Simonovska (2015) use a Stone-Geary utility function, which can be obtained by setting  $\zeta = \sigma = 1$  and  $\bar{c} > 0$  in (25). In that case,  $v'(c) = (c + \bar{c})^{-1}$ , and  $v'(0) = 1/\bar{c} < \infty$  such that the reservation price is finite and non-negativity constraints could bind. Melitz and Ottaviano (2008) use a quadratic utility function, which can be obtained from (25) by setting  $\zeta = -\sigma = 1 > 0$  and defining  $s \equiv -\bar{c} > 0$ . In that case,  $v'(c) = s - c$ , and  $v'(0) = s$  is again finite. The 0-1 utility assumed in our paper is derived by setting  $\bar{c} = -1$  and  $\zeta = -\sigma$ , and letting  $\zeta \rightarrow \infty$ , as can be easily seen  $v'(0) \equiv v(1) - v(0) = 1$  with 0-1 preferences.

The basic economic forces captured by the different assumption on preferences can be summarized as follows. Generally, if income per capita increases, single-product firms will both increase quantities and prices if you hold the number of firms and products constant. In the homothetic CES case, only quantities increase and prices/markups stay constant. With 0-1 utility, only prices increase and quantities stay constant. For intermediate cases, e.g. quadratic utility giving rise to linear demand, both quantities and prices increase.

## A.2 Alternative forms of international knowledge spillovers

Our baseline case assumes perfect international knowledge spillovers. We think this is a reasonable starting point since knowledge can in principle overcome borders relatively easy

(e.g. through professional and scientific journals, conferences or trade shows, or the internet). However, whether this is a reasonable assumption is essentially an empirical question. There is some evidence that more open economies benefit more from foreign R&D (e.g. Coe and Helpman 1995; Coe, Helpman and Hoffmaister 2009).

Hence, we sketch how different degrees of (international) knowledge spillovers affect steady state growth in our model framework. Suppose, labor input in South is inversely proportional to  $\tilde{N}(t) \equiv (N^S(t))^\gamma (N_S(t))^\delta (N(t))^{1-\gamma-\delta}$  with  $0 \leq \gamma, \delta \leq 1$  and  $\gamma + \delta \leq 1$ . In other words,  $F^S(t) = F^S/\tilde{N}(t)$  and  $b^S(t) = b^S/\tilde{N}(t)$ . The parameter  $\gamma$  governs the relative degree of learning from products already copied whereas  $\delta$  governs the relative degree of learning from products produced and consumed in South. Hence, South's labor market condition (13) in steady state can be written as follows

$$\beta L = \phi(m, n) (gF^S + b^S L), \quad \text{where } \phi(m, n) \equiv m^{1-\gamma}/n^\delta.$$

This formulation can represent any degree of spillovers between no international spillovers to perfect international spillovers. The polar cases are given by

$$\phi(m, n) \equiv \begin{cases} 1, & \text{if } \gamma = 1, \delta = 0 \text{ (no international spillovers)} \\ m/n, & \text{if } \gamma = 0, \delta = 1 \text{ (learning by importing)} \\ m, & \text{if } \gamma = 0, \delta = 0 \text{ (perfect international spillovers [baseline])}. \end{cases}$$

Note that since  $m > n$ , we have  $1 > m/n > m > 0$ . No international spillovers imply here that South only benefits from knowledge embedded in varieties that are produced domestically, i.e. they already imitated in the past. Learning by importing means that South benefits from knowledge contained in imported varieties plus from varieties imitated in the past (note that due to symmetric utility this also includes some varieties that skipped the export stage and thus, have never been imported by South). While the idea of learning by importing has been picked up by models studying technology diffusion (e.g. Buera and Oberfield 2016), we are not aware that this has been studied in the context of product cycles. Eventually, perfect international knowledge spillovers refer to the baseline case discussed in the text.

How does the degree of international knowledge spillovers affect steady state growth? From South's reformulated labor market condition above follows that  $g$  decreases in  $\phi$ . Moreover,  $\phi$  is increasing in  $\gamma$  and  $\delta$  and hence, decreasing in the degree of international knowledge spillovers. We conclude that there is a positive relationship between the degree of international knowledge spillovers and steady state growth.

### A.3 Balance of payments

The intertemporal budget constraint of households in South, the resource constraint in South, and the zero-profit condition in South imply the balance of payments as stated in the text. Due to Walras' law, the intertemporal budget constraint of North is redundant. We drop the time index  $t$  where no confusion arises.

