# Prediction of extreme price occurrences in the German day-ahead electricity market

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## Abstract

Understanding the mechanisms that drive extreme negative and positive prices in day-ahead electricity prices is crucial for managing risk and market design. In this paper, we consider the problem of understanding how fundamental drivers impact the probability of extreme price occurrences in the German day-ahead electricity market. We develop models using fundamental variables to predict the probability of extreme prices. The dynamics of negative prices and positive price spikes differ greatly. Positive spikes are related to high demand, low supply, and high prices the previous days, and mainly occur during the morning and afternoon peak hours. Negative prices occur mainly during the night, and are closely related to low demand combined with high wind production levels. Furthermore, we do a closer analysis of how renewable energy sources, hereby photovoltaic and wind power, impact the probability of negative prices and positive spikes. The models confirm that extremely high and negative prices have different drivers, and that wind power is particularly important in relation to negative price occurrences. The models capture the main drivers of both positive and negative extreme price occurrences, and perform well with respect to accurately forecasting the probability with high levels of confidence. Our results suggests that probability models are well suited to aid in risk management for market participants in day-ahead electricity markets.


*Keywords:* Energy Markets; Fundamental Analysis; Spikes; EPEX

# 1 Introduction

Electricity spot prices exhibit seasonality, mean-reversion, time-varying and at times high volatility, as well as occasional price jumps. The spot price is determined by the intersection between the demand and the supply curves, and the price for each period is set by the most expensive generator required to satisfy demand. Electricity markets have highly inelastic short term demand and a nonlinear convex supply curve. Consequently, the relationship between fundamentals and prices is complex and nonlinear. Electricity itself is a unique commodity since it is produced and consumed simultaneously, and offers no possibilities of storage of any significant capacity. Available reserves are therefore always limited, and in times of scarcity and high demand the producers with available capacity have market power. These producers can set asking prices well above marginal costs contributing to the extremely high prices occasionally observed in electricity markets (Bunn et al. (2016)). The following unique characteristics occasionally result in extreme prices; the prices may be very low and negative when supply exceeds demand, or extremely high when supply cannot fulfill the inelastic demand.

At each point in time there is a certain amount of supplying generators at different price levels available, thus making up the merit order curve. Until recently, the merit order curve has consisted of base load from nuclear and coal first, up until the most expensive peak-load units, e.g., gas and oil. However, recently the share of renewable energy sources has increased in many markets. Due to low marginal costs, renewable production places before traditional large-scale base load plants when actively producing. For market participants such as producers, retailers, risk managers and regulators it is vital being able to model the tails of the prices, to understand and adapt to the price related risk. Much of the risk related to trading in power markets is due to the extreme tail price occurrences, either extremely low or high prices. Therefore, predicting and understanding the tail behaviour of electricity prices is often more important than forecasting the expected price of a given period. The German electricity market has seen a large increase of renewable energy sources, while at the same time also having an increasing frequency of negative price occurrences and positive spikes. This makes the tail behaviour of the German market particularly interesting to analyze.

The intermittent nature of renewable energy sources is challenging from a risk-management perspective, an increasingly relevant issue in recent years. Lower than expected production from renewable sources may cause deficits and correspondingly high prices, which is a large source of risk for retailers. On the contrary, excessively high production may cause oversupply and low, or even negative, prices. When the spot price is negative, producers essentially pay for dumping electricity on the market. Opportunity cost of shutting/ramping down may exceed the negative price of produced electricity; for example, there has been instances where utilities were willing to pay up to €120/MWh or more to get rid of the excess electricity produced (Keles et al., 2011).

Our paper identifies main drivers behind the occurrence of extreme prices at the German day-ahead electricity market (European Power Exchange, EPEX), as well as predicting these occurrences. Literature on prediction of extreme electricity price behaviour is lacking, particularly for markets with a large share of intermittent renewable energy and negative price occurrences. This suggests that tail behaviour of electricity spot prices and causes of extreme price movements is currently not well understood. The first contribution of this paper is estimating logit models for forecasting the probability of an extreme price as a function of selected fundamental variables. This analysis reveals which fundamental variables drive the probability of extreme price occurrences, and quantifies the impact on the probability of observing extreme prices. Our study is the first, to our knowledge, which provides such a detailed overview of the link between fundamental variables which impact the probability of spike occurrences. This marks a clear contribution to the literature stream on fundamental modeling and comes with complementary insights to existing VaR papers by Byström (2005) and Paraschiv et al. (2016). Currently, it is assumed that high levels of wind production is the main driver behind negative prices. Positive spikes are likely to be closely related to the relationship between demand and supply, in which renewable energy sources are playing an increasingly important role. The second contribution of this paper is therefore a further exploration of how forecasts of photovoltaic and wind power production affect the probability of extreme prices.

## 2 Literature Review

Our paper can be placed in the context of two main research areas in the field of electricity spot price markets: (i) modeling of spike occurrences and behaviour, and (ii) the impact of renewable energy sources on electricity prices. There is an extensive amount of research on modeling and forecasting electricity spot prices. Yet, the literature seems to be at the beginning of understanding the economic drivers behind extreme price occurrences – particularly in markets with a large share of renewable energy sources.

Different modelling techniques have been applied in order to capture and model the distribution of extreme price behaviour. Bunn and Karakatsani (2003) gives an overview of several methods used for forecasting electricity prices. Bunn et al. (2016) use a multifactor, dynamic, quantile regression formulation and show how the price elasticities of the fundamentals vary extensively across quantiles. However, the elastisities of gas, coal and carbon prices exhibit no specific pattern across quantiles hence they hardly have any influence on the peak price distribution. Thomas et al. (2011) develop an autoregressive (AR) model to capture the effects of individual spikes while controlling for seasonality in spot price returns in the Australian electricity market. They conclude that incorporation of supply and demand information is necessary to adequately capture negative prices. Christensen et al. (2012) extend the research on AR models by looking at the prediction of spikes using an autoregressive conditional hazard (ACH) model on the Australian electricity market. As a benchmark, the logit model is used, yielding similar results. Focusing on the short-term forecasts of spike occurrences, Eichler et al. (2013) develop variations of the dynamic binary response model, e.g. with regime-switching mechanisms, proposed by Kauppi and Saikkonen (2008). The models have a superior fit on the Australian market data and they suggest to replace the logistic function by an asymmetric link function leading to significant improvements. Eichler et al. (2013) also extend the ACH model used in Christensen et al. (2012) by incorporating past price information with that improving the performance of the model. Karakatsani and Bunn (2010) apply various statistical models to investigate the relationship between fundamental drivers and electricity price volatility in the UK market, which is a different piece of the puzzle for understanding extreme price behaviour.

In the litterature, the focus has shifted from forecasting prices based on the entire price distribution to isolating normal range prices from the price spikes. Byström (2005) and Paraschiv et al. (2016) investigate the performance of EVT on accurately modeling and forecasting the extreme tails of electricity price distributions. Both studies conclude that EVT is a powerful tool for this purpose. Lu et al. (2005), Zhao et al. (2007b) and Zhao et al. (2007a) model normal and extreme prices separately to achieve more complete and robust models and more accurate forecasts. Zhao et al. (2007b) use a method based on data mining by applying two algorithms to the data - support vector machine and probability classifier - to predict the spike occurrence. The results are highly accurate and provide improved risk management practices related to extreme price prediction, but provide limited economic insights. Higgs and Worthington (2008) study the Australian spot electricity market, which exhibits frequent price spikes, and employ three models to capture these effects; a stochastic, a mean-reverting and a regime-switching part. The regime-switching model outperforms the other two because the allowance of price spikes is better. A shortcoming of the model is the unrealistic assumption of constant transition probabilities. Mount et al. (2006) solves this issue by adjusting the regime-switching model by modeling the transition probabilities as a function of the load and/or the implicit reserve margin. By modeling the volatile behaviour of the electricity prices in the Pennsylvania-New Jersey-Maryland (PJM) Power Pool, they provide accurate spike predictions. However, the model is dependent on precise reserve margin measurements, which are not easily obtained.

Regime switching models are frequently used to model spike behaviour (Arvesen et al. (2013), Huisman and de Jong (2003), Keles et al. (2011), Weron et al. (2004),Weron (2009), Weron and Misiorek (2005)). They allow the spot price to switch between a base regime and higher/lower jump regimes. Paraschiv et al. (2015) propose a regime-switching approach to simulate price paths and forecasts on the EPEX. They extend the approach from Kovacevic and Paraschiv (2014) to include serial dependencies and transition probability for spike clustering. Christensen et al. (2009) perform a Poisson AR framework to identify spikes defined as threshold exceedances. Keles et al. (2011) consider positive price spikes and negative prices at the EPEX by implementing a regime-switching model. All of the above-mentioned models focus mainly on positive spikes in markets dominated by conventional energy sources. Consequently, the models are not including the impact of renewable energy production. Literature including the impact of renewable energy sources on spikes and negative prices is limited. This particularly applies to negative prices, which have become increasingly more common in latter years (Paraschiv et al. (2014), Schneider and Schneider (2010) and Keles et al. (2011)).

Huisman et al. (2013) investigate the effect of renewable energy sources on electricity prices indirectly by studying hydro power in the Nord Pool market. They argue that theoretical and simulation studies show declining electricity prices when introducing sustainable energy supply, but empirical studies supporting this result are scarce. Paraschiv et al. (2014) investigate directly the influence of renewable energy sources on the German electricity market. By analyzing the impact of wind and photovoltaic on day-ahead spot prices at the EPEX, they conclude that the introduction of renewable energy sources increase the extreme price behaviour and influence the fuel mix for electricity production. Hagfors et al. (2016) expand the literature on renewable energy to the German market by looking at the effect of renewable power sources on EPEX price formation using quantile regression. They analyze the effect of wind and photovoltaic, and confirm the results from Paraschiv et al. (2014) that renewable energies have a dampening effect on the spot prices. Further they find that negative prices, mostly due to wind power, mainly occur during night hours when demand is low.

Our paper extends the research regarding how renewable energy sources impact the extreme electricity price behaviour, in particular on the EPEX. In doing so, we complement the research by Paraschiv et al. (2014) by focussing on extreme price movements.

## 3 The German Electricity Market

The German electricity spot market has gone through large regulatory changes and energy input mix changes, as shown in Table 1. The regulatory changes have contributed to incentivizing an increased share of renewable energy sources and contributed to a large expansion of renewable production. As the successor to the Electricity Feed Act ("Stromeinspeisungsgesetz", StrEG) from 1991, the EEG was issued in 2000 and guarantees producers of renewable energy a minimum compensation price per kWh. The compensation is secured through feed-in tariffs that come on top of the spot market prices. The EEG has stimulated a strong increase of installed production of renewable sources, changing the energy input mix vastly to include 14,6% renewable energy (8,9% wind, 5,7% photovoltaic) in 2014 (Table 1). The target of the EEG is to have 35% renewable production in 2035; consequently, renewable energy will likely continue to be incentivized, although the feed-in-tariffs have been lowered in later years (Paraschiv et al. (2014)). Frondel et al. (2010) and Fanone et al. (2013) argue that increase of renewable energy sources has reduced the average spot electricity prices, and even led to a number of occurrences of negative prices. However, consumers have not experienced lower prices; the feed-in-tariffs exceed the price reduction from increased renewable capacity. Further, Frondel et al. (2010) claim that government policy has failed in introducing renewable energy in a cost-effective and viable way. On 1$^{st}$ of January 2010 the last significant regulatory change took place, the Equalization Mechanism Ordinance (AusglMechV). The AusglMechV obliges the Transmission System Operators (TSOs) to market the EEG-electricity on the day-ahead market, thus entirely changing the market mechanisms of trading energy from renewable sources.

