Structural model for electricity forward prices

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Outlook

- Structural models for forward electricity prices are highly relevant: major structural changes in the market due to the infeed from renewable energy
- Renewable energies infeed reflected in the market expectation – **impact on futures (forward) prices?**
Structural changes in seasonality
Excursus price forward curves (PFCs)

- We observe futures prices:
  - We take as input forward prices of arbitrary maturities, but only prices of a limited set of standard maturities can be observed for electricity!
  - Observable prices: (weekly, monthly, quarterly, yearly)
- We derive and forecast one seasonality shape
- We optimize to align the forecasted shape to the observed futures prices, resulting the Price Forward Curve
Excursus price forward curves (PFCs)

The shape allows the derivation of a daily price-forward curve from a few observed market prices from the solution of an optimization model whose constraints enforce the absence of arbitrage.
Excursus price forward curves (PFCs)
Literature review

- Models for forward prices in commodity/energy:
  - Specify one model for the spot price and from this derive for forwards: Lucia and Schwartz (2002); Cartea and Figueroa (2005); Benth, Kallsen, and Mayer-Brandis (2007);
  - Heath-Jarrow-Morton approach – price forward prices directly, by multifactor models: Roncoroni, Guiotto (2000); Benth and Koekebakker (2008); Kiesel, Schindlmayr, and Boerger (2009);

- Critical view of Koekebakker and Ollmar (2005), Frestad (2008)
  - Few common factors cannot explain the substantial amount of variation in forward prices
  - Non-Gaussian noise

- Random-field models for forward prices:
  - Roncoroni, Guiotto (2000);
  - Andresen, Koekebakker, and Westgaard (2010);

- Derivation of seasonality shapes and price forward curves for electricity:
  - Fleten and Lemming (2003);
Questions to be answered

- We will refer to a panel of daily price forward curves derived over time

- We deseasonalize and aim at a structural model for the stochastic component of PFCs
  - Examine and model the dynamics of risk premia, the distribution of noise (non-Gaussian, stochastic volatility), spatial correlations
  - The analysis is spatio-temporal: cross-section analysis with respect to the time dimension and the maturity “space”
Overview of modeling procedure

Modeling assumptions – p.9

Is it realistic?

We validate assumptions

Empirical analysis:
- Fit the model to 2’386 PFCs
- Examine statistics of:
  - Risk premia
  - Distribution of noise
  - Volatility term structure
  - Spatial correlations

Refine the model:
- Volatility term structure
- Model coloured noise
- Spatial correlations

Fine Tuning
Problem statement

- Previous models model forward prices evolving over time (time-series) along the 
  **time at maturity** $T$: Andresen, Koekebakker, and Westgaard (2010)

- Let $F_t(T)$ denote the forward price at time $t \geq 0$ for delivery of a commodity at time 
  $T \geq t$

- Random field in $t$: 
  \[ t \mapsto F_t(T), \quad t \geq 0 \]  
  (1)

- Random field in both $t$ and $T$: 
  \[ (t, T) \mapsto F_t(T), \quad t \geq 0, \quad t \leq T \]  
  (2)

- Get rid of the second condition: **Musiela parametrization** $x = T - t, \ x \geq 0$. 
  \[ F_t(t + x) = F_t(T), \quad t \geq 0 \]  
  (3)

- Let $G_t(x)$ be the forward price for a contract with time to maturity $x \geq 0$. Note that: 
  \[ G_t(x) = F_t(t + x) \]  
  (4)
Graphical interpretation

$$(t,T) \mapsto F_t(T), \quad t \leq T$$

$$x = T - t$$

$$t \mapsto G_k(X)$$
Influence of the “time to maturity”
Model formulation: Heath-Jarrow-Morton (HJM)

- The stochastic process $t \mapsto G_t(x), \quad t \geq 0$ is the solution to:

$$dG_t(x) = (\partial_x G_t(x) + \beta(t, x)) \, dt + dW_t(x)$$

(5)

- Space of curves are endowed with a Hilbert space structure $\mathcal{H}$
- $\partial_x$ differential operator with respect to time to maturity
- $\beta$ spatio-temporal random field describing the market price of risk
- $W$ Spatio-temporal random field describing the randomly evolving residuals

- Discrete structure:

$$G_t(x) = f_t(x) + s_t(x),$$

(6)

- $s_t(x)$ deterministic seasonality function $\mathbb{R}_+^2 \ni (t, x) \mapsto s_t(x) \mathbb{R}$
Model formulation (cont)

We furthermore assume that the deseasonalized forward price curve, denoted by $f_t(x)$, has the dynamics:

$$df_t(x) = (\partial_x f_t(x) + \theta f_t(x)) \, dt + dW_t(x),$$

with $\theta \in \mathbb{R}$ being a constant. With this definition, we note that

$$dF_t(x) = df_t(x) + ds_t(x)$$

$$= (\partial_x f_t(x) + \theta f_t(x)) \, dt + \partial_t s_t(x) \, dt + dW_t(x)$$

$$= (\partial_x F_t(x) + (\partial_t s_t(x) - \partial_x s_t(x)) + \theta(F_t(x) - s_t(x))) \, dt + dW_t(x).$$

In the natural case, $\partial_t s_t(x) = \partial_x s_t(x)$, and therefore we see that $F_t(x)$ satisfy (5) with $\beta(t, x) := \theta f_t(x)$.

