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REGULAR ARTICLE

Is inequality harmful for innovation and growth? Price versus market size effects

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Abstract We introduce non-homothetic preferences into an R&D based growth model to study how demand forces shape the impact of inequality on innovation and growth. Inequality affects the incentive to innovate via a price effect and a market size effect. When innovators have a large productivity advantage over traditional producers a higher extent of inequality tends to increase innovators' prices and mark-ups. When this productivity gap is small, however, a redistribution from the rich to the poor increases market sizes and speeds up growth.

Keywords Inequality · Growth · Demand composition · Price distortion

JEL Classification O15 · O31 · D30 · D40 · L16

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1 Introduction

The distribution of income and wealth across households may affect incentives to undertake R&D investments through price- and market-size effects. On the one hand, innovations are fostered if there are rich consumers willing to pay high prices for new products. On the other hand, profitable innovations require sufficiently large markets, which may be lacking when incomes are concentrated among a small number of rich households. Several previous writers have mentioned the importance of price and market size effects for innovation and growth. A prominent advocate for the importance of price effects is von Hayek (1953) who argues that progressive taxation would be detrimental for innovation incentives by reducing rich consumers' willingness to pay for new goods. In contrast, Schmookler (1966) provides a forceful statement emphasizing the relevance of market size effect in fostering R&D investments.

To study these competing forces of income inequality, we introduce non-homothetic preferences into a standard R&D based growth model. Households either consume one unit of a particular product or do not consume it. The assumption of such binary consumption implies that consumer get quickly satiated within a product line. (By assumption, consuming more than one unit of a product does not generate utility.) In such a framework, demand can only keep pace with income growth when new products are invented or when there are other (non-innovative) sectors that can absorb the residual demand. We assume that goods are produced by monopolistic firms who, after having made an innovation, have a monopoly on their product. We also assume that there is a non-innovative sector (which could, alternatively, be leisure or home production) that captures the residual demand of rich consumers. Within this framework, we can characterize the relative importance of price and market size effects which affect innovation incentives in opposite directions. Higher inequality increases the demand for non-innovative goods, because some rich consumers are already satiated with innovative products, redistributing income towards them diverts demand from the innovative to the non-innovative sector. This "market size" effect reduces the incentive to innovate. However, higher inequality may also raise the willingness to pay for innovative products for those who are not yet satiated with innovative products. The higher willingnesses to pay allow innovators to increase the mark-ups. These "price" effects increase the incentive to innovate.

Our paper builds upon the mainstream endogenous growth literature. On the consumption side, we assume that consumers are rational, forward looking and have time-invariant preferences. They have the same preferences which are unrelated to the economic role of the individuals. (This is different from e.g. the Post-Keynesian literature where propensities to save and consume differ by economic classes.) However, our approach differs from standard models because we assume that preferences are non-homothetic. This assumption implies that the distribution of income and wealth affects the demand for innovative products. On the production side, we follow the mainstream literature. Producers have fixed set-up costs and operate in an environment of monopolistic competition. Restricting ourselves to a parsimonious set of assumptions, we end up with a tractable framework that has interesting implications for the relationship between inequality, innovation and growth. In line with a large part of the mainstream literature our analysis focuses on balanced growth paths. The advantage of this approach is that the economic impact of changing an exogenous policy parameter can be understood well by comparative statics. Clearly, the drawback is that balanced growth analysis does not allow for feedback loops which are an important feature of the evolutionary literature. We come back to this issue in the discussion section.

A few previous papers have studied the role of income distribution on incentives to innovate via price- and/or market-size effects. In Murphy et al. (1989) a more egalitarian distribution increases the expenditure share for innovative goods and reduces the share of traditional products thus fostering industrialization. Their model emphasizes the market size effect, while potential effects on prices are ruled out by the assumption of constant prices and mark-ups. Moreover, their model is static and does not study the impact of inequality on growth. Falkinger (1994) studies a model of growth along a hierarchy of wants. He shows that the inequality growth relation depends on the nature of the technical progress but his model does not consider price effects. The present paper differs from our own previous work because of its emphasis on the relative importance of price and market size effects for the inequality-growth relationship. The analysis in Foellmi and Zweimüller (2006) does not allow for market-size effects. In that framework monopolistic producers do not face any restrictions in their pricing behavior because of the absence of a sector absorbing the residual demand of satiated rich consumers. This implies very high willingnesses to pay for new products and hence stronger innovation incentives when inequality increases. In Foellmi et al. (2014) we elaborate the potential impact of inequality on the introduction of high-quality luxury products versus low-quality mass products. In that framework, the inequality growth relationship depends on the nature of technical progress, i.e. whether new product innovations or the introduction of mass production technologies drive technical progress. This is quite different from the present model where price and market-size effects arise in a more parsimonious framework of horizontal innovations (and without any quality differentiation).[1]

There is an established heterodox literature that studies the interrelationship between consumption/savings, inequality and growth. We come back to seminal contribution of Kaldor (1966) in the discussion section. For recent contributions in the Post-Keynesian tradition, see Salvadori (2006), Kurz and Salvadori (2010),

[1]Non-homothetic preferences have turned out important to explain the structural changes in employment and output in long-run growth, see Matsuyama (1992, 2008), Buera and Kaboski (2006), Foellmi and Zweimüller (2008), and Boppart (2014). Other papers have studied how inequality affects growth via non-homotheticities have also emphasized market size effects. In Matsuyama (2002) technical progress is driven by learning by doing and an intermediate degree of inequality is required to realize the full learning potential. Falkinger (1994) studies the impact on inequality on market sizes under the assumption of exogenously given profit-margins. In Chou and Talmain (1996) consumers have non-homothetic preferences over a homogenous consumption good and a (CES) bundle of differentiated goods which affects the market size (but not the mark-up) of innovators. Zweimüller (2000) provides a dynamic version of Murphy et al. (1989). In Galor and Moav (2004) non-homotheticities affect growth via savings rates that differ by income. A very different strand of the literature studies the interrelationship between consumption/savings, inequality and growth in the Post-Keynesian tradition. For recent contributions, see Salvadori (2006) and Kurz and Salvadori (2010).

and Araujo (2013). An important issue in the evolutionary literature is the question how consumer wants emerge and generate sufficient demand in an environment of growing incomes (Witt 2001). The problem of demand growth is crucial in our framework, where consumers are quickly satiated with existing products and demand can only keep pace with growing incomes when new goods are introduced.[2] Several recent studies explore agent-based frameworks to understand the relation (and feedback loops) of growth, inequality and consumption patterns. Ciarli et al. (2010) study an agent-based framework where technological changes lead to changes in the income distribution that feed back to consumption patterns. Bernardino and Araujo (2013) explore the importance of inequality when technical progress is driven by the demand for positional goods. Lorentz et al. (2015) study how the micro-dynamics of consumption behavior are related to inequality and they show that increased heterogeneity in consumer's reaction to income changes affect firm selection and the dynamics of market structure.