The balance of payments in present value form at  $t = 0$  is given by

$$\begin{aligned}
0 &= \left\{ \int_0^\infty [(1 - \beta)LN^S\omega^N b^N - \beta L(N_S - N^S)z_S] \exp\left(-\int_0^t r(s)ds\right) dt \right\} \\
&+ \int_0^\infty \beta LT_S \exp\left(-\int_0^t r(s)ds\right) dt \\
&+ \left\{ \beta La_S(0) - \int_0^\infty N^S [\pi^S - g^S v^S] \exp\left(-\int_0^t r(s)ds\right) dt \right\}
\end{aligned}$$

where we used  $\beta LN = \dot{N}^S F^S + N^S b^S L$  from the resource constraint,  $v^S = \omega^S F^S$  from the zero-profit condition, and a no-Ponzi game condition. The first two lines denote the current account, which consists of the trade balance and net transfer payments. The third line denotes net foreign asset holdings. In the steady state, we have that  $r$  and  $\pi^S$  are constant,  $N^S$  grows at a constant rate  $g^S = g$ , and  $v^S = \pi^S/r$ . This implies that net foreign assets become  $\{\beta La_S(0) - N^S(t)\pi^S/r\}$ . Hence, the balance of payments in the steady state is determined by

$$0 = \{N^S(t)(1 - \beta)L\omega^N b^N - (N_S(t) - N^S(t))z_S(t)\beta L\} + \beta LT_S(t) + \{\beta La_S(t) - N^S(t)\pi^S/r\}$$

which holds for all  $t$  in steady state, in particular at  $t = 0$ . Hence, it becomes obvious that if we assume initial wealth at time  $t = 0$  of households in South  $\beta La_S(t)$  to be exactly equal to the present discounted value of aggregate firm profits in South  $N^S(t)v^S(t)$ , net foreign assets will remain zero in steady state. We see that if Southern households would inherit sufficiently large asset holdings they could run a permanent trade deficit (even in the absence of transfers from North).

#### A.4 Budget constraints

The intertemporal budget constraint of households in North is in the steady state given by

$$N(t) \{m + (n - m) [\beta + (1 - \beta)z_N] + (1 - n)z_N\} = (r - g)a_N(t) + w^N(t) - T_N(t)$$

where  $y_N(t) = a_N(t) + w^N(t)/(r - g) - T_N(t)/(r - g)$  denotes the lifetime income of a Northern household. We observe that Northern households save only out of their capital income (note that  $r - g = \rho$ ), and consume all their labor income (and possible transfer income). In other words, the marginal propensity to consume out of labor and transfer income is one. Similarly, in the steady state the intertemporal budget constraint of households in South becomes

$$N(t) \{m + (n - m) [\beta + (1 - \beta)z_N]\} = (r - g)a_S(t) + w^S(t) + T_S(t)$$

where  $y_S(t) = a_S(t) + w^S(t)/(r - g) + T_S(t)/(r - g)$  denotes the lifetime income of a household in South. Similarly to Northern households, Southern households save only out of capital income and consume all labor income. Hence, relative lifetime incomes per capita in the steady state

are (endogenously) determined by

$$\frac{y_S(t)}{y_N(t)} = \frac{\rho a_S(t) + w^S(t) + T_S(t)}{\rho a_N(t) + w^N(t) - T_N(t)}.$$

### A.5 Derivation of transitional dynamics

Using the resource constraint of South, the relationship between  $g$  and  $g^S$ , the resource constraint of North to substitute for  $g$ , and the balance of payments to substitute for  $n$  (assuming that it is balanced period by period), we obtain the  $\dot{m}$  - schedule

$$\frac{\dot{m}}{m} = \left( \frac{1}{F^N/(b^N L)} \right) \left\{ \frac{\lambda \beta z_N}{\beta + (1 - \beta) z_N} - \left[ (1 - \beta) \left( \frac{1}{b^N} - 1 \right) + m - F \left( \frac{\beta/b^S}{m} - 1 \right) \right] \right\}$$

where  $\lambda = \lambda_N/\lambda_S$  which is constant and equal to its steady state value, and  $F = \frac{F^N/(b^N L)}{F^S/(b^S L)}$ . The first-order conditions of the static problem still apply along the transitional path because they rely on the separability of preferences only. The  $\dot{m} = 0$  locus is determined by

$$\frac{\beta \lambda z_N}{\beta + (1 - \beta) z_N} = (1 - \beta) \left( \frac{1}{b^N} - 1 \right) + m - F \left( \frac{\beta/b^S}{m} - 1 \right).$$