The market price of risk is proportional to the deseasonalized forward prices.
We discretize the dynamics in Eq. (7) by an Euler discretization

\[ df_t(x) = (\partial_x f_t(x) + \theta f_t(x)) \, dt + dW_t(x) \]

\[ \partial_x f_t(x) \approx \frac{f_t(x + \Delta x) - f_t(x)}{\Delta x} \]

\[ f_{t+\Delta t}(x) = (f_t(x) + \frac{\Delta t}{\Delta x} (f_t(x + \Delta x) - f_t(x)) + \theta f_t(x) \Delta t + \epsilon_t(x) \] (8)

with \( x \in \{x_1, \ldots, x_N\} \) and \( t = \Delta t, \ldots, (M - 1)\Delta t \), where \( \epsilon_t(x) := W_{t+\Delta t}(x) - W_t(x) \).

\[ Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x} (f_t(x + \Delta x) - f_t(x)) \] (9)

which implies

\[ Z_t(x) = \theta f_t(x) \Delta t + \epsilon_t(x) , \] (10)

\[ \epsilon_t(x) = \sigma(x) \tilde{\epsilon}_t(x) \] (11)
Deseasonalization approach

- We firstly remove the long-term trend from the hourly electricity prices
- Follow Blöchlinger (2008) for the derivation of the seasonality shape for EPEX power prices: very „data specific”; removes seasonal effects and autocorrelation!
- In a first step, we identify the seasonal structure during a year with daily prices: factor-to-year ($f^{2y}$)
- In the second step, the patterns during a day are analyzed using hourly prices: factor-to-day ($f^{2d}$)
- Forecasting models for the factors are derived, such that the resulting shape can be predicted
- The shape is aligned to the level of futures prices
Factor to year

\[ f2y_d = \frac{S^{\text{day}}(d)}{\sum_{k \in \text{year}(d)} S^{\text{day}}(k) \frac{1}{K(d)}} \]  \hspace{1cm} (12)

To explain the \( f2y \), we use a multiple regression model:

\[ f2y_d = \alpha_0 + \sum_{i=1}^{6} b_i D_{di} + \sum_{i=1}^{12} c_i M_{di} + \sum_{i=1}^{3} d_i CDD_{di} + \sum_{i=1}^{3} e_i HDD_{di} + \varepsilon \]  \hspace{1cm} (13)

- \( f2y_d \): Factor to year, daily-base-price/yearly-base-price
- \( D_{di} \): 6 daily dummy variables (for Mo-Sat)
- \( M_{di} \): 12 monthly dummy variables (for Feb-Dec); August will be subdivided in two parts, due to summer vacation
- \( CDD_{di} \): Cooling degree days for 3 different German cities – \( \max(T - 18.3^\circ C, 0) \)
- \( HDD_{di} \): Heating degree days for 3 different German cities – \( \max(18.3^\circ C - T, 0) \)

where \( CDD_{i}/HDD_{i} \) are estimated based on the temperature in Berlin, Hannover and Munich.
Regression model for the temperature

• For temperature, we propose a forecasting model based on fourier series:

\[ T_t = a_0 + \sum_{i=1}^{3} b_{1,i} \cos\left(\frac{2\pi}{365} Y T_t\right) + \sum_{i=1}^{3} b_{2,i} \sin\left(\frac{2\pi}{365} Y T_t\right) + \varepsilon_t \]  

(14)

where \( T_p \) is the average daily temperature and \( Y T \) the observation time within one year

• Once the coefficients in the above model are estimated, the temperature can be easily predicted since the only exogenous factor \( Y T \) is deterministic!

• Forecasts for CDD and HDD are also straightforward
Factor to day

- The $f2d$, in contrast, indicates the weight of the price of a particular hour compared to the daily base price.

$$f2d_t = \frac{S^{\text{hour}}(t)}{\sum_{k \in \text{day}(t)} S^{\text{hour}}(k) \frac{1}{24}}$$ (15)

- with $S^{\text{hour}}(t)$ being the hourly spot price at the hour $t$.