Our analysis provides a theoretical framework for a better understanding of the role of demand forces in the inequality-growth literature relationship. The initial empirical literature provided support for the idea that inequality is harmful for growth (Alesina and Rodrik 1994; Persson and Tabellini 1994; Deininger and Squire 1996), while more recent studies do not support such a clear-cut relationship. Barro (2000, 2008) shows that there is a positive relationship for rich countries but a negative one for poor countries. Forbes (2000), using panel data, finds a positive relationship. The more recent literature uses new and better data and also tries to overcome methodological shortcomings of previous studies. However, the new studies were not able to come up with clear-cut results. Banerjee and Duflo (2003) find nonlinear relationships between inequality and growth, while Voitchovsky (2005) shows that inequality at the top is positively and inequality at the bottom is negatively related to subsequent growth. More recent work by Berg and Ostry (2011) finds that more equal counties have significantly longer growth spells. Halter et al. (2014) argue that inequality may affect growth negatively in the short run, but positively in the long run. Ostry et al. (2014) point to the important distinction of the effects on growth of pre-tax inequality and of redistribution through the tax transfer-system. However, the new empirical literature remains inconclusive (see Voitchovsky 2009 and Boushey and Price 2014 for recent surveys). It is therefore important to understand the mechanisms through which ambiguities may arise.

In Section 2 we present the main assumptions of the model. Section 3 discusses price determination and market sizes and their implications for the incentives to undertake R&D investments. In Section 4 we look at the balanced growth path. Section 5 studies the impact of inequality on long-run growth and Section 6 provides a discussion of important assumptions and how they affect the inequality-growth relationship.

[2]Empirical evidence suggests that the diversity of consumption is closely linked to household income, see e.g. Jackson (1984), Falkinger and Zweimüller (1996), Chai and Rohde (2012).

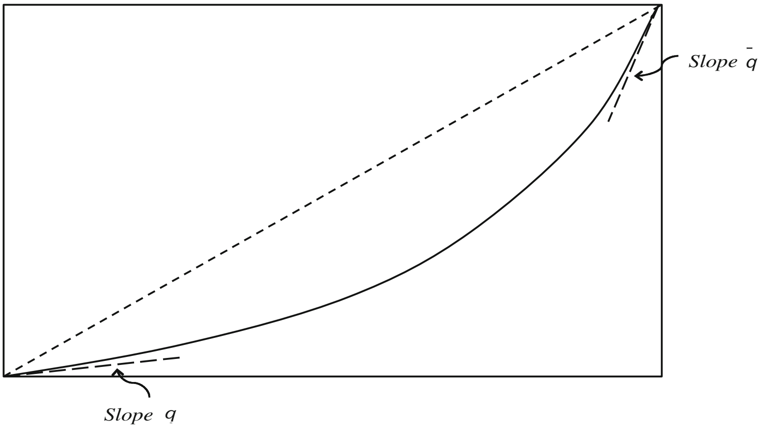

Fig. 1 Lorenz curve

2 The model

Endowments and distribution Consider an economy with a unit measure of households whose aggregate supply of labor is L, constant over time. Households get income from wages and profits. At date t there are $N(t)$ monopolistic firms generating positive profits. There is a nondegenerate distribution of income reflecting both skill differences and differences in capital ownership. A household is endowed with θ units of labor and $\theta N(t)$ shares of profitable firms, where θ is distributed across households with the cdf $G(\theta)$ with support $\left[\underline{\theta}, \bar{\theta}\right]$. A household with endowment θ earns labor income $\theta w(t)L$ and capital income $\theta r(t)V(t)$ where $w(t)$ is the wage rate per unit of effective labor, $r(t)$ is the interest rate, and $V(t)$ is the aggregate value of assets (i.e. the capitalized value of all existing firms). The resulting distribution is shown in the Lorenz curve of Fig. 1 below.[3]

Preferences and consumption choices All households have the same preferences. There is an infinitely large number of potentially producible goods, $j \in [0, \infty)$. All goods are equally valued by the household. Goods are consumed in discrete amounts and the household is saturated after consuming one unit. We denote by $x(j, t)$ the indicator function such that $x(j, t) = 1$ if good j is consumed at date t,and $x(j, t) = 0$ if not.

Households have an infinite horizon. A household with endowment θ chooses $\{x(\theta, j, t)\}_{j=0,t=\tau}^{j=\infty,t=\infty}$ to maximize

$$\int_{\tau}^{\infty} \log\left[\int_{0}^{\infty} x(\theta, j, t)dj\right] e^{-\rho(t-\tau)}dt$$

[3]The assumption that labor and capital endowments are perfectly correlated and identically distributed is made for analytical convenience. Below we assume additive and logarithmic *intertemporal* preferences generating equal optimal savings rates for all households. This assumption (and the absence of income shocks) ensures that the initial distribution of θ persists over time. Hence time indices for θ are omitted.

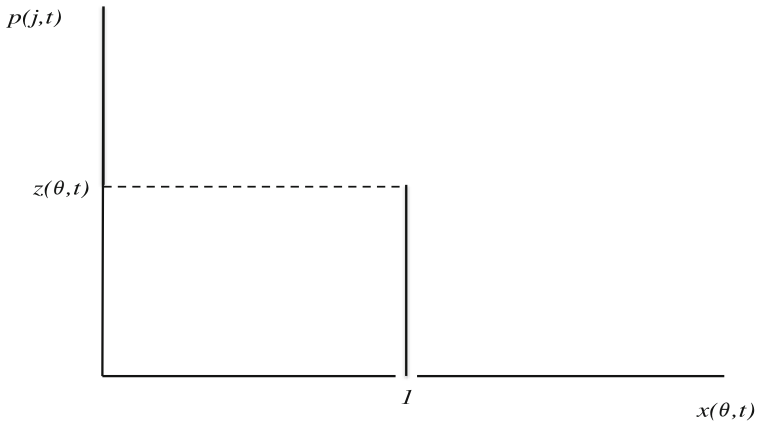

Fig. 2 Individual demand

subject to

$$\int_{\tau}^{\infty}\left[\int_{0}^{\infty} p(j,t)x(\theta,j,t)dj\right]e^{-R(t,\tau)}dt \leq \theta\left[\int_{\tau}^{\infty} w(t)Le^{-R(t,\tau)}dt + V(\tau)\right]$$

