CREDITOR GOVERNANCE AND CORPORATE POLICIES: THE ROLE OF DEBT COVENANT RENEGOTIATIONS

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The Role of Debt Covenant Renegotiations*

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ABSTRACT

This paper analyzes the impact of debt covenant renegotiations on corporate policies. We develop a structural model of a levered firm that can renegotiate debt both at investment and in corporate distress. Covenant renegotiation at investment disciplines equity holders in their financing and investment decisions and, hence, mitigates the agency cost of debt. Our model explains the empirical intensity and patterns of the occurrence of debt renegotiation. We also quantify the role of debt covenant renegotiations as a governance channel on corporate financial policies and on the value of corporate securities. Additionally, the model offers a rich set of novel testable predictions.

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Keywords: Debt Covenants, Corporate Investment, Renegotiation, Capital Structure.

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1. Introduction

The traditional governance view presumes that managers and equity holders determine firm policies, while debt holders remain silent unless a firm is in distress. The recent empirical literature, however, shows that creditors frequently influence corporate decisions through debt renegotiations, and that the plurality of these renegotiations is not associated with distress (see, e.g., Roberts and Sufi, 2009b; Denis and Wang, 2014; Roberts, 2015). Despite this insight, the theoretical literature lacks a model to rationalize both the intensity and patterns of the occurrence of debt renegotiations. A model that captures how frequently and in which situations renegotiations occur is, however, crucial to understand the impact of the debt governance channel on firms. The void is troubling because, empirically, renegotiations with debt holders or their mere possibility have a profound effect on corporate investment policies, financing decisions, loan contract terms, security design, CEO turnover, and firm value (Nini et al., 2009; Roberts and Sufi, 2009a; Nini et al., 2012; Denis and Wang, 2014).

This paper provides a model that allows equity holders to renegotiate debt both outside and in distress. Specifically, we develop a dynamic structural model of a levered firm in the spirit of Mello and Parsons (1992). We incorporate distressed reorganization as introduced by Fan and Sundaresan (2000) and investment as modeled in Hackbarth and Mauer (2012). The novel feature of our model is that initial debt carries a subsequent financing covenant that can be renegotiated between equity and debt holders at investment. Equity holders implement the covenant to mitigate the agency cost of debt. The reason for this mitigation is that the possibility to renegotiate the covenant disciplines equity holders in their ex-post investment and the corresponding financing decisions. The incorporation of non-distressed renegotiation as a channel for creditor influence is motivated by the empirical literature. In particular, Dichev and Skinner (2002), Chava and Roberts (2008), and Bradley and Roberts (2015) conclude that covenants are primarily installed to allow creditors to affect firm policies that are subject to agency conflicts. Based on this view, Denis and Wang (2014) and Roberts (2015) show that non-distressed covenant renegotiations represent an important channel through which debt holders exert control outside of default over firm decisions such as changes in investment, operating, or financing policies.

Our model improves the understanding of creditors’ influence on firm policies in four important dimensions. First, we explain the empirically observed occurrence patterns of renegotiation that cannot be rationalized with existing corporate finance models. Understanding these patterns is a crucial step toward quantifying the impact of the possibility of creditors to affect firm decisions. We start our illustration with the real firms sample of private credit agreements from Nini, Smith, and Sufi (2009) that is frequently analyzed in the literature. For instance, Denis and Wang (2014) and Roberts (2015) show that non-distressed covenant renegotiations represent an important channel through which debt holders exert control outside of default over firm decisions such as changes in investment, operating, or financing policies.
ples of model-implied firms that are structurally similar to the real firms sample, we find that existing structural corporate finance models fail to reflect the reported occurrence patterns of renegotiation. The state-of-the-art debt renegotiation framework of Fan and Sundaresan (2000), for example, implies that only 12.9% of the debt contracts are renegotiated before maturity, 100% of debt renegotiations occur due to corporate distress, and the average effective loan duration is more than 90% of the initially stated maturity. By incorporating equity holders’ endogenous decision to renegotiate at investment, our model yields that about 42% of the contracts are renegotiated, but only 23.9% of renegotiations are associated with distress. Further, it predicts a reduction of the average effective loan duration to 63%. Denis and Wang (2014) calculate additional statistics on the renegotiation frequency. We also improve in explaining these patterns compared to a model with only distressed reorganization. Thus, our model provides an important step towards rationalizing the empirically observed renegotiation occurrence patterns.

Second, we find that incorporating creditors’ influence outside distress is crucial to assess the quantitative impact of the possibility to renegotiate debt on firm value. We start by analyzing the benefit and cost from non-distressed renegotiation. Issuing new debt at investment to finance the investment cost dilutes the claim of initial debt holders. Without covenant protection of the initial debt, equity holders overlever at investment compared to a policy that maximizes the value of the firm. The wealth transfer from initial debt holders to equity holders associated with this overleverage results in debt holder expropriation. As a consequence, equity holders also invest too early compared to the firm value maximizing policy, i.e., there is overinvestment (Hackbarth and Mauer, 2012).

Our model shows that the inclusion of a financing covenant that can be renegotiated at investment is motivated by firms’ desire to mitigate these agency costs of debt. The covenant protects initial debt holders against the excessive dilution of their claim through the issuance of additional debt. In case the covenant is enforced, equity holders must finance the investment cost by issuing additional equity. At investment, however, equity holders renegotiate the covenant with initial debt holders. We model this renegotiation as a Nash bargaining game, which allows us to assign different levels of bargaining power to the two parties. Equity holders offer initial debt holders an increase in the promised interest rate in exchange for waiving the covenant. The increase in the promised interest rate and the amount of new debt issued are determined as the Nash bargaining solution. We show that the Nash bargaining solution to the covenant renegotiation game leads to the firm value maximizing leverage at investment. The surplus from renegotiation in this solution is split between equity and debt holders such that each party receives a fraction corresponding to its bargaining power. Renegotiation also mitigates the overinvestment problem because equity holders do not expropriate initial debt holders at investment by issuing too much new debt. Thus, debt covenant renegotiation mitigates the agency costs of debt due to overleverage and overinvestment. Further, the reduction in the agency costs of debt allows equity holders to implement a higher initial coupon compared to the case without covenant protection. This higher coupon leads to an additional increase in the firm value.
We find, however, that covenant inclusion also carries a cost. Specifically, due to the higher initial coupon and because equity holders cannot expropriate initial debt holders upon investment, the distressed reorganization risk before investment is larger in the firm with a financing covenant. A higher reorganization risk is costly as the entire tax shield is lost at distressed reorganization. Further, this insight on the cost of a covenant implies that renegotiation at investment and distressed reorganization are inherently linked. Hence, it can be misleading to analyze each of the two types of renegotiation in isolation.

Importantly, we show that the benefit of covenant inclusion and renegotiation typically dominates the cost. Hence, our model provides a rationale for the frequent use of financing covenants in practice. For example, the value of a representative baseline firm increases by 1.78% if a financing covenant is included. We also find that neglecting the possibility to renegotiate debt outside distress leads to a strong underestimation of the value from debt renegotiation to firms. In particular, incorporating only distressed reorganization increases the value of the baseline firm with an investment opportunity by only 1.58%. Additionally considering renegotiation at investment rises the value from debt renegotiation to firms by 3.40%. Similarly, the bargaining power of equity holders is more important for the value of firms in our model than in existing renegotiation frameworks that severely underestimate the occurrence of debt renegotiation.

Third, the possibility of non-distressed renegotiations affects corporate financial policies. Including a renegotiable financing covenant increases the optimal market leverage of the baseline firm. The inclusion also reduces the sensitivity of the market leverage to the value of the growth opportunity, which rationalizes the corresponding empirical finding in Billett, King, and Mauer (2007). With respect to the timing of investment, our model implies that a financing covenant delays investment because equity holders cannot expropriate debt holders upon investment. We additionally analyze the dependence of the benefit and cost of a renegotiable financing covenant on firm parameters, which allows us to explain empirically observed covenant structures such as the impact of leverage and investment opportunities on the propensity of covenant inclusion. This analysis also implies that the cost of covenant inclusion is positively related to equity holders’ bargaining power. The reason is that a higher bargaining power increases the distressed reorganization risk. Hence, a financing covenant is less valuable to firms with a stronger bargaining power of equity holders. The recognition of this channel generates new testable predictions on the relation between determinants of bargaining power and the probability of the inclusion of financing covenants. It also suggests that the impact of firm characteristics on the value from covenant inclusion critically depends on the bargaining power of equity and debt holders.

Finally, we show that incorporating renegotiation outside distress is crucial to evaluate the impact of renegotiation on credit spreads. Solely considering distressed reorganization implies that credit spreads increase in the baseline firm compared to a firm without debt reorganization. Adding non-distressed renegotiation, however, shows that credit spreads actually decrease with the possibility to renegotiate debt. Our analysis also suggests that the covenant structure and the bargaining power of equity holders are important aspects to explain empirically observed credit
spreads. This insight helps to understand the cross section of credit spreads that is difficult to explain with traditional parameters associated with credit risk (Zhang et al., 2009).

Our paper contributes to different veins of the literature. Most importantly, we show that by expanding models with distressed reorganization to renegotiation outside distress, we are able to explain the stylized renegotiation patterns reported in the recent, fast growing empirical literature on covenant renegotiation. In particular, our model rationalizes the finding in Roberts and Sufi (2009b) and Roberts (2015) that most debt renegotiations have little to do with corporate distress or default. Similarly, Wang (2013) shows that 80% of the contracts that experience some renegotiation do not report covenant violations within the same year. Even covenant violations rarely lead to firm liquidation or an acceleration of the loan but rather entail renegotiation resulting in stronger contractual restrictions (Smith, Jr. and Warner, 1979; Nini et al., 2009; Gopalakrishnan and Parkash, 1995). Hence, covenants are not primarily implemented to avoid default. Instead, they are set tightly to allow creditors to frequently intervene in firm policies that are subject to conflicts of interest (Dichev and Skinner, 2002; Bradley and Roberts, 2015; Chava and Roberts, 2008; Roberts, 2015; Denis and Wang, 2014). Renegotiation outside distress also explains why, empirically, most debt contracts are renegotiated before maturity, the mean time to renegotiation is considerably shorter than the initially stated maturity, and many contracts are even renegotiated multiple times before maturity (Roberts and Sufi, 2009b; Nikolaev, 2013; Denis and Wang, 2014; Roberts, 2015). To the best of our knowledge, we are the first to explain the empirically reported patterns regarding the occurrence of renegotiation.

The extension to renegotiation at investment complements the literature that focuses on renegotiation associated with distress or default. Giammarino (1989), for instance, argues that because of the private information of equity holders about the firm’s type, financial distress costs cannot be avoided by costless renegotiation. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) show that due to costly bankruptcy threats, equity holders’ ability to make take-it-or-leave-it offers to their creditors allows them to deviate from contractual coupon payments. Mella-Barral (1999) develops a continuous time pricing model of dynamic debt restructuring. Due to equity holders’ discretion over the timing of default, equity holders can reduce debt service obligations or force concessions on debt holders’ collateral claim in liquidation through contract renegotiations when the firm’s status deteriorates. The implications of distressed reorganization on corporate financial policies are also analyzed in a continuous time capital structure model by Fan and Sundaresan (2000), and in a static capital structure model by Gorton and Kahn (2000). Empirical studies by Gilson, John, and Lang (1990), Gilson (1990), Smith, Jr. (1993), Davydenko and Strebulaev (2007), and Benmelech and Bergman (2008) analyze the outcome and implications of ex-post bargaining in payment default, covenant violation, and bankruptcy.

Our work is also related to a stream of literature that links investment to renegotiation in static models. Bergman and Callen (1991) show that when the realization of a firm’s profit is low, equity holders can credibly threaten debt holders to adapt a suboptimal investment policy that saps firm value. This threat allows them to renegotiate the debt to their advantage. In the model
of Berlin and Mester (1992), the firm manager has an incentive to underinvest in safe activities, and to overinvest in growth activities. Covenants oblige to a minimum required investment in the safe activities. The value of the option to renegotiate stems from the ability to invest less in safe activities and, hence, more in growth activities when all parties realize that an unfavorable outcome is unlikely. Gorton and Kahn (2000) examine a moral hazard problem between the borrower and lender to analyze the design, pricing, and renegotiation of loan contracts. The borrower can undertake a costly, risk-increasing action (asset substitution), while the lender can demand collateral upon the arrival of bad news. The role of the initial debt contract is to allocate bargaining power in the subsequent contract renegotiation. Similar to our model, lenders extract some surplus from equity holders’ investment decision by increasing interest rates. Dessein (2005) argues that good borrowers are willing to shift formal control rights over investment to the less informed investor because they want to signal congruent preferences. Bad borrowers find similar concessions too costly. Garleanu and Zwiebel (2009) show that as the borrower is better informed about the potential of future wealth transfer, lenders receive strong ex-ante decision rights. When information is revealed, renegotiation occurs to transfer some control rights back to the borrower.

The notion that renegotiation can mitigate the agency conflicts of debt is already discussed in the static models literature. We differ from this literature by three important contributions that cannot be addressed with static models. First, our continuous time approach enables us to explain empirically observed renegotiation timing patterns. Second, we endogenize the relation between covenant renegotiation, the financing of investment, the investment timing, and distressed reorganization. This endogenization is important because Hackbarth and Mauer (2012) find that the financing of investment has important implications for the timing of investment, and that this timing itself is a potential source of the agency cost of debt. With our model, we can analyze how renegotiation affects the timing of both investment and distressed reorganization, and to what extent these effects depend on the bargaining power of equity holders. We, thereby, continue a line of research that uses dynamic structural models to investigate corporate policy decisions. Among these papers, our work is most closely related to a number of studies that analyze the impact of the financial structure on investment and financing decisions (e.g., Childs, Mauer, and Ott, 2005; Hackbarth and Mauer, 2012; Favara, Morelec, Schroth, and Valta, 2015). Third, the structural approach allows us to calibrate the model to real data to generate not only qualitative but also quantitative predictions on the values of corporate securities. We show that understanding the occurrence pattern and the implications of covenant renegotiation outside corporate distress is a crucial step towards quantitatively explaining debt yields and corporate credit spreads. Thus, we contribute to the literature on corporate debt pricing that finds that not simply leverage, but the particular debt and ownership structures are important to explain observed debt yields. Datta, Datta-Iskandar, and Patel (1999), for example, demonstrate that firms with bank loans have lower bond yield spreads at the time of issuance. Lin, Ma, Malatesta, and Xuan (2011) show that a wider divergence between cash flow and control rights increases the cost of debt financing. Similarly, He and Xiong (2012) emphasize the importance of the debt maturity for rollover risk.
The remainder of the paper is organized as follows. In Section 2, we present our model that is solved in Section 3. Section 4 describes the main implications of covenant renegotiation at investment for firms. In Section 5, we use our model to explain empirically observed renegotiation patterns, covenant structures, and financial policies. Finally, we conclude in Section 6.