The exchange based trading in the day-ahead market in Germany is conducted at the EPEX. Trading for delivery on the next day opens at 12:00, and the exchange is active all year. The market is divided in 24 hourly intervals, where a trading period is defined as a one hour period, e.g. 12:00-13:00. Market participants bid a given quantity MWh in a trading period at a set price, where offered price must be in the range -€500/MWh to €3000/MWh. Negative prices have been allowed at the EPEX since 1st September 2008 to ensure market clearing in situations where supply exceeds demand (Genoese et al. (2010)).

Table 1: Percentagewise contribution of various fuel sources and production technologies out of the total energy production, making out the German energy input mix from 2009 to 2014, Energiebilanzen (2015).

| | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| Coal | 42.6% | 41.5% | 42.8% | 44.0% | 45.2% | 43.2% |
| Nuclear | 22.6% | 22.2% | 17.6% | 15.8% | 15.4% | 15.8% |
| Natural Gas | 13.6% | 14.1% | 14.0% | 12.1% | 10.5% | 9.5% |
| Oil | 1.7% | 1.4% | 1.2% | 1.2% | 1.0% | 1.0% |
| Wind | 6.5% | 6.0% | 8.0% | 8.1% | 8.4% | 8.9% |
| Hydro power | 3.2% | 3.3% | 2.9% | 3.5% | 3.2% | 3.3% |
| Biomass | 4.4% | 4.7% | 5.3% | 6.3% | 6.7% | 7.0% |
| Photovoltaic | 1.1% | 1.8% | 3.2% | 4.2% | 4.7% | 5.7% |
| Waste-to-energy | 0.7% | 0.7% | 0.8% | 0.8% | 0.8% | 1.0% |
| Other | 3.6% | 4.2% | 4.2% | 4.1% | 4.0% | 4.3% |

# 4 Data Analysis

## 4.1 Choice of Data and Descriptive Statistics

We analyze hourly PHELIX spot prices in the German electricity market observed between $4^{th}$ of January 2010 and $31^{st}$ of May 2014, and model each hourly time series separately. At the beginning of 2010, large changes in the energy input mix were coupled with the last significant regulatory change (AusglMechV). The properties of electricity prices differ considerably from those of financial assets. Electricity prices are seasonal, and exhibit high volatility that might result in extremely high (or low) prices. Unlike most stock and commodity prices, spot electricity prices are stationary. An augmented Dickey Fuller (ADF) test with two lags on the spot prices in our sample yields a statistic of -45.04, confirming the electricity price series are stationary.[1]

The descriptive statistics of spot prices are given in Table 3. The spot price ranges from negative €222/MWh to positive €210/MWh. However, the standard deviation of €16.6/MWh shows that large share of prices are relatively close to the mean. The excess kurtosis is high, indicating a possibility of extreme prices in either direction. The skewness is low and negative, implying prices below mean are more likely than prices above mean.

Electricity spot prices are complex to model, and can be linearly or non-linearly related to many variables. Table 2 shows the fundamental variables considered to explain the dynamics of electricity spot prices, as these variables are expected to impact the price formation. Lagged price 1 (day before) and 7 (a week before) have been included because of the high correlation between spot prices and lagged prices. Electricity demand exhibits predictable weekly patterns, and this is handled through the inclusion of seven-day lagged price. A test statistic of 138 from a Ljung-Box test conducted with seven lags, confirms auto-correlation is present. To model the volatility, spot prices were first regressed on the seven first lags, and the regression residuals were used in an EWMA model with smoothing constant $\lambda = 0.94$. This method captures the seasonal effects, weekly trends and auto-correlation between the first seven lags, and is a pragmatic representation of the volatility at each point. Demand forecasts are not consistently published by the four TSOs in Germany, and thus need to be modeled. Forecast data used in our analysis has been modeled according to the method described in Paraschiv et al. (2014), thus seasonality is here handled. The analysis and modeling focuses on the effect of different supply sources, and the modeling therefore separately considers power plant availability, wind, and photovoltaic. Each trading period has unique price dynamics, and demand/supply has been modelled accordingly.

Descriptive statistics of fundamental variables are provided in Table 3. It is worth noting that the wind forecasts and photovoltaic forecasts have higher standard deviation than the other fundamental variables, as well

[1] All variables have been tested and found to be stationary with an ADF test, except for $CO_2$ price and coal price. Intuitively they both have an upper and lower bound, and there is reason to expect that a longer time series of these prices would be found to be stationary.

Table 2: Overview of the variables chosen for modeling.

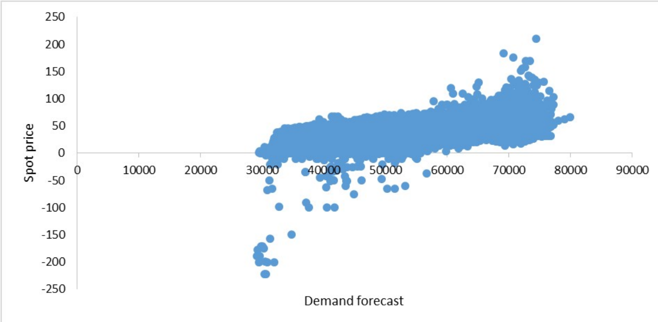
| Variable, units | Description | Data source | Granularity |
|---|---|---|---|
| Lag spot price | Market clearing price for the same hour of the last relevant delivery day – lag 1 and 7 | European Energy Exchange (EPEX) | Hourly |
| Expected demand, MWh | Demand forecast for the relevant hour on the delivery day as modelled in Paraschiv et al. (2014) | Own data, German Weather Service | Hourly |
| Expected wind power infeed, MWh | Expected infeed published by German transmission system operators following the electricity price auction | Transmission system operators http://www.50hertz.com/de/ http://amprion.de/ https://www.transnetbw.com/en http://www.tennet.eu/nl/home.html | Hourly |
| Expected photovoltaic infeed, MWh | Expected infeed published by German transmission system operators following the electricity price auction | Transmission system operators http://www.50hertz.com/de/ http://amprion.de/ https://www.transnetbw.com/en http://www.tennet.eu/nl/home.html | Hourly |
| Expected power plant availability, MWh | Ex ante expected power plant availability for electricity production (voluntary publication) on the delivery day, published daily at 10:00 am | European Energy Exchange and transmission system operators: ftp://infoproducts.eex.com | Hourly |
| Coal price, EUR/12,000 t | Latest available price (daily auctioned) of the front-month Amsterdam-Rotterdam-Antwerp (ARA) futures contract before the electricity auction takes place | European Energy Exchange | Daily |
| Gas price, EUR/MWh | Last price of the NCG Day Ahead Natural Gas Spot Price on the day before the electricity price auction takes place | Bloomberg, Ticker: GTHDAHD Index | Daily |
| Oil price, EUR/bbl | Last price of the active ICE BrentCrude futures contracton the day before the electricity price auction takes place | Bloomberg, Ticker: COA Comdty | Daily |
| Price for EUA, EUR 0.01/EUA 1000 t $CO_2$ | Latest available price of the EPEX Carbon Index (Carbix), daily auctioned at 10:30 am | European Energy Exchange (EPEX) | Daily |
| Spot price volatility | Volatility at each data point based on an EWMA model | European Energy Exchange(EPEX) | Hourly |

Table 3: Descriptive statistics of variables used for modeling. Lag 1/lag 7 have similar characteristics as spot price, volatility is not included.

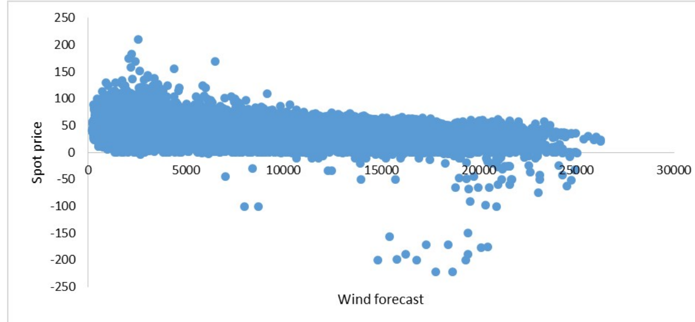
| | Spot price | Demand | Wind | PV | PPA | Coal | Gas | Oil | $CO_2$ |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 42.9 | 54852 | 5297 | 2514 | 55146 | 71.2 | 23.2 | 74.2 | 9.3 |
| Standard deviation | 16.6 | 10081 | 4432 | 4280 | 4894 | 11.6 | 4.1 | 4.8 | 4.3 |
| Min | -222.0 | 29201 | 229 | 0 | 40016 | 51.5 | 11.2 | 61.0 | 2.5 |
| Max | 210.0 | 79884 | 26256 | 24525 | 64169 | 99.0 | 39.5 | 83.8 | 16.8 |
| Skewness | -1.02 | -0.05 | 1.52 | 2.04 | -0.27 | 0.24 | -0.63 | -0.72 | 0.28 |
| Excess kurtosis | 16.44 | -1.04 | 2.30 | 3.75 | -0.77 | -1.08 | 0.88 | -0.35 | -1.48 |

as positive excess kurtosis and positive skew. These characteristics indicate there is a higher chance of very high forecasts of wind/photovoltaic production, and higher variability than, e.g. demand and power plant availability. Demand forecasts have negative excess kurtosis, indicating there is a low probability for very extreme values in either direction.

The scatter plot of wind forecasts versus spot prices shown in Figure 1 confirms that extremely low spot prices are seen nearly exclusively when wind production is high, in accordance with the negative correlation. This indicates that high forecasts of wind are a driver behind negative prices. The demand versus spot prices confirm that very high prices occur when demand is high, and that low prices occur when demand is low, in accordance with the positive correlation. Very low prices are observed when demand forecasts are low, and wind forecasts are high – implying negative prices are due to either low demand, excess supply from wind, or a combination of factors.




(a) PHELIX spot price plotted against demand forecast.




(b) PHELIX spot price plotted against wind forecast.

Figure 1: Scatter plots for demand forecast (a) and wind forecast (b) against the PHELIX spot price. The plots show that negative prices usually occur in combination with high wind production and low demand.

### 4.2 Definition of Extreme Prices and Analysis of Occurrences

Filtering extreme observations from normal is a challenging task that can be solved in various ways. Advanced examples include using Markov regime switching models (Janczura and Weron (2010), Weron (2009)), wavelet filtering (Stevenson et al. (2006)), and thresholds implied by Gaussian prediction intervals (Borovkova and Permana (2006)). Simpler methods include setting a subjectively chosen fixed threshold (Eichler et al. (2013)), use a variable price threshold determined by a certain percentage of the highest prices classified as extreme (Trück et al. (2007)), classifying prices as spikes if they exceed the mean price by three standard deviations (Cartea and Figueroa (2005)) or determining the threshold using a joint maximum likelihood approach (Paraschiv et al. (2015)).