where $p(j,t)$ is the price of good j at date t,and $R(t,\tau) = \int_{\tau}^{t} r(s)ds$ is the cumulative interest factor.[4] The optimal solution to this intertemporal choice problem satisfies the first-order condition

$$x(\theta,j,t) = \left\{\begin{array}{ll} 1, & p(j,t) \leq z(\theta,t) \\ 0, & p(j,t) > z(\theta,t) \end{array}\right\} \text{ where } z(\theta,t) \equiv \frac{e^{R(\tau,t)-\rho(t-\tau)}}{\mu(\theta)N(\theta,t)}, \quad (1)$$

where $z(\theta,t)$ is the household's willingness to pay. Here $z(\theta,t)$ is inversely related to $\mu(\theta)$, the household's time-0 marginal value of wealth (the Lagrangian multiplier), and to $N(\theta,t) \equiv \left[\int_0^{\infty} x(\theta,j,t)dj\right]$, the optimal quantity consumed at date t. Equation 1 represents a simple consumption rule: A household with endowment θ purchases good j at date t if the price of this good, $p(j,t)$, does not exceed the consumer's willingness to pay at that date. Consequently, individual demand is a simple step function, see Fig. 2.

Production and technical progress The supply side of the model is very simple. All goods are produced with identical technologies and labor is the only production factor. Each good can be produced in two different ways, with a traditional and with an innovative technology. The *traditional* technology has productivity $\beta(t)$ and operates under constant returns to scale. and the *innovative* technology has productivity $\alpha(t)$ with $\beta(t) < \alpha(t)$. Unlike the traditional technology (that produces under constant returns to scale. The innovative technology requires an initial (one-time) set-up effort equal to $\Phi(t)$ units of labor. A firm that incurs this set-up cost makes an

[4] The log intertemporal utility is used for ease of exposition. The same results would hold true (in particular the invariance of distribution) if the utility would be CRRA in the consumption aggregator $\int_0^{\infty} x(\theta,j,t)dj$.

"innovation". Think of an innovation as a completely new good that crowds out traditional products or, alternatively, as an improved way to produce an already existing good.[5] (Both interpretations are valid as both existing and new goods enter the utility function symmetrically). In line with endogenous growth theories, we assume that the knowledge stock of this economy equals the number of innovations that have taken place up to date t,denoted by $N(t)$. It is assumed that $\Phi(t) = F/N(t)$, $\alpha(t) = aN(t)$ and $\beta(t) = bN(t)$ where $F > 0$ and $a > b > 0$ are exogenous parameters. We normalize labor costs in the traditional sector to unity $w(t)/\beta(t) = 1$. Under our assumption on the knowledge stock, this implies that the growth rate of wages equals the rate of innovation, $w(t) = bN(t)$. This implies that production costs in the innovative sector are $w(t)/\alpha(t) = b/a < 1$, and the innovation cost are $w(t)\Phi(t) = bF$, constant over time.

3 Prices, market sizes, and innovation incentives

Price setting of monopolistic firms There is a measure of $N(t)$ monopolistic firms, equal to the number of innovations, on the market. By symmetry, all firms face the same cost- and demand-curves. The representative firm faces a trade-off between setting a high price and selling to a small group of consumers (and vice versa). The traditional technology is freely accessible, hence the innovative firm has to choose a price lower than (or equal to) unity to prevent the competitive fringe from entering the market. Consumers purchase the goods with the lowest prices until they exhaust their budget.

The equilibrium outcome is most easy to grasp when there are only two groups, rich and poor. The representative firm faces the choice between selling only to the rich at a high price; or selling to all households at a price low price. For obvious reasons, a situation where *all* firms sell to *all* consumers or where *all* firms sell *only to the rich* can not be an equilibrium. In the former case, both groups would have identical expenditures and in the latter case, the poor would have no expenditures implying that one of the two groups does not exhaust its budget. As left-over budgets are associated with very high willingnesses to pay for the marginal good, firms have an incentive to deviate in both cases. In equilibrium, an (endogenous) fraction of firms sells exclusively to the rich and the remaining fraction of firms sells to all households. The former set a price that equals the willingness to pay of the rich and the latter set a price that equals the willingness to pay of the poor. In equilibrium, both types of firms make the same profit and both types of households exhaust their budget. Obviously, these arguments generalize to $K > 2$ discrete groups.[6]

[5]An isomorphic case would the situation where innovative firms produce a better product, yielding higher utility, with the same production technology as traditional firms (or some combination of productivity/quality gain).

[6]With K groups of consumers, there are K firm types such that type 1 sells to the richest group (and charges their willingness to pay), the second type sells to the richest and second richest (and charges the willingness to pay of the second richtest group), ..., and the Kth type sells to all households (charging the willingness to pay of the poorest group). In equilibrium the distribution of firms across types is endogenous and satisfies conditions (i) and (ii) mentioned in text.

A continuous distribution $G(\theta)$ generates a continuous distribution of prices and firm sizes. We label of a firm as type θ when it sells to all households with endowment θ or richer (and has market size $[1-G(\theta)]L$). A firm of type θ charges a price $p(\theta)=z(\theta)$, the willingness to pay of household θ.[7] Notice that $p(\theta)$ is increasing in θ, reflecting the basic trade-off that firms face: either they sell at low prices and high quantity (the market size $[1-G(\theta)]L$ is decreasing in θ) or they set higher prices but have a smaller market size. Due to the competitive fringe, modern firms are limited to types $\theta\in[\underline{\theta},\hat{\theta}]$ where firm $\hat{\theta}$ charges a price $p(\hat{\theta})=1$ and has profits $[1-G(\hat{\theta})]L[1-b/a]=\pi$.

The equilibrium firm size distribution is given as follows: A measure $n(\underline{\theta})$ of monopolistic firms sells to all consumers and produces L units. The firm size distribution is continuous and determined by the endowment distribution $G(\theta)$ (see Lemma 1 below). Each firm makes the same profit $\pi=[1-G(\theta)]\,L\,[p(\theta)-b/a]$ for all $\theta\in[\underline{\theta},\hat{\theta}]$. Consumers with $\theta<\hat{\theta}$ consume only a subset of innovative goods and consumers with $\theta>\hat{\theta}$ consume all innovative goods and some traditional products supplied by the competitive fringe. Our analysis below treats $\hat{\theta}$ as the crucial endogenous variable (in addition to the endogenously determined growth rate).