2. The model

Our structural model is in the spirit of Mello and Parsons (1992). We incorporate distressed reorganization as suggested by Fan and Sundaresan (2000), and investment as proposed by Hackbarth and Mauer (2012). Initial debt carries a covenant that prohibits additional debt issues. The novel feature of our model is that equity holders can renegotiate this covenant to finance part of the new investment with additional debt. In case of renegotiation, equity holders and debt holders bargain over the surplus stemming from the issue of a mix of new equity and debt.

We first describe the firm’s assets in place and the investment opportunity and subsequently discuss debt renegotiation and the financing of investment.

2.1. Assumptions

Assets are continuously traded in complete and arbitrage-free markets. Investors may lend and borrow at the risk-free rate $r$. Corporate taxes are paid at a constant rate $\tau$ on operating cash flows, and full offsets of corporate losses are allowed.

The firm’s assets in place and investment opportunity. We consider an infinitely-lived firm with assets in place and an investment opportunity. At each time $t$, assets in place generate a cash flow $X_t$. The cash flow $X_t$ constitutes the exogenous state variable in our model. We assume that $X_t$ is observable, but not verifiable by courts or other outside parties.\footnote{This contracting friction is in the spirit of Grossman and Hart (1986), Hart and Moore (1988), and Bolton and Scharfstein (1996). Specifically, simple contracts contingent on cash flow that specify, for example, financing or investment policies cannot be written because a court is unable to enforce such a contract. In particular, the assumption that investment choices are not contractible ex-ante is standard in the debt overhang literature (e.g., Bhattacharya and Faure-Grimaud, 2001).} The cash flow $X_t$ of the firm follows a geometric Brownian motion under the risk-neutral probability measure $Q$

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 > 0,$$

in which $\mu$ and $\sigma$ are the drift and volatility, respectively, and $W_t$ is a Brownian motion under $Q$.

The investment opportunity of the firm is modeled as an American call option on the cash flow, analogous to Arnold, Wagner, and Westermann (2013). Specifically, if the firm invests at time $\bar{t}$, it pays the exercise cost $I$ and receives an additional future cash flow of $(s - 1)X_{\bar{t}}$, for
some factor $s > 1$ for all future times $t \geq \bar{t}$. After investment, the firm consists of only invested assets. The investment decision is irreversible.

Initially, the firm is financed by issuing equity and infinite maturity private debt. Private debt is one of the largest if not the largest source of funds for U.S. corporations (Krishnaswami and Subramaniam, 1999; Denis and Mihov, 2003). While renegotiation may also occur with public bonds in practice, it is less difficult and costly with private debt agreements (Krishnaswami and Subramaniam, 1999). After debt has been issued, the firm pays a total coupon rate $c_o$ to initial debt holders until it defaults or invests. The firm also pays corporate taxes at a constant rate $\tau$. In case the required debt service exceeds the cash flow, shareholders can inject funds to finance the coupon. Alternatively, shareholders have the possibility to default on their debt obligations (Leland, 1998). If equity holders decide to default, the firm is immediately liquidated.

Debt holders enjoy absolute priority of their claims. Hence, they obtain the unlevered asset and investment opportunity values times the recovery rate $\alpha$. This setup implies that the default costs correspond to a fraction $1 - \alpha$ of the unlevered value of the assets in place and the investment opportunity. While tax benefits encourage debt financing by way of shielding part of the firm’s cash flow from taxation, costly default reduces the incentive to issue debt.

Debt covenant renegotiation and financing. Covenants that restrict financing and the issuance of additional debt are ubiquitous in the debt contracts of real firms (e.g., Smith, Jr. and Warner, 1979, Bradley and Roberts, 2015). In the model, we therefore allow debt to carry a covenant that prevents equity holders from issuing additional debt. We later show that the inclusion of such a debt issuance covenant is optimal for most realistic parameter combinations. As new debt financing increases the value of equity, equity holders renegotiate this covenant with debt holders upon investment. Specifically, equity holders induce debt holders to waive the covenant in exchange for a compensation. The compensation to initial debt holders consists of an additional compensation coupon such that the total coupon (after investment) to initial debt holders is $\hat{c}_o \geq c_o$. Changes in the interest rate at renegotiation are very common in practice. Roberts and Sufi (2009b), for example, find that 55% of renegotiations entail an adaption in the coupon, with an average change in the interest rate of 64 bps, or 40% of the initial coupon. We show in Section 3.3 that the mean of compensation does not alter the model’s solution and results, such that the assumption of an interest rate compensation is without loss of generality.

Upon renegotiation, equity holders issue a mix of equity and new debt to finance the investment. The coupon to new debt holders after renegotiation is denoted by $\hat{c}_n$. We consider equal priority of all debt claims in case of default, i.e., initial [new] debt holders receive a fraction $\frac{\hat{c}_n}{\hat{c}_n + c_o} \alpha$ $\frac{\hat{c}_n}{\hat{c}_n + c_o} \alpha$ of the unlevered asset value in case default occurs after investment. Assuming different priority structures in default does not alter the model’s solution. The sum of the values of new debt and equity corresponds to $I$, the investment cost. We do not model covenant violations. While covenant violations occur in practice (Roberts and Sufi, 2009a), covenant renegotiations

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As in Arnold, Wagner, and Westermann (2013), we assume that also a fraction $\alpha$ of the unlevered investment opportunity is recovered at default.
are more frequent than covenant violations (Denis and Wang, 2014). One reason that firms avoid violations is that violations impose serious consequences on firms’ investment and financing policies, collateral requirements, monitoring and reporting frequencies, ratings, CEO turnover, and interest rate spreads (Chava and Roberts, 2008; Nini, Smith, and Sufi, 2009).

Equity holders control the investment decision. Once they decide to invest, the new capital structure is determined as the Nash bargaining solution of the renegotiation game. The renegotiation game is characterized as follows. Debt holders can enforce the prevailing covenant and prevent equity holders from issuing additional debt, which constitutes the outside option or disagreement point of the renegotiation game. We assume that the irreversibility of the investment decision also holds in case of disagreement. Therefore, equity holders’ outside option is to finance the investment cost by issuing equity only. Hence, Nash bargaining determines a sharing rule between equity and debt holders over the surplus from financing the investment cost by issuing a mix of debt and equity as opposed to financing it with an equity issuance only. Equity holders’ bargaining power is denoted by $\eta$, and, hence, debt holders’ bargaining power is given by $1 - \eta$.

Reorganization: Debt renegotiation in corporate distress. We model distressed reorganization as a debt-equity swap following Fan and Sundaresan (2000). In particular, if cash flows deteriorate, equity holders offer debt holders to swap their original debt against equity. Hence, at distressed reorganization, the firm becomes an all-equity firm. Disagreement triggers immediate default, inducing a loss of a fraction $1 - \alpha$ of the unlevered firm value. Thus, the firm’s claim holders have an incentive to reorganize to avoid these default costs.

The fraction of the firm’s equity that is offered to debt holders, denoted by $1 - \theta$, corresponds to the Nash bargaining solution. At reorganization, the value of the firm’s equity equals the unlevered firm value. Reorganization can occur both before or after investment. Before investment, the unlevered firm value at reorganization is the sum of the unlevered asset value and the unlevered value of the investment opportunity. The fraction $1 - \theta$ of this firm value is offered to debt holders in exchange for their debt. After investment, the unlevered firm value at reorganization corresponds to the unlevered asset value. The fraction $1 - \theta$ of this firm value is offered jointly to initial and new debt holders. We assume that, at reorganization after investment, the fraction is shared between initial and new debt holders proportionally to the value of their claims. That is, initial debt holders receive a fraction $(1 - \theta) \frac{c_o}{c_o + c_n}$, and new debt holders a fraction $(1 - \theta) \frac{c_n}{c_o + c_n}$ of the unlevered firm value.

\footnote{Roberts and Sufi (2009a) report that despite unfavorable terms offered by existing lenders after a violation, very few borrowers actually replace the violated agreement with financing from other lenders. Thus, covenant violations have a large impact on firms because violaters seem unable to obtain funding from alternative lenders.}
3. Model solution

The model is solved by backward induction. First, we present reorganization in corporate distress. Next, the value functions and the default policy after investment are derived (Subsection 3.2). Subsequently, Subsection 3.3 states and solves the bargaining problem at investment that determines the compensation coupon to initial debt holders and the coupon of new debt. Finally, we solve for the value functions of corporate securities before investment (Subsection 3.4), and then show how to find the reorganization threshold, the investment boundary, and the optimal initial capital structure (Subsection 3.5).

3.1. The bargaining game at reorganization

Denote the reorganization boundary after investment by $\hat{X}_S$. After investment, the firm’s unlevered after-tax asset value is given by

$$\hat{V}(X) = \frac{1-\tau}{r-\mu}X. \quad (2)$$

Following Fan and Sundaresan (2000), the sharing rule $\hat{\theta}$, i.e., the fraction of the unlevered assets that is offered to debt holders in exchange for their claim, is determined as the Nash bargaining solution

$$\hat{\theta} = \arg \max_{0 \leq \tilde{\theta} \leq 1-\alpha} \left\{ \tilde{\theta} \hat{V}(\hat{X}_S) - 0 \right\}^{\eta} \left\{ (1-\tilde{\theta}) \hat{V}(\hat{X}_S) - \alpha \hat{V}(\hat{X}_S) \right\}^{1-\eta} \quad (3)$$

$$= \eta (1-\alpha). \quad (4)$$

Using equity holders’ optimality condition, the reorganization boundary $\hat{X}_S$ can be calculated in closed-form as

$$\hat{X}_S = \frac{r-\mu}{r} \frac{\beta_2}{\beta_2 - 1} \frac{c}{1-\tilde{\theta}} \quad (5)$$

$$= : \hat{X}_D \frac{1}{1-\tilde{\theta}}, \quad (6)$$

in which

$$\beta_2 = 1 - \frac{\mu}{\sigma^2} - \sqrt{\left(1 - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (7)$$

and $\hat{X}_D$ denotes the firm’s default boundary in the absence of reorganization. Further details including the value functions for debt and equity are presented in Appendix A.
Similarly, we denote the reorganization boundary before investment by $X_S$. Before investment, the fraction $1 - \theta$ of the unlevered assets and investment opportunity that equity holders offer to debt holders in exchange for their debt is determined as the Nash bargaining solution

$$\theta = \arg \max_{0 \leq \tilde{\theta} \leq 1 - \alpha} \left\{ \tilde{\theta} \left( V(X_S) + G^{unlev}(X_S) \right) - 0 \right\}^{\eta} \cdot \left\{ \left(1 - \tilde{\theta} \right) \left( V(X_S) + G^{unlev}(X_S) \right) - \alpha \left( V(X_S) + G^{unlev}(X_S) \right) \right\}^{1-\eta} \quad (8)$$

in which $G^{unlev}(X)$ denotes the value of the unlevered investment opportunity, calculated in closed form in Appendix B.

### 3.2. Value functions after investment

Let $\hat{d}(X; c)$ and $\hat{e}(X; c)$ denote the values of corporate debt and equity, respectively, after investment at cash flow level $X$ given coupon $c$. $\hat{X}_S$ is the reorganization boundary for the debt-equity swap after investment. The value functions are calculated in Fan and Sundaresan (2000). Details are provided in Appendix A.

### 3.3. Debt covenant renegotiation and investment

The threshold that triggers debt investment and covenant renegotiation is denoted by $X_R$. The sharing rule is defined as $\{\hat{c}_o, \hat{c}_n\}$, i.e., as the total coupon to initial debt holders and the coupon of the new debt for which equity holders receive issue proceeds. $\{\hat{c}_o, \hat{c}_n\}$ is characterized as the Nash bargaining solution:

$$\{\hat{c}_o, \hat{c}_n\} = \arg \max_{\{\hat{c}_o, \hat{c}_n\}} \left\{ \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{e} \left( sX_R; c_o \right) \right\}^{\eta} \cdot \left\{ \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} \left( sX_R; c_o \right) \right\}^{1-\eta}. \quad (10)$$

The surplus to equity from renegotiation is the difference between the value of equity in renegotiation and the value of equity in disagreement. The value of equity at renegotiation is the total value of equity given the enhanced cash flows from investment and the total new coupon plus the issue proceeds from the new debt less the investment cost. The value of equity in disagreement corresponds to the value of equity given investment, and a coupon equal to the initial coupon,
less the investment costs. Hence, denoting the surplus to equity holders as $SE (X_R; \hat{c}_o, \hat{c}_n)$, we have that

$$SE (X_R; \hat{c}_o, \hat{c}_n) = \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - I - \left( \hat{e} (sX_R; c_o) - I \right)$$

$$= \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \left( \hat{e} (sX_R; c_o) \right).$$  \quad (11)$$

Eq. (11) corresponds to the terms in the brackets on the right hand side of the first line of Eq. (10). Similarly, the surplus to debt holders is calculated as the difference between the value of debt at renegotiation and the value of debt in disagreement. Upon renegotiation, equity holders promise initial debt holders an additional compensation coupon of $\hat{c}_o - c_o > 0$. In disagreement, the coupon to debt holders remains unchanged at the initial coupon. Hence, the surplus to debt holders, $SD (X_R; \hat{c}_o, \hat{c}_n)$, is given by

$$SD (X_R; \hat{c}_o, \hat{c}_n) = \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} (sX_R; c_o),$$  \quad (12)$$

which corresponds to the terms in the bracket in the second line of Eq. (10). Consequently, the total surplus, $ST (X_R; \hat{c}_o, \hat{c}_n)$, is calculated as

$$ST (X_R; \hat{c}_o, \hat{c}_n) = SE (X_R; \hat{c}_o, \hat{c}_n) + SD (X_R; \hat{c}_o, \hat{c}_n)$$

$$= \hat{e} (sX_R; \hat{c}_o + \hat{c}_n) + \hat{d} (sX_R; \hat{c}_o) + \hat{d} (sX_R; \hat{c}_n) - \hat{e} (sX_R; c_o) - \hat{d} (sX_R; c_o)$$

$$= \hat{F}V (sX_R; \hat{c}_o + \hat{c}_n) - \hat{e} (sX_R; c_o) - \hat{d} (sX_R; c_o),$$  \quad (13)$$

in which $\hat{F}V (\cdot)$ denotes the firm value. The following Proposition 1 presents the basic properties of the Nash bargaining solution in the model.

**Proposition 1.** If the initial debt carries a renegotiable covenant that prevents the issuance of new debt, the Nash bargaining solution

$$\{ \hat{c}_o, \hat{c}_n \} = \arg \max \left\{ \hat{c}_o, \hat{c}_n \right\} \left\{ \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \hat{e} (sX_R; c_o) \right\}^\eta$$

$$\cdot \left\{ \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} (sX_R; c_o) \right\}^{1-\eta}$$  \quad (14)$$

exhibits the following properties:

(i) The total coupon determined by Nash bargaining $\hat{C} := \hat{c}_o + \hat{c}_n$ corresponds to the first-best coupon $\hat{C}^{fb}$ that maximizes the value of the firm, i.e.,

$$\hat{C} = \hat{C}^{fb} = \frac{r}{r - \mu} \left( \frac{\beta_2 - 1}{\beta_2} \right) (1 - \theta) \frac{1}{\beta_2} X_R.$$  \quad (15)$$

Further, the total surplus from renegotiation, $ST (X_R; \hat{c}_o, \hat{c}_n)$, depends only on the sum of $\hat{c}_o$ and $\hat{c}_n$. It is given by

$$ST (X_R; \hat{c}_o, \hat{c}_n) = ST (X_R; \hat{c}_o + \hat{c}_n) = \hat{F}V^{fb} (sX_R) - \hat{e} (sX_R; c_o) - \hat{d} (sX_R; c_o),$$  \quad (16)$$

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in which $FV^{fb}(sX_R)$ denotes the first-best firm value at the cash flow level $X_R$.

