A structural model for electricity forward prices

Florentina Paraschiv, University of St. Gallen, ior/cf
with Fred Espen Benth, University of Oslo

ECOMFIN2016, Paris
Outlook

• Structural models for forward electricity prices are highly relevant: major structural changes in the market due to the infeed from renewable energy

• Renewable energies impact the market price expectation – **impact on futures (forward) prices?**

• We will refer to a panel of daily price forward curves derived over time: cross-section analysis with respect to the *time dimension* and the *maturity space*

• Examine and model the dynamics of *risk premia*, the *volatility term structure*, *spatial correlations*
Literature review

• Models for forward prices in commodity/energy:
  – Specify one model for the spot price and from this derive for forwards: *Lucia and Schwartz* (2002); *Cartea and Figueroa* (2005); *Benth, Kallsen, and Mayer-Brandis* (2007);
  – Heath-Jarrow-Morton approach – price forward prices directly, by multifactor models: *Roncoroni, Guiotto* (2000); *Benth and Koekebakker* (2008); *Kiesel, Schindlmayr, and Boerger* (2009);

  – Few common factors cannot explain the substantial amount of variation in forward prices
  – Non-Gaussian noise

• Random-field models for forward prices:
  – *Roncoroni, Guiotto* (2000);
  – *Andresen, Koekebakker, and Westgaard* (2010);

• Derivation of seasonality shapes and price forward curves for electricity:
  – *Fleten and Lemming* (2003);
Problem statement

- Previous models model forward prices evolving over time (time-series) along the
  time at maturity $T$: Andresen, Koekebakker, and Westgaard (2010)

- Let $F_t(T)$ denote the forward price at time $t \geq 0$ for delivery of a commodity at time $T \geq t$

- Random field in $t$:
  \[ t \mapsto F_t(T), \quad t \geq 0 \quad (1) \]

- Random field in both $t$ and $T$:
  \[ (t, T) \mapsto F_t(T), \quad t \geq 0, \quad t \leq T \quad (2) \]

- Get rid of the second condition: **Musiela parametrization** $x = T - t, \ x \geq 0$.
  \[ F_t(t + x) = F_t(T), \quad t \geq 0 \quad (3) \]

- Let $G_t(x)$ be the forward price for a contract with time to maturity $x \geq 0$. Note that:
  \[ G_t(x) = F_t(t + x) \quad (4) \]
Graphical interpretation

\[(t, T) \mapsto F_t(T), \quad t \leq T\]

\[x = T - t\]

\[t \mapsto G_t(X)\]
Influence of the “time to maturity”

Change in the market expectation ($\Delta t$)

Change due to decreasing time to maturity ($\Delta x$)

$t_1$, $t_2$
Model formulation: Heath-Jarrow-Morton (HJM)

- The stochastic process $t \mapsto G_t(x), \quad t \geq 0$ is the solution to:

$$dG_t(x) = (\partial_x G_t(x) + \beta(t, x)) \, dt + dW_t(x)$$  \hspace{1cm} (5)

- Space of curves are endowed with a Hilbert space structure $\mathcal{H}$
- $\partial_x$ differential operator with respect to time to maturity
- $\beta$ spatio-temporal random field describing the market price of risk
- $W$ Spatio-temporal random field describing the randomly evolving residuals

- Discrete structure:

$$G_t(x) = f_t(x) + s_t(x), \quad (6)$$

- $s_t(x)$ deterministic seasonality function $\mathbb{R}_+^2 \ni (t, x) \mapsto s_t(x) \in \mathbb{R}$
Model formulation (cont)

We furthermore assume that the deseasonalized forward price curve, denoted by $f_t(x)$, has the dynamics:

$$
df_t(x) = (\partial_x f_t(x) + \theta(x)f_t(x)) \, dt + dW_t(x),
$$

with $\theta \in \mathbb{R}$ being a constant. With this definition, we note that

$$
dF_t(x) = df_t(x) + ds_t(x) = (\partial_x f_t(x) + \theta(x)f_t(x)) \, dt + \partial_t s_t(x) \, dt + dW_t(x)
= (\partial_x F_t(x) + (\partial_t s_t(x) - \partial_x s_t(x)) + \theta(x)(F_t(x) - s_t(x))) \, dt + dW_t(x).
$$

In the natural case, $\partial_t s_t(x) = \partial_x s_t(x)$, and therefore we see that $F_t(x)$ satisfy (5) with

$$
\beta(t, x) := \theta(x)f_t(x).
$$

The market price of risk is proportional to the deseasonalized forward prices.
We discretize the dynamics in Eq. (7) by an Euler discretization

\[ df_t(x) = (\partial_x f_t(x) + \theta(x)f_t(x))\ dt + dW_t(x) \]

\[ \partial_x f_t(x) \approx \frac{f_t(x + \Delta x) - f_t(x)}{\Delta x} \]

\[ f_{t+\Delta t}(x) = (f_t(x) + \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) + \theta(x)f_t(x)\Delta t + \epsilon_t(x) \] (8)

with \( x \in \{x_1, \ldots, x_N\} \) and \( t = \Delta t, \ldots, (M - 1)\Delta t \), where \( \epsilon_t(x) := W_{t+\Delta t}(x) - W_t(x) \).

\[ Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) \] (9)

which implies

\[ Z_t(x) = \theta(x)f_t(x)\Delta t + \epsilon_t(x) \] ,

\[ \epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \] (11)
Overview of modeling procedure

Theoretical model: Spatio-temporal random field of forward prices

Empirical analysis:
- Fit the model to 2'386 PFCs
- Examine statistics of:
  - Risk premia
  - Distribution of noise
  - Volatility term structure
  - Spatial correlations

We validate assumptions

Refine the model:
- Volatility term structure
- Model coloured noise
- Spatial correlations

Is it realistic?