Zero profit condition The costs of an innovation are bF, constant over time (see above). The value of an innovation equals the profit flow associated with a monopoly position. We assume an innovator gets a patent that lasts forever. The profit flow is equal to $\pi=[1-G(\hat{\theta})]L[1-b/a]$, independent of t as long as $\hat{\theta}$ is time-invariant which is the case along a balanced growth path. Along this path the interest rate is constant, $r(t)=r$, and the value of the innovation is given by $\int_t^\infty \pi\exp(-r\,(s-t))dt=\pi/r$. In equilibrium, the value of an innovation may not exceed innovation costs, $\pi/r\leq bF$. This condition holds with equality when innovation takes place. The zero-profit condition can then be written as

$$rbF=[1-G(\hat{\theta})]L[1-b/a]. \tag{2}$$

4 The balanced growth path

Along the balanced growth path, the economy's resources are fully utilized. Labor demand in research is $\dot{N}(t)\Phi(t)$. Using $\Phi(t)=F/N(t)$ and the definition $g\equiv\dot{N}(t)/N(t)$ we get $\dot{N}(t)\Phi(t)=gF$. Labor demand in production comes either from innovative monopolistic producers or from traditional competitive producers. A consumer with endowment $\theta\leq\hat{\theta}$ purchases $N(\theta,t)$ goods supplied by innovative producers which requires $N(\theta,t)/\alpha(t)=[N(\theta,t)/N(t)]/a$ units of labor. A consumer with endowment $\theta>\hat{\theta}$ purchases all $N(t)$ goods by innovative producers and $N(\theta,t)-N(t)$ goods by traditional producers which requires $[N(\theta,t)-N(t)]/\beta(t)+N(t)/\alpha(t)=1/a+[N(\theta,t)/N(t)-1]/b$ units of labor.

[7] Instead of writing consumption expenditures of a household with endowment $\tilde{\theta}$ as $\int_0^{N(\tilde{\theta})}p(j)dj$ we can write $\int_{\underline{\theta}}^{\tilde{\theta}}p(\theta)dN(\theta)+p(\underline{\theta})N(\underline{\theta})$ where $N(\underline{\theta})$ and $p(\underline{\theta})$ are the menu and the price of the goods that the poorest household can afford.

In a steady state, where the distribution of income and wealth is stationary, consumption $N(\theta, t)$ grows at the same constant rate g for all consumers. Defining $n(\theta) \equiv N(\theta, t)/N(t)$, we see that $n(\theta)/a$ units of labor are needed to produce the goods consumed by household $\theta \leq \hat{\theta}$; and $1/a + (n(\theta) - 1)/b$ for household $\theta > \hat{\theta}$. Summing up labor demands and setting them equal to aggregate labor supply yields the full employment condition

$$L = gF + \frac{L}{a}\left(\int_{\underline{\theta}}^{\hat{\theta}} n(\theta) dG(\theta) + 1 - G(\hat{\theta})\right) + \frac{L}{b}\int_{\hat{\theta}}^{\bar{\theta}} (n(\theta) - 1)\, dG(\theta).$$

Lemma 1 *a. The stationary distribution of endowments $G(\theta)$ is associated with stationary prices $p(\theta)$; consumption expenditures equal to $N(t)\theta b\,[1 + \rho F/L]$ and consumption growth $g = r - \rho$; b. The optimal consumption levels are:*

$$n(\theta) = \begin{cases} a\underline{\theta}\frac{L+\rho F}{(g+\rho)aF+L} & \theta = \underline{\theta} \\ a\int_0^{\theta} \frac{(L+\rho F)(1-G(\xi))}{(g+\rho)aF+L(1-G(\xi))} d\xi & \underline{\theta} < \theta < \hat{\theta} \\ 1 + (L + \rho F)\,(\theta - \hat{\theta})b/L & \theta \geq \hat{\theta} \end{cases} \quad (3)$$

Proof See Appendix. □

In steady state, consumers with relative wealth $\hat{\theta}$ consume all innovative goods, hence $n(\hat{\theta}) = 1$ or

$$1 = a\int_0^{\hat{\theta}} \frac{(L + \rho F)\,(1 - G(\theta))}{(g + \rho)aF + L\,(1 - G(\theta))} d\theta. \quad (4)$$

The resulting consumption structure of innovative goods is depicted in Fig. 3. The poorest consumers buy only a fraction $n(\underline{\theta})$ of the innovative products, while households with wealth $\theta > \hat{\theta}$ buy all innovative products.

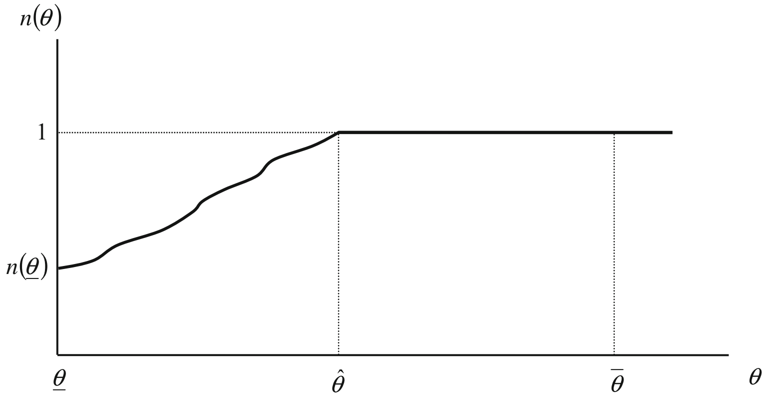

Fig. 3 Share of innovative products consumed

This gives rise to a firm type and size distribution (Fig. 4). Denote the poorest consumer served by a distinct firm as critical consumer. There is a continuum of firm where the critical consumers have wealth $\theta > \underline{\theta}$, and a positive mass of firms sell to all consumers. Correspondingly, there is a positive mass of firms with size L (Fig. 4b).

The general equilibrium of the model is characterized by the two equations (2) and (4) in the two unknowns g and $\hat{\theta}$. Note that the resource constraint holds when both Eqs. 2 and 4 simultaneously are satisfied. We analyze the equilibrium graphically (Fig. 5).