It is straightforward to show that  $dz_N/dm > 0$ ,  $z_N(m) \rightarrow -\infty$  as  $m \rightarrow 0$ , and  $z_N(m)$  equals a positive constant larger than one as  $m \rightarrow \beta/b^S$  if and only if  $(1 - \beta) (1/b^N - 1) + \beta/b^S > \lambda$  (this simply requires inequality between North and South to be sufficiently high). Thus, the  $\dot{m} = 0$  locus is increasing in the  $(z_N, m)$ -space. For values of  $z_N$  above the  $\dot{m} = 0$  locus,  $\dot{m} > 0$  and for values of  $z_N$  below the  $\dot{m} = 0$  locus,  $\dot{m} < 0$ .

The  $\dot{z}_N$  - schedule is obtained by using the balance of payments, the Northern and Southern resource constraints, the definition of the hazard rate, the Euler equation, and North's zero-profit condition

$$\begin{aligned} \frac{\dot{z}_N}{z_N} &= \left( \frac{1}{F^N/(b^N L)} \right) \left\{ (z_N - 1) (1 - \beta) + \frac{\lambda \beta z_N}{\beta + (1 - \beta) z_N} \right. \\ &\quad \left. - \left[ (1 - \beta) \left( \frac{1}{b^N} - 1 \right) + m + \frac{\rho F^N}{b^N L} + F \left( \frac{m}{1 - m} \right) \left( \frac{\beta/b^S}{m} - 1 \right) \right] \right\}. \end{aligned}$$

The  $\dot{z}_N = 0$  locus is determined by

$$\begin{aligned} (1 - \beta) (z_N - 1) + \frac{\beta \lambda z_N}{\beta + (1 - \beta) z_N} &= (1 - \beta) \left( \frac{1}{b^N} - 1 \right) \\ &\quad + m + \frac{\rho F^N}{b^N L} + F \left( \frac{m}{1 - m} \right) \left( \frac{\beta/b^S}{m} - 1 \right). \end{aligned}$$

The slope of the  $\dot{z}_N = 0$  locus is given by

$$\frac{dz_N}{dm} = \frac{[\beta + (1 - \beta) z_N]^2}{\beta^2 \lambda + (1 - \beta) [\beta + (1 - \beta) z_N]^2} \left[ 1 - \frac{F}{(1 - m)^2} (1 - \beta/b^S) \right].$$

We define  $\tilde{m} \equiv 1 - \sqrt{F(1 - \beta/b^S)} > 0$  with  $\beta/b^S < 1$ , and where  $\tilde{m} > 0$  requires that  $(1 - \beta/b^S)^{-1} > F$ , which holds e.g. in the case of identical technology, i.e.  $F = 1$ . It follows that  $dz_N/dm > 0$  if  $m < \tilde{m}$ , and vice versa. In other words, the  $\dot{z}_N = 0$  locus is decreasing for  $m \in (\tilde{m}, \beta/b^S)$ , and increasing for  $m \in (0, \tilde{m})$  in the  $(z_N, m)$ -space. We note that  $z_N(m) \rightarrow -\infty$  as  $m \rightarrow 1$  and  $z_N(m)$  converges to a constant as  $m \rightarrow 0$ . We conclude, for values of  $z_N$  above the  $\dot{z}_N = 0$  locus,  $\dot{z}_N > 0$  and for values of  $z_N$  below the  $\dot{z}_N = 0$  locus,  $\dot{z}_N < 0$ .

Hence, we have a system of two differential equations in  $m$  (state variable) and  $z_N$  (choice variable), whose solution is saddle-path stable. Figure 6 below shows the phase diagram. We see that if  $m$  is below (above) its steady state value  $m^*$  it converges monotonically towards the steady state along the saddle path.