(ii) $\{\hat{c}_o, \hat{c}_n\}$ is such that the surplus from renegotiation to initial debt holders, $SD(X_R; \hat{c}_o, \hat{c}_n)$, and the surplus from renegotiation to equity holders, $SE(X_R; \hat{c}_o, \hat{c}_n)$, satisfy

\[
SE(X_R; \hat{c}_o, \hat{c}_n) = \eta ST(X_R; \hat{c}_o + \hat{c}_n) \quad (17)
\]
\[
SD(X_R; \hat{c}_o, \hat{c}_n) = (1 - \eta) ST(X_R; \hat{c}_o + \hat{c}_n) \quad (18)
\]

i.e., the two parties receive a fraction of the total surplus that corresponds to their respective bargaining power.

Proof. See Appendix C.

Proposition 1 is intuitive. Property (i) states that renegotiation leads to the first-best leverage at investment, which occurs due to the Pareto efficiency of the Nash bargaining solution. Property (ii) shows that the total surplus is shared according to the bargaining power of the two parties.

The following Remark 1 states that the properties of the Nash bargaining solution of the covenant renegotiation game are robust to our assumptions concerning the priority rule for initial and new debt in default, the sharing rule of equity from the debt-equity swap in reorganization between initial and new debt holders, and the mean of compensation to initial debt holders upon renegotiation.

Remark 1. Proposition 1 is robust to alternative assumptions regarding:

(i) Debt priority rules in default;

(ii) Sharing rules of equity from the debt-equity swap in reorganization between initial and new debt holders;

(ii) The mean of compensation to initial debt holders.

Specifically, the presented model assumes that (i) at default after covenant renegotiation, initial [new] debt holders receive a fraction $\alpha \frac{\hat{c}_o}{\hat{c}_o + \hat{c}_n} [\alpha \frac{\hat{c}_n}{\hat{c}_o + \hat{c}_n}]$ of the unlevered asset value; (ii) at reorganization after covenant renegotiation, the equity is assigned to debt holders according to the fraction of their respective coupon, i.e., a fraction $\alpha \frac{\hat{c}_o}{\hat{c}_o + \hat{c}_n} [\alpha \frac{\hat{c}_n}{\hat{c}_o + \hat{c}_n}]$ of the new equity is attributed to initial [new] debt holders; (iii) initial debt holders are compensated with an additional coupon $\hat{c}_o - c_o$. Remark 1 emphasizes that none of these assumptions influences our results. In particular, the implied leverage at investment of the Nash bargaining game is the first-best leverage, independent of priority and sharing rules or the mean of compensation. Similarly, both parties always receive a fraction of the total surplus equal to their bargaining power, independent of the mean by which this surplus is shared. For example, alternatively assuming that initial debt holders are compensated by a one-time payment at renegotiation does not affect the properties.
3.4. Value functions before investment

In this subsection, we present the value functions before investment for equity and corporate debt, denoted by $e(X; c_o)$ and $d(X; c_o)$, respectively, for a given initial coupon $c_o$. The reorganization threshold is denoted by $X_S$, and the covenant renegotiation boundary by $X_R$.

**Proposition 2.** (i) The value of equity in the continuation region $X_S \leq X \leq X_R$ is given by

$$e(X; c_o) = A_e^0 + A_e^1 X + A_e^2 X^{\beta_1} + A_e^3 X^{\beta_2},$$

in which

$$\beta_{1,2} = \frac{1}{2} - \mu \sigma^2 \pm \sqrt{\left(\frac{1}{2} - \mu \sigma^2\right)^2 + 2r \sigma^2}$$

$$A_e^0 = -\frac{(1-\tau)c_o}{r}$$

$$A_e^1 = \frac{1-\tau}{r - \mu}.$$  

$A_e^2, A_e^3$ jointly solve the system

$$M \begin{bmatrix} A_e^3 \\ A_e^1 \end{bmatrix}^T = b^e,$$

in which

$$M = \begin{bmatrix} X_S^{\beta_1} & X_S^{\beta_2} \\ X_R^{\beta_1} & X_R^{\beta_2} \end{bmatrix},$$

and

$$b^e = \begin{bmatrix} -A_e^0 - A_e^1 X_S + \theta \left(\frac{1-\tau}{r - \mu} X_S + A_e^G X_S^{\beta_1}\right) \\ \eta \hat{f}_{FV} s X_R + (1-\eta) \hat{e}(s X_R; c_o) - \eta \hat{d}(s X_R; c_o) - A_e^0 - A_e^1 X_R \end{bmatrix}. $$

$\hat{f}_{FV}$ is the factor to calculate the first-best firm value ($FV^f(X) = \hat{f}_{FV} X$), i.e.,

$$\hat{f}_{FV} = \left[1 - \tau + \tau (1 - \beta_2) \frac{1}{r - \mu} (1 - \theta)\right] \frac{1}{r - \mu},$$

and

$$A_e^G = \frac{\beta_1 - \beta_2}{\beta_1 - 1} \frac{1}{\beta_1 - 1} \left(\frac{r - \mu}{1 - \tau}\right)^{-\beta_1} \left(\frac{I}{s - 1}\right)^{-\beta_1} I$$

is the coefficient in the value function of the unlevered investment opportunity.

(ii) The value of corporate debt in the continuation region is given by

$$d(X; c_o) = A_d^0 + A_d^2 X^{\beta_1} + A_d^3 X^{\beta_2},$$
in which \( \beta_{1,2} \) are defined in Eq. (20) and

\[
A_0^d = \frac{c_o}{r}. \tag{29}
\]

\( A_2^d, A_3^d \) jointly solve the system

\[
M \begin{bmatrix} A_3^d \\ A_4^d \end{bmatrix}^T = b^d, \tag{30}
\]

in which

\[
M = \begin{bmatrix} X_S^{\beta_1} & X_S^{\beta_2} \\ X_R^{\beta_1} & X_R^{\beta_2} \end{bmatrix}, \tag{31}
\]

and

\[
b^d = \begin{bmatrix} -A_0^d + (1 - \theta) \left( \frac{1}{r-\mu} X_S + A_1^G X_S^{\beta_1} \right) \\ (1 - \eta) \hat{f}_{FV} s X_R + (1 - \eta) \hat{e} (s X_R; c_o) - \eta \hat{d} (s X_R; c_o) - A_0^d \end{bmatrix}. \tag{32}
\]

\( \hat{f}_{FV} \) is defined in Eq. (26), and \( A_1^G \) is given in Eq. (27).

**Proof.** See Appendix D.

The following Corollary 1 states that the values of debt and equity are independent of our assumptions concerning the priority rule for initial and new debt in default, the sharing rule of equity from the debt-equity swap in reorganization between initial and new debt holders, and the mean of compensation to initial debt holders upon renegotiation.

**Corollary 1.** The value functions of equity and debt as stated in Proposition 2, Eqs. (19) and (28), remain unchanged under alternative assumptions regarding

(i) Debt priority rules in default;

(ii) Sharing rules of equity from the debt-equity swap in reorganization between initial and new debt holders;

(iii) The mean of compensation to initial debt holders.

Corollary 1 follows directly from Remark 1 stating that the properties of the Nash bargaining solution are unaffected by these assumptions. Hence, the values of initial debt and equity are also unaffected. The reason is that Nash bargaining determines the surplus to equity and debt holders upon investment. For example, the alternative assumption of new senior debt leads to an increase in the compensation coupon compared to the case of equal priority debt. Nash bargaining requires that this increase in the compensation coupon lead to the same surplus as in the case of equal priority debt. Therefore, the values of equity and debt remain unchanged.
3.5. Reorganization threshold, renegotiation boundary, and capital structure

For a given initial coupon $c_0$, equity holders solve

$$\{X_S, X_R\} = \arg \max_{X_S, X_R} e(X; c_0). \quad (33)$$

Hence, the smooth-pasting conditions are

$$\frac{\partial}{\partial X} e(X; c_0) \big|_{X=X_S} = \theta \left( \frac{1-\tau}{r-\mu} + \beta_1 A^G_1 X_S^{\beta_1-1} \right) \quad (34)$$

$$\frac{\partial}{\partial X} e(X; c_0) \big|_{X=X_R} = \eta f_{FS} + (1-\eta) \frac{\partial}{\partial X} \hat{e}(X; c_0) \big|_{X=sX_R} - \eta \frac{\partial}{\partial X} \hat{d}(X; c_0) \big|_{X=sX_R}, \quad (35)$$

in which $A^G_1$ is defined in Eq. (27).

Finally, equity holders choose the initial capital structure by maximizing the value of their objective function ex-ante. Therefore, equity holders solve

$$c_0 = \arg \max_{c_0} \{ e(X_0; \hat{c}_0) + d(X_0; \hat{c}_0) \}. \quad (36)$$

Thus, equity holders’ problem consists of solving Eq. (36) subject to Eqs. (34)-(35). A closed-form solution does not exist. We use numerical procedures and verify the optimality of the investment and reorganization boundaries numerically.

4. Results

In this section, we derive the main implications of our model with covenant renegotiation at investment for corporate policies and firm values.

4.1. Calibration

We use baseline parameter values to reflect a typical S&P 500 firm. The risk free interest rate is $r = 5\%$, the risk-neutral growth rate of the cash flows $\mu = 3\%$, the volatility of the cash flow $\sigma = 23\%$, the tax advantage of debt $\tau = 15\%$, and the recovery rate $\alpha = 60\%$. The initial cash flow is normalized to $X_0 = 1$. Equity holders’ bargaining power is set to $\eta = 0.5$ as in Fan and Sundaresan (2000).

For the investment opportunity, we choose an investment cost of $I = 20$. A scale parameter $s = 1.8$ implies an initial market to book ratio for our baseline firm of 1.62 that closely reflects the average in our empirical sample of firms with private credit agreements from Nini, Smith,
and Sufi (2009). The market to book ratio of a model firm is calculated by dividing the market value of the firm by the value of the invested assets.

4.2. Analysis of firms with reorganization and financing covenant renegotiation at investment

We start by discussing the base case with reorganization and financing covenant renegotiation at investment. Table I gives an overview of the impact of renegotiation at investment on firms.

In both the first- and second-best, we incorporate distressed reorganization, but no covenant renegotiation upon investment. Panel A compares the corporate policies and values in the first-best, second-best, and our model for an initial coupon that is fixed at \( c_0 = 1.04 \), i.e., at the level that is optimal in the second-best. We first fix the initial coupon to isolate from the impact of the initial financing. The first-best firm value is reached if firm-value maximizing investment and investment financing policies are applied, given that equity holders decide about reorganization. With the baseline parameters in Panel A, it is 68.28. In the second-best case, equity holders choose investment and financing policies that maximize the ex-post value of their claim. Panel A shows that equity holders issue too much new debt at investment (overleverage): leverage at investment is 71% in the second-best case, whereas it is only 63% in the first-best. The reason is that equity holders do not internalize the impact of the increased risk of reorganization from issuing new debt on the value of the initial debt, but fully benefit from the additional tax shield. Because the investment and restructuring surplus at investment accrues to equity holders, and due to the transfer of wealth from initial debt to equity holders associated with overleverage, equity holders also invest too early (overinvestment). In particular, the renegotiation (investment) boundary declines from 2.43 in the first-best to 1.94 in the second-best. The last column in Panel A shows that due to these overleverage and overinvestment problems, the first-best firm value is 0.85% above the second-best value.5

The benefit to firms of including a financing covenant in the initial debt contract is that its renegotiation solves the overleverage problem at investment, as shown in Proposition 1. In fact, the leverage of 63% implemented at investment in our model with non-distressed covenant renegotiation corresponds to the first-best leverage at investment (see Panel A of Table I). Additionally, the overinvestment problem is mitigated mainly because renegotiation prevents the expropriation of initial debt holders. The renegotiation (investment) boundary in Panel A increases from 1.94 in the second-best to 2.48 in our model. We call the reduction in the agency costs from overleverage and overinvestment through renegotiation at investment the “agency costs effect.”

4We do not consider the case in which there is a financing covenant that can not be renegotiated. The reason is that, ex-post, both the equity and initial debt holders prefer to renegotiate the covenant. Hence, a setting without renegotiation is not renegotiation-proof.

5Hackbarth and Mauer (2012) also consider the second-best case with an investment that is only equity financed. We do not consider this case. The reason is that an ex-ante commitment by equity holders to use only equity financing at investment is not ex-post incentive compatible. Even if equity holders initially implement a covenant that prohibits new debt financing, they have an incentive to ex-post renegotiate this covenant.
The impact of financing covenant renegotiation

This table shows the impact of financing covenant renegotiation at investment on firm value and policies for different parameters. In Panel A, results are reported for a fixed coupon. Panel B displays the results for the initial coupon that maximizes the value of equity holders’ claim. In Panel C–G, one parameter at a time is changed, and the initial coupon is chosen optimally by equity holders. $s$ is the scale parameter of the investment opportunity, $\sigma$ the volatility of the cash flow, $\tau$ the corporate tax rate, $\alpha$ the recovery rate at default, and $\mu$ the cash flow drift. In the first-best, financing and investment policies are chosen to maximize the value of the firm. In the second-best, equity holders choose these policies to maximize the value of their claim. The first- and second-best solutions incorporate distressed reorganization. Our model includes renegotiation at distress and investment. $X_S$ is the reorganization boundary, $X_R$ is the investment boundary, which initiates covenant renegotiation in our model but not in the first- and second-best case, and $c_0$ is the initial coupon. $\text{lev}_R$ and $\text{lev}_0$ are leverage at investment and initial leverage, respectively, $FV_0$ is the initial firm value, and the value gain from covenant renegotiation, $VG$, is defined as the percentage increase compared to the second-best case, i.e., $VG = 100 \cdot \left(1 - \frac{FV}{FV_{sb}}\right)$.

<table>
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<tr>
<th>Panel</th>
<th>Parameters</th>
<th>$X_S$</th>
<th>$X_R$</th>
<th>$c_0$</th>
<th>$\text{lev}_R$</th>
<th>$\text{lev}_0$</th>
<th>$FV_0$</th>
<th>$VG$</th>
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<td>2.43</td>
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<td>0.26</td>
<td>67.70</td>
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<td>Panel D: Higher volatility of cash flows ($\sigma = 0.25$)</td>
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<td>Panel F: Higher recovery rate ($\alpha = 0.9$)</td>
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<td>0.63</td>
<td>0.53</td>
<td>41.54</td>
<td>1.45</td>
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</table>

The cost to firms of including a financing covenant is that the distressed reorganization risk before investment increases. The reason is that equity holders cannot expropriate debt holders upon investment. Hence, equity holders anticipate a lower value of their claim at investment than in the second-best, which increases their propensity to reorganize the firm before investment. Panel A of Table I shows that for the fixed initial coupon of 1.04, the reorganization boundary increases from 0.21 in the second-best case to 0.22 in our model. A higher distressed reorganization risk reduces the value of a firm because the entire tax shield is lost at reorganization. We call
this channel from the interaction of covenant renegotiation with distressed reorganization the “interaction effect.”