To estimate a logit model for extreme price prediction it is required to determine which price is considered to be extreme. A reasonable approach is to filter out some percentage of the highest prices, typically 1%. This method is simple to implement, but sufficiently powerful to be used for modeling, and yields a positive spike threshold at €79.2/MWh. The drawback is that the threshold will vary as the data sample changes. Another option is defining a fixed threshold based on a subjective perception of what classifies as extreme for market participants, e.g., €80/MWh, €90/MWh. Zhao et al. (2007a) argue the threshold can be set as the mean of the price series plus a multiple of the historically based volatility. Although the choice of threshold may seem arbitrary, 1% of the highest prices is considered to be in the extreme end of the price distribution. We show in Section 5.4, that estimating the logit model using different thresholds yields similar results. The exact choice of threshold does therefore not impact the modelling performance excessively, as long as there are sufficiently many occurrences to estimate the model while isolating extreme prices from the normal range. Negative extreme prices are defined as prices below zero, with the assumption that these occurrences are considered extreme and have been highly unusual in electricity spot price markets until recent years. These definitions of extreme prices yield 387 positive spikes and 177 negative prices – 1.00% and 0.46% of total observations.Base case is defined as prices in the normal range, meaning above zero and below €79.2.

Our analysis focuses on tail behaviour in electricity prices, where each probability model is based on a single trading period. An overview of spike occurrences in Figure 2 clearly shows that most positive spikes occur when demand is high, between 17:00 – 19:00. Although demand is high during midday, there are far fewer positive spikes; this might be due to the significantly higher solar production during those trading periods. Negative prices exhibit a slightly less predictable behaviour, and are observed in most of the trading periods with the exception of the afternoon peak demand period. Negative prices are clearly most common during the night, and occur when wind production is high and demand low compared to average values of each trading period (Tables 14 and 13).Negative prices occurring during midday are, on average, much closer to zero than observations during night, implying that prices during the night are more likely to be extreme. Negative daytime prices seem to occur while the photovoltaic production forecast is very high, thus leading to oversupply in the system. It is also noteworthy that demand forecast is lower than the overall mean of 54 852 MW for

Table 4: Overview of extreme price occurrences year 2010-14 (values for 2014 are linearly extrapolated based on data from first 5 months).

| | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|
| Positive spikes | 91 | 65 | 131 | 93 | 17 |
| Negative spikes | 12 | 15 | 56 | 64 | 72 |

entire sample when negative prices occur in daytime. Compared with Table 14, demand is also lower than the mean of each trading period when negative prices occur, indicating the supply side is not adjusting according to demand, and prices can become extremely low.

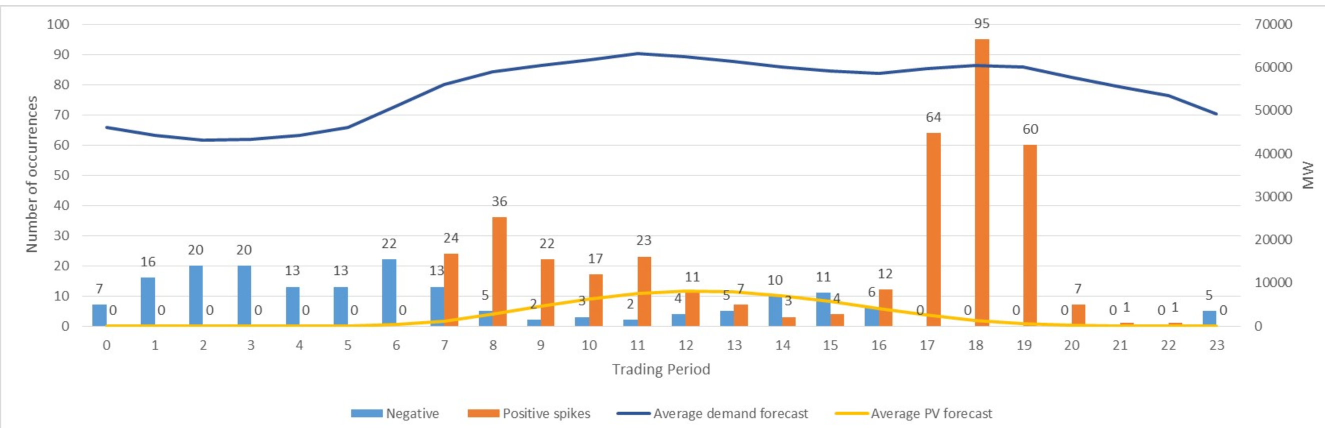



Figure 2: Extreme price occurrences and average demand forecast and PV forecast per trading period.

The spikes are to some extent seasonally distributed, and occur primarily in winter. This might be related to electricity prices in winter being higher in general, combined with higher demand. As shown in Table 4, negative prices are becoming more frequent towards the end of the sample. If 2014 values are extrapolated for the entire year, the number of negative prices would increase compared to all previous years. Positive spikes are, however, seemingly becoming less frequent, with peak number of occurrences in 2012 of 131. The price dampening may be explained by several factors; it may be a direct consequence of the increased share of renewable energy sources in the energy input mix. It is also possible the market participants have improved the ability to manage their positions in the market, thus avoiding price spikes. Another possible explanation is improved transfer capacity, reducing congestion in the transmission system and thus lowering the prices.

A block is defined as a consecutive sequence of prices above (positive) or below (negative) the threshold (Eichler et al. (2013)). Blocks can occur in consecutive trading periods within a day (intra-daily blocks), or from day to day for the same trading period (daily blocks). Positive spikes and negative prices occur in blocks of varying durations, as shown in Table 5. Adjusting the occurrence of negative prices to the same level as positive spikes (originally 387 positive spikes and 177 negative prices), the intra-daily blocks are approximately equally common. Although a large number of spike occurrences limit themselves to a one-hour duration, spikes have a tendency to be followed by further spikes the next trading period(s), implying similar drivers across different trading periods. Analysis of the data confirms daily blocks of extreme prices are common, particularly for positive spikes. This implies the lagged price is likely to be a significant explanatory variable for predicting the positive spike probability. Although negative prices also occur in daily blocks, the tendency is much weaker than for positive spikes. The implication is that lagged prices are not good predictors of negative price occurrences. Positive spikes and negative prices tend to cluster, but the phenomenon is particularly evident for positive spikes. As these periods are likely to have volatile prices, the estimated volatility variable will likely capture this effect and thus contribute to predicting extreme price occurrences. The clustering observations are

Table 5: Overview of duration of extreme prices and number of consecutive days of negative prices/positive spikes.

| Intra-daily blocks | | | | | |
|---|---|---|---|---|---|
| Duration [hours per block] | Negative prices | Positive spikes | Duration [hours prt block] | Negative prices | Positive spikes |
| 1 | 23 | 101 | 9 | 1 | 0 |
| 2 | 12 | 57 | 10 | 2 | 0 |
| 3 | 8 | 15 | 11 | 0 | 0 |
| 4 | 7 | 6 | 12 | 0 | 0 |
| 5 | 4 | 8 | 13 | 0 | 0 |
| 6 | 1 | 2 | 14 | 0 | 1 |
| 7 | 1 | 3 | 15 | 0 | 0 |
| 8 | 2 | 0 | 16 | 0 | 1 |

| Daily blocks (negative) | | | |
|---|---|---|---|
| Duration (2) | Duration (3) | Duration (4) | Duration (5) |
| 12 | 8 | 7 | 4 |

| Daily blocks (positive) | | | |
|---|---|---|---|
| Duration (2) | Duration (3) | Duration (4) | Duration (5) |
| 74 | 36 | 22 | 0 |

supported by inelasticity of demand and high correlation between spot price and lagged prices, as the demand level will not immediately adjust to higher prices.

The increase of negative prices and decrease in positive spikes is observed simultaneously as the share of intermittent renewable energy is increasing in the energy input mix. The descriptive statistics of the spot prices in the base case, positive spikes, and negative prices are shown in Table 6. Average value of negative prices and positive spikes are not close to the threshold values, implying that when extreme prices occur the values can become extremely low/high. The high, positive excess kurtosis further supports that prices can become extremely low/high. The negative skew of the base case and negative prices indicates prices below mean are more likely to occur than above mean, meaning there is a higher probability of observing prices in the lower tail of the price distribution. The positive skew of the positive spikes indicates a higher probability of prices much higher than the mean. The standard deviation is quite high for the negative price sample subset, because of the large spread of few observed prices.

Table 6: Descriptive statistics of spot prices in base case, positive spike, and negative price.

| | Base case | Positive spike | Negative price |
|---|---|---|---|
| Mean | 42.76 | 93.87 | -28.77 |
| Standard deviation | 14.62 | 17.58 | 52.16 |
| Skewness | -0.19 | 2.57 | -2.37 |
| Excess kurtosis | -0.23 | 9.36 | 4.85 |

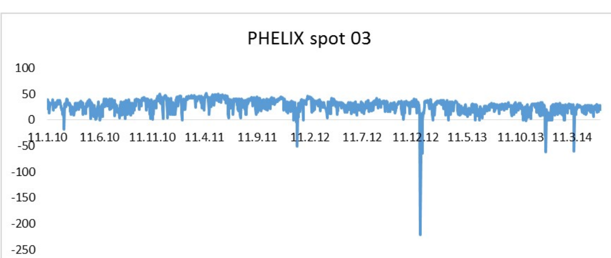
### 4.3 Specific Analysis of Trading Periods 3 and 18

We have chosen to make models for one trading period with a large amount of negative prices and another with a large amount of positive spikes. Hence trading period 3 (03:00-04:00) and 18 (18:00-19:00) are chosen for extreme price prediction. As seen in Table 13, trading period 3 has the highest number of negative prices (20). The average spot price for period 3 overall is €27.4/MWh, while the average negative spot price very low at -€38.8/MWh, as shown in Table 7. This implies that negative prices do occur well below zero; in contrast, the negative prices observed in trading periods during the day are barely below zero on average. Power plant availability forecasts greatly exceed the demand forecasts, likely due to the high level of low marginal cost wind production during this time, and is a likely driver behind the negative prices. Trading period 18 has the highest number of observations of positive spikes, in total 95 spikes. In this period, very high demand forecasts are coupled with low power plant availability forecasts when spikes occur. The lack of supply is exacerbated by the low average wind forecast and lack of sun during this period. Fuel prices of coal, gas and oil are higher during peak demand, and might be contributing to the low supply forecast. The low average value of photovoltaic forecast when positive spikes occur in trading period 18 suggests that higher photovoltaic production reduces the probability of extremely high prices. Hence, photovoltaic production forecasts will be included in the modelling. It is likely that photovoltaic production is more relevant in trading periods with less extremely high prices (typically between 06:00-18:00), as there is more sun during these hours. The lack of extremely high prices may be partially explained by the increase in supply from photovoltaic production. If demand does not increase accordingly due to higher availability, the risk of observing extremely high prices is reduced. On the contrary, high photovoltaic production is observed when negative prices occur during daytime. Since we have chosen to model the negative price prediction model outside this interval, photovoltaic will not be included in the modelling of negative prices.