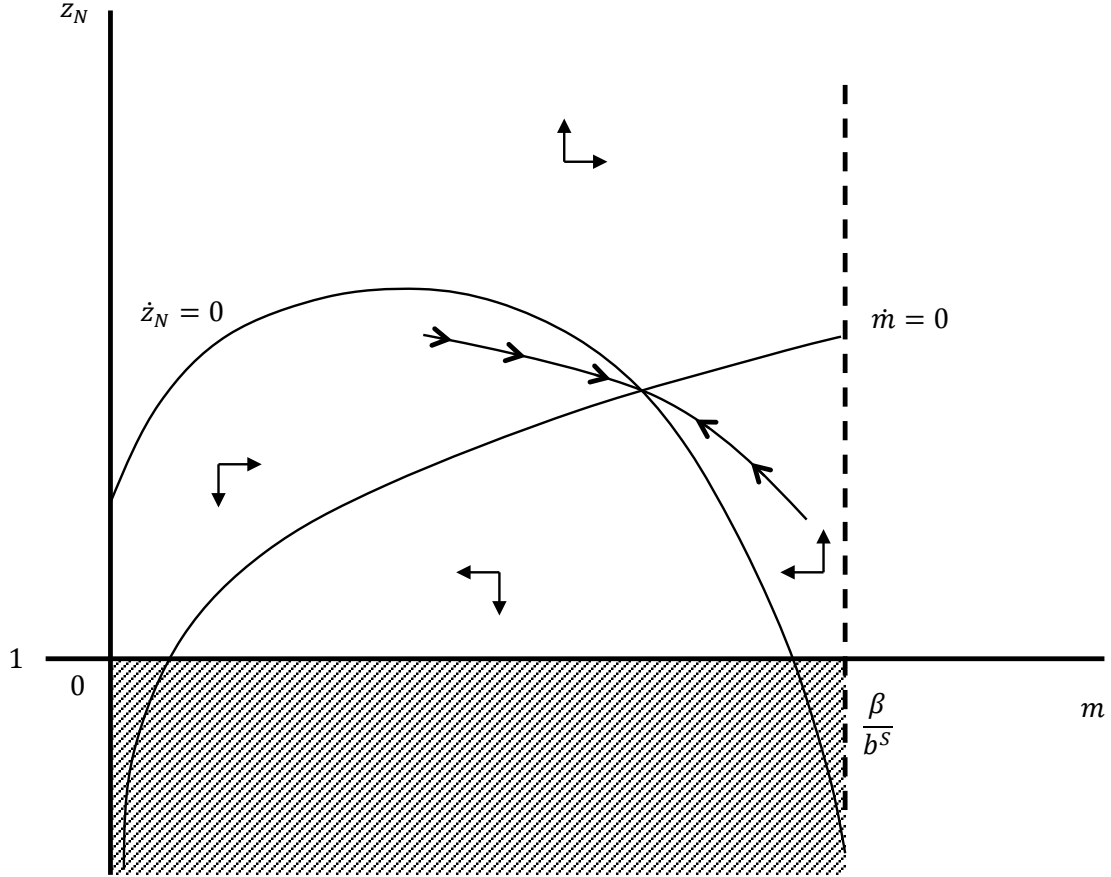


Figure 6: Phase diagram

## A.6 Simulations

We choose the following parameter configuration for our baseline simulation:  $L = 1$ ,  $F^N = F^S = 5$ ,  $b^N = b^S = 0.75$ ,  $\beta = 0.5$ ,  $\rho = 0.04$ , and  $T = 0$ .

### A.6.1 Change in Southern labor productivity

Figures 7-8 show the comparative statics results of a change in labor productivity in production in the South. Figures 9-10 show the comparative statics results of a change in labor productivity in R&D in South.

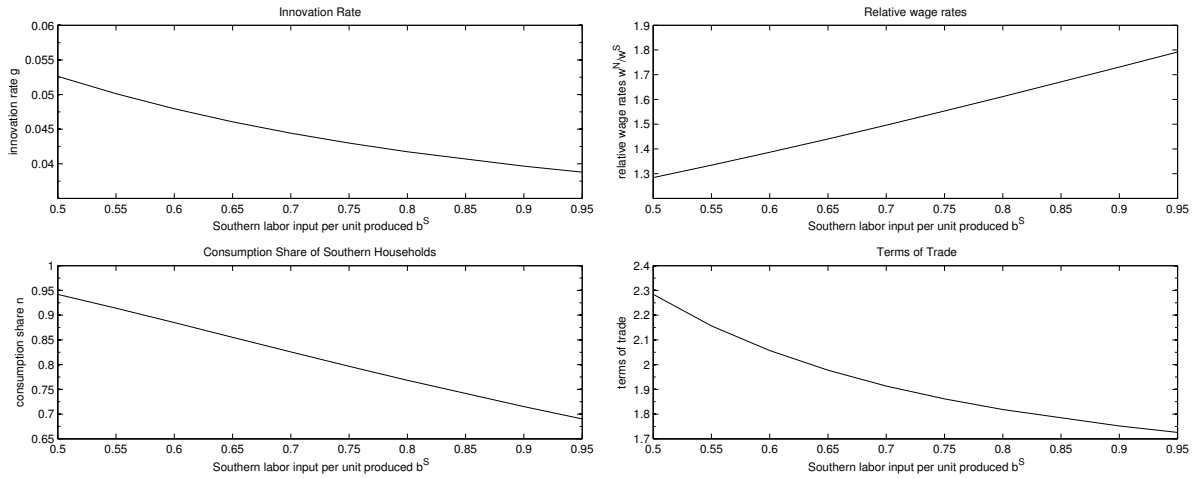


Figure 7: Upper panel: Effect on innovation rate and relative wages. Lower panel: Effect on consumption share of South and terms of trade.