The zero profit condition is a decreasing curve in the $(g, \hat{\theta})$-space. If $\hat{\theta}$ increases, market size is smaller (recall that $p(\hat{\theta}) = 1$ which is constant), hence the real interest rate $r = \rho + g$ must be smaller to guarantee a zero profit equilibrium. The consumption equation (4) is an increasing curve in the $(g, \hat{\theta})$-space. In general equilibrium, higher growth rates must go hand in hand with larger prices and lower expenditures. Therefore, the relative wealth $\hat{\theta}$ must be larger such that $n(\hat{\theta}) = 1$ holds.

5 The impact of inequality on growth

We concentrate on the relevant case where only sufficiently rich consumers are able to purchase *all* modern products. Such an equilibrium emerges when

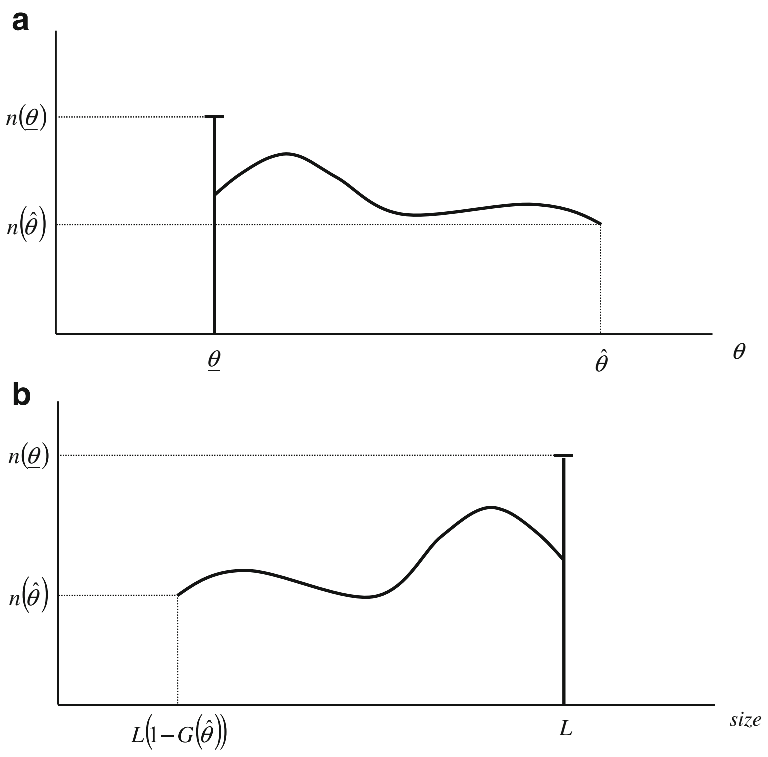

Fig. 4 **a** Firm-type distribution. **b** Firm-size distribution

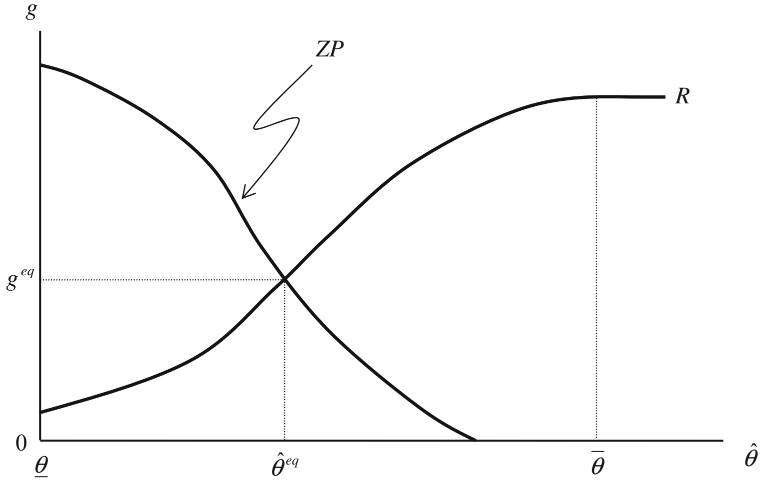

Fig. 5 General equilibrium

$\underline{\theta}\,(1+\rho F/L) < 1/b$. To see this, notice that an equilibrium with $\hat{\theta} > \underline{\theta}$ (such that some households cannot afford to buy all goods) requires that, at $\theta = \underline{\theta}$, the value of g that satisfies the zero profit constraint (2) has to exceed the value of g in the consumption equation (4). If this condition is violated even the poorest consumer buys all innovative products. In such an equilibrium, the prices of all innovative goods are unity (all monopolistic firms have to charge a price that deters entry from competitive producers) and market size is at its highest possible level. In such an equilibrium, changes in inequality do not have an impact on growth.

Assuming $\underline{\theta}\,(1+\rho F/L) < 1/b$,we are now ready to analyze the effect of more inequality.

Proposition 1 *a. A regressive transfer among consumers with $\theta < \hat{\theta}$ increases the growth rate. b. A regressive transfer from a consumer with $\theta < \hat{\theta}$ to a consumer with $\theta > \hat{\theta}$ reduces the growth rate. c. A regressive transfer among consumers with $\theta > \hat{\theta}$ leaves the growth rate unaffected.*

Proof See Appendix. □

The reason behind part a. of the above proposition is the dominance of the *price effect*. A regressive transfer among consumers with $\theta < \hat{\theta}$ is a transfer from consumers who pay a lower price to consumers who pay on average a higher price. Therefore a regressive transfer among consumers with $\theta < \hat{\theta}$ increases average prices and mark-ups. In the new equilibrium the profit flow π of innovative producers is larger which increases the incentive for further innovation. Figure 6a characterizes part a. of the proposition graphically.

Part b. of the proposition reflects the dominance of the *market size effect*. A regressive transfer from a consumer with $\theta_0 < \hat{\theta}$ to a consumer with $\theta_1 > \hat{\theta}$ implies an decrease in demand for monopolistic producers and an increase in demand for traditional producers. In particular, household θ_0 has a lower willingness to pay and hence firm θ_0 experiences a fall in its price. In equilibrium all firms earn the same profit. Hence the reduction in the price of one firm must decrease the prices of other firms. The result is a reduced incentive to innovate. Figure 6b shows the change in the equilibrium curves for a regressive transfer from the consumer of the "middle class" with $\theta_0 < \hat{\theta}$ to a rich consumer with $\theta_1 > \hat{\theta}$.

Finally, part c. of the proposition results from the fact that a redistribution among households with $\theta > \hat{\theta}$ does not affect the demand for innovative products at all and leave prices and market size of innovative firms unaffected.