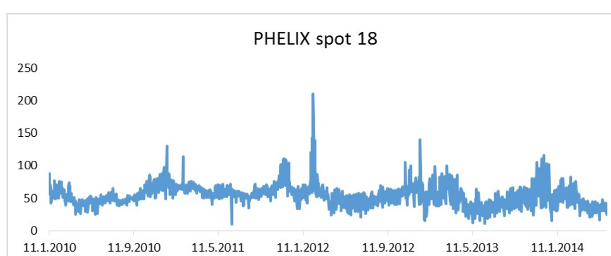
Table 7: Average values of spot price and explanatory variables when extreme prices occur in trading period 3 and 18.

| Average of: | Trading period 3 | Trading period 18 |
|---|---|---|
| Spot price | -38.8 | 96.9 |
| Lagged Price (1) | -13.5 | 83.1 |
| Lagged Price (7) | 20.5 | 79.6 |
| Demand forecast | 38373 | 70818 |
| Wind forecast | 17865 | 3295 |
| PV forecast | 0 | 21 |
| PPA forecast | 57276 | 59544 |
| Coal price | 67.7 | 74.0 |
| Gas price | 24.4 | 25.8 |
| Oil price | 75.8 | 74.2 |
| $CO_2$ price | 6.6 | 8.5 |
| Volatility | 27.1 | 12.5 |

Figure 3 shows how differently the spot prices behave in these two periods. The prices in trading period 3 appear more stable overall with the exception occasional negative prices. The descriptive statistics of all variables in period 3 and 18 are shown in Table 8. The excess kurtosis is extremely high for period 3, meaning the price distribution is very peaked with thick tails. The skewness is also negative, meaning the probability of very low prices is prevalent. Spot price in trading period 18 exhibits entirely different properties; the excess kurtosis is much lower, and the skew is positive due to the increased risk of very high prices. The range of spot prices in trading period 3 is much higher than for 18, but standard deviation is lower, as expected based on plot in Figure 3.




(a) PHELIX spot period 3 over time.




(b) PHELIX spot period 18 over time.

Figure 3: Spot prices over time of modeled trading periods.

Table 8: Descriptive statistics for all explanatory variables (except volatility) for trading period 3 and 18.

| Descriptive statistics period 3 | | | | | | |
|---|---|---|---|---|---|---|
| | Spot price | Lagged price (t-1) | Lagged price (t-7) | Demand forecast | Wind Forecast | |
| Mean | 27.35 | 27.26 | 27.33 | 43 259.42 | 5 149.61 | |
| Std.Dev | 14.46 | 14.53 | 14.53 | 5 165.90 | 4 206.47 | |
| Min | -221.94 | -221.94 | -221.94 | 29 201.00 | 286.16 | |
| Max | 51.08 | 51.08 | 51.08 | 59 149.00 | 24 525.45 | |
| Skew | -6.21 | -6.12 | -6.13 | 0.17 | 1.63 | |
| Excess kurtosis | 89.37 | 87.45 | 87.57 | -0.25 | 2.91 | |
| | PV Forecast | PPA Forecast | Coal price | Gas price | Oil price | $CO_2$ price |
| Mean | 0.11 | 55 128.20 | 71.28 | 23.25 | 74.22 | 9.33 |
| Std.Dev | 0.53 | 4 898.73 | 11.60 | 4.10 | 4.75 | 4.34 |
| Min | - | 40 015.50 | 51.49 | 11.15 | 61.03 | 2.48 |
| Max | 4.52 | 64 168.50 | 99.02 | 39.50 | 83.78 | 16.84 |
| Skew | 5.39 | -0.26 | 0.23 | -0.63 | -0.73 | 0.29 |
| Excess kurtosis | 29.87 | -0.78 | -1.08 | 0.96 | -0.31 | -1.47 |

| Descriptive statistics period 18 | | | | | | |
|---|---|---|---|---|---|---|
| | Spot price | Lagged price (t-1) | Lagged price (t-7) | Demand forecast | Wind Forecast | |
| Mean | 55.02 | 55.02 | 54.91 | 60 466.70 | 5 452.92 | |
| Std.Dev | 16.83 | 16.83 | 16.85 | 8 050.92 | 4 435.30 | |
| Min | 10.33 | 10.33 | 7.62 | 38 192.00 | 290.63 | |
| Max | 210.00 | 210.00 | 210.00 | 79 019.00 | 24 448.44 | |
| Skew | 1.52 | 1.52 | 1.51 | -0.21 | 1.46 | |
| Excess kurtosis | 8.34 | 8.34 | 8.35 | -0.60 | 2.12 | |
| | PV Forecast | PPA Forecast | Coal price | Gas price | Oil price | $CO_2$ price |
| Mean | 1342.10 | 55 128.25 | 71.26 | 23.25 | 74.21 | 9.33 |
| Std.Dev | 1682.02 | 4 897.20 | 11.61 | 4.10 | 4.76 | 4.34 |
| Min | - | 40 015.50 | 51.49 | 11.15 | 61.03 | 2.48 |
| Max | 7120.10 | 64 168.50 | 99.02 | 39.50 | 83.78 | 16.84 |
| Skew | 1.18 | -0.26 | 0.23 | -0.64 | -0.73 | 0.29 |
| Excess kurtosis | 0.36 | -0.77 | -1.08 | 0.95 | -0.31 | -1.47 |

## 4.4 Model design and evaluation

We will be using a standard logit model to investigate the relationship between the fundamental factors and spike occurrences for the same trading period in the following day.

To test the model prediction ability we will use in-sample tests on the modelled data sample, as well as testing the estimated models on data from other similar trading periods as an out-of-sample test.

The in-sample test assesses the ability of the models to truly and falsely predict extreme price occurrences. The probability cutoff value for prediction of an occurrence is commonly set to 50% in most software packages, where estimated probabilities at or above the cutoff are considered occurrences of extreme prices. Adjusting the threshold may make the models more useful, as it is not given that a 50% threshold is the most appropriate for electricity spot markets. Failure to forecast a spike is more detrimental to profits for many market participants – particularly retailers that purchase power at the unregulated market spot price, as argued by Christensen et al. (2012). Consequently, false predictions are preferred over failure to predict, and the cut-off may be set lower for models predicting positive spikes. The chosen cutoff value strongly depends on the purpose of the forecaster, and will be a trade-off between the costs of not detecting a spike versus falsely forecasting one (Eichler et al. (2013)). We argue for evaluating prediction results of the estimated models with different cutoffs, to get a more complete picture of the predictive power.

If the probability of extremely high prices is high, the bidders can attempt to increase profits by increasing their bids, or ramping up production in time to increase volume in those trading periods. For negative price occurrences the dynamics might be different, as there is a trade-off between continuing operations at a loss versus incurring ramping down/shut-down and ramping up/start-up costs. For these producers it might be more relevant knowing the probability of the magnitude of the low price, which a logit model is unable to do. The cut-off probability for evaluating the prediction ability for negative prices might therefore benefit from a higher cut-off than the model for positive spikes. What is defined as an occurrence in the input to the logit modeling is a positive spike for time period 18, and price below zero for time period 3. The threshold can be varied to test the model performance with different extreme price specifications, as long as there are sufficiently many occurrences to estimate a robust model.

# 5 Results and Discussion

## 5.1 Estimated logit models

Few explanatory variables seemed to drive negative prices in trading period 3, as highly insignificant variables include lagged prices, power plant availability, oil price, and $CO_2$ price. This is similar to the results in Kiesel and Paraschiv (2015). Photovoltaic is not included as the production during early morning hours is negligible. Price lag seven and volatility had higher significances, and results from testing models with and without these variables are presented in Table 9. The best performing model with the lowest Bayesian Information Criterion (BIC) value was the model including only demand and wind forecast, as well as coal and gas price – this is likely due to the BIC criterion heavily penalizing excess variables.

Table 9: Modeling trading period 3.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Model A | Model without lag 7 and volatility | | | | | | | |
| Model B | Model A including volatility | | | | t-ratio of volatility: 0.342 | | | |
| Model C | Model A including lag 7 | | | | t-ratio of lag 7: 0.161 | | | |
| Model D | Model A including lag 7 and volatility | | | | t-ratio of volatility: 0.105, lag 7: 0.057 | | | |
| | AIC | | BIC | | Average prediction success | | | |
| Model A | 87.52 | | 114.41 | | 49.23% | | | |
| Model B | 88.53 | | 120.80 | | 48.08% | | | |
| Model C | 87.26 | | 119.53 | | 48.85% | | | |
| Model D | 86.48 | | 124.13 | | 50.77% | | | |
| | True predictions | | | | False predictions | | | |
| Cutoff | Model A | Model B | Model C | Model D | Model A | Model B | Model C | Model D |
| 10 % | 16 | 17 | 16 | 0 | 29 | 27 | 25 | 0 |
| 20% | 14 | 13 | 13 | 0 | 16 | 17 | 16 | 0 |
| 25% | 12 | 13 | 12 | 0 | 12 | 11 | 14 | 0 |
| 30% | 12 | 12 | 12 | 0 | 10 | 7 | 9 | 0 |
| 35% | 12 | 12 | 11 | 0 | 5 | 4 | 7 | 0 |
| 40% | 11 | 11 | 11 | 0 | 3 | 3 | 3 | 0 |
| 45% | 11 | 11 | 10 | 0 | 2 | 3 | 2 | 0 |
| 50% | 10 | 10 | 10 | 0 | 2 | 3 | 1 | 0 |
| 55% | 9 | 8 | 9 | 0 | 2 | 2 | 1 | 0 |
| 60% | 7 | 6 | 8 | 0 | 2 | 2 | 1 | 0 |
| 70% | 6 | 5 | 6 | 0 | 2 | 1 | 1 | 0 |
| 80% | 4 | 4 | 5 | 0 | 1 | 1 | 0 | 0 |
| 90% | 4 | 3 | 4 | 0 | 0 | 1 | 0 | 0 |

The selection criteria and logit coefficients are shown in Table 10. The final model is chosen based on relatively low BIC/AIC, as well as the exclusion of insignificant and collinear variables.

## 5.2 Discussion on estimated coefficients

Coefficient estimates, as shown in Table 10, are reasonable for both models. In trading period 3, the negative coefficient of demand implies that high demand reduces the probability of a negative price, while high wind production increases the probability and thus has a positive coefficient. This is in accordance with previously observed and described market dynamics. Volatility is insignificant in the estimated model for trading period 3, suggesting that negative prices do not primarily occur in volatile periods. High fuel prices seem to reduce the probability of observing negative day-ahead prices. This might be explained by producers being more conservative when trading on the power exchange, and preferring shutting down or ramping down production rather than continuing operation at a higher loss due to high fuel prices. Consequently, cheap fuel drives producers to rather continue production at negative prices as the opportunity cost of doing so is lower than reducing output/shutting down.

Trading period 18 shows different dynamics from period 3; demand forecast coefficient is positive as high demand is seen in concordance with increased probability of extremely high prices. A higher level of wind production, on the other hand, reduces the probability of positive spikes. Spikes tend to occur in daily blocks, and it is thus expected that the lagged price coefficients are significant and positive. We note that the first price lag coefficient is approximately twice as high as for lag seven, meaning the price of the previous day is weighed more than the weekly pattern. Volatility also has a positive coefficient, which is reasonable as spikes tend to occur in highly volatile periods. High gas price is a natural driver behind extremely high prices, as gas is related to peak supply in the day-ahead market. All supply parameters have negative coefficients, as it is reasonable that high levels of supply will decrease the chance of positive price spikes.