The agency costs effect dominates the interaction effect such that the value of the firm increases from including a financing covenant. With the baseline parameters in Panel A of Table I and a fixed initial coupon of $c_o = 1.04$, for example, including a covenant that is renegotiated at investment raises the initial firm value by 0.81% compared to the second-best (see the last column in line three of Panel A). Hence, the agency costs decline to only 0.04% in our model. The percentage increase in the firm value of our model compared to the second-best value measures the importance to firms of including a covenant that is renegotiated at investment.

In Panel B, we incorporate that equity holders also adapt the initial coupon to maximize the value of their claim when the firm deviates from the second-best. The first-best initial coupon is 2.26, more than twice of the one in the second-best. It is determined by trading off the reorganization cost against the tax shield. With an optimal leverage, the first-best firm value is 2.04% higher than the second-best value due to the (levered) agency costs. The agency costs of debt are larger than in Hackbarth and Mauer (2012) or in Childs, Mauer, and Ott (2005). The main reason is that, in our framework, firms in all cases endogenously choose a higher leverage ex-ante than in these papers due to the ability to avoid costly default through distressed reorganization. The higher initial leverage enlarges the agency costs of debt.

Because covenant inclusion mitigates the agency costs of debt, equity holders implement a higher initial coupon in our model than in the second-best. Panel B of Table I shows that the coupon is 2.08, which corresponds to an important increase compared to the second-best initial coupon of 1.04. The larger leverage in our model increases both the interaction effect and the agency costs effect. The interaction effect increases because a larger coupon implies earlier distressed reorganization. De facto, the reorganization boundary in Panel B rises from 0.21 in the second-best to 0.43 in our model. The agency costs effect becomes stronger since overleverage and overinvestment are more severe with a higher initial leverage. In particular, the agency costs in the second-best of Panel B of 2.04% are considerably larger than the ones of 0.85% in Panel A. Overall, the agency costs effect is more sensitive to the increase in leverage than the interaction effect. Hence, the inclusion of a renegotiable covenant is more important to firms when we incorporate the optimal adaption of the initial leverage. In particular, the percentage firm value increase in our model with covenant renegotiation at investment compared to the second-best is 1.78% in Panel B, and only 0.81% in Panel A. Panel B also shows that covenant renegotiation at investment virtually eliminates the agency costs of debt. The first-best firm value outreaches the one with renegotiation at investment by only 0.26%.

We also calculate the total value from the possibility to renegotiate debt to firms. In the Hackbarth and Mauer (2012) model with investment but without any renegotiation, the firm

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6 The reorganization boundary with a coupon of 2.08 in the second-best is 0.39.  
7 The remaining difference stems from equity holders’ investment timing disincentives, early reorganization incentives, and the consequential lower coupon in the firm with renegotiation at investment.
value is 66.65. Adding distressed reorganization increases the value of the firm by 1.58% to 67.70 (see the second-best in Panel B), mainly because reorganization avoids the bankruptcy cost. Our model shows that neglecting the possibility that firms also renegotiate at investment strongly underestimates the value from debt renegotiation to firms. Specifically, with additional renegotiation at investment, the firm value increases by 3.40% to 68.91. Hence, incorporating non-distressed renegotiation more than doubles the impact of debt renegotiation on firm value.

Firm parameters influence the size of the agency costs effect and the interaction effect and, hence, the added value to firms from including a financing covenant. To explore this influence, we change one parameter in each Panel C–G of Table I and report the resulting corporate policies and firm values. The differences in each panel compared to Panel B do not only occur because of the pure change in a firm parameter, but also because equity holders optimally adapt the leverage if a parameter is altered. For completeness, we also report the pure marginal effect from a change in a parameter in Table 1 in Appendix F for the Panels of Table I. To investigate the pure marginal effect, we keep the leverage constant at the level of the baseline firm in Panel B.

We start by analyzing how the scale parameter $s$ affects our results. A larger $s$ enhances the value gain from the investment. Additionally, the investment opportunity is exercised earlier (the renegotiation thresholds in Panel C of Table 1 decline compared to the ones in Panel B). Therefore, the agency costs increase from 2.04% in Panel B to 3.39% in Panel C of Table 1. Higher agency costs promotes the importance of the agency costs effect. At the same time, the interaction effect is weaker because equity holders are less likely to reorganize the firm before investment when the investment opportunity has a larger value. The propensity to reorganize can be expressed as the ratio of the reorganization threshold to the initial coupon. This ratio declines to 0.17 in line three of Panel C in Table 1 compared to a ratio of 0.21 in line three of Panel B. As a result, the pure marginal effect of a higher $s$ is that renegotiation at investment becomes more important to firms. In fact, the firm value with renegotiation in Panel C of Table 1 increases by 3.16% compared to the second-best. A firm with a larger scale parameter, however, optimally implements a lower initial leverage than the baseline firm. The impact of this leverage reduction dominates the marginal effect, such that the importance to firms from covenant inclusion with $s = 2.2$ is smaller than in the base case. In particular, the value of the firm with debt renegotiation at investment in Panel C of Table I is only 1.60% larger than the one in the second-best.

Next, we investigate the relation between the cash flow volatility and the value of financing covenant renegotiation to firms. The marginal effect of a higher volatility is to delay the exercise of the investment opportunity (see the investment thresholds in Panel D of Table 1 compared to Panel B), which mitigates the importance of the agency costs effect. It also increases the riskiness of debt, which raises both the agency costs and the interaction effects. On the whole, the marginal effect of a higher volatility is to slightly increase the importance of covenant renegotiation to firms. Specifically, the firm value with renegotiation at investment in Panel D of Table 1 increases by 1.81% compared to the second-best, which is larger than the increase in Panel B of Table I. At the same time, however, equity holders decrease the initial leverage when the volatility amplifies,
which lowers the importance of renegotiation. The marginal effect and the impact from the leverage adaption almost cancel out such that the value to firms from covenant renegotiation in Panel D of Table I is similar to the one in Panel B.

We also analyze the impact of an increase in the tax rate. A larger tax rate raises the value of the additional tax shield from upstructuring debt at investment and, hence, also the surplus from renegotiation. Therefore, equity holders have a stronger tendency to overinvest. The differences between the investment thresholds in the second-best and our model to the first-best increase in Panel E of Table 1 compared to Panel B. At the same time, Panel E in Table I shows that equity holders implement a larger leverage with a higher tax rate than in Panel B because more of the firm value derives from the tax shield. As a consequence, the agency costs increase. Specifically, compared to the second-best, the first-best rises by 2.23% in Panel E, but only by 2.04% in Panel B. The value to firms of including a covenant in Panel E, however, is only 1.52% and, hence, smaller than the 1.78% with the baseline parameters. There are two reasons for this decline in the added value. First, renegotiation at investment does not entirely solve the overinvestment problem that is more severe with a higher tax rate. Second, equity holders reduce the initial coupon due to this agency cost in our model compared to the first-best. The need to reduce the initial coupon is more costly in Panel E than in Panel B because the tax shield is more valuable.

The recovery rate influences equity holders’ incentive for distressed reorganization (Fan and Sundaresan, 2000). If the recovery rate in default rises, the total reorganization surplus and the relative surplus that equity holders can extract through distressed reorganization become smaller, and equity holders reorganize later. Therefore, the ratio of the reorganization threshold to the initial coupon in line three of Panel F in Table 1 of 0.18 is smaller than the ratio of 0.21 in line three of Panel B. The interaction effect is, therefore, less important for the value of firms with renegotiation at investment. Hence, the marginal effect of an increase in the recovery rate is to enhance the value to firms of renegotiation at investment to 2.09%. The increase in the initial leverage caused by a higher recovery rate also renders renegotiation more important. Thus, covenant renegotiation is particularly valuable to firms with a high recovery rate. In fact, the firm value with renegotiation at investment in Panel F of Table I increases by 2.25% compared to the second-best.

Finally, we investigate the impact of a reduction in the cash flow drift. A lower drift increases equity holders’ propensity to reorganize, which raises the interaction effect (see the higher ratio of the reorganization threshold to the initial coupon in line three of Panel G in Table 1 of 0.33 compared to the ratio of 0.21 in line three of Panel B). Hence, the marginal effect of a lower cash flow drift reduces the importance of including a financing covenant. In fact, the value from covenant renegotiation at investment to firms in Panel G of Table 1 is only 1.59%. Additionally, the increase in the optimal leverage from the second-best to our model in Panel G of Table I is smaller than in Panel B. Hence, the importance to firms of covenant renegotiation at investment additionally declines to only 1.45% compared to the case with the baseline parameters.
4.3. Bargaining power and the value from covenant renegotiation

To analyze how the bargaining power $\eta$ of equity holders affects the value of covenant renegotiation to firms, we first plot the reorganization boundaries in Figure 1 against the bargaining power. The solid line is the reorganization boundary for a baseline firm with a renegotiable covenant, and the dashed line the one of the second-best firm. We fix the initial coupon for each firm at the level that is optimal for $\eta = 0.5$. A larger bargaining power entails a higher surplus to equity holders at distressed reorganization, which increases their propensity to reorganize the firm early (Fan and Sundaresan, 2000). Hence, both lines are increasing in $\eta$. Additionally, the solid line lies above the dashed line because of the interaction effect and the higher coupon in our model.

Distressed reorganization is costly to firms because the entire tax shield is lost at reorganization. The higher the reorganization boundary, the more important is this concern for firm valuation. Hence, the optimal response of firms to larger $\eta$ is to reduce the initial coupon, as shown in Figure 2. The solid line plots the optimal initial coupon for a firm with renegotiation at investment against the bargaining power of equity holders, the dashed line the one in the second-best firm. Both lines are decreasing in the bargaining power.

Figure 2 also implies that the difference in the optimal initial coupon between the firm with renegotiation at investment and the second-best firm declines with $\eta$. The reason is that as shown in Figure 1, the importance of the interaction effect increases with $\eta$. Hence, the advantage to firms with covenant renegotiation from implementing a higher initial leverage due to the lower agency costs of debt than in the Second-Best decreases with the bargaining power.

In Figure 3, we additionally plot the value of the firm with renegotiation (solid line) and the value of the second-best firm (dashed line) against the bargaining power of equity holders. As the distressed reorganization risk increases with $\eta$ both lines are declining with the bargaining power. The decline is, however, stronger for the firm with renegotiation at investment because the interaction effect increases with the bargaining power.

In Table II, we explore the impact of firm parameters on the relation between the value from covenant inclusion to firms and the bargaining power. In each panel, we change one parameter, and adapt the optimal initial coupon. Hence, besides the marginal effect of the change in a parameter, we also incorporate the impact of the optimal leverage adaption on our results.
An increase in the initial leverage renders the distressed reorganization risk more severe and, therefore, enlarges the negative impact of \( \eta \) on the value from including a renegotiable covenant.

### Table II

**Bargaining power and the value of financing covenant renegotiation**

This table shows the impact of bargaining power on firm value and policies. The bargaining power of equity holders is set to \( \eta = 0.1 \) and to \( \eta = 0.9 \). In each panel, one parameter is changed, and the initial coupon is chosen optimally by equity holders. \( s \) is the scale parameter of the investment opportunity, \( \sigma \) the volatility of the cash flow, \( \tau \) the corporate tax rate, \( \alpha \) the recovery rate at default, and \( \mu \) the cash flow drift. The second-best solution incorporates distressed reorganization. Our model additionally includes renegotiation at distress and investment. \( X_S \) is the reorganization boundary, \( X_R \) is the investment boundary, which initiates covenant renegotiation in our model but not in the first- and second-best case, and \( c_0 \) is the initial coupon. \( lev_R \) and \( lev_0 \) are leverage at investment and initial leverage, respectively. \( FV_0 \) is the initial firm value, and the value gain from covenant renegotiation, \( VG \), is defined as the percentage increase compared to the second-best case, i.e., \( VG = 100 \cdot \left(1 - \frac{FV}{FV_{sb}}\right) \).

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Baseline parameters, optimal initial coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.20 1.87 1.13 0.84 0.28 68.71</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.63 2.44 0.75 0.63 70.21 2.18</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.23 2.02 0.91 0.58 0.23 66.67</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.73 1.69 0.52 0.43 67.58 1.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Higher scale parameter ( (s = 2.2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.10 1.36 0.63 0.81 0.12 83.66</td>
</tr>
<tr>
<td></td>
<td>Our model 0.40 1.83 2.69 0.75 0.59 85.25 1.90</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.13 1.43 0.54 0.56 0.11 81.10</td>
</tr>
<tr>
<td></td>
<td>Our model 0.39 1.89 1.80 0.52 0.39 82.13 1.26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C</th>
<th>Higher volatility of cash flows ( (\sigma = 0.25) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.18 1.96 1.09 0.84 0.26 68.97</td>
</tr>
<tr>
<td></td>
<td>Our model 0.41 2.82 2.52 0.75 0.62 70.49 2.21</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.21 2.11 0.90 0.58 0.22 66.99</td>
</tr>
<tr>
<td></td>
<td>Our model 0.41 2.92 1.73 0.51 0.42 67.92 1.38</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Panel D</th>
<th>Higher tax rate ( (\tau = 0.25) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.33 2.02 2.08 0.86 0.48 66.56</td>
</tr>
<tr>
<td></td>
<td>Our model 0.44 2.97 2.48 0.77 0.67 67.74 1.77</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.36 2.23 1.53 0.60 0.37 62.73</td>
</tr>
<tr>
<td></td>
<td>Our model 0.44 3.17 1.79 0.54 0.47 63.50 1.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel E</th>
<th>Higher recovery rate ( (\alpha = 0.9) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.19 1.86 1.14 0.87 0.28 68.90</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.63 2.52 0.77 0.65 70.46 2.27</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.20 1.89 1.10 0.80 0.27 68.40</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.81 2.41 0.72 0.59 69.91 2.20</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Panel F</th>
<th>Lower cash flow drift ( (\mu = 0.02) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 0.1 )</td>
<td>Second-best 0.25 2. 0.91 0.85 0.35 41.54</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.75 1.57 0.75 0.64 42.29 1.81</td>
</tr>
<tr>
<td>( \eta = 0.9 )</td>
<td>Second-best 0.29 2.16 0.71 0.59 0.28 40.33</td>
</tr>
<tr>
<td></td>
<td>Our model 0.43 2.72 1.08 0.51 0.43 40.78 1.10</td>
</tr>
</tbody>
</table>

Panel A of Table II shows that covenant renegotiation is more important to firms with low bargaining power of equity holders. In particular, the firm value with renegotiation at investment increases by 2.18% with \( \eta = 0.1 \) and by 1.37% with \( \eta = 0.9 \) compared to the Second-Best. The reason for the dependence of the value to firms from covenant inclusion on the bargaining power is that the interaction effect increases with \( \eta \). Panel A also shows that bargaining power is more important to the value of firms when we incorporate renegotiation outside corporate distress.
Specifically, decreasing the bargaining power from 0.9 to 0.1 increases the value of the firm by 3.9% in our model, but only by 3.1% in the model without a renegotiable covenant.