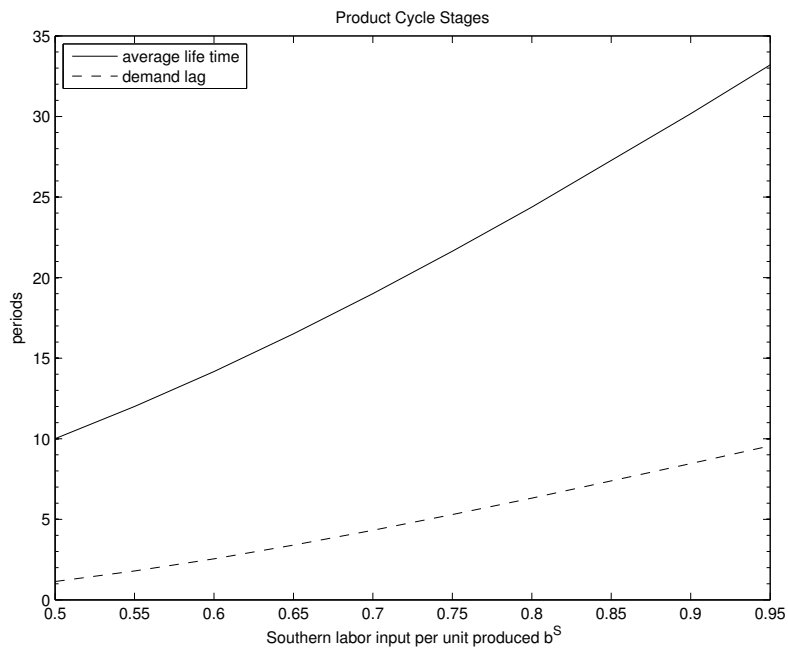


Figure 8: Effect on the stages of the product cycle

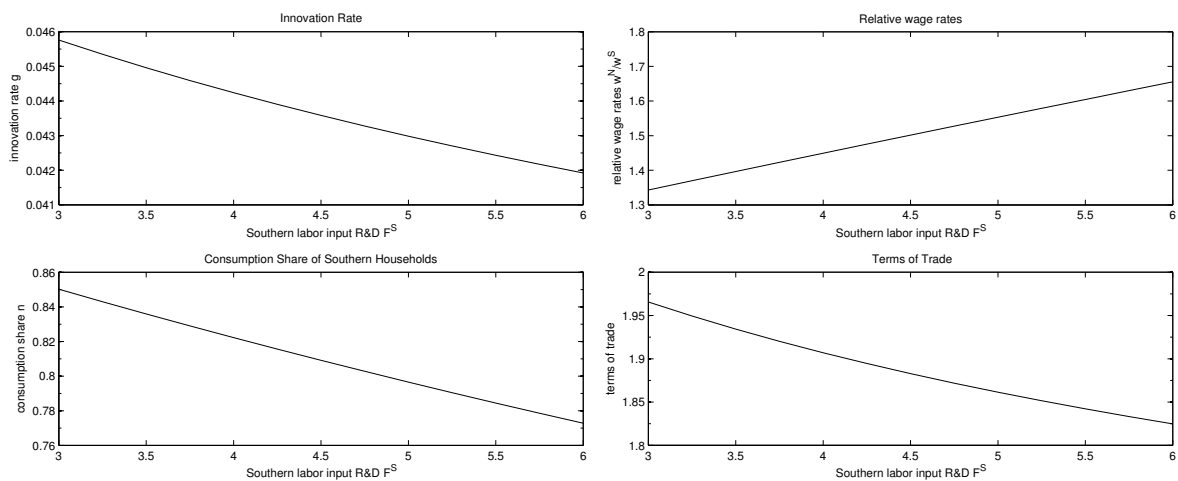


Figure 9: Upper panel: Effect on innovation rate and relative wages. Lower panel: Effect on consumption share of South and terms of trade.