Table 10: Estimated logit coefficients of model trading period 3 and 18.

| | | | |
|---|---|---|---|
| AIC | 87.52 | AIC | 302.95 |
| BIC | 114.41 | BIC | 351.37 |
| Logit coefficients (period 3) | | Logit coefficients (period 18) | |
| Constant | 102.67*** | Constant | -140.135*** |
| Lagged price (1) | | Lagged price (1) | 0.0334191*** |
| Lagged price (7) | | Lagged price (7) | 0.016962** |
| Demand forecast | -13.4864*** | Demand forecast | 21.7591*** |
| Wind forecast | 7.02236*** | Wind forecast | -1.58368*** |
| PV forecast | | PV forecast | -2.37763** |
| PPA Forecast | | PPA Forecast | -10.4955*** |
| Coal | -4.03566* | Coal | |
| Gas | -3.83327** | Gas | 6.14538*** |
| Vol | | Vol | 0.127951*** |

* 10% level of significance ** 5% level of significance *** 1% level of significance

## 5.3 Evaluation of model performance

When testing the model performance we use the definitions from (Zhao et al. (2007a)) and divide observations into four categories: true positive (TP), false positive (FP), true negative (TN) and false negative (FN)

A good prediction model should be highly accurate, thus the value of Equation 1 should be as high as possible. A high prediction accuracy is often obtained at the expense of the confidence level, as defined in Equation 2. A high prediction confidence level means the spike forecast is credible. An ideal forecasting model achieves both high accuracy and good confidence simultaneously, but a trade-off is normally required. In-sample model predictions results of both trading periods are shown in Figure 4.

$$Prediction\_accuracy = \frac{TP}{TP + FN} \tag{1}$$

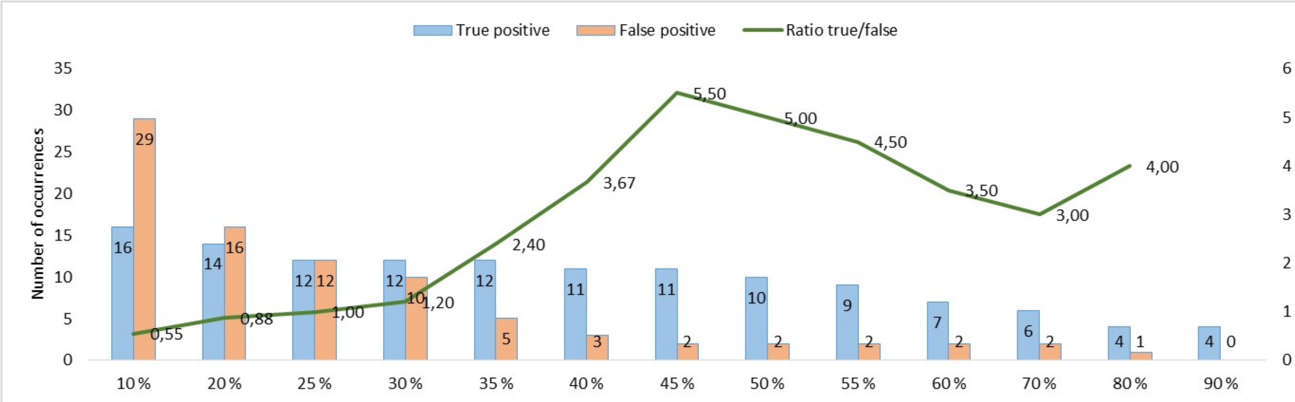
$$Prediction\_confidence = \frac{TP}{TP + FP} \tag{2}$$

### 5.3.1 Trading period 3 prediction results

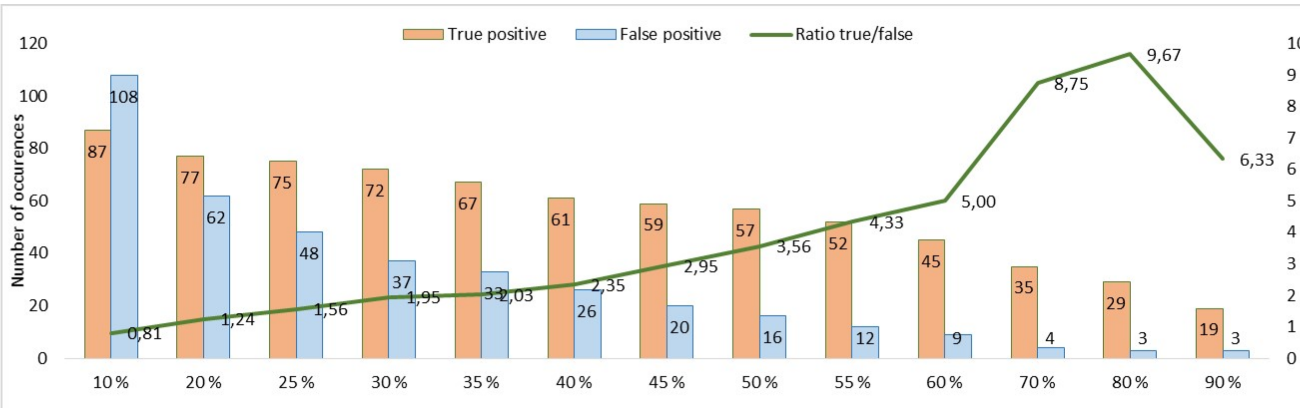
At the commonly applied cutoff of 50% probability, the model predicts 50% of the negative prices that occurred (true positives), with a confidence of 83%, as shown in Figure 4a. Depending on the cost of failing to predict a negative price, the cutoff can be lowered at the expense of more false positive predictions. At the lowest tested cutoff of 10%, the estimated model predicts 16 (80%) of true positive occurrences, in addition to 29 false positives at a confidence of 36%. At higher cutoffs the model is close to 100% accurate. False negatives exhibit a combination of low wind forecasts compared to the forecasts observed with true positives, and high demand relative to the true positive. False positives have common characteristics of demand well below the average of trading period 3 (Table 8), combined with high wind production relative to the true negatives. Fuel prices tend to be low when false positives are predicted, which partially explains the incorrect model predictions. Not surprisingly, true negatives with the lowest probabilities tend to have highly different characteristics compared to true positives: average/high demand, and low/average wind production forecast. The data points with probabilities larger than zero but below lowest cutoff of 10% tend to have either lower than average demand, or higher than average wind production. The analysis implies that negative prices that exhibit typical features for the modelled trading period are easily captured, but that it is more challenging predicting the negative price occurrences that exhibit uncommon characteristics.

### 5.3.2 Trading period 18 prediction results

At the standard cutoff value of 50%, the model is 60% accurate, and predicts 57 true positives as well as 16 false positives, as shown in Figure 4b. Higher cutoffs reduce the number of false positives relative to true positives as cutoff increases, implying that higher cutoffs make the estimated conditional probability more trustworthy in terms of avoiding false positive spike predictions. The chosen cutoff depends on user preference; if the opportunity cost of a false spike is significantly lower than for a missed true spike, a lower cutoff is beneficial. With a cutoff of 10%, the model predicts 87 out of 95 spikes (91.58%). The false negatives are predicted when the photovoltaic forecast is well above average of the spikes that occurred during period 18 (Table 7). Demand/wind forecasts are slightly lower/higher than the average of the spike occurrences, but still above/below average of the entire period (Table 8). The false negatives also tend to have much lower lagged prices that the average for spike occurrences. The lagged prices are, in some cases, lower than the average lagged prices of period 18. Volatility estimates are also quite low for false negatives, implying not all spikes occur in volatile periods. False positives, on the other hand, exhibit some typical characteristics of true spikes; the lagged prices are far above average, the demand is high and supply low, volatility is high, and fuel prices tend to be above average of that period. True negative observations exhibit low lagged prices, low demands, and in some cases high forecasted production from renewable sources. True negative observations with probabilities below and close to 10% typically have demand forecasts well above average of the trading period, as well as quite low production from renewable sources. Interestingly, the gas price was above average, implying that peak power plants were not cheaper to run during these periods. The logit model may struggle to capture the first spike, as the lagged price is not necessarily high. Conversely, it may also falsely predict spikes following a daily blocks of spikes in highly volatile periods, and is unable adjust immediately to a lower price period that follows. However, the model easily captures those occurrences that exhibit typical features, and thus identifies the majority of occurrences.




(a) Prediction evaluation trading period 3.




(b) Prediction evaluation trading period 18.

Figure 4: Number of true/false positive predictions for a variety of cutoff values, and prediction accuracy.

#### 5.3.3 Overall prediction results and discussion

The dynamics of the prediction models are very different. Positive spikes behave largely as expected, and the model predicts a large share of true positives as the cutoff increases. The model results confirm that positive spikes are mainly dependent on demand and supply, as well as gas price and volatility, and tend to occur if demand is high/supply is low during volatile periods. The negative price behaviour is less predictable, but clearly strongly dependent on two variables: demand and wind production forecasts. Fuel prices also have an impact, but the demand/supply parameters are much more decisive. The negative price prediction model is less successful, and at most predicts 80% of negative prices compared to 92% of positive spikes. As noted, a higher cutoff increases the prediction confidence at the expense of a lower prediction accuracy for both models. The logit model thus mainly captures extreme prices that exhibit the typical characteristics for either positive spikes or negative prices, and is less successful in predicting extreme prices with uncommon characteristics.

#### 5.3.4 Sensitivity of probabilities to changes in fundamental variables

Results from sensitivity analysis of both models are shown in Table 11 (trading period 3) and 12 (trading period 18). The probability change row represents the change in the average probability of the data set for each trading period, as the input variables are incremented. Negative changes reduce the accuracy of the model, while positive changes increase it - at the expense of reduced prediction confidence. For example, if demand is increased by 2000 MW in trading period 18 the overall probability of spike predictions in the sample increases by 1.82%. Furthermore, the model accuracy increases from 71% (initial case) to 82%, while confidence decreases from 67% to 53%. The sensitivity analysis of trading period 3 studies the effect of varying the forecasted demand and wind production, where the increments are varied from zero (initial case) to higher/lower values - the negative 286 MW limit on the wind forecast increment is to avoid a negative production. The overall probability of predicting a negative price using the estimated model changes as expected: the change is negative when demand increases or wind production decreases, implying a higher demand or lower wind production reduces the probability of observing negative prices. Oppositely, the probability change is positive if demand decrease or wind production increases. The number of true and false positive predictions with a cutoff of 50% changes with the increments, and the change is particularly observable when the wind forecast is strongly increased; the accuracy decreases from 83% to as low as 22% when wind is increased by 7500 MW. We further tested incrementing the coal and gas prices; the overall probability of predicting a negative price decreased when fuel prices increase, as expected. The model for trading period 3 is mainly sensitive to changes in wind production forecast.

Table 11: Sensitivity of probability and prediction accuracy due to changes in demand or wind forecast (trading period 3).

| Demand forecast changes | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Initial case | 1000 | 3000 | 5000 | -1000 | -3000 | -5000 |
| Probability change | 0.00% | -0.21% | -0.55% | -0.79% | 0.25% | 0.90% | 1.81% |
| True positive | 10 | 8 | 5 | 4 | 11 | 13 | 15 |
| False negative | 10 | 12 | 15 | 16 | 9 | 7 | 5 |
| False positive | 2 | 2 | 1 | 0 | 2 | 12 | 20 |
| Accuracy | 0.50 | 0.40 | 0.25 | 0.20 | 0.55 | 0.65 | 0.75 |
| Confidence | 0.83 | 0.80 | 0.83 | 1.00 | 0.85 | 0.52 | 0.43 |
| Wind forecast changes | | | | | | | |
| | Initial case | 500 | 1500 | 2500 | 4000 | 7500 | -286 |
| Probability change | 0.00% | 0.17% | 0.59% | 1.12% | 2.23% | 7.14% | -0.09% |
| True positive | 10 | 10 | 11 | 13 | 15 | 19 | 10 |
| False negative | 10 | 10 | 9 | 7 | 5 | 1 | 10 |
| False positive | 2 | 2 | 6 | 13 | 19 | 68 | 2 |
| Accuracy | 0.50 | 0.50 | 0.55 | 0.65 | 0.75 | 0.95 | 0.50 |
| Confidence | 0.83 | 0.83 | 0.65 | 0.50 | 0.44 | 0.22 | 0.83 |

Trading period 18 sensitivity analysis encompasses varying the first lagged price, demand, wind, and photovoltaic forecast. The variables are chosen because they are the most influential in the model, and due to the focus on renewable energy source effects on extreme prices. Incrementing the lagged price did not strongly influence the spike probability for increments up to 20, implying that rather extreme changes are needed to significantly increase the spike probability. Increasing the wind forecast reduced the accuracy of the model, however, the marginal reduction was clearly decreasing with increasing increment. Changing the demand had a large impact on the estimated probability; particularly positive increments had a strong effect, as prediction accuracy increased and confidence decreased. It is noteworthy that the change in probability was close to linear as the positive increment increased (for increments up to 5000 MW). Gradually increasing the photovoltaic forecast had a large impact on the spike predictions: the accuracy was reduced, as even small positive increments strongly reduced the probability of predicting spikes. Consequently, the model is quite sensitive to changes in input that are close to the average value of each case of spike/no spike, except in the case of demand where the magnitude of change increased approximately linearly even for high, positive increments.