A higher scale parameter in Panel B mitigates the distressed reorganization risk compared to Panel A. In particular, even though the initial coupon increases for the firms with renegotiation at investment, the reorganization thresholds decline. The reason is that equity holders anticipate a higher value from realizing the investment opportunity, which decreases their propensity to reorganize before investment. As the importance of the interaction effect declines with a lower distressed reorganization risk, the marginal effect of a higher $s$ is to dampen the impact of $\eta$ on the value from including a financing covenant. Additionally, firms optimally implement a lower initial leverage. Overall, the lower distressed reorganization risk reduces the dependence of the value from covenant inclusion on the bargaining power. Specifically, the value from including a renegotiable covenant is 1.90% with $\eta = 0.1$ compared to the Second-Best, and 1.26% with $\eta = 0.9$, the difference between the two being only 0.64% as opposed to 0.81% in the base case.

The marginal effect of a higher cash flow volatility in Panel C renders debt riskier. At the same time, however, the investment opportunity becomes more valuable, which mitigates the distressed reorganization risk. These two channels almost cancel out such that the marginal effect on the dependence of the value to firms from including a covenant on $\eta$ is relatively small. Because the optimal leverage is also similar than in Panel A, a higher volatility does hardly affect the dependence of the value from a renegotiable covenant on the bargaining power of equity holders.

The interaction effect increases with $\eta$ because a stronger bargaining power of equity holders rises the distressed reorganization risk. Equity holders, however, obtain a higher portion of the surplus from renegotiation at investment with a larger $\eta$. A higher portion of this surplus reduces their propensity to reorganize before investment, which mitigates the increase in the interaction effect with $\eta$. For a higher tax rate such as in Panel D, the surplus from renegotiation at investment is larger than in Panel A. As a consequence, the reduction in the positive dependence of the interaction effect on $\eta$ is more pronounced than in Panel A. Hence, the marginal effect of increasing the tax rate reduces the sensitivity of the value from covenant inclusion to the bargaining power. A higher tax rate also entails a larger initial leverage. The marginal effect dominates such that $\eta$ is less important for the value from covenant inclusion than in Panel A. In particular, with $\eta = 0.1$ the firm value increases by 1.77% from including a renegotiable covenant, and with $\eta = 0.9$ by 1.23%.

A higher recovery rate $\alpha$ mitigates the distressed reorganization concern of firms (Fan and Sundaresan, 2000). Hence, the marginal effect reduces the dependence of the value from covenant inclusion on the bargaining power. Firms also increase their leverage with a larger $\alpha$. The marginal effect dominates, such that the dependence of the additional firm value from covenant inclusion on $\eta$ becomes smaller than in Panel A. In particular, Panel E shows that covenant inclusion entails a value gain of 2.27% with $\eta = 0.1$, and 2.20% with $\eta = 0.9$. 

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In Panel F of Table II, we decrease the cash flow drift. This change hardly affects the sensitivity of the value from covenant inclusion to the bargaining power.

Our finding that covenant renegotiation at investment is less valuable to firms with a stronger bargaining power of equity holders maintains throughout Table II. The increase in the value of the firm with covenant renegotiation at investment compared to the second-best is larger with $\eta = 0.1$ than with $\eta = 0.9$ in each Panel.

5. Empirical predictions

The recognition that firms can renegotiate both at distress and upon investment allows us to explain empirically observed renegotiation patterns, covenant structures, and financial policies. It also yields novel testable predictions.

5.1. Waiting times to renegotiation

We first investigate the model-implied expected waiting time from initiation to any renegotiation (i.e., covenant renegotiation or distressed reorganization). To this end, we calculate the expected waiting time in closed form (see Appendix E). The expected waiting time to renegotiation for the baseline firm is 3.45 years.

As shown in Table I of the Appendix, firms with a more valuable investment opportunity renegotiate the financing covenant at a lower threshold than firms with a less valuable investment opportunity. Hence, the expected waiting time to covenant renegotiation decreases when the scale parameter $s$ increases. Firms with a more valuable investment opportunity, however, also reorganize at a lower boundary than firms with a less valuable investment opportunity (see Table I of the Appendix). Hence, the expected waiting time to distressed reorganization increases with the scale parameter $s$. Thus, the question of whether the expected waiting time to any renegotiation (covenant renegotiation or distressed reorganization) decreases or increases with $s$ is not trivial.

Figure 4 plots the expected waiting time to any renegotiation against the market to book ratio, which is determined by the scale parameter $s$. It shows that the expected waiting time is decreasing in the market to book ratio. The reason is that the negative impact of $s$ on the expected waiting time to covenant renegotiation dominates.

Because both the reorganization threshold and the covenant renegotiation boundary are decreasing in $s$, the conditional probability of covenant renegotiation at investment given any renegotiation is increasing in $s$. To assess the importance of covenant renegotiation relative to distressed
reorganization, we calculate the conditional probability in closed form (see Appendix E). As illustrated in Figure 5, the conditional probability of covenant renegotiation is 49.14% in the baseline firm, which has a market to book ratio of 1.62.

INSERT FIGURE 5 HERE

With a market to book ratio of 2.4, the conditional probability of covenant renegotiation is 77.72% and, hence, much higher than the one for distressed reorganization. Even for low market to book ratios, covenant renegotiations remain important. For example, a market to book ratio of 1.2 still implies a conditional probability of covenant renegotiation of almost one third (29.4%).

Overall, the analysis suggests that covenant renegotiation at investment contributes substantially to the expected waiting time to renegotiation. This insight naturally leads us to the question of whether our model can explain empirically observed renegotiation patterns. We address this question in the next section.

5.2. Explaining empirical renegotiation patterns

We investigate our model’s ability to explain the renegotiation patterns reported in the empirical literature. To this end, we adapt a simulation approach in the spirit of Strebulaev (2007). The reasons are twofold. First, the expected waiting times in the previous section are derived for optimally financed firms at initiation. Firms in the real world, however, issue renegotiable debt at other times than initiation. Hence, when a firm issues renegotiable debt, its capital structure, distance to reorganization, and distance to covenant renegotiation typically differ from the ones of a firm at initiation. These deviations can be captured by a simulation-based approach, in which cash flows fluctuate over time. Second, Figs. 4 and 5 show that the expected waiting time to renegotiation as well as the conditional probability of covenant renegotiation are non-linear in the market to book ratio. Due to these non-linearities, deviations from the average market to book ratio do not translate linearly into deviations from the average waiting time. Hence, it can be misleading to predict the renegotiation patterns in the cross section of empirical firms based on an average, representative model-implied firm.8

To conduct the simulation, we first construct an empirical sample of a cross section of firms. We collect the 3720 private credit agreements covered in Nini, Smith, and Sufi (2009) from their home page. This sample is analyzed by several papers with regard to renegotiations (e.g., Roberts and Sufi, 2009b; Wang, 2013; Denis and Wang, 2014). It consists of all private loans in the Loan Pricing Corporation Dealscan database extended to nonfinancial public firms from 1996 to 2005 with available Compustat information, and for which the original credit agreements are available.

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8Fig. 5 suggests that the conditional probability of covenant renegotiation is concave in the market to book ratio. Using concavity, it follows by Jensen’s inequality that the conditional probability of covenant renegotiation of the average firm (i.e., 54.01%) constitutes a lower bound for the conditional probability of covenant renegotiation in the cross section.
in SEC filings. We merge this set with Compustat Data and calculate for each observation the 
average market to book and market leverage ratios over the four previous quarters. The market to 
book ratio is constructed as Total Assets (item 44) minus Book Equity plus Market Equity, scaled 
by Total Assets. Book Equity is Total Assets minus Total Liabilities (item 54) minus Preferred 
Stock (item 55) plus deferred taxes (item 52). Market Equity is the Equity Price (item 14) times 
Common Shares Outstanding (item 61). The market leverage ratio is Book Debt, divided by 
Total Assets minus Book Equity plus Market Equity. The market leverage and market to book 
ratio are Winsorized at the upper and lower 2.5 percentiles. Our final empirical sample contains 
3,070 private credit agreements (“true cross section”) characterized by the market to book and 
leverage ratios of the corresponding firm.

Next, we generate a model-implied sample of firms that reflects the true cross section, following 
the simulation-based approach of Strebulaev (2007) as extended by Arnold, Wagner, and 
Westermann (2013). For this purpose, we generate a large universe of optimally financed model-
implied firms. Specifically, we consider firms for which the initial scale parameter ranges from 
s = 1.05 to s = 3 using a step size of 0.05. Firms with a scale parameter of s = 3 invest at 
a threshold of \( X_t = 1.13 \). Larger scale parameters imply a high likelihood of almost immediate 
investment after initiation. To avoid biased results due to the initial selection of scale parameters, 
we use the stable distribution of s in the initial universe. The stable distribution of s is obtained 
by simulation. Our universe consists of approximately 28,000 firms. It is simulated forward for 
ten years using a quarterly frequency. We denote this procedure “pre-simulation.” Firms that 
reorganize or invest are replaced by an optimally financed firm with an identical investment op-
portunity that is not yet exercised. For each model-implied firm, the market to book ratio is 
calculated as the value of the firm divided by the value of the invested assets, and the market 
leverage is given by the value of debt divided by the value of the firm.

We now match the 3,070 observations in the true cross section with simulated, model-implied 
firms. Specifically, for each observation in the true cross section we select the simulated firm at 
the end of the pre-simulation with the smallest squared sum of distances with respect to leverage 
and market to book ratios. Hence, we obtain a sample of 3,070 model-implied matched firms.9

Using this model-implied matched sample, we investigate time patterns of renegotiation. To 
this end, we simulate the matched sample forward with a quarterly frequency. To maintain a 
balanced sample over time, we replace a reorganized or invested firm by an optimally financed firm 
that has the same scale parameter as the replaced firm. We reflect the average contract maturity 
of 46.7 months in Denis and Wang (2014) by choosing a horizon of 48 months for the post-
simulation. The procedure is denoted as “post-simulation.” Using a number of post-simulations, 
we report statistics about the time patterns of renegotiations in the simulated samples.

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9We also repeat the procedure with matching by book leverage and market to book ratios. The impact on the 
results is minor.
The pre-simulation is conducted 50 times. For each pre-simulation, 500 post-simulations are considered, resulting in a total of 25,000 simulations.

In general, our matching procedure is accurate. At matching, the square root of the sum of squared distances between real firms and simulated sample firms with respect to leverage and market to book ratios is 0.17 with a standard deviation of 0.392. Table III shows summary statistics for the true cross section from Nini, Smith, and Sufi (2009), and for our simulated samples at matching. The latter are structurally similar to the empirical sample. The main differences occur due to observations in the cross section with very large market to book ratios that we are not able to match closely with our model.

Table III

Summary statistics of empirical and simulated samples at matching

<table>
<thead>
<tr>
<th></th>
<th>True cross section</th>
<th>Simulated samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market to book ratio mean</td>
<td>1.69</td>
<td>1.61</td>
</tr>
<tr>
<td>Market to book ratio median</td>
<td>1.39</td>
<td>1.40</td>
</tr>
<tr>
<td>Market to book ratio standard deviation</td>
<td>0.90</td>
<td>0.59</td>
</tr>
<tr>
<td>Market leverage mean</td>
<td>0.41</td>
<td>0.45</td>
</tr>
<tr>
<td>Market leverage median</td>
<td>0.40</td>
<td>0.39</td>
</tr>
<tr>
<td>Market leverage standard deviation</td>
<td>0.22</td>
<td>0.18</td>
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</tbody>
</table>

Denis and Wang (2014) and Roberts and Sufi (2009b) report stylized facts about empirical renegotiation patterns in large, randomly selected subsamples of the sample of Nini, Smith, and Sufi (2009). The statistics are summarized in columns one and two of Table IV. On average, slightly over 60% of contracts are renegotiated at least once, and only around 18% of the debt renegotiations are associated with corporate distress.\(^\text{10}\) We calculate the average effective duration of an initial contract, i.e., the mean waiting time to the first renegotiation or to the final maturity (in case a contract is not renegotiated) from the numbers presented in the papers mentioned above. The average effective duration is 2.0 years in Denis and Wang (2014) and 1.5 years in Roberts and Sufi (2009b).\(^\text{11}\) The reported times to the first renegotiation of the contracts that are renegotiated are around 1.4 years. We obtain the effective duration in percent of the initially stated maturity, and the duration in percent of the initially stated maturity from the numbers presented in Table two of Denis and Wang (2014). The former measure is the average time to the first renegotiation or to maturity, expressed in terms of the initially stated maturity. The latter measure is the average time to any renegotiation or to maturity divided by the initially

\(^{10}\)Roberts (2015) and Wang (2013) confirm in their sample that most renegotiations are initiated by borrowers in response to changing conditions, rather than because of lender interventions due to a close default.

\(^{11}\)Roberts and Sufi (2009b) report a relatively large fraction of loans in their sample that disappear or reach the end of the sample without a renegotiation. They are, on average, characterized by long stated maturities, which could explain part of the difference between the average effective durations in Denis and Wang (2014) and Roberts and Sufi (2009b).
stated maturity, including renegotiations that occur after the first renegotiation. Roberts and Sufi (2009b) directly report this measure for their sample. In both papers, the durations are around 60% of the initially stated maturity. We additionally depict the statistics on the renegotiation frequency from Denis and Wang (2014). Conditional on being renegotiated, the average contract is renegotiated 2.7 times. Additionally, their Figure one shows that around 37% of contracts are renegotiated just once, 22% twice, and close to 13% three times. About 53% of contracts are renegotiated between two and five times during their live span.

Table IV

Empirical and model-implied renegotiation occurrence patterns

This table summarizes key statistics about renegotiation frequencies. The first line shows the portion of debt contracts that are renegotiated before maturity. % distressed renegotiations is the percentage of total debt renegotiations that are associated with corporate distress. The effective duration is the mean waiting time to the first renegotiation or to the final maturity in years. Time to first renegotiation is the average time in years to the first renegotiation of the contracts that are renegotiated. Effective duration % of stated maturity is the average time to the first renegotiation or to maturity, expressed in terms of the initially stated maturity. Duration % of stated maturity is the average time to any renegotiation or to maturity divided by the initially stated maturity, including renegotiations that occur after the first renegotiation. Average renegotiation frequency is the mean number of renegotiation rounds of the contracts that are renegotiated. The last four lines summarize the frequencies of renegotiation rounds. The empirical samples are the loan contract samples of Denis and Wang (2014), and Roberts and Sufi (2009b). In the simulated samples of Fan and Sundaresan (2000), firms only reorganize to avoid default but also have an investment option in the spirit of Hackbarth and Mauer (2012). In the simulated samples of our model, firms renegotiate at investment and reorganize to avoid default.