- We know that there are considerable differences both in the daily profiles of workdays, Saturdays and Sundays, but also between daily profiles during winter and summer season.

- We classify the days by weekdays and seasons and choose the classification scheme presented in Table 1.
Profile classes for each day

Table 1: The table indicates the assignment of each day to one out of the twenty profile classes. The daily pattern is held constant for the workdays Monday to Friday within a month, and for Saturday and Sunday, respectively, within three months.

<table>
<thead>
<tr>
<th>Week day</th>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>M</th>
<th>J</th>
<th>J</th>
<th>A</th>
<th>S</th>
<th>O</th>
<th>N</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Sun</td>
<td>13</td>
<td>13</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>13</td>
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<tr>
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<td>18</td>
<td>18</td>
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<td>19</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>17</td>
</tr>
</tbody>
</table>
Profile classes for each day

- The regression model for each class is built quite similarly to the one for the yearly seasonality. For each profile class $c = \{1, \ldots, 20\}$ defined in table 1, a model of the following type is formulated:

$$f2d_t = a^c_o + \sum_{i=1}^{23} b^c_i H_{t,i} + \varepsilon_t \quad \text{for all } t \in c.$$  \hspace{1cm} (16)

where $H_i = \{0, \ldots, 23\}$ represents dummy variables for the hours of one day

- The seasonality shape $s_t$ can be calculated by $s_t = f2y_t \cdot f2d_t$.

- $s_t$ is the forecast of the relative hourly weights and it is additionally multiplied by the yearly average prices, in order to align the shape at the prices level

- This yields the seasonality shape $s_t$ which is finally used to deseasonalize the electricity prices
Deseasonalization result

The deseasonalized series is assumed to contain only the stochastic component of electricity prices, such as the volatility and randomly occurring jumps and peaks.

Figure 1: Autocorrelation function before and after deseasonalization.

- Let $f_t$ be the price of the forward contract with delivery at time $t$, where time is measured in hours, and let $F(T_1, T_2)$ be the price of forward contract with delivery in the interval $[T_1, T_2]$. Since only bid/ask prices can be observed, we have:

$$F(T_1, T_2)_{\text{bid}} \leq \frac{1}{\sum_{t=T_1}^{T_2} \exp(-rt/a)} \sum_{t=T_1}^{T_2} \exp(-rt/a)f_t \leq F(T_1, T_2)_{\text{ask}}$$

(17)

where $r$ is the continuously compounded rate for discounting per annum and $a$ is the number of hours per year.

$$\min \left[ \sum_{t=1}^{T} (f_t - s_t)^2 \right] + \lambda \sum_{t=2}^{T-1} (f_{t-1} - 2f_t + f_{t+1})^2$$

(18)

- In the original model of *Fleten & Lemming* (2003), applied for daily steps, a smoothing factor prevents large jumps in the forward curve. However, in the case of HPFCs, *Blöchlinger* (2008) (p. 154), concludes that the higher the relative weight of the smoothing term, the more the hourly structure disappears.
## Input mix for electricity production Germany

<table>
<thead>
<tr>
<th>Source</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>42.6</td>
<td>41.5</td>
<td>42.8</td>
<td>44</td>
<td>45.2</td>
<td>43.2</td>
</tr>
<tr>
<td>Nuclear</td>
<td>22.6</td>
<td>22.2</td>
<td>17.6</td>
<td>15.8</td>
<td>15.4</td>
<td>15.8</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>13.6</td>
<td>14.1</td>
<td>14</td>
<td>12.1</td>
<td>10.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Oil</td>
<td>1.7</td>
<td>1.4</td>
<td>1.2</td>
<td>1.2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Renewable energies from which</td>
<td>15.9</td>
<td>16.6</td>
<td>20.2</td>
<td>22.8</td>
<td>23.9</td>
<td>25.9</td>
</tr>
<tr>
<td>Wind</td>
<td>6.5</td>
<td>6</td>
<td>8</td>
<td>8.1</td>
<td>8.4</td>
<td>8.9</td>
</tr>
<tr>
<td>Hydro power</td>
<td>3.2</td>
<td>3.3</td>
<td>2.9</td>
<td>3.5</td>
<td>3.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Biomass</td>
<td>4.4</td>
<td>4.7</td>
<td>5.3</td>
<td>6.3</td>
<td>6.7</td>
<td>7.0</td>
</tr>
<tr>
<td>Photovoltaic</td>
<td>1.1</td>
<td>1.8</td>
<td>3.2</td>
<td>4.2</td>
<td>4.7</td>
<td>5.7</td>
</tr>
<tr>
<td>Waste-to-energy</td>
<td>0.7</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>Other</td>
<td>3.6</td>
<td>4.2</td>
<td>4.2</td>
<td>4.1</td>
<td>4</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Table 2: Electricity production in Germany by source (%), as shown in Paraschiv, Bunn and Westgaard (2016).
Data