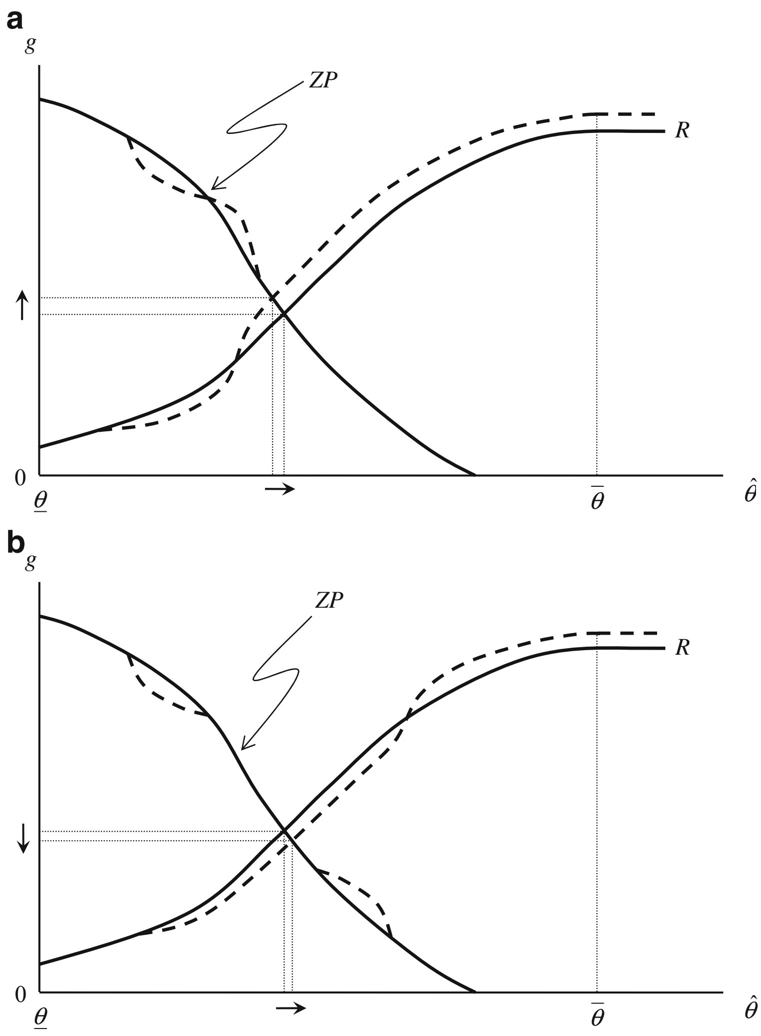

Fig. 6 **a** Regressive redistribution among the middle class. **b** Redistribution from the middle class to the rich

How do our results relate to existing demand explanations of the relationship between inequality and growth? It is crucial for the inequality-growth relationship to which innovative firms are constrained in their price setting behavior. In the polar case where no competitive fringe exists (as in Foellmi and Zweimüller 2006), only price effects are at work and inequality is beneficial for growth. In the presence of a competitive fringe, it is the cost advantage of innovators that determines the scope for price setting. If the technology gap is small, price effects are weak and market size effects dominate implying that inequality is harmful for growth. Our model encompasses both mechanisms. When the productivity gap between the innovative sector and the traditional sector $a/b > 1$ is very high, $\hat{\theta}$ approaches $\bar{\theta}$. In that case, only the very rich can afford all innovative products, all households (except the richest) consume only a subset of innovative goods, and the competitive fringe has only a tiny market share. Consequently, regressive transfers increase growth.

Foellmi et al. (2014) allow for quality differentiation to generate market size effects. Innovators supply high-quality products to the rich and mass products to the poor. In a highly unequal economy, most firms supply only high-quality to serve rich consumers while in an egalitarian society, mass production is more prevalent. The extent of inequality determines the relative attractiveness of introducing new products versus implementing mass production of existing products. Whether inequality is beneficial or harmful for growth depends on the nature of technology: When the introduction of mass production technology is the driver of technical progress, high inequality decreases growth. The opposite is true when the product innovations (i.e. the invention of high-quality goods for the rich) are the engine of growth. In contrast to Foellmi et al. (2014), the present paper generates price- and market size effects even in the absence of quality differentiation.

Our model also encompasses the situation studied in Murphy et al. (1989). Similar to their model, a regressive transfer from households who cannot purchase all innovative goods to consumers who can afford all these goods, decreases innovators' market size and depresses growth. However, in Murphy et al. (1989) only the market size effect is at work and price effects are ruled out by assumption. Hence a redistribution among consumers $\theta < \hat{\theta}$ (all of whom cannot purchase the entire menu of innovative goods) leaves the the number of industrializing sectors unaffected in their model as only the market size effect is at work. In contrast, in our analysis such a redistribution generates price effects that lead to a positive impact of inequality on growth. In sum, our model predicts that redistributions towards consumers just below $\hat{\theta}$, both from below and from above, increase growth.

As changes in inequality affect the growth rate, it is interesting to ask whether growth-increasing redistributions can lead to Pareto-improving outcomes. Interestingly, the answer is a qualified yes for both types of redistributions. The group of donating households loses on impact but gains from a steeper consumption path in the long run. Pareto-improvements may occur if the rate of time preference is very low, so that the donators values dynamic gains more strongly than static losses. Pareto-improvements are also more likely the higher the standard of living of the donators. In that case, static losses are smaller (due to lower marginal utilities).

6 Discussion

The above relationship between inequality and growth has focused on a balanced growth path and was derived under simplifying (and restrictive) assumptions on the distribution of wages and profits. The purpose of this section is to discuss the robustness of our results and their relations to the related evolutionary and Post-Keynesian literature. In particular, we discuss what happens when we allow for (i) asymmetric distributions of labor earnings and capital incomes and (ii) non-stationarities and feedback effects from growth to inequality. The main message is that adding realism to our distributional assumptions makes the model much more complex but does not change the results qualitatively. Moreover our model is also a useful starting point for more general non-stationary environments. In this sense, the assumptions are made for simplicity, but carry over to more general situations.