<table>
<thead>
<tr>
<th></th>
<th>Empirical DW 2014</th>
<th>Empirical RS 2009b</th>
<th>Simulated samples FS 2000 + Investment</th>
<th>Simulated samples our model</th>
</tr>
</thead>
<tbody>
<tr>
<td>% renegotiated</td>
<td>60.6</td>
<td>64.5</td>
<td>12.9</td>
<td>41.6</td>
</tr>
<tr>
<td>% distressed renegotiations</td>
<td>18</td>
<td>100</td>
<td>23.9</td>
<td>23.9</td>
</tr>
<tr>
<td>Effective duration</td>
<td>2.0</td>
<td>1.5</td>
<td>3.6</td>
<td>2.8</td>
</tr>
<tr>
<td>Time to first renegotiation</td>
<td>1.3</td>
<td>1.5</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Effective duration % of stated maturity</td>
<td>58</td>
<td>90.8</td>
<td>68.66</td>
<td>63.1</td>
</tr>
<tr>
<td>Duration % of stated maturity</td>
<td>61</td>
<td>57</td>
<td>90.6</td>
<td>63.1</td>
</tr>
<tr>
<td>Average renegotiation frequency</td>
<td>2.7</td>
<td>-</td>
<td>1</td>
<td>1.4</td>
</tr>
<tr>
<td>% renegotiated once</td>
<td>37</td>
<td>-</td>
<td>95.3</td>
<td>76.3</td>
</tr>
<tr>
<td>% renegotiated twice</td>
<td>22</td>
<td>-</td>
<td>4.7</td>
<td>14.0</td>
</tr>
<tr>
<td>% renegotiated three times</td>
<td>13</td>
<td>-</td>
<td>0</td>
<td>5.3</td>
</tr>
<tr>
<td>% renegotiated two to five times</td>
<td>53</td>
<td>-</td>
<td>4.7</td>
<td>23.1</td>
</tr>
</tbody>
</table>

Next, we present the renegotiation patterns in the post-simulations of model-implied samples. We calculate all measures for a contract maturity of 48 months to reflect the average maturity of the contracts in the empirical sample of Denis and Wang (2014). Denis and Wang (2014) and Roberts and Sufi (2009b) show that the maturity and amount of debt are frequently changed upon renegotiation but do not report the structural adaption of each firm. To calculate the renegotiation frequency in our simulated samples, we choose to replace each firm after renegotiation with an optimally structured firm subject to the same scale parameter as the replaced firm.\textsuperscript{12}

\textsuperscript{12}As we do not know the exact firm structure after renegotiation, we assume that a new firm is structured optimally, and is of the same type as the replaced firm.
To assess the marginal contribution of covenant renegotiation at investment, we first run our entire simulation for firms that can only renegotiate at distress, given an identical investment opportunity as in our model. That is, we use firms in the spirit of Fan and Sundaresan (2000), extended for an investment opportunity that is modeled following Hackbarth and Mauer (2012). In the third column of Table IV, we summarize the results for the simulated samples. Only 12.9% of contracts are, on average, renegotiated. All renegotiations occur due to corporate distress. The durations are way longer than their empirical counterparts and the frequency pattern of the renegotiation rounds per contract fails to reflect the one in the empirical samples. For example, hardly any simulated contract is renegotiated more than once.

The final column four of Table IV shows the renegotiation patterns implied by our model that includes both reorganization to avoid default and renegotiation at investment. 41.6% of contracts are renegotiated, and 23.9% of these debt renegotiations are distressed reorganizations. The effective duration is 2.8 years. Also the durations in percent of the initially stated maturity are much closer to their empirical counterparts than the ones implied by the extended Fan and Sundaresan (2000) model. In particular, the average duration to any renegotiation or to maturity of 63.1% of the initially stated maturity reflects its empirical counterparts of around 60%. Overall, our model explains the occurrence and duration patterns in the empirical sample quite well, thereby providing a distinctive improvement over traditional renegotiation models.

We also considerably improve in explaining the empirically observed renegotiation frequencies. For example, 14% of contracts are renegotiated twice, which is much closer to the empirical fraction of 22% than the 4.7% implied by the extended Fan and Sundaresan (2000) model. While we improve the matching of all our frequency measures, however, we are still unable to explain the occurrence of very high renegotiation frequencies. In Denis and Wang (2014), for example, some contracts are renegotiated up to 17 times.

The only measure for which our model does not provide an improvement compared to the extended Fan and Sundaresan (2000) model is the average time to the first renegotiation conditional on the occurrence of a renegotiation. Conditional on renegotiation, the extended Fan and Sundaresan (2000) model predicts a waiting time to the first renegotiation of 1.1 years, which closely matches the empirical counterpart of 1.3 years. While this matching is quite accurate, its importance is limited because the conditional waiting time only includes the 12.9% of the contracts that are renegotiated in the extended Fan and Sundaresan (2000) model (as opposed to more than 60% empirically).

5.3. Explaining covenant structures

We analyze the model in Table II of Section 4.2, and Table I of Section 4.3 under optimal initial leverage. Hence, the change in a parameter in each Panel does not only affect the firm value directly through the marginal effect, but also indirectly through the adaption in the optimal
initial leverage. Empirical regression approaches, however, mostly investigate the marginal effect of a parameter change after controlling for leverage. Hence, we only compare the marginal effect of a change in a parameter of our model to the results in empirical studies.

Our model implies that firms with a higher leverage have a stronger incentive to implement a new debt issuance covenant. This empirical pattern is reported by several studies (e.g., Bradley and Roberts, 2015, Begley and Feltham, 1999).

There is conflicting evidence concerning the relation between financing covenants and investment opportunities. Kahan and Yermack (1998), Nash, Netter, and Poulsen (2003), Reisel (2014), Goyal (2005), and Chava and Roberts (2008) suggest that public debt covenants that restrict additional debt financing are less likely to be included in firms with large investment opportunities. In contrast, Bradley and Roberts (2015) report a positive relation between growth opportunities and covenants in private debt contracts. Our theory speaks to these findings. In particular, public bonds are harder to renegotiate than private debt because a bondholder receives only public information, it is difficult to contact and coordinate dispersed bondholders, and any renegotiation suffers from free-rider problems (Rajan, 1992). Hence, non-renegotiable covenants of public bonds tend to severely restrict the flexibility of firms. Firms should, therefore, reduce their use particularly if they have a high growth potential. The covenants of private debt, however, do not restrict firms but rather, as we suggest, serve to mitigate the agency costs of debt via renegotiation. As the agency costs aggravate with the scale parameter, the marginal effect of an increase in the growth opportunity in our model is to raise the value to firms from including a renegotiable covenant. Hence, our model explains the seemingly puzzling difference between public and private debt with respect to the relation of investment opportunities to covenant inclusion.

The effects discussed in Sections 4.2 and 4.3 also generate several novel empirical predictions. First, the usage of a financing covenant should be increasing with the asset volatility and the recovery rate, and decreasing with the tax rate. Second, firms should be more likely to include a financing covenant if equity holders’ [borrowers’] bargaining power is weaker [stronger]. Third, we suggest that bargaining power and firm parameters jointly determine the value of covenant renegotiation to firms. Hence, empirical studies that analyze the impact of firm characteristics on observed covenant structures should also incorporate the bargaining power of equity holders.

5.4. Explaining financial policies

Figure 6 plots the optimal initial leverage of the baseline firm with a renegotiable financing covenant (solid line), and the one of the second-best firm (dashed line) against the bargaining power of equity holders.

INSERT FIGURE 6 HERE

Bradley and Roberts (2015) show that debt sweeps are positively associated with the cash flow volatility, but negatively with the scaled value of property plant and equipments.
As renegotiation at investment reduces the agency costs of debt, the optimal initial leverage of the baseline firm with renegotiation is larger than the one in the Second-Best. This difference is particularly pronounced for a lower $\eta$ mainly because of the impact of the bargaining power on the interaction effect discussed in Section 4.3. Our analysis implies that the choice of the capital structure strongly depends on whether a financing covenant is included. A novel empirical prediction is that the impact of covenants on the capital structure should be particularly pronounced for firms in which equity holders have weak bargaining power.

Our analysis of the implications of covenant renegotiation at investment on the capital structure also speaks to the empirical observation in Billett, King, and Mauer (2007) that covenant protection significantly reduces the negative relation between leverage and growth opportunities. Figure 7 plots the market to book ratio against the market leverage ratio for our model firm with covenant renegotiation at investment (solid line), and for the Second-Best (dashed line), both with $\eta = 0.5$. The solid line is less steep than the dashed line. The reason is that the second-best firm optimally implements a much lower initial leverage for a higher market to book ratio mainly because the agency costs due to overleverage and overinvestment increase with the importance of the growth opportunity. As including a financing covenant mitigates these agency costs of debt, the negative relation between the initial leverage and the value of the growth opportunity is less pronounced in the firm with covenant renegotiation at investment.

INSERT FIGURE 7 HERE

Besides rationalizing the finding in Billett, King, and Mauer (2007), Figure 7 generates a new testable prediction. The dashed-dotted line plots the relation between the market to book ratio and the market leverage ratio for a firm with covenant renegotiation at investment that has a bargaining power of equity holders of $\eta = 0.9$, and the dotted line the one for the second-best firm with $\eta = 0.5$. The difference in the slopes of these two lines is less pronounced than with $\eta = 0.5$ for two reasons. First, the interaction effect is stronger with higher $\eta$, which renders the reduction in the agency costs of debt from overleverage and overinvestment with a covenant more expensive. Second, firms reduce the initial leverage if $\eta$ increases due to the higher reorganization risk. With a lower leverage, the agency costs due to overleverage and overinvestment in the second-best firm are less sensitive to the importance of the growth opportunity. Hence, including a covenant has a lower impact on this sensitivity than with $\eta = 0.5$. The finding in Billett et al. (2007) that covenant protection reduces the negative relation between leverage and growth opportunities should, therefore, be less pronounced for firms with a higher bargaining power.

In Figure 8, we investigate the impact of covenants and bargaining power on the investment timing in the baseline firm. The solid line depicts the optimal investment threshold of the firm with renegotiation at investment, and the dashed line the one of the second-best firm. Both lines are plotted for an optimal initial coupon.
A comparison of the solid to the dashed lines yields our empirical prediction for the investment timing: The model implies that including a financing covenant in the debt contract delays investment because equity holders cannot expropriate debt holders upon investment. Additionally, we find that this investment delay should be more distinct for low levels of $\eta$. The reasons are that for a weaker bargaining power, the surplus to equity holders at investment is particularly low with a financing covenant, and the initial leverage is larger which increases the wealth transfer upon investment in the Second-Best.

### Table V
**Predicted Credit Spreads**

This table summarizes the impact of renegotiation on corporate credit spreads. Credit spreads are calculated by dividing the initial coupon by the market value of debt, and subtracting the risk free interest rate. In the first column, we calculate these spreads in the model of Hackbarth and Mauer (2012) without any renegotiation. For the second column, we use the distressed reorganization framework of Fan and Sundaresan (2000) augmented for investment. The last column shows the credit spreads in our model with both distressed reorganization and covenant renegotiation at investment. The coupon is 2.08 in each model.

<table>
<thead>
<tr>
<th></th>
<th>HM 2012</th>
<th>FS 2000</th>
<th>Our model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recovery rate $\alpha = 0.9$</td>
<td>124</td>
<td>229</td>
<td>70</td>
</tr>
<tr>
<td>Volatility $\sigma = 0.3$</td>
<td>135</td>
<td>191</td>
<td>44</td>
</tr>
<tr>
<td>Bargaining power $\eta = 0.9$</td>
<td>199</td>
<td>319</td>
<td>124</td>
</tr>
<tr>
<td>+Investment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table V, we summarize the implications of renegotiation for credit spreads. The credit spread is calculated by dividing the initial coupon by the market value of debt and subtracting the risk free interest rate. To compare the results across models, we keep the coupon constant at 2.08. In the model of Hackbarth and Mauer (2012) with investment but without any renegotiation, the credit spread is 124 basis points (bps). Adding renegotiation (distressed reorganization) to obtain the framework of Fan and Sundaresan (2000) extended for investment in column two increases the credit spread to 229 bps. The main reason is that equity holders reorganize the firm earlier than they default in column one because they can extract concessions from debt holders at reorganization. The last column shows that to predict the impact of renegotiation on credit spreads, it is crucial to incorporate that renegotiation can also occur outside corporate distress. In fact, non-distressed renegotiation reduces the initial credit spread for two reasons. First, it decreases the leverage at investment. Second, debt holders anticipate that they will obtain part of the covenant renegotiation surplus upon investment, which reduces the debt yield that needs to be promised by equity holders. This reduction is stronger than the increase from distressed reorganization, such that the credit spread in our model is smaller than in column one. Hence, renegotiation mitigates and does not increase credit spreads.

---

14 Both predictions also hold with a fixed instead of an optimal initial coupon.

15 This coupon corresponds to the optimal coupon in our model.
Renegotiation outside corporate distress also has implications for the sensitivity of credit spreads to firm parameters that are typically associated with credit risk. Without renegotiation in column one, the credit spread slightly declines when the recovery rate increases from to baseline parameters to 0.9. The main reason is that with a higher recovery rate, equity holders issue more new debt at investment, and, hence, also invest earlier. In the second column with distressed reorganization, a higher recovery rate entails less expropriation of debt holders at reorganization. Therefore, equity holders reorganize later than with the baseline parameter, which reduces the credit spread by 16.59% compared to the baseline parameters. In our model with renegotiation at distress and investment, the credit spread even declines by 37.14%. The reason for this stronger sensitivity than in column two is that the interaction effect of Section 4.2 becomes less important when the recovery rate rises. A higher cash flow volatility increases the credit spread in each model compared to the baseline parameters. The increase is, however, particularly pronounced (+77.14%) in our model because the interaction effect is stronger when the volatility is larger.

Finally, we also investigate the impact of bargaining power on credit spreads in the last line of Table V. With distressed reorganization in column two, the credit spread increases with the bargaining power because the stronger ability to expropriate debt holders induces equity holders to reorganize earlier (+18.78%). This relative increase is more pronounced in our model (+32.86%) because the interaction effect becomes stronger with the bargaining power (see Section 4.3).

Recent structural models are successful in replicating average observed credit spreads (e.g., Bhamra et al., 2010). They, however, still struggle to explain the cross-firm variation in credit spreads (e.g., Eom et al., 2004; Zhang et al., 2009). Our analysis provides an important step toward improving the understanding of the cross section of observed credit spreads in several dimensions. First, it shows that renegotiation has quite a distinct impact on credit spreads, depending on whether renegotiation occurs only at distress or also at investment. In the former case credit spreads increase with renegotiation, in the latter case they decline. Second, small variations in the debt covenant structure such as the inclusion of a financing covenant can have a tremendous impact on both absolute credit spreads and the sensitivity of credit spreads to firm parameters. In fact, the first line of Table V implies that including a financing covenant decreases credit spreads from 229 bps to 70 bps. This change is much stronger than the one resulting from a realistic variation in firm parameters that are traditionally associated with credit risk. Finally, because renegotiation is a crucial aspect of credit risk, we suggest to incorporate the bargaining power of equity holders when explaining cross sectional differences in observed credit spreads.