- We employed a unique data set of 2'386 daily price forward curves \(F_t(x_1), \ldots, F_t(x_N)\) generated each day between 01/01/2009 and 15/07/2015 based on the latest information from the observed futures prices for the German electricity Phelix price index.
- We firstly deseasonalize the prices:
  \[
  F_t(x) = f_t(x) + s_t(x),
  \]
  (19)
- Estimate the risk premia \((\theta)\), the volatility term structure \((\sigma(x))\) and analyse the noise \(\tilde{\epsilon}_t(x)\)

  \[
  Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x))
  \]
  (20)

  which implies

  \[
  Z_t(x) = \theta f_t(x) \Delta t + \epsilon_t(x),
  \]
  (21)

  \[
  \epsilon_t(x) = \sigma(x) \tilde{\epsilon}_t(x)
  \]
  (22)
Increase in renewables: increase in the PFC’s volatility

Figure 2: Stochastic component of PFCs generated at 01/17/2010 (upper graph) and 01/07/2011 second.
Increase in renewables: increase in the PFC’s volatility

Figure 3: *Stochastic component of PFCs generated at 01/17/2013 (upper graph) and 01/07/2014 second.*
Risk premia

- Short-term: it oscillates around zero and has higher volatility (similar in Pietz (2009), Paraschiv et al. (2015))
- Long-term: it becomes negative and has more constant volatility (Burger et al. (2007)): In the long-run power generators accept lower futures prices, as they need to make sure that their investment costs are covered.

![Graph showing the magnitude of the risk premia over time.](image-url)
Term structure volatility

- We observe Samuelson effect: overall higher volatility for shorter time to maturity
- Volatility bumps (front month; second/third quarters) explained by increased volume of trades
- Jigsaw pattern: weekend effect; volatility smaller in the weekend versus working days
Explaining volatility bumps

Figure 5: The sum of traded contracts for the monthly futures, evidence from EPEX, own calculations (source of data ems.eex.com).
Figure 6: The sum of traded contracts for the quarterly futures, evidence from EPEX, own calculations (source of data ems.eex.com).
Statistical properties of the noise

- We examined the statistical properties of the noise time-series $\tilde{\epsilon}_t(x)$

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \quad (23)$$

- We found: Overall we conclude that the model residuals are **coloured noise**, with **heavy tails** (leptokurtic distribution) and with a tendency for **conditional volatility**.

<table>
<thead>
<tr>
<th>$\tilde{\epsilon}_t(x_k)$</th>
<th>Stationarity</th>
<th>Autocorrelation $\tilde{\epsilon}_t(x_k)$</th>
<th>Autocorrelation $\tilde{\epsilon}_t(x_k)^2$</th>
<th>ARCH/GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h$</td>
<td>$h1$</td>
<td>$h1$</td>
<td>$h2$</td>
</tr>
<tr>
<td>Q0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q7</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 1: $\Delta x = 1 \text{day}$. In column stationarity, if $h = 0$ we fail to reject the null that series are stationary. For autocorrelation $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series.
Autocorrelation structure of noise time series

Figure 7: Autocorrelation function in the level of the noise time series $\tilde{\epsilon}_t(x_k)$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1 \text{ day}$. 
Autocorrelation structure of noise time series (squared)

Figure 8: Autocorrelation function in the squared time series of the noise $\tilde{\epsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1$ day.
Leptokurtic distribution

![Graph of leptokurtic distribution with normal density and kernel (empirical) density for epsilon t (k=7) and epsilon t (k=10).]
Normal Inverse Gaussian (NIG) distribution for coloured noise

Empirical results – p.36
Spatial dependence structure

Figure 9: Correlation matrix with respect to different maturity points along one curve.
Conclusion and future work

- We developed a spatio-temporal dynamical arbitrage free model for electricity forward prices based on the Heath-Jarrow-Morton (HJM) approach under Musiela parametrization
- We examined a unique data set of price forward curves derived each day in the market between 2009–2015
- We examined the spatio-temporal structure of our data set
  - **Risk premia**: higher volatility short-term, oscillating around zero; constant volatility on the long-term, turning into negative
  - **Term structure volatility**: Samuelson effect, volatility bumps explained by increased volume of trades
  - **Coloured (leptokurtic) noise** with evidence of conditional volatility
  - **Spatial correlations structure**: decaying fast for short-term maturities; constant (white noise) afterwards with a bump around 1 year
- Last step (future work): test for a realistic Lévy stochastic process in Hilbert space for the noise time series, with NIG marginals and spatial covariance operator.
References