Endogeneity of the income distribution The first point we want to emphasize is that the stationarity of the income distribution is *not* an assumption about exogenous parameters, but an endogenous outcome of the model. A household has labor endowment $\theta_L(t)L$ and wealth endowment $\theta_\Pi(t)V(t)$ and the particular assumption we made to solve the model was that $\theta_L(t)L(t) = \theta L$ and $\theta_\Pi(\tau)V(\tau) = \theta V(\tau)$ where τ is the initial period when the economy starts. Only the distribution of labor endowments is truly exogenous (and time-invariant) implying that households' labor incomes $w(t)\theta L$ grow pari passu with the competitive wage $w(t)$ and its distribution stays constant by assumption. However, the assumption on the wealth distribution refers only to *initial* wealth endowments (i.e. the distribution of $V(t)$ at the initial date τ). To keep the analysis simple, we assumed that the initial wealth endowment distribution coincides with the labor endowment distribution, so that $\theta_\Pi(\tau) = \theta_L(\tau) = \theta$. Notice also that $\theta_\Pi(t) = \theta$ in later periods $t > \tau$ is an endogenous outcome, resulting from optimal savings choices. The stationarity result $\theta_\Pi(t) = \theta$ hinges on the assumption of intertemporal CRRA preferences. This assumption implies that, in steady-state, all households save at the same rate from capital income, implying that individual wealth levels of rich and poor consumers grow pari passu (see Bertola et al. 2006). We discuss below that deviating from CRRA generates non-stationarities and feedback effects from growth to inequality.

Asymmetric labor- and wealth-endowment distributions The assumption that labor earnings and wealth are symmetrically distributed is clearly very stylized and not realistic from an empirical point of view. In empirical data there is no perfect correlation between labor earnings and capital incomes in a cross section of households (although the correlation is positive: high-wage households tend to also have higher wealth than low-wage households). More importantly, wealth and capital incomes are much more unequally distributed than labor incomes. It is therefore important to ask: Does the result of Proposition 1 hinge upon symmetry of the earnings and wealth distributions?

It turns out that the answer is "no". The symmetry assumption is a simplifying assumption and deviating from this assumption does not generate substantive new insights. An increase in income inequality (from whatever source) will affect (i) the distribution of households' willingness to pay for innovative products and (ii) the composition of demand for innovative versus backstop products. Asymmetric income sources complicate the analysis since (i) we need to specify how a change in inequality of a particular income source translates into a more unequal distribution of income before we can discuss (ii) the impact on growth. In the symmetric case, step (i) is trivial and allows us to move to step (ii) immediately; in the asymmetric case step (i) is more complicated to solve – although is can be done and we briefly sketch here how this changes Proposition 1.

With symmetry, step (i) is trivial because there is one critical relative endowment, $\hat{\theta}$, leading to the critical willingness to pay, $\hat{z}$, at which the household purchases all innovative products but no backstop goods. With asymmetric labor and wealth distributions, $\hat{z}$ depends on the mix of (θ_L, θ_Π) and there are, in general, many different combinations of (θ_L, θ_Π) that lead to the same $\hat{z}$. However, once we have determined $\hat{z}$, the impact on inequality on growth along the lines of proposition 1: When a change in the distribution of income source $j = (L, V(\tau))$ translates into an regressive income transfer among consumers with $z(\theta_L, \theta_H) < \hat{z}$ innovation and growth increases. A change in the distribution of incomes source j that translates into a regressive transfer from a consumer with $z(\theta_L, \theta_H) < \hat{z}$ to a consumer with $z(\theta_L, \theta_H) > \hat{z}$ decreases growth. Finally, when the change in the distribution is associated with a regressive transfer among consumers with $z(\theta_L, \theta_H) > \hat{z}$ innovative activity and growth are unaffected.

Nonstationarities, feedback effects and relation to the heterodox literature Both the evolutionary and Post-Keynesian literature have emphasized that scale economies and increasing returns are central to understand economic growth. This argument, dating back at least to Young (1928), or even to Adam Smith's pin factory, has been taken up by the mainstream endogenous growth literature as well. In the Post-Keynesian tradition, Kaldor (1966, 1981) developed the principle of cumulative causation, again building on earlier contributions by Veblen and Myrdal (see Berger and Elsner 2007). Kaldor argues that the structure of demand is affected by the distribution of income and wealth across households and the social structure. An expansion of demand sets a process of productivity gains in motion: A larger market size allows for learning-by-doing and manufacturers can increase vertical specialization. The implied reduction in the cost of production implies a reduction in prices. This lets the product market expand further, generating a virtuous cycle. This argument is central for the structuralist school as well. Cumulative causation is important also in our setting, albeit in a reduced form. The wealth distribution affects aggregate demand which in turn determines innovation activities and productivity increases through spillover effects. Because of symmetry, however, our model is silent about different markets and the productivity-growth potential of e.g. manufacturing versus services. In our extension discussed above, the functional distribution between labor incomes and profits affect outcomes, but through a different mechanism than put forward by the Post-Keynesian literature.

The evolutionary literature addresses similar topics, in particular the importance of demand growth and market scope for innovative products, as well as the importance of scale economies through cumulative causation, see e.g. Argyrous (1996).

By assuming non-homothetic preferences and market power, our analysis allows for structural change as well (e.g. Saviotti 2001). This is a further element that our analysis shares with the evolutionary literature. In our approach, structural change is captured in a simple way as all adjustment occurs in the intensive margin.[8] The most important difference from the evolutionary economics is that our analysis focuses on the balanced growth path. Evolutionary economics understands growth essentially as a disequilibrium process where Darwinian selection, e.g. on a firm level, plays an important role with an emphasis of out-of-equilibrium dynamics due to feedback loops.

Our analysis focuses on the balanced growth path. As mentioned in the introduction, this allows for comparative static analysis that is relatively easy to perform and understand. Clearly, reality is far from such a "golden age", and feedback effects from growth to inequality are potentially very important. In fact, an important literature started by Kuznets (1955) argues that income inequality increases at low levels of development (in a transition process from the traditional to the modern sector) and then decreases again at later stages of development (when the economy is increasingly dominated by the modern sector). This lead to the famous "Kuznets-curve", a hump-shaped feedback relation from growth to inequality. Notice however, that empirical evidence does not support Kuznets' predictions. While inequality has been decreasing during most of the 20th century – consistent with the Kuznets hypothesis – it has been on the rise in many countries since the late 1970s and a large literature discusses causes and possible cures (for influential recent contributions see Piketty 2014 and Atkinson 2015).