Demand is much more influential in the trading period 18 compared to period 3, and the effect of incremental changes also differs: an increase in demand lowers accuracy and increases confidence for trading period 3, while in trading period 18 a higher demand increases accuracy and reduces confidence - as expected. Wind is also, somewhat more surprising, more influential in the model for trading period 18 when considering proba-

Table 12: Sensitivity of probability and prediction accuracy due to changes in fundamental variables (trading period 18).

| | Lagged price 1 changes | | | | | Demand changes | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Initial case | 10 | 20 | -10 | -20 | 2000 | 5000 | -2000 | -5000 |
| Probability change | 0.00% | 0.93% | 1.98% | -1.57% | -0.43% | 1.82% | 5.19% | -1.49% | -3.15% |
| True positive | 57 | 60 | 67 | 36 | 55 | 67 | 78 | 37 | 26 |
| False negative | 38 | 35 | 28 | 59 | 40 | 28 | 17 | 58 | 69 |
| False positive | 16 | 22 | 33 | 7 | 12 | 33 | 68 | 7 | 3 |
| Accuracy | 0.60 | 0.63 | 0.71 | 0.38 | 0.58 | 0.71 | 0.82 | 0.39 | 0.27 |
| Confidence | 0.78 | 0.75 | 0.73 | 0.82 | 0.83 | 0.67 | 0.53 | 0.84 | 0.90 |
| | Wind forecast changes | | | | | PV forecast changes | | | |
| | Initial case | 1000 | 2000 | 3000 | 4000 | 500 | 1000 | 1500 | 2000 |
| Probability change | 0.00% | -1.23% | -1.99% | -2.51% | -2.91% | -2.53% | -4.06% | -4.93% | -5.39% |
| True positive | 57 | 39 | 35 | 30 | 26 | 30 | 18 | 8 | 6 |
| False negative | 38 | 56 | 60 | 65 | 69 | 65 | 77 | 87 | 89 |
| False positive | 16 | 7 | 3 | 3 | 2 | 4 | 1 | 0 | 0 |
| Accuracy | 0.60 | 0.41 | 0.37 | 0.32 | 0.27 | 0.32 | 0.19 | 0.08 | 0.06 |
| Confidence | 0.78 | 0.73 | 0.67 | 0.84 | 0.82 | 0.88 | 0.95 | 1.00 | 1.00 |

bility percentage changes. This may be due to the lower average wind production during this period, causing the increments to have a larger impact. It may also imply the model for positive spikes is more sensitive, which may be explained by the larger number of variables or more extreme price occurrences as basis for modeling (Figure 2). The results from the sensitivity analysis yield expected results, and further confirms the main drivers behind extremely high and low prices.

#### 5.3.5 Out-of-sample test

A comparison of trading period 3 with period 2 and 4 shows that these periods exhibit many similarities. As seen in Table 13, there is a similar number of negative price occurrences; 20, 20, and 14 observations for period 2, 3, and 4 respectively. The average negative spot prices were quite low in all cases, although the absolute magnitude was slightly higher for period 4. The demand forecasts and wind forecasts average values are similar between the three periods, implying that the price formation dynamics are similar. For the entire set of values observed in these periods, the characteristics are even more similar; the average spot price, demand forecast and average wind forecast are essentially the same, and photovoltaic forecast is zero for these night-time periods. Prices of fuel and $CO_2$ are given on a per-day basis, as noted in Table 2, and will thus have the same impact in the modeling of these periods. Trading period 18 exhibits much of the same characteristics as period 17 and 19, as seen in Table 14. Table 15 shows that the number of spikes is higher for trading period 18 (95 observations), relative to 64 and 60 respectively for period 17 and 19. The average spot price and average demand forecast for spike occurrences is slightly lower for period 19. In addition, the average photovoltaic forecast is very low for trading period 19, and hardly has an impact on extremely high prices. Photovoltaic forecasts are on average higher for period 17, as there naturally is more sunlight during this period, and this constitutes the largest difference compared with period 18. However, when spikes occur the difference is insignificant.

The general price formation dynamics for trading period 2, 3 and 4 are quite similar, and the extreme prices appear to have similar drivers. For trading period 18, the implication is that period 17 is very similar and likely to behave similarly in the tails. Although each hourly time series is unique, there are strong similarities in adjacent trading periods. Because of the lack of sufficiently many normal range and extreme price observations in each trading period, conducting a traditional out-of-sample test for the modelled trading periods was challenging. We therefore use data from trading period 2 and 4 to test the model for predicting negative prices, and trading period 17 data to test the model for predicting positive spikes.

Table 13: Overview values of average price, demand and supply for when negative prices occurred.

| Trading period | Negative prices | Avg. spot price | Avg. demand forecast | Avg. wind forecast | Avg. PV forecast | Avg. PPA forecast |
|---|---|---|---|---|---|---|
| 0 | 7 | -35.5 | 41518 | 21308 | 0 | 59454 |
| 1 | 16 | -30.5 | 39368 | 18390 | 0 | 57273 |
| 2 | 20 | -34.8 | 38783 | 18300 | 0 | 57776 |
| 3 | 20 | -38.8 | 38373 | 17865 | 0 | 57276 |
| 4 | 13 | -46.6 | 37691 | 17509 | 14 | 56639 |
| 5 | 13 | -38.7 | 37480 | 17532 | 75 | 56511 |
| 6 | 22 | -30.4 | 36740 | 15233 | 162 | 56389 |
| 7 | 13 | -22.2 | 38937 | 16443 | 364 | 57116 |
| 8 | 5 | -1.4 | 38378 | 13203 | 2415 | 57969 |
| 9 | 2 | -1.4 | 45295 | 15663 | 4114 | 55523 |
| 10 | 3 | -1.9 | 49795 | 17870 | 11073 | 54677 |
| 11 | 2 | -4.2 | 55233 | 22820 | 9355 | 55375 |
| 12 | 4 | -2.7 | 51022 | 17254 | 12851 | 55677 |
| 13 | 5 | -19.8 | 49004 | 15800 | 10274 | 54905 |
| 14 | 10 | -24.6 | 46599 | 12353 | 13225 | 53185 |
| 15 | 11 | -17.6 | 46544 | 13612 | 10585 | 53150 |
| 16 | 6 | -15.3 | 44966 | 13651 | 10072 | 51415 |
| 23 | 5 | -30.4 | 44868 | 20668 | 0 | 59150 |

Results from the tests are shown in Tables 16 and 17. Intra-daily blocks are observed in the data, as discussed in section 4.2, which means many of the extreme price occurrences within a single day have very similar characteristics. Results from running the estimated models on the adjacent data is likely to be affected, and will likely contribute to prediction accuracy and confidence being similar to the original in-sample tests. However, we argue that using this data will provide an indication of overall model performance and can be a partial substitute of a true out-of-sample test.

The model estimated based on data from trading period 3 performed well on predicting spikes on the data set of period 2 and 4. The levels of prediction accuracy and confidence were, as expected, very similar to the original data from period 3, but with slightly better performance with the new data. Trading period 4 accuracy was consistently the highest, at the expense of a somewhat lower confidence. The prediction accuracies of trading period 2 and 3 were quite similar, but period 2 in general outperformed period 3 with exceptions for cutoffs close to 50%. However, it is important to note that there are relatively few negative price observations in each trading period, and a single true/false positive has a visible impact. The test results illustrate that the logit model is able to recognize which states of the fundamental variables are most likely to lead to negative prices. Even considering that many of the negative prices do occur in intra-daily blocks, the tests strongly suggest the logit model is well-suited to identify negative prices that exhibit the typical characteristics. The estimated logit model will likely perform well while the share of renewable energy and transfer capacities are approximately at current levels. As discussed in section 4.2, negative prices have become more frequent in latter years, an increase that is happening simultaneously as the share of wind power in the energy input mix increases (section 3). Until significant changes to transmission capacity are implemented, either within Germany or through improved connections to other markets, the estimated logit model will continue successfully predicting negative prices while the drivers are high wind production and low demand.

The test on the model estimated for trading period 18 performed quite well with data from trading period 17, in terms of prediction accuracy and confidence, implying the model captures the main drivers behind positive price spikes. The prediction accuracy was higher than for the original data, at the expense of lower confidence levels. At high cutoffs, the difference in accuracy is even clearer, combined with a slightly higher confidence for the new data at the highest cutoff of 90%. The results suggest that, as for the model for trading period 3, the model would likely have performed well on an actual out-of-sample test while market conditions are relatively similar. A difference from the data for negative prices is that the number of positive spike observations

Table 14: Overview values of average price, demand and supply for all trading periods.

| Trading period | Avg. spot price | Avg. demand forecast | Avg. wind forecast | Avg. PV forecast | Avg. PPA forecast |
|---|---|---|---|---|---|
| 0 | 35.1 | 46027 | 5241 | 0 | 55146 |
| 1 | 31.8 | 44153 | 5205 | 0 | 55146 |
| 2 | 29.2 | 43213 | 5173 | 0 | 55146 |
| 3 | 27.4 | 43284 | 5147 | 0 | 55146 |
| 4 | 27.8 | 44159 | 5134 | 5 | 55148 |
| 5 | 31.1 | 46138 | 5109 | 59 | 55145 |
| 6 | 38.0 | 51135 | 5098 | 318 | 55146 |
| 7 | 46.9 | 56028 | 5092 | 1150 | 55146 |
| 8 | 50.4 | 58922 | 5086 | 2696 | 55146 |
| 9 | 50.6 | 60365 | 5121 | 4616 | 55146 |
| 10 | 50.0 | 61822 | 5190 | 6365 | 55146 |
| 11 | 50.3 | 63153 | 5313 | 7593 | 55146 |
| 12 | 47.9 | 62527 | 5445 | 8097 | 55146 |
| 13 | 45.3 | 61365 | 5555 | 7898 | 55146 |
| 14 | 43.4 | 60065 | 5613 | 7079 | 55146 |
| 15 | 42.8 | 59181 | 5611 | 5746 | 55146 |
| 16 | 43.7 | 58638 | 5578 | 4144 | 55146 |
| 17 | 49.7 | 59707 | 5529 | 2600 | 55146 |
| 18 | 55.0 | 60493 | 5453 | 1336 | 55146 |
| 19 | 55.0 | 60074 | 5362 | 511 | 55146 |
| 20 | 50.4 | 57723 | 5292 | 103 | 55146 |
| 21 | 45.8 | 55524 | 5269 | 8 | 55146 |
| 22 | 44.7 | 53432 | 5260 | 0 | 55146 |
| 23 | 38.3 | 49301 | 5249 | 0 | 55146 |

decreases over time, based on the latest trend in the data (section 4.2).