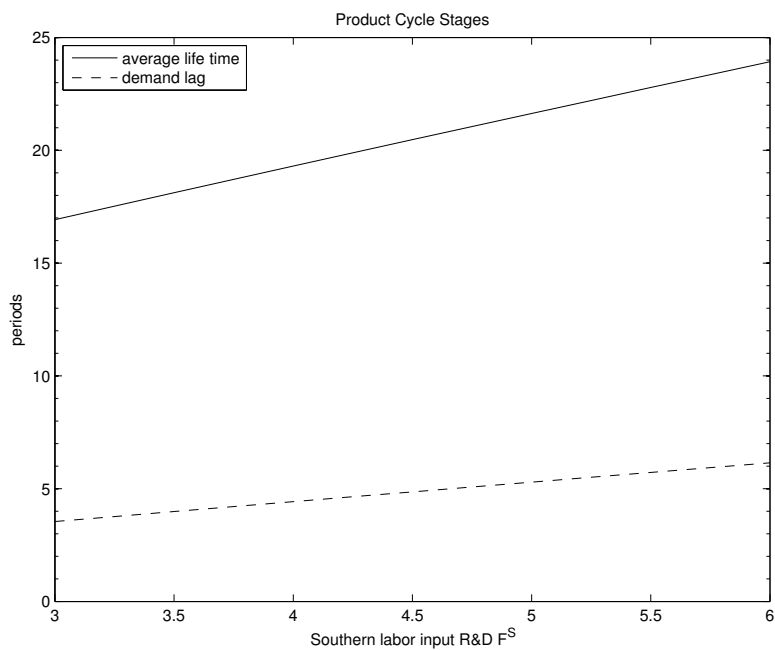


Figure 10: Effect on the stages of the product cycle

### A.6.2 Changes in Southern population

Figures 11-12 show the effects of an increase in Southern population.



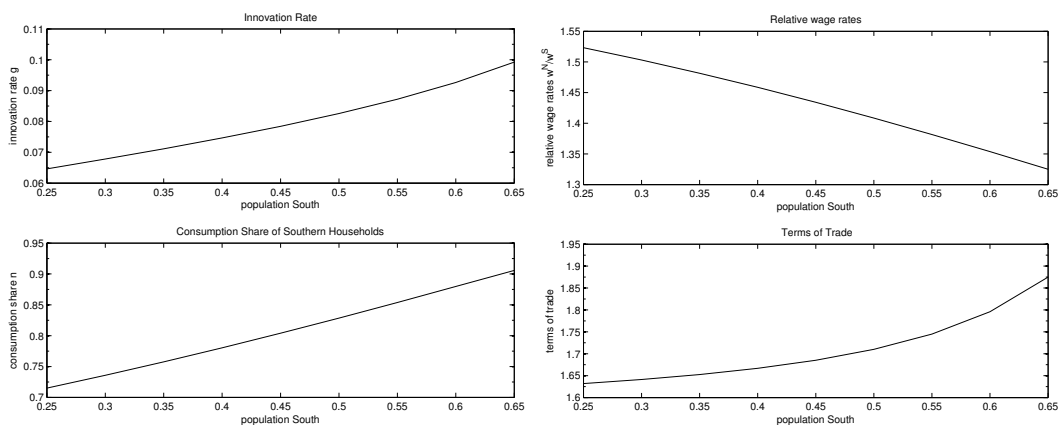


Figure 11: Upper panel: Effect on innovation rate and relative wages. Lower panel: Effect on consumption share of South and terms of trade.

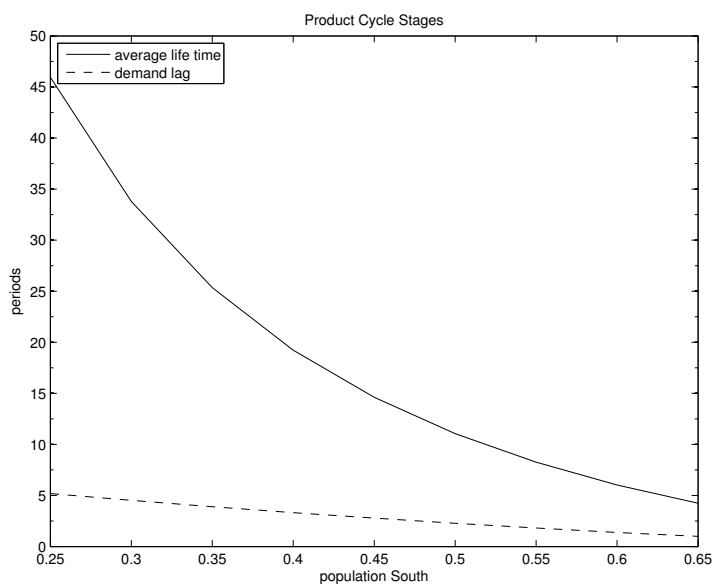


Figure 12: Effect on stages of the product cycle

### A.6.3 Changes in inequality across regions

Figures 13-14 depict the effects of a decrease in inequality across regions due to a transfer from rich North to poor South.