Fine Tuning
Risk premia

- Short-term: it oscillates around zero and has higher volatility (similar in Pietz (2009), Paraschiv et al. (2015))

- Long-term: it becomes negative and has more constant volatility (Burger et al. (2007)): In the long-run power generators accept lower futures prices, as they need to make sure that their investment costs are covered.
Term structure volatility

- We observe Samuelson effect: overall higher volatility for shorter time to maturity
- Volatility bumps (front month; second/third quarters) explained by increased volume of trades
- Jigsaw pattern: weekend effect; volatility smaller in the weekend versus working days
Explaining volatility bumps

Figure 2: The sum of traded contracts for the monthly futures, evidence from EPEX, own calculations (source of data ems.eex.com).
Figure 3: The sum of traded contracts for the quarterly futures, evidence from EPEX, own calculations (source of data ems.eex.com).
Statistical properties of the noise

- We examined the statistical properties of the noise time-series $\tilde{\epsilon}_t(x)$

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x)$$  \hspace{1cm} (12)

- We found: Overall we conclude that the model residuals are **coloured noise**, with **heavy tails** (leptokurtic distribution) and with a tendency for **conditional volatility**.

<table>
<thead>
<tr>
<th>$\tilde{\epsilon}_t(x_k)$</th>
<th>Stationarity</th>
<th>Autocorrelation $\tilde{\epsilon}_t(x_k)$</th>
<th>Autocorrelation $\tilde{\epsilon}_t(x_k)^2$</th>
<th>ARCH/GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h$</td>
<td>$h_1$</td>
<td>$h_1$</td>
<td>$h_2$</td>
</tr>
<tr>
<td>Q0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Q5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Q7</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 1:** The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 1: $\Delta x = 1\text{ day}$. In column stationarity, if $h = 0$ we fail to reject the null that series are stationary. For autocorrelation $h_1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column $h_2 = 1$ indicates that there are significant ARCH effects in the noise time-series.
Autocorrelation structure of noise time series (squared)

Figure 4: Autocorrelation function in the squared time series of the noise $\tilde{\varepsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1 \text{day}$. 
Normal Inverse Gaussian (NIG) distribution for coloured noise
Spatial dependence structure

Figure 5: Correlation matrix with respect to different maturity points along one curve.
Revisiting the model

- We have analysed empirically the noise residual $dW_t(x)$ expressed as $\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x)$ in a discrete form.
- Recover an infinite dimensional model for $W_t(x)$ based on our findings:

$$W_t = \int_0^t \Sigma_s \, dL_s ,$$

(13)

where $s \mapsto \Sigma_s$ is an $L(\mathcal{U}, \mathcal{H})$-valued predictable process and $L$ is a $\mathcal{U}$-valued Lévy process with zero mean and finite variance.
- As a first case, we can choose $\Sigma_s \equiv \Psi$ time-independent:

$$W_{t+\Delta t} - W_t \approx \Psi(L_{t+\Delta t} - L_t)$$

(14)

Choose now $\mathcal{U} = L^2(\mathbb{R})$, the space of square integrable functions on the real line equipped with the Lebesgue measure, and assume $\Psi$ is an integral operator on $L^2(\mathbb{R})$

$$\mathbb{R}_+ \ni x \mapsto \Psi(g)(x) = \int_{\mathbb{R}} \tilde{\sigma}(x,y)g(y) \, dy$$

(15)

- we can further make the approximation $\Psi(g)(x) \approx \tilde{\sigma}(x,x)g(x)$, which gives

$$W_{t+\Delta t}(x) - W_t(x) \approx \tilde{\sigma}(x,x)(L_{t+\Delta t}(x) - L_t(x)).$$

(16)
Revisiting the model (cont)

• Recall the spatial correlation structure of $\tilde{\epsilon}_t(x)$. This provides the empirical foundation for defining a covariance functional $Q$ associated with the Lévy process $L$.

• In general, we know that for any $g, h \in L^2(\mathbb{R})$,

$$\mathbb{E}[(L_t, g)_2(L_t, h)_2] = (Qg, h)_2$$

where $(\cdot, \cdot)_2$ denotes the inner product in $L^2(\mathbb{R})$

$$Qg(x) = \int_\mathbb{R} q(x, y)g(y) \, dy , \quad (17)$$

• If we assume $g \in L^2(\mathbb{R})$ to be close to $\delta_x$, the Dirac $\delta$-function, and likewise, $h \in L^2(\mathbb{R})$ being close to $\delta_y$, $(x, y) \in \mathbb{R}^2$, we find approximately

$$\mathbb{E}[L_t(x)L_t(y)] = q(x, y)$$

• A simple choice resembling to some degree the fast decaying property is $q(|x - y|) = \exp(-\gamma|x - y|)$ for a constant $\gamma > 0$.

• It follows that $t \mapsto (L_t, g)_2$ is a NIG Lévy process on the real line.
Conclusion

• We developed a spatio-temporal dynamical arbitrage free model for electricity forward prices based on the Heath-Jarrow-Morton (HJM) approach under Musiela parametrization

• We examined a unique data set of price forward curves derived each day in the market between 2009–2015

• We examined the spatio-temporal structure of our data set
  – Risk premia: higher volatility short-term, oscillating around zero; constant volatility on the long-term, turning into negative
  – Term structure volatility: Samuelson effect, volatility bumps explained by increased volume of trades
  – Coloured (leptokurtic) noise with evidence of conditional volatility
  – Spatial correlations structure: decaying fast for short-term maturities; constant (white noise) afterwards with a bump around 1 year

• After explaining the Samuelson effect in the volatility term structure, the residuals are modeled by an infinite dimensional NIG Lévy process, which allows for a natural formulation of a covariance functional.