It is therefore suggestive to ask how accounting for feedback effects from growth to inequality qualifies the predictions of our analysis. It turns out that feedback effects can be captured within our theoretical framework. The important assumption generating a balanced growth path is the assumption of intertemporal CRRA preferences. As mentioned above, this assumption generates a situation where all labor incomes are consumed and where there is a constant (time-invariant) optimal savings rate from capital income. However, if preferences are not CRRA, this does not longer hold and there feedback effects from growth to inequality through systematic changes in individual savings rates. As shown in Bertola et al. (2006), chap. 3, preferences that feature decreasing relative risk aversion generate the empirically relevant situation where the savings rate increases with income. This implies that inequality increases with economic growth.[9] What are the consequences of such feedback effects for the inequality-growth relationship studied here? Our model still provides predictions how inequality affects growth even in such a non-stationary environment. In contrast to growth along a balanced growth path, the above feedback effects imply that the distribution of households' willingnesses to pay is become more unequal as overall

[8]In a model with representative agents, we analysed the extensive margin to study demand-driven structural change (Foellmi and Zweimüller 2008).

[9]The reference to "risk" is misleading here since risk and uncertainty do not play a role in our analysis. CRRA features a constant elasticity of marginal utility with respect to the level of consumption. In the present context, decreasing relative risk aversion (DRRA) means that the elasticity of marginal utility is increasing with consumption, implying that rich households save a larger fraction of their income that the poor.

inequality increases. Increases in inequality among households who cannot afford all innovative products will generate price effects, i.e. rising mark-ups in the innovative sector increasing the incentive to innovate. On the other hand, increases in inequality will also divert demand from the innovative to the backstop sector (or, more generally, to sectors that do not contribute to technical progress) and this market-size effect reduces the impact of inequality on growth. Whether growth increases or decreases depends on the relative importance of these price- and market size effects and the relative important of these effects may vary over time.

Feedback effects from growth to inequality may not only occur through savings. It may also be that, as the economy develops, poorer individuals get better access to economic resources, they may become able to invest (more) in human capital, get better access to capital markets and investment opportunities etc. These mechanisms generate feedback effects that can reduce rather than increase inequality, generating less dispersed willingnesses to pay and lower relative demand for the backstop sectors. Again, such feedback mechanisms may generate non-stationarities and lead to continuous changes in willingnesses to pay and demand compositions over time. However, innovation incentives will then be driven by the evolution of the willingness to pay and market sizes. We conclude that the above analysis, despite its focus on a balanced growth path, provides a useful framework also for non-stationary environments. Price and market size effects affect innovation and growth in qualitatively similar ways when the economy operates off the balanced growth path.

Our analysis has focused on a closed economy. An important direction for future research is to bring the international perspective into the picture. Technological changes and globalization lead to increased inequalities within countries but it also allowed low-income countries to grow, leading to a more even distribution of incomes across countries. As markets become increasingly global, the role of these inequality changes will generate important price and market size effects on innovations on world markets.

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Appendix

Proof of Lemma 1

Proof a. Differentiating $z(\theta, t)$ with respect to t, using $\dot{N}(\theta, t)/N(\theta, t) = g$, yields $\dot{z}(\theta, t)/z(\theta, t) = r - \rho - g$. Guess that $g = r - \rho$, so $z(\theta)$ is stationary. In that case household θ purchases all goods with prices lower than or equal $p(\theta)$ at all times and pays average price $\bar{p}(\theta)$,constant over time, and has expenditures $\bar{p}(\theta)N(\theta, t)$. Notice further that wages evolve according to $w(t) = bN(\tau)e^{-(r-g)(t-\tau)}$ and that $V(\tau) = bFN(\tau)$ (since the value of each firm is $\pi/r = bF$). This allows us to rewrite the household's lifetime budget constraint as $\bar{p}(\theta)N(\theta, t) = N(t)\theta b(1 + \rho F/L)$, confirming our guess that $N(\theta, t)$ and $N(t)$ grow pari passu.

b. The budget constraint of the poorest consumer is $p(\underline{\theta})N(\underline{\theta}, t) = N(t)\underline{\theta}b(1 + \rho F/L)$. We calculate $p(\underline{\theta})$ using $p(\underline{\theta}) - b/a = [1 - G(\hat{\theta})][1 - b/a]$ (all firms make the same profit), (2) and $r = g + \rho$. This yields $p(\underline{\theta}) = (g + \rho)bF/L + b/a$. Substituting into the budget constraint and solving for $N(\theta, t)$ yields the first claim of part b). The budget constraint of household $\theta \in (\underline{\theta}, \hat{\theta})$ is $p(\underline{\theta})N(\underline{\theta}, t) + \int_{\underline{\theta}}^{\theta} p(\xi)dN(\xi, t) = N(t)\theta b(1 + \rho F/L)$. Differentiating with respect to θ yields $p(\theta)[dN(\theta, t)/d\theta] = N(t)b(1 + \rho F/L)$. Solving for $dN(\theta, t)/d\theta$ and integrating yields $N(t)b(1 + \rho F/L)\int_{\underline{\theta}}^{\theta}(1/p(\xi))d\xi + N(\underline{\theta}, t)$. Calculating $p(\xi) = b[(g + \rho)aF + (1 - G(\xi))L] / [a(1 - G(\xi))L]$ from Eq. 2 and substituting into the previous equation yields the second claim of part b). By definition, household $\hat{\theta}$ purchases all goods produced by monopolistic firms but no goods produced by the competitive fringe. A household $\theta > \hat{\theta}$ spends $N(t)\hat{\theta}b(1+\rho F/L)$ for the $N(t)$ monopolistic goods and $N(t)(\theta - \hat{\theta})b(1+\rho F/L)$ for the remaining $N(\theta, t) - N(t)$ goods produced by the competitive fringe. This yields the third claim of part b). □

Proof of Proposition 1

Proof a. The integrand in Eq. 4 is a concave function of $G(\bullet)$. If $G(\bullet)$ undergoes a second order stochastically dominated transfer, where $\int_0^{\hat{\theta}} G(\theta)d\theta$ remains unchanged, the value of the integral in Eq. 4 must increase due to Jensen's inequality. Hence, the consumption curve (4) shifts up at $\theta = \hat{\theta}$. Further, with $G(\hat{\theta})$ unchanged, the zero profit constraint does not change at $\theta = \hat{\theta}$, therefore the equilibrium growth rate rises.

b. The integrand in Eq. 4 takes lower values at the values of θ involved in the transfer. Hence the value of the integral in Eq. 4 decreases meaning that less purchasing power is left in the hands of households with $\theta < \hat{\theta}$. Around $\theta = \hat{\theta}$, the consumption curve shifts down and the zero profit constraint remains unaffected, the growth rate decreases.

c. Neither (2) nor (4) are affected for $\theta \leq \hat{\theta}$. □

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