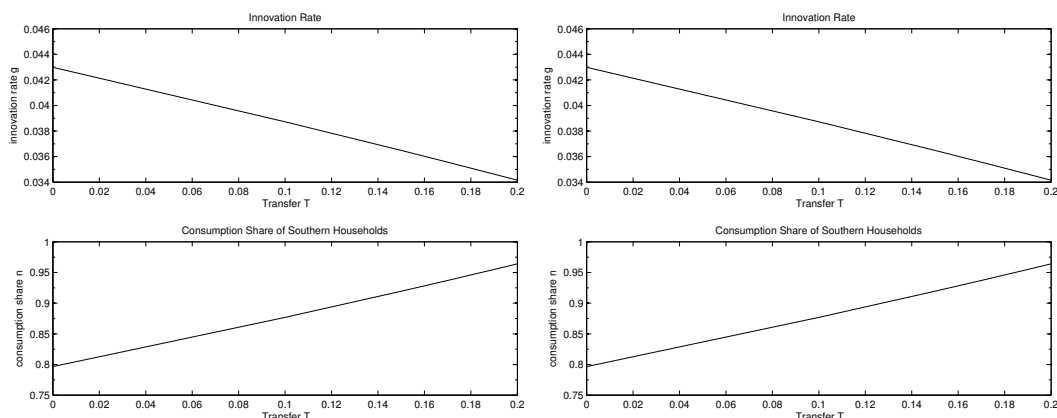


Figure 13: Upper panel: Effect on innovation rate and relative wages. Lower panel: Effect on consumption share of South and terms of trade.

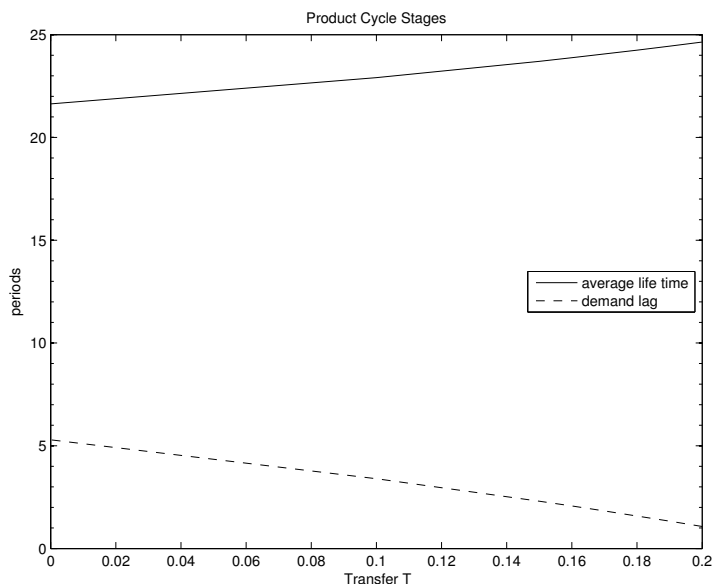


Figure 14: Effect on stages of the product cycle

## A.7 Suggestive evidence

### A.7.1 Demand lags

A web search readily shows that all of the products in Table 2 were first introduced in the United States: the electric dishwasher in 1950 by Hobart Corp., the automatic electric clothes dryer in 1949 by Hamilton Manufacturing Corp. and General Electric, the domestic deep freezer in 1949 by General Electric, the countertop microwave oven in 1967 by Amana Corp., the VCR in 1965 by Sony, Ampex, and RCA, and the automatic electric washing machine in 1947 by Bendix and General Electric.

We estimate the following model with OLS:

$$\log(\Delta_{ij}) = \beta_0 + \beta_1 \log\left(\frac{GDP_{pc_{ij}}}{GDP_{pc_{US}}}\right) + \beta_2 \log\left(\frac{Pop_{ij}}{Pop_{US}}\right) + \varepsilon_{ij}$$

where  $\Delta_{ij}$  denotes the introduction lag, defined as the number of years that elapsed until product  $j$  was introduced in country  $i$ ,  $GDPpc_{ij}$  and  $Pop_{ij}$  denote GDP per capita and population size in country  $i$  at the time product  $j$  was introduced in the US, respectively. The coefficient  $\beta_1$  shows the importance of (relative) GDP per capita, holding relative population sizes constant. We expect  $\beta_1$  to be negative.