As previously mentioned, the share of renewable energy source input will probably increase in coming years, while authorities focus on strengthening the grid. The effects of these changes are not yet known, but a likely consequence is that the dynamics in the day-ahead electricity market change. Consequently, extremely high prices may be more or less frequent, and some drivers may become increasingly important. If, for example, storage opportunities are cost-effectively introduced into the market, the intermittent nature of wind and photovoltaic power plants will have less impact. In this case high demand forecasts may be a more influential driver behind positive price spikes. For negative prices, this may imply the frequency would decrease, as the excess energy produced to some degree could be stored when the supply excessively exceeds demand.

### 5.4 Effect of renewable energy sources as explanatory variables

When removing wind as explanatory variable from the model for trading period 3, it performs significantly worse. The results of the modeling are displayed in Table 20, and show that the model performed worse on all criteria, including AIC, BIC, and prediction accuracy/confidence. The finding confirms that high wind production is the main driver behind negative prices during nightly trading periods. Coefficients of variables are lower in the model without wind forecasts included, as shown in Table 19. The model estimation is unable to make a sufficiently powerful logit model for predicting negative prices if wind forecasts are not included.

Table 15: Overview values of average price, demand and supply for when positive spikes occurred.

| Trading period | Positive spikes | Avg. spot price | Avg. demand forecast | Avg. wind forecast | Avg. PV forecast | Avg. PPA forecast |
|---|---|---|---|---|---|---|
| 7 | 24 | 93.7 | 66414 | 2460 | 361 | 58962 |
| 8 | 36 | 94.8 | 67121 | 2425 | 1757 | 57669 |
| 9 | 22 | 93.4 | 69392 | 2805 | 2946 | 57903 |
| 10 | 17 | 93.0 | 71744 | 2936 | 3084 | 59355 |
| 11 | 23 | 89.8 | 70982 | 2673 | 5183 | 56639 |
| 12 | 11 | 89.7 | 71764 | 2249 | 5138 | 58231 |
| 13 | 7 | 88.3 | 71468 | 2173 | 4970 | 59083 |
| 14 | 3 | 94.5 | 70783 | 2077 | 3101 | 60103 |
| 15 | 4 | 89.1 | 71528 | 2615 | 1577 | 60464 |
| 16 | 12 | 88.3 | 70637 | 2166 | 406 | 60132 |
| 17 | 64 | 95.4 | 72123 | 3378 | 57 | 59468 |
| 18 | 95 | 96.9 | 70818 | 3295 | 21 | 59544 |
| 19 | 60 | 91.8 | 68067 | 3409 | 16 | 58751 |
| 20 | 7 | 95.2 | 67368 | 4295 | 0 | 58521 |
| 21 | 1 | 94.9 | 67678 | 2032 | 0 | 60571 |
| 22 | 1 | 79.7 | 66366 | 1892 | 0 | 60571 |

Table 18: Estimated coefficients with/without wind/photovoltaic forecast for trading period 18.

| Coefficient | Model A | Model B | Model C | Model D |
|---|---|---|---|---|
| Constant | -140.135*** | -140.78*** | -252.715*** | -205.421*** |
| Lagged price (1) | 0.0334191*** | 0.0448175*** | 0.0403652*** | 0.0491872*** |
| Lagged price (7) | 0.016962** | 0.0149543** | 0.0214126*** | 0.01178986*** |
| Demand forecast | 21.7591*** | 17.315*** | 25.2775*** | 19.3292*** |
| Wind forecast | -1.58368*** | 0 | -1.49916 | 0 |
| PV forecast | -2.37763** | -1.63813* | 0 | 0 |
| PPA Forecast | -10.4955*** | -6.91747** | -3.95059 | -3.1017 |
| Gas | 6.14538*** | 5.30204*** | 6.01466*** | 5.14221*** |
| Volatility | 0.127951*** | 0.108976*** | 0.136867*** | 0.118718*** |

* for 10% level of significance ** for 5% level of significance
*** for 1% level of significance

Table 19: Estimated coefficients with/without wind forecast for trading period 3.

| Coefficient | Model A (with wind) | Model B (without wind) |
|---|---|---|
| Constant | 102.67*** | 103.734*** |
| Demand forecast | -13.4864*** | -9.11115*** |
| Wind forecast | 7.02236*** | |
| Coal price | -4.03566* | -2.76219* |
| Gas price | -3.83327** | 0.0611909 |

* for 10% level of significance ** for 5% level of significance
*** for 1% level of significance

For trading period 18, the prediction results of models estimated without the renewable energy sources are significantly less accurate than the basic model. As seen in Table 21, Model A clearly has the best prediction accuracy for price spikes, and in addition has relatively few occurrences of false positives. Model B predicts the

Table 16: Prediction results from using estimated model for predicting negative prices on data from trading period 2 and 4.

| | Data 2 | | | | | Data 4 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Cutoff | TP | FN | FP | Accuracy | Confidence | TP | FN | FP | Accuracy | Confidence |
| 10% | 19 | 1 | 30 | 0.95 | 0.39 | 10 | 3 | 27 | 0.77 | 0.27 |
| 20% | 17 | 3 | 13 | 0.85 | 0.57 | 10 | 3 | 15 | 0.77 | 0.40 |
| 25% | 14 | 6 | 13 | 0.70 | 0.52 | 10 | 3 | 11 | 0.77 | 0.48 |
| 30% | 13 | 7 | 9 | 0.65 | 0.59 | 10 | 3 | 9 | 0.77 | 0.53 |
| 35% | 12 | 8 | 7 | 0.60 | 0.63 | 9 | 4 | 6 | 0.69 | 0.60 |
| 40% | 11 | 9 | 5 | 0.55 | 0.69 | 9 | 4 | 4 | 0.69 | 0.69 |
| 45% | 10 | 10 | 2 | 0.50 | 0.83 | 8 | 5 | 4 | 0.62 | 0.67 |
| 50% | 9 | 11 | 2 | 0.45 | 0.82 | 8 | 5 | 4 | 0.62 | 0.67 |
| 55% | 9 | 11 | 1 | 0.45 | 0.90 | 7 | 6 | 3 | 0.54 | 0.70 |
| 60% | 8 | 12 | 1 | 0.40 | 0.89 | 6 | 7 | 3 | 0.46 | 0.67 |
| 70% | 6 | 14 | 1 | 0.30 | 0.86 | 5 | 8 | 3 | 0.38 | 0.63 |
| 80% | 4 | 16 | 1 | 0.20 | 0.80 | 4 | 9 | 1 | 0.31 | 0.80 |
| 90% | 4 | 16 | 0 | 0.20 | 1.00 | 2 | 11 | 0 | 0.15 | 1.00 |

Table 17: Prediction results from using estimated model for predicting spikes on data from trading period 17.

| Cutoff | TP | FN | FP | Prediction accuracy | Prediction confidence |
|---|---|---|---|---|---|
| 10% | 60 | 3 | 97 | 0.95 | 0.38 |
| 20% | 54 | 9 | 58 | 0.86 | 0.48 |
| 25% | 52 | 11 | 46 | 0.83 | 0.53 |
| 30% | 49 | 14 | 36 | 0.78 | 0.58 |
| 35% | 46 | 17 | 31 | 0.73 | 0.60 |
| 40% | 44 | 19 | 27 | 0.70 | 0.62 |
| 45% | 41 | 22 | 21 | 0.65 | 0.66 |
| 50% | 39 | 24 | 15 | 0.62 | 0.72 |
| 55% | 37 | 26 | 14 | 0.59 | 0.73 |
| 60% | 36 | 27 | 12 | 0.57 | 0.75 |
| 70% | 32 | 31 | 7 | 0.51 | 0.82 |
| 80% | 26 | 37 | 4 | 0.41 | 0.87 |
| 90% | 18 | 45 | 2 | 0.29 | 0.90 |

highest number of false positives, but performs better than model D on predicting true positives. We conclude that removing the renewable energy sources as explanatory variables makes the logit models less able to successfully predict spike occurrences. Coefficients of estimated models are relatively similar for all models, and are given in Table 18. The lagged price coefficients are somewhat higher for Model B and C, indicating first price lag is weighed more relative to the model excluding renewable supply forecast. This implies the price the previous day has a larger impact on predicted spike probability when renewable energy sources are not included. Consequently, estimated models rely more on the lagged price, and may be less able to predict the first spike in a potential series of spikes for trading period 18 over several days.

### 5.5 Robustness Analysis

As discussed in section 4, determining the spike threshold is important to be able to estimate a logit model. Table 22 shows how the number of spike occurrences changes for different thresholds. Estimating models using various spike thresholds yields the coefficients shown in Table 23. Lagged prices, demand forecast and supply

Table 20: Modelling results and predictions of basic model with/without wind for trading period 3.

| | | | | |
|---|---|---|---|---|
| Model A Model with wind included | | | | |
| Model B Model without wind included | | | | |
| | | AIC | BIC | |
| Model A | | 87.52 | 114.41 | |
| Model B | | 197.56 | 226.45 | |
| | True predictions | | False predictions | |
| Cutoff | Model A | Model B | Model A | Model B |
| 10% | 16 | 6 | 29 | 6 |
| 20% | 14 | 0 | 16 | 0 |
| 25% | 12 | 0 | 12 | 0 |
| 30% | 12 | 0 | 10 | 0 |
| 35% | 12 | 0 | 5 | 0 |
| 40% | 11 | 0 | 3 | 0 |
| 45% | 11 | 0 | 2 | 0 |
| 50% | 10 | 0 | 2 | 0 |
| 55% | 9 | 0 | 2 | 0 |
| 60% | 7 | 0 | 2 | 0 |
| 70% | 6 | 0 | 2 | 0 |
| 80% | 4 | 0 | 1 | 0 |
| 90% | 4 | 0 | 0 | 0 |

Table 21: Modelling results and predictions of basic model without wind and/or PV for trading period 18.

| | | | | | | | AIC | BIC |
|---|---|---|---|---|---|---|---|---|
| Model A | Final model for trading period 18 | | | | | | 302.9 | 351.37 |
| Model B | Model for trading period 18 without wind forecast | | | | | | 363.91 | 406.94 |
| Model C | Model for trading period 18 without PV forecast | | | | | | 314.30 | 357.33 |
| Model D | Model for trading period 18 without wind and PV forecast | | | | | | 369.04 | 406.70 |
| | True predictions | | | | False predictions | | | |
| Cutoff | Model A | Model B | Model C | Model D | Model A | Model B | Model C | Model D |
| 10% | 87 | 81 | 85 | 75 | 108 | 162 | 109 | 94 |
| 20% | 77 | 72 | 73 | 55 | 62 | 76 | 58 | 49 |
| 25% | 75 | 62 | 71 | 53 | 48 | 54 | 50 | 29 |
| 30% | 72 | 57 | 66 | 50 | 37 | 43 | 39 | 24 |
| 35% | 67 | 53 | 62 | 43 | 33 | 32 | 27 | 19 |
| 40% | 61 | 49 | 60 | 39 | 26 | 23 | 24 | 16 |
| 45% | 59 | 46 | 57 | 31 | 20 | 20 | 20 | 15 |
| 50% | 57 | 43 | 56 | 29 | 16 | 14 | 14 | 12 |
| 55% | 52 | 36 | 51 | 27 | 12 | 12 | 11 | 7 |
| 60% | 45 | 31 | 44 | 25 | 9 | 9 | 9 | 5 |
| 70% | 35 | 23 | 36 | 18 | 4 | 4 | 5 | 2 |
| 80% | 29 | 18 | 30 | 12 | 3 | 1 | 3 | 1 |
| 90% | 19 | 10 | 21 | 9 | 3 | 1 | 3 | 1 |

forecasts are the dominating parameters for all thresholds. Photovoltaic forecasts and seventh lagged price eventually become less significant as the threshold increases. Oil price significance also decreases as threshold is increased, while coal and $CO_2$ continue to be highly insignificant in all the estimated models. The seventh lag coefficient is approximately the same as the first lag coefficient if the threshold is €70, in contrast to all the other thresholds. The first price lag coefficient is larger in magnitude than the seventh lag coefficient as the threshold increases, and its impact thus dominates. This implies that the weekday-effect is not a main driver behind extremely high prices, but is more strongly related to moderate/very high prices.