We find support for our implications in the empirical literature that argues that not simply leverage in general, but the debt structure is important to explain the cost of debt. Bradley and Roberts (2015) and Reisel (2014), for example, show that including financing covenants reduces the cost of private debt and bond issues.
6. Conclusion

In this paper, we develop a model in which creditors can influence firm policies by renegotiating debt with equity holders either at investment to waive a financing covenant, or at corporate distress to reorganize the capital structure. We find that renegotiation outside corporate distress is crucial to explain empirically observed debt renegotiation occurrence patterns. Our theory also implies that covenant renegotiation at investment mitigates the agency costs of debt. In particular, we show that it solves the overleverage problem at investment, and mitigates overinvestment. The cost of covenant renegotiation is that the distressed reorganization risk before investment increases. Exploring the dependence of the cost and benefit of covenant renegotiation on firm characteristics and on the bargaining power of equity holders allows us to rationalize observed covenant structures, financial policies, and credit spreads. We also derive a rich set of novel empirical predictions.

Our model provides a fundamental step toward understanding the quantitative consequences for firms of an important aspect of governance, i.e., the possibility of non-distressed debt renegotiation. The recognition of the intensity of this channel implies that bargaining power is a crucial determinant of firm policies and the values of corporate securities. This insight should motivate the empirical literature to incorporate bargaining power when investigating governance questions. We also believe that covenant renegotiations should help to rationalize observed corporate debt structures, such as the choice between private and public debt or the selection of debt owners.

We only analyze the renegotiation of a financing covenant. Considering alternative covenants could generate supplementary insights on the impact of creditors’ influence on firms. Additionally, a call feature may be an alternative to renegotiation for firms with a high renegotiation cost, or with easy access to funding from alternative lenders. We look forward to future research addressing these extensions.
References


Figures

Figure 1. The impact of bargaining power on distressed reorganization. This figure shows the relation between the reorganization boundaries and the bargaining power of equity holders for a baseline firm with renegotiation at investment (solid line), and for the second-best (dashed line). The initial oupon for each firm is fixed at the level that is optimal for \( \eta = 0.5 \).

Figure 2. The impact of bargaining power on the initial coupon. This figure shows the relation between the optimal initial coupon and the bargaining power of equity holders for the firm with renegotiation at investment (solid line), and for the second-best (dashed line).
Figure 3. The impact of bargaining power on the firm value. This figure shows the relation between the firm value and the bargaining power of equity holders for the firm with renegotiation at investment (solid line), and for the second-best (dashed line). The optimal initial coupon is used for each firm.

Figure 4. Expected waiting time to renegotiation. This figure plots the relation between the market to book ratio of a firm with renegotiation at investment and the expected waiting time to any renegotiation.
Figure 5. Probability of covenant renegotiation conditional on any renegotiation. This figure shows the relation between the market to book ratio of a firm and the probability of covenant renegotiation at investment given any renegotiation occurs.

Figure 6. Optimal initial leverage choice. This figure shows the relation between the optimal initial leverage and the bargaining power of equity holders for the firm with renegotiation at investment (solid line), and for the second-best (dashed line).
**Figure 7. Relation between market to book ratio and leverage.** This figure plots the relation between the market to book ratio and the market leverage ratio for a baseline firm with renegotiation at investment (solid line), and for the second-best firm (dashed line). The dashed-dotted and the dotted lines show the corresponding relations with a bargaining power of 0.9 for a firm with renegotiation at investment and for the second-best, respectively.

**Figure 8. Investment timing.** This figure shows the relation between the optimal investment threshold and the bargaining power of equity holders for the firm with renegotiation at investment (solid line), and for the second-best (dashed line). The optimal initial coupon is used for each firm.
Appendix

A. Value functions after investment

The value functions and corporate policies after investment are derived in Fan and Sundaresan (2000), but are presented to the reader for the sake of completeness.

Unlevered firm value. The unlevered firm value, \( V(X) \), corresponds to

\[
V = V(X) = \frac{1 - \tau}{r - \mu} X. \tag{37}
\]

The valuation of equity. The value of equity after investment, denoted by \( \hat{e}(X) \), corresponds to the present value of the expected payoffs to shareholders until reorganization plus the payoff at reorganization, i.e.,

\[
\hat{e}(X) = \mathbb{E}^Q \left[ \int_t^{\hat{X}_S^e} e^{-r(u-t)} (1 - \tau) (X_u - \hat{c}) du + e^{-r \hat{X}_S^e} \theta V(\hat{X}_S) | X_t = X \right], \tag{38}
\]

in which \( \hat{X}_S^e \) is the first time the firm reaches the reorganization threshold, i.e.,

\[
\hat{X}_S^e = \inf \left\{ t \geq 0 : X_t \leq \hat{X}_S \right\}, \tag{39}
\]

and \( \hat{X}_S \) is the firm’s reorganization threshold. \( \hat{c} \) is the total coupon of the firm after investment. Solving the corresponding Hamilton-Jacobi-Belman equation yields

\[
\hat{e}(X) = \hat{A}_0^e + \hat{A}_1^e X + \hat{A}_2^e X^{\beta_2}, \tag{40}
\]

in which

\[
\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \tag{41}
\]

\[
\hat{A}_0^e = -\frac{r}{\tau} \hat{c} \tag{42}
\]

\[
\hat{A}_1^e = \frac{1 - \tau}{r - \mu} \tag{43}
\]

\[
\hat{A}_2^e = \frac{1 - \tau}{r - \mu} \left( \frac{r - \mu}{r} \right)^{1 - \beta_2} \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} \frac{1}{1 - \beta_2} (1 - \theta)^{1 - \beta_2} \hat{c}^{1 - \beta_2}. \tag{44}
\]

The optimal reorganization threshold is calculated as

\[
\hat{X}_S = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{r} \frac{1}{1 - \theta} \hat{c}. \tag{45}
\]
The valuation of corporate debt. Similarly, the value of total corporate debt after investment, denoted by $\hat{d}(X)$, corresponds to the present value of the expected payoffs to debt holders until reorganization plus the payoff at reorganization, i.e.,

$$
\hat{d}(X) = \mathbb{E}^Q \left[ \int_t^{X_S} e^{-r(u-t)}cdu + e^{-rX_S} (1 - \theta) V(\hat{X}_S) | X_t = X \right].
$$

(46)

Solving the associated Hamilton-Jacobi-Belman equation, we find that

$$
\hat{d}(X) = \hat{A}_0^d + \hat{A}_2^d X^{\beta_2},
$$

(47)

in which

$$
\hat{A}_0^d = \frac{\hat{c}}{r},
$$

(48)

$$
\hat{A}_2^d = \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} \frac{1}{r} \left( \frac{r - \mu}{r} \right)^{-\beta_2} \hat{c}_o \hat{c}^{1-\beta_2} \left( \frac{1}{1 - \theta} \right)^{1 - \beta_2} \left( (1 - \tau) \frac{\beta_2}{\beta_2 - 1} - 1 \right),
$$

(49)

and $\beta_2$ is as defined in Eq. (41).

The value function of the corporate debt of initial debt holders, who have a claim on the coupon $c_o$, is given by:

$$
\hat{d}(X; c_o) = \hat{A}_0^d + \hat{A}_2^d X^{\beta_2},
$$

(50)

in which

$$
\hat{A}_0^d = \frac{\hat{c}_o}{r},
$$

(51)

$$
\hat{A}_2^d = \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} \frac{1}{r} \left( \frac{r - \mu}{r} \right)^{-\beta_2} \hat{c}_o \hat{c}^{1-\beta_2} \left( \frac{1}{1 - \theta} \right)^{1 - \beta_2} \left( (1 - \tau) \frac{\beta_2}{\beta_2 - 1} - 1 \right).
$$

(52)

Analogously, the value function of the corporate debt of new debt holders, who have a claim on the coupon $\hat{c}_n = \hat{c} - \hat{c}_o$, can be written as:

$$
\hat{d}(X; c_n) = \hat{A}_0^d + \hat{A}_2^d X^{\beta_2},
$$

(53)

in which

$$
\hat{A}_0^d = \frac{\hat{c}_n}{r},
$$

(54)

$$
\hat{A}_2^d = \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} \frac{1}{r} \left( \frac{r - \mu}{r} \right)^{-\beta_2} \hat{c}_n \hat{c}^{1-\beta_2} \left( \frac{1}{1 - \theta} \right)^{1 - \beta_2} \left( (1 - \tau) \frac{\beta_2}{\beta_2 - 1} - 1 \right).
$$

(55)

Capital structure. The optimal (first-best) capital structure is obtained by maximizing the firm value, which yields the first-best coupon

$$
\hat{c}^{fb} = \frac{r}{r - \mu} \left( 1 - \beta_2 \right)^{1 - \beta_2} \left( 1 - \theta \right) X.
$$

(56)

The corresponding first-best firm value is calculated as

$$
FV^{fb} = \left[ 1 - \tau + \tau \left( 1 - \beta_2 \right)^{1 - \beta_2} \left( 1 - \theta \right) \right] \frac{X}{r - \mu}.
$$

(57)
The firm value can be decomposed into the unlevered asset value and the value of the tax shield, in which the latter also incorporates the loss of the tax shield due to the debt-equity-swap in case of reorganization.

B. The value of the unlevered investment opportunity

Investment entitles its holder to a stream of (taxable) cash flows \((s - 1)X\) in exchange for a fixed investment cost \(I\). Denote the investment boundary of the unlevered investment opportunity by \(X_{IU}\) and the unlevered value of the investment opportunity by \(G(X)\). Standard arguments imply that the value of the investment opportunity in the continuation region is given by

\[
G(X) = A_1^G X^{\beta_1},
\]

in which

\[
\beta_1 = \frac{1}{2} + \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}},
\]

\[
A_1^G = \frac{\beta_1}{\beta_1 - 1} 1^{-\beta_1} \left(1 - \frac{r - \mu}{1 - \tau}\right)^{^{-\beta_1}} \left(1 - \tau\right)^{-\beta_1} I, \tag{60}
\]

and the unlevered investment boundary is given by

\[
X_{IU} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{1 - \tau} I. \tag{62}
\]

C. The renegotiation problem

Proof of Proposition 1. As stated in the main text (Eq. (10)), the renegotiation problem is given by:

\[
\{\hat{c}_o, \hat{c}_n\} = \arg \max_{\{\hat{c}_o, \hat{c}_n\}} \left\{ \hat{e} \left(sX_R; \hat{c}_o + \hat{c}_n\right) + \hat{d} \left(sX_R; \hat{c}_n\right) - \hat{e} \left(sX_R; c_o\right) \right\}^{\eta} \cdot \left\{ \hat{d} \left(sX_R; \hat{c}_o\right) - \hat{d} \left(sX_R; c_o\right) \right\}^{1-\eta}. \tag{63}
\]
Define the total new coupon after investment by \( \hat{c} := \hat{c}_o + \hat{c}_n \). A change of variables yields

\[
\max_{\{\hat{c}_o, \hat{c}_n\}} \left( \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \hat{e} \left( sX_R; c_o \right) \right)^\eta \\
\left( \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} \left( sX_R; c_o \right) \right)^{1-\eta}
\]

\[
\max_{\{\hat{c}_o, \hat{c}_n\}} \left( \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \hat{e} \left( sX_R; c_o \right) \right)^\eta \\
\left( \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} \left( sX_R; c_o \right) \right)^{1-\eta}
\]

The last equality uses the fact that \( \hat{c} \) is bounded by the firm’s debt capacity from above and by zero from below; \( \hat{c}_o \) is bounded by \( \hat{c} \) above and by zero from below; hence, the range of \( \{\hat{c}_o, \hat{c}\} \) is a convex and compact set in \( \mathbb{R}^2 \). Note that this fact also guarantees the existence of the maximum. Therefore, the first step in the proof consists of solving the following problem for an arbitrary, but fixed, \( \hat{c} \):

\[
\{\hat{c}_o\} = \arg \max_{\{\hat{c}_o\}} \left( \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \hat{e} \left( sX_R; c_o \right) \right)^\eta \\
\left( \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} \left( sX_R; c_o \right) \right)^{1-\eta}.
\]

We define the total surplus from renegotiation \( ST(\hat{c}, \hat{c}_o) \), the surplus to equity holders from renegotiation \( SE(\hat{c}, \hat{c}_o) \), and the surplus to debt holders from renegotiation \( SD(\hat{c}, \hat{c}_o) \):

\[
ST(\hat{c}, \hat{c}_o) = \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{e} \left( sX_R; c_o \right)
\]

\[
SE(\hat{c}, \hat{c}_o) = \hat{e} \left( sX_R; \hat{c}_o + \hat{c}_n \right) + \hat{d} \left( sX_R; \hat{c}_n \right) - \hat{e} \left( sX_R; c_o \right)
\]

\[
SD(\hat{c}, \hat{c}_o) = \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{d} \left( sX_R; c_o \right)
\]

For any given total coupon \( \hat{c} \), the total firm value and total surplus are independent of the compensation coupon \( \hat{c}_o \):

\[
ST \left( \hat{c}, \hat{c}_o \right) = \hat{e} \left( sX_R; \hat{c} \right) + \hat{d} \left( sX_R; \hat{c} - \hat{c}_o \right) + \hat{d} \left( sX_R; \hat{c}_o \right) - \hat{e} \left( sX_R; c_o \right) - \hat{d} \left( sX_R; c_o \right)
\]

\[
FV \left( sX_R; \hat{c} \right) = \hat{e} \left( sX_R; c_o \right) - \hat{d} \left( sX_R; c_o \right)
\]

\[
FV \left( sX_R; \hat{c} \right) \text{ denotes the value of the firm given total coupon } \hat{c}. \text{ Note that this calculation holds for any sharing rule in default. The total firm value does not depend on } \hat{c}_o. \text{ Intuitively, the firm value is determined by the total coupon, but not by the allocation of the total coupon to new and existing debt holders. Because the disagreement functions depend only on the already existing coupon, the total surplus is also independent of the compensation coupon } \hat{c}_o:
\]

\[
ST \left( \hat{c}, \hat{c}_o \right) = ST \left( \hat{c} \right)
\]
Hence, re-writing Eq. (65) yields that for any given \( \tilde{c}, \hat{c}_o \) is the solution to the maximization problem

\[
\{ \hat{c}_o \} = \arg \max \begin{pmatrix} \tilde{c}, \hat{c}_o \end{pmatrix} \left( SE \left( \tilde{c}, \hat{c}_o \right) \right)^\eta \left( ST \left( \tilde{c} \right) - SE \left( \tilde{c}, \hat{c}_o \right) \right)^{1-\eta}.
\]  

(71)

The first order condition with respect to \( \hat{c}_o \) writes:

\[
\eta \frac{\partial SE(\tilde{c}, \hat{c}_o)}{\partial \hat{c}_o} + (1 - \eta) \frac{-\partial SE(\tilde{c}, \hat{c}_o)}{ST(\tilde{c}) - SE(\tilde{c}, \hat{c}_o)} = 0,
\]  

(72)

which is equivalent to

\[
SE(\tilde{c}, \hat{c}_o) = \eta ST(\tilde{c}).
\]  

(73)

Consequently,

\[
SD(\tilde{c}, \hat{c}_o) = (1 - \eta) \frac{ST(\tilde{c})}{ST(\tilde{c}) - SE(\tilde{c}, \hat{c}_o)},
\]  

(74)

which completes the proof of (ii). Intuitively, the surplus to equity holders is a fraction \( \eta \) of the total surplus, in which \( \eta \) corresponds to equity holders’ bargaining power. In turn, the surplus to debt holders is a fraction \( 1 - \eta \) of the total surplus, in which \( 1 - \eta \) corresponds to debt holders’ bargaining power.