Table 2: Introduction of major consumer durables across Europe and the U.S.

	Dishwasher		Dryer		Freezer		Microwave		VCR		Washing Machine	
	Yr	GDPpc	Yr	GDPpc	Yr	GDPpc	Yr	GDPpc	Yr	GDPpc	Yr	GDPpc
Austria	1962	6296	1965	6296	1953	6296	1974	13575	1977	12651	1962	6296
Belgium	1960	7992	1960	7992	1956	7992	1974	13602	1975	12820	1955	7992
Denmark	1960	9366	1965	9366	1954	9366	1974	15980	1977	15145	1958	9366
Finland	1964	6192	1973	6192	1961	6192	1975	11590	1978	11203	1960	6192
France	1959	7107	1963	7107	1960	7107	1975	13578	1974	12522	1954	7107
Germany	1960	6251	1966	6251	1956	6251	1969	14348	1974	14377	1952	6251
Greece	1966	2576	1990	2576	1972	2576	1982	9742	na	na	1964	2576
Ireland	1965	5880	1963	5880	1958	5880	1976	8789	na	na	1966	5880
Italy	1961	5361	1968	5361	1965	5361	1975	12305	1976	11015	1957	5361
Netherlands	1960	9961	1968	9961	1960	9961	1971	16356	na	na	1960	9961
Norway	1961	9434	1970	9434	1957	9434	1976	16366	1977	14966	1960	9434
Portugal	1966	2614	1973	2614	1956	2614	1982	5937	na	na	1956	2614
Spain	na	na	1973	3796	1972	3796	1973	10215	1977	9321	1964	3796
Sweden	1959	10301	1969	10301	1953	10301	1973	17043	1977	16380	1958	10301
Switzerland	na	na	1966	13712	na	na	1973	22880	1978	22056	na	na
United Kingdom	1958	10447	1960	10447	1954	10447	1971	14886	1974	14365	1954	10447
United States	1950	13119	1949	13119	1949	13119	1967	19522	1965	18364	1947	13119
Spearman's $\rho$	-0.86		-0.55		-0.68		-0.65		-0.01		-0.46	

Notes: Spearman's  $\rho$  denotes Spearman's rank correlation coefficient. Heston, Summers and Aten (2012) provide data from 1950-2009. GDP per capita in 1947 and 1949 is approximated by GDP per capita in 1950. Sources: Tellis, Stremersch and Yin (2003); Heston, Summers and Aten (2012).

## A.7.2 Export and production patterns

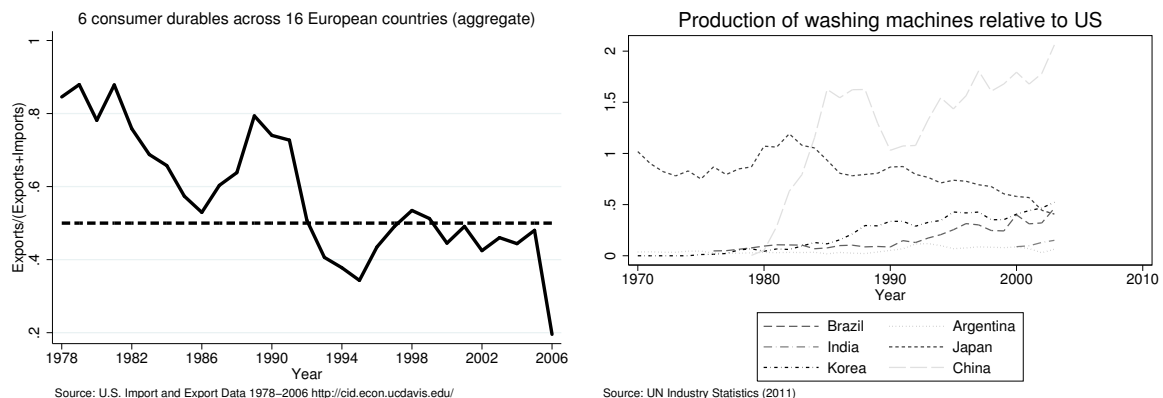


Figure 15: US export performance across all 6 consumer durables and 16 European countries (left-hand side), and production of the washing machine relative to the US (right-hand side)