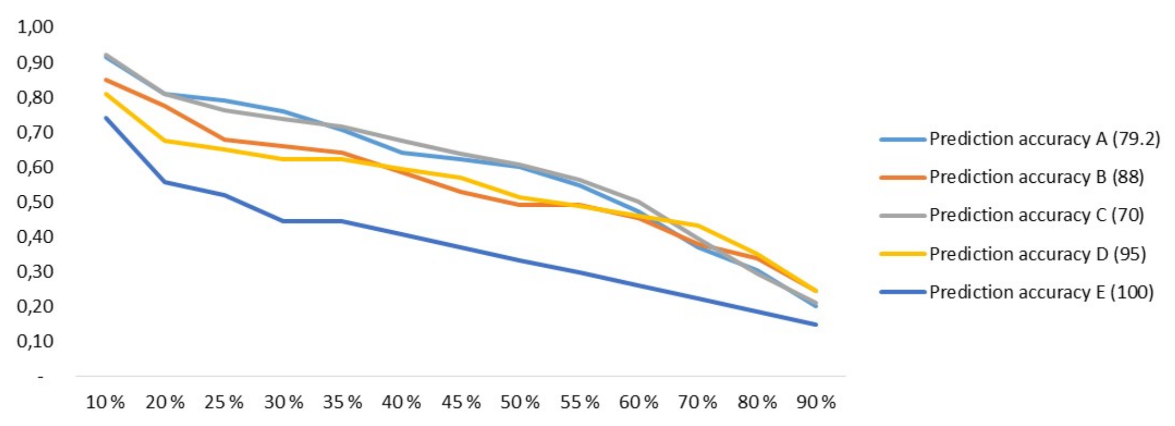
Table 22: Variety of spike threshold values and corresponding count of spikes.

| | Threshold | Spike count | Spike count period 18 |
|---|---|---|---|
| Model A: benchmark model | 79,2 | 387 | 95 |
| Model B: 0.5% of highest prices | 88,0 | 194 | 53 |
| Model C: fixed threshold | 70 | 1204 | 191 |
| Model D: fixed threshold | 95 | 124 | 37 |
| Model E: fixed threshold | 100 | 95 | 27 |

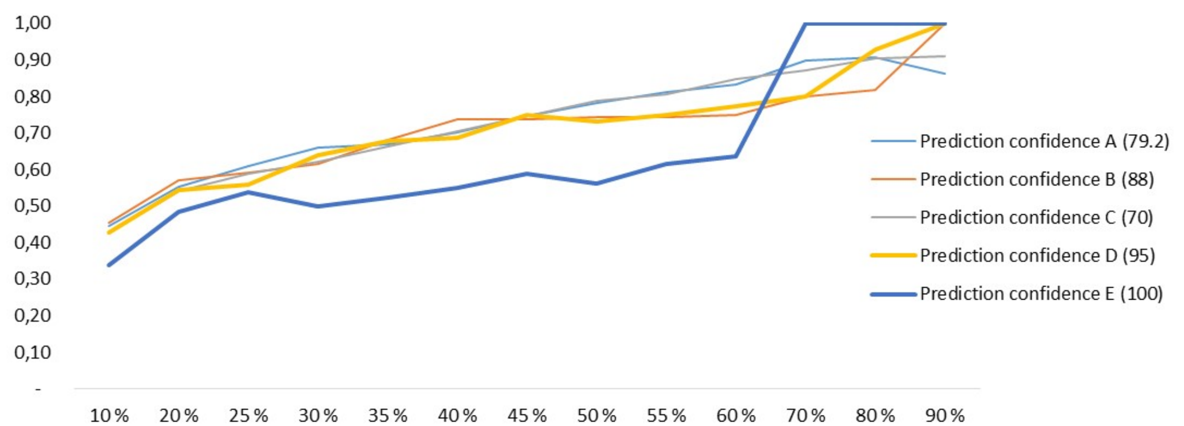
As the threshold is set higher, fewer explanatory variables are significant in the model, suggesting that very extreme price spikes have slightly different dynamics than slightly less extreme prices. Hence, how the modeller sets the spike threshold impacts the model; however, some variables continue to be important regardless of threshold. Demand and supply forecasts, first lagged price, and volatility are consistent drivers behind high prices, and are the main driving force behind extreme price occurrences. Spike prediction performance for various thresholds is shown in Figure 5. Models with higher thresholds consistently perform slightly worse on prediction accuracy, as well as the confidence of the predictions. This may be explained by fewer spike occurrences used to estimate the model, and thus less accurate predictions from the model. It could also imply that a larger share of the extreme price occurrences are less related to fundamental variables, and have a larger random component. The model with the lowest threshold performs quite well, which supports the theory that the model parameters are better if there is a relatively high number of occurrences in the sample.

Table 23: Estimated coefficients of models with varying threshold. All model parameters are significant at 5% confidence level.

| | Model A (79.2) | Model B (88.0) | Model C (70) | Model D (95) | Model E (100) |
|---|---|---|---|---|---|
| Constant | -140.135 | -234.516 | -99.928 | -228.891 | -219.605 |
| Lagged price (1) | 0.033 | 0.055 | 0.030 | 0.050 | 0.029 |
| Lagged price (7) | 0.017 | | 0.030 | | |
| Demand forecast | 21.759 | 29.415 | 15.775 | 29.905 | 28.247 |
| Wind forecast | -1.584 | -1.576 | -1.428 | -1.994 | -1.431 |
| PV forecast | -2.378 | | -1.004 | | |
| PPA Forecast | -10.496 | -10.163 | -4.298 | -11.774 | -10.662 |
| Gas | 6.145 | 7.057 | 4.913 | 10.081 | 8.310 |
| Oil | | | -8.780 | | |
| Vol | 0.128 | 0.166 | 0.083 | 0.139 | 0.098 |




(a) Prediction accuracy.




(b) Prediction confidence.

Figure 5: Trading period 18 prediction results for different thresholds.

# 6 Conclusion and Recommendations for Further Work

The estimated models are able to explain extreme prices quite well, as confirmed by the in-sample test in sections 5.3.1, and 5.3.2 and the out-of-sample test in section 5.3.5). The results suggest that extreme price occurrences have clear drivers, and that logit models are an appropriate tool for assessing the impacts of fundamental variables. Our analysis shows that probability models can identify and quantify these drivers, both for negative prices and positive spikes, and can thus be a powerful tool contributing to risk management. The benefit of using logit models is that they are easy to estimate and are flexible, as each modeller can customize thresholds to define extreme prices and change the probability cutoff to adjust the weighing of accuracy versus confidence. The model performance was quite good for all thresholds, with somewhat lower accuracy and confidence for very high thresholds (€100). Very high thresholds have the drawback of fewer data points for model estimation, and the choice of threshold should balance the need to filter extreme prices from normal range, while ensuring robust model specification. Thus, a challenge in estimating a robust model in this field is to obtain sufficiently many data points of extreme price occurrences. This complicates the testing procedure of the model, as out-of-sample tests require enough data points to be credible. In general, as for all probability models estimated on a data set, the model should and will have the highest predictive power in a market situation similar to the original data sample. Consequently, as the German day-ahead electricity market fundamentals change over time, as described in Chapter 3, new models should be estimated to ensure good performance.

As initial study of the data suggested, positive spikes and negative prices have very different dynamics and fundamental drivers. In addition, each trading period has unique price dynamics, especially when comparing day/night-time. Positive spikes are mainly observed when demand forecast levels are high and supply forecasts are low, as well as high lagged price the previous day. The estimated logit model confirms these dynamics, and in addition implies that the weekday effect measured through the seven-day lag is not a very significant driver behind extreme prices. Negative prices are, on the other hand, driven mainly by wind power combined with very low demand. As shown in section 5.4, model performance was poor when wind was not included as fundamental variable. The results imply that negative prices were also not driven by low lagged prices, as was suggested in the initial data study (section 4.2). The effect of renewable energy sources is clear in the study of extreme price behaviour. Positive spikes are less frequent if wind/photovoltaic production forecasts are high, as the spikes occur when production forecasts are low causing demand to exceed supply. The sensitivity analysis of parameters strongly suggests that increased renewable energy production forecasts lower the probabilities of observing spikes. Negative prices are strongly dependent on renewable energy, and are most common during night as a consequence of high levels of wind production and low demand. Daytime negative prices are seen when photovoltaic production forecasts are very high, although these observations are quite few. The negative prices are, on average, much lower during night than during day, implying the effect of renewable energy sources is strongest at night.

As a main conclusion, the extreme price behavior in a single trading period is captured well with a logit model. Identifying the main drivers behind extreme prices is highly useful in risk management for market participants. The effect of renewable energy sources is clear, and will continue to be important in this market. Intermittency of renewable energy sources is unlikely to decrease in near future, and it is important to use predictive models that account for these dynamics is important for participants in the electricity spot market. We expect the negative price observations to decrease with future improvements of the transmission gird, as balance between the power production in the north and power demand in the south is improved.

To further model the occurrences of spikes in the German EPEX market an obvious step is to model the likely magnitude of the predicted extreme prices. One suggested method is incorporating the binomial logit model with a quantile regression. The estimated logit probability may be used as an input to determine which quantile the price is most likely to be; a 90% logit probability may be used as an indicator that the forecasted price is in an upper quantile. Another option would be to include the logit probability as a fundamental variable in the quantile regression model, thus utilizing the probability of a spike to forecast the magnitude. An Extreme Value Theory (EVT) model may be used to more accurately model the tail distribution of prices. A combined approach, using a logit model for the probability of spike occurences combined with an EVT model on top for shaping the extreme tails can bring an additional contribution methodologically. For a more comprehensive study, the analysis may not be limited to two trading periods, but be estimated for different times of the day. Alternatives are closer analysis of daytime negative prices, as well as morning hours where both negative prices and positive spikes occur, e.g. trading period 7, 8, and 9. To further improve the estimated logit models accuracy and confidence, a more advanced volatility model, e.g., GARCH, may improve the model fitting.

Testing of the model may be done on different simulated fundamental variable scenarios and corresponding spot price. Demand is strongly seasonal and has clear weekly patterns, while the supply is highly dependent on fuel prices for conventional power plants, as well as weather forecasts. The logit model performance may be done on the simulated data. Probability of extreme price occurrences in interesting in many markets, not only the German EPEX, and the methodology may be employed on other markets. This could, for instance, be done on the Nord Pool or UK APX. Comparisons of extreme price dynamics in these markets may be conducted, to study if spikes are always driven by the same parameters. A possible approach may be studying the energy input mix and its impact on the extreme price occurrences and magnitude, as well as drivers.

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