To prove (i), consider the maximization problem

\[
\{ \hat{c} \} = \arg \max \begin{pmatrix} \tilde{c}, \hat{c}_o \end{pmatrix} \left( SE \left( \tilde{c}, \hat{c}_o \right) \right)^\eta \left( ST \left( \tilde{c} \right) - SE \left( \tilde{c}, \hat{c}_o \right) \right)^{1-\eta}
\]

\[
= \arg \max \begin{pmatrix} \tilde{c} \end{pmatrix} \left(\eta ST(\tilde{c}, \hat{c}_o) \right)^\eta \left( 1 - \eta \right) ST(\tilde{c})^{1-\eta},
\]

(75)

in which we use Eqs. (73) and (74). Then, the first order condition with respect to \( \hat{c} \) is given by:

\[
\eta \frac{\partial ST(\tilde{c}, \hat{c}_o)}{\partial \hat{c}} + (1 - \eta) \frac{(1-\eta) \partial ST(\tilde{c}, \hat{c}_o)}{(1 - \eta) ST(\tilde{c})} = 0,
\]

(76)

which is equivalent to

\[
\frac{\partial}{\partial \hat{c}} ST(\hat{c}) = 0,
\]

(77)

which proves part (i) using concavity of the total surplus. In words, the total new coupon from renegotiation is determined such that the total surplus is maximized. Because the total surplus is maximized at the optimal first-best capital structure, the total coupon is given by

\[
\hat{c}^{fb} = \frac{\beta_2 - 1}{\beta_2} \left( 1 - \beta_2 \right)^\frac{1}{\beta_2} \left( 1 - \theta \right) X_R
\]

(78)

according to Eq. (56). Hence,

\[
\hat{c}_n + \hat{c}_o = \hat{c}^{fb}.
\]

(79)

The corresponding first-best firm value is, according to Eq. (57):

\[
\hat{FV}^{fb} = \left[ 1 - \tau + \tau \left( 1 - \beta_2 \right)^\frac{1}{\beta_2} \left( 1 - \theta \right) \right] \frac{X_R}{r - \mu}
\]

(80)
D. Value functions and corporate policies before investment

The valuation of equity. The value of equity in the base case, \( e(X) \), corresponds to the present value of expected future cash flows to equity holders, i.e.,

\[
e(X) = \mathbb{E}^Q \left[ \int_t^T e^{-r(u-t)} (1 - \tau) (X_u - c_o) \, du \mid X_t = X \right] \
+ \mathbb{E}^Q \left[ 1_{T=X^*_S} e^{-r(X^*_S-t)} (V(X_S) + G^{unlev}(X_S)) \mid X_t = X \right] \
+ \mathbb{E}^Q \left[ 1_{T=X^*_R} \int_T^{X^*_R} e^{-r(u-t)} (1 - \tau) (sX_u - (\hat{c}_o + \hat{c}_n)) \, du \mid X_t = X \right] \
+ \mathbb{E}^Q \left[ 1_{T=X^*_R} \left( -I + \int_T^{X^*_R} e^{-r(u-t)} c_n \, du \right) \mid X_t = X \right] \
+ \mathbb{E}^Q \left[ 1_{T=X^*_R} e^{-r(X^*_R-t)} \theta V \left( \frac{1}{s} \hat{X}_S \right) \mid X_t = X \right],
\]

in which \( R \) denotes the stopping time of reorganization or investment, whichever occurs first, i.e.,

\[
T := \inf \{ X^*_S, X^*_R \mid X_0 = X \}. \tag{82}
\]

Here, \( X^*_S \) is the first time the firm reaches the reorganization threshold, i.e.,

\[
X^*_S := \inf \{ t \geq 0 : X_t \leq X_S \mid X_0 = X \}, \tag{83}
\]

and \( X^*_R \) denotes the first time it reaches the investment threshold, i.e.,

\[
X^*_R := \inf \{ t \geq 0 : X_t \geq X_R \mid X_0 = X \}. \tag{84}
\]

\( \hat{c}_o \) and \( c_n \) are determined by Eq. (10). The first line of Eq. (81) corresponds to the free cash flow to equity holders until reorganization or covenant renegotiation. In case of distressed reorganization, the cash flow corresponds to the second line. The third and fourth lines show the additional cash flows at covenant renegotiation. Finally, the last line captures the cash flows in case of reorganization after covenant renegotiation. The associated ordinary differential equation (ODE) reads

\[
\begin{cases}
\{ e(X) &= \theta (V(X_S) + G^{unlev}(X_S)) &0 \leq X \leq X_S \\
res(X) &= (1 - \tau) (X - c_o) + \mu X e'(X) + \frac{1}{2} \sigma^2 X^2 e''(X) &X_S < X < X_R \\
e(X) &= \hat{e} (sX; \hat{c}_o + \hat{c}_n) - (I - \hat{d} (sX; \hat{c}_n)) &X \geq X_R,
\end{cases}
\]

subject to the boundary conditions

\[
e(X_S; c_o) = (1 - \theta) (V(X_S) + G^{unlev}(X_S)) \tag{86}
\]

\[
e(X_R; c_o) = \hat{e} (sX_R; \hat{c}_o + \hat{c}_n) - (I - \hat{d} (sX_R; \hat{c}_n)). \tag{87}
\]

\( \hat{c}_o \) and \( \hat{c}_n \) are determined by Eq. (10). Standard arguments and Proposition 1 imply the solution as stated in the main text, Eqs. (19)–(25).
The valuation of corporate debt. The value of debt in the base case, \( d(X) \), corresponds to the present value of expected future cash flows to debt holders, i.e.,

\[
d(X) = \mathbb{E} \left[ \int_0^T e^{-r(u-t)}c_0 du | X_t = X \right] + \mathbb{E} \left[ 1_{T=X_S^g} e^{-r(X_S^g-t)} (1-\theta) \left( V(X_S) + G^{unlev}(X_S) \right) | X_t = X \right] + \mathbb{E} \left[ 1_{T=X_R^g} \left( \int_0^{X_S^g} e^{-r(u-t)}\tilde{c}_0 du + e^{-r(X_S^g-t)} (1-\theta) \tilde{V} \left( \frac{1}{s} \tilde{X}_S \right) | X_t = X \right) \right].
\]

The first line of Eq. (88) shows the value of coupon payments to debt holders until investment or default. In case of default, debt holders receive the recovery rate times the unlevered value of the firm’s assets, line two of Eq. (88). In case of investment, the value of coupon payments to debt holders after investment correspond to the third line of Eq. (88), determined as the sum of the coupon payments until default and the payment in default after investment. The ODE for the value of debt is given by

\[
\begin{align*}
\{ & \quad d(X) = (1-\theta) \left( V(X_S) + G^{unlev}(X_S) \right) \quad 0 \leq X \leq X_S \\
& \quad rd(X) = c_0 + \mu Xd'(X) + \frac{1}{2} \sigma^2 X^2d''(X) \quad X_S < X < X_R \\
& \quad d(X) = \hat{d} \left( sX; \tilde{c}_0, \alpha \frac{\hat{c}_0}{\tilde{c}_0 + \tilde{c}_n} \right) \quad X \geq X_R,
\end{align*}
\]

subject to the boundary conditions

\[
\begin{align*}
d(X_S; c_0) &= (1-\theta) \left( V(X_S) + G^{unlev}(X_S) \right) \\
d(X_R; c_0) &= \hat{d} \left( sX; \tilde{c}_0, \alpha \frac{\hat{c}_0}{\tilde{c}_0 + \tilde{c}_n} \right).
\end{align*}
\]

Standard arguments and Proposition 1 imply the solution as stated in the main text, Eqs. (28)–(32).

E. Waiting time to renegotiation

Recall the definitions of the stopping times

\[
\begin{align*}
T &= \inf \{ X_S^g, X_R^g | X_0 = X \} \\
X_S^g &= \inf \{ t \geq 0 : X_t \leq X_S | X_0 = X \} \\
X_R^g &= \inf \{ t \geq 0 : X_t \geq X_R | X_0 = X \}
\end{align*}
\]

as in Eqs. (82)-(84). Standard arguments imply that

\[
M_t := \mathbb{E} [T | \mathcal{F}_t]
\]

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is a martingale. Further:

\[
M_t = \mathbb{E} [T \cdot 1_{T \leq t} + T \cdot 1_{T > t} | \mathcal{F}_t] 
= T \cdot 1_{T \leq t} + \mathbb{E} [(T - t) \cdot 1_{T \leq t} + t \cdot 1_{T > t} | \mathcal{F}_t] 
= T \cdot 1_{T \leq t} + t \cdot 1_{T > t} + \mathbb{E} [(T - t) \cdot 1_{T > t} | \mathcal{F}_t] 
= T \cdot 1_{T \leq t} + t \cdot 1_{T > t} + 1_{t - T} w(X_t) 
= T \cdot 1_{T \leq t} + 1_{T > t} (t + w(X_t)),
\]

in which we use the Markov property, and

\[
w(X_t) = \mathbb{E} [(T - t) | \mathcal{F}_t]
\]
denotes the waiting time to renegotiation any renegotiation (either distressed or covenant renegotiation) from time \( t \) on.

Hence, an application of Ito’s lemma shows that for \( T \leq t \), \( w(X) \) solves the ODE

\[
\mu X w'(X) + \frac{1}{2} \sigma^2 X^2 w''(X) + 1 = 0,
\]

subject to the boundary conditions

\[
w(X_S) = 0 \quad \quad \quad (101)
\]
\[
w(X_R) = 0. \quad \quad \quad (102)
\]

The solution is given by

\[
w(X) = \frac{2 \sigma^2}{2 \mu - \sigma^2} - 2 \log (X) - 2 \left(2 \mu \right) \frac{2 \mu}{2 \mu - \sigma^2} \mu \log (X) - 2 \left(2 \mu \right) \frac{2 \mu}{2 \mu - \sigma^2} \mu \log (X_S) + X_S \frac{2 \mu}{2 \mu - \sigma^2} \sigma^2 \exp \left(\frac{1}{\sigma^2} \left(2 \mu - \sigma^2\right) \left(\log (X) + \log (X_R)\right)\right) - X_R \frac{2 \mu}{2 \mu - \sigma^2} \sigma^2 \exp \left(\frac{1}{\sigma^2} \left(2 \mu - \sigma^2\right) \left(\log (X) + \log (X_R)\right)\right) - X_S \frac{2 \mu}{2 \mu - \sigma^2} \sigma^2 \log (X_R) + X_R \frac{2 \mu}{2 \mu - \sigma^2} \sigma^2 \log (X_S) - \left(2 \mu - \sigma^2\right) \left(X_S \frac{2 \mu}{2 \mu - \sigma^2} - X_R \frac{2 \mu}{2 \mu - \sigma^2}\right)^{-1} + (2 \log (X_R) - 2 \log (X)) \left(X_S \frac{2 \mu}{2 \mu - \sigma^2} - X_R \frac{2 \mu}{2 \mu - \sigma^2}\right)^{-1}. \quad \quad \quad (103)
\]

Similarly,

\[
N_t := \mathbb{P} (T = X^\nu_R | \mathcal{F}_t)
\]
is a martingale. Using analogous arguments, the ODE for the probability of covenant renegotiation, \( f(X_t) \), defined as

\[
f(X_t) = \mathbb{E} [1_{X_T = R} | \mathcal{F}_t],
\]
reads

\[
\mu X f'(X) + \frac{1}{2} \sigma^2 X^2 f''(X) = 0,
\]

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subject to the boundary conditions

\begin{align}
  f(X_S) &= 0 \quad \text{(107)} \\
  f(X_R) &= 1. \quad \text{(108)}
\end{align}

The solution is given by

\begin{align}
  f(X) &= \frac{1}{X_S^{2\mu/\nu - 1} - X_R^{2\mu/\nu - 1}} \left( \left( \frac{X}{X_SX_R} \right)^{1-\frac{2\mu}{\nu}} - X_R^{\frac{2\mu}{\nu} - 1} \right). \quad \text{(109)}
\end{align}
F. Table: The impact of financing covenant renegotiation for fixed leverage

Table 1
The impact of financing covenant renegotiation, fixed leverage

This table shows the impact of financing covenant renegotiation at investment on firm value and policies for different parameters. In Panel A, the results are reported for a fixed coupon. Panels B–G take the optimal leverage in the base case as given. Specifically, initial leverage is 52% in the first-best case, 26% in the second-best case and 53% in our model. Panel B displays the results for the initial coupon that maximizes the value of equity holders’ claim. In Panel C–G, one parameter at a time is changed, and the initial coupon is chosen such that the target leverage of each case is matched. \( s \) is the scale parameter of the investment opportunity, \( \sigma \) the volatility of the cash flow, \( \tau \) the corporate tax rate, \( \alpha \) the recovery rate of the corporate assets at default, and \( \mu \) the cash flow drift. In the first-best, financing and investment policies are chosen to maximize the value of the firm. In the second-best, equity holders choose these policies to maximize the value of their claim. The first- and second-best solutions incorporate distressed reorganization. Our model includes renegotiation at distress and investment. \( X_S \) is the reorganization boundary, \( X_R \) is the the investment boundary, which initiates covenant renegotiation in our model but not in the first- and second-best case, and \( c_o \) is the initial coupon. \( lev_R \) and \( lev_0 \) are leverage at investment and initial leverage, respectively, \( FV_0 \) is the initial firm value, and the value gain from covenant renegotiation, \( VG \), is defined as the percentage increase compared to the second-best case, i.e., 

\[
VG = 100 \cdot \left(1 - \frac{c_o}{FV_{sb}}\right)
\]

<table>
<thead>
<tr>
<th>Panel</th>
<th>( X_S )</th>
<th>( X_R )</th>
<th>( c_o )</th>
<th>( lev_R )</th>
<th>( lev_0 )</th>
<th>( FV_0 )</th>
<th>( VG )</th>
</tr>
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<tbody>
<tr>
<td><strong>Panel A</strong>: Baseline parameters, fixed initial coupon (( c_o = 1.04 ))</td>
<td></td>
<td></td>
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<tr>
<td>First-best</td>
<td>0.22</td>
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<td>0.27</td>
<td>68.28</td>
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<tr>
<td>Second-best</td>
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<td>2.48</td>
<td>1.04</td>
<td>0.63</td>
<td>0.31</td>
<td>68.25</td>
<td>0.81</td>
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<tr>
<td><strong>Panel B</strong>: Baseline parameters, optimal initial coupon</td>
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</tr>
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<td>1.03</td>
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<td>0.26</td>
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</tr>
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<td>0.53</td>
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<td><strong>Panel C</strong>: Higher scale parameter (( s = 2.2 ))</td>
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<td>First-best</td>
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<td><strong>Panel D</strong>: Higher volatility of cash flows (( \sigma = 0.25 ))</td>
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