The Liability Regime of Insurance Pools and Its Impact on Pricing

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Insurance Pools in Practice

According to Farny (2011) insurance pools are mutual organizations of several insurance companies having been founded for the purpose of insuring a special type of risk and appear either as co-insurance pools or reinsurance pools.

European Commission (2013) gives an overview about the pool landscape in Europe:
- The study figured out 51 pools in the EU; most of them (11) cover energy risk
- Among interviewed pools (44): 30% reinsurance pools, rest co-insurance or both
### Subject of Our Study

#### Previous Literature Explicitly Dealing with Pools

<table>
<thead>
<tr>
<th>Optimal/Fair Risk Sharing</th>
<th>Diversification Benefits</th>
<th>Legal/Organizational</th>
<th>State Pools/Public-Private Partnership</th>
</tr>
</thead>
</table>

#### Motivation

Our study is motivated from a legal point of view:
What happens if one or more pool insurers default on the policyholders’ claims?

#### Market Observation

**Regime of Several Liability**
A default of one insurer is not compensated by other insurers

**Regime of Joint Liability**
Policyholders have access to the insurers’ aggregated funds

#### Research Subject

Quantify the regimes’ effects on pricing as well as equity requirements and discuss risk-shifting problems in both regimes
A contingent claims model based on *Doherty and Garven (1986)*

**Model assumptions**

- Pool is composed of insurers $i = 1, \ldots, n$
- ICs organized as stock companies and equity holders seek profit maximization
- Pool holds no own balance sheet or funds, business is disclosed in the ICs’ balance sheets; separated investments
- ICs underwrite only pool business
- Complete market and no arbitrage opportunity $\rightarrow$ pricing under an unique risk-neutral measure
- Pricing is net of administrative costs, deductibles and any reinsurance
- Two-period-consideration: Premium and equity payments at time $t = 0$; claims payment and investment return at time $t = 1$
- Insurer defaults if liabilities at $t = 1$ exceed available assets
- $\alpha_i$ denotes the risk share of insurer $i$, $\beta_i$ the premium share
Our Model – Positions at Time $t = 0$

A contingent claims model based on *Doherty and Garven (1986)*

At time $t = 0$ insurance company $i$ has assets available amounting to

$$A_i^0 = E_i^0 + \beta_i P_0$$

Fair positions of policyholders and equity holders

In the context of risk-neutral pricing we presume fairness for policyholders and equity holders if their positions at time $t = 0$ equals the present values of the payoffs at time $t = 1$, i.e.

$$\left( P_0, E_0^1, \ldots, E_0^n \right) = \left( PV(P_1), PV(E_1^1), \ldots, PV(E_1^n) \right)$$

→ Net present value of zero for all stakeholders
→ Equilibrium: no wealth transfer between stakeholders
Our Model – Payoffs at Time $t = 1$

**A contingent claims model** based on *Doherty and Garven (1986)*

![Diagram](image)

At time $t = 1$ the policyholders receive an indemnification amounting to

$$P_i = C_i - D_i$$

$D_i$ is the pool’s shortfall. Its size depends on the liability regime at hand:

- **Several Liability**
  $$D_i = \sum_{i=1}^{n} \left[ \alpha_i C_{i} - A_{i}^i \right]^+$$

- **Joint Liability**
  $$D_i = \left[ C_i - \sum_{i=1}^{n} A_i^i \right]^+$$

The payoff of IC $i$ to its equity holders at time $t = 1$ is

$$E_{i} = \left[ A_i^i - \alpha_i C_{i} \right]^+$$

$$E_{i} = \left[ A_i^i - \alpha_i C_{i} - G_i^i \right]^+$$

We have formally defined $G_i^i$ to be in line with joint liability’s default mechanism in practice.
Numerical Example – Setting (1/2)

- Pool claims are modelled as jump-diffusion process (GBM & Poisson Process, Merton (1976))
- Assets returns are modelled as ordinary GBM
- The claims’ face value at time $t = 0$ is fixed at $PV(C_1) = 100(A1)$
- We assume identical risk and premium share, i.e. $\alpha_i = \beta_i$
- To reduce numerical and illustrative complexity we set $n = 2$
- $\alpha_1 \in \{0, 0.1, \ldots, 1\}$; for ensuring comparability of different shares and liability regimes we focus on a reference case for which the contract safety level is throughout $PV(D_1) = 0.5$ (A2)
- $(A1)$ & $(A2) \Rightarrow$ Fair pool premium in reference case $P_0 = PV(P_1) = PV(C_1) - PV(D_1) = 99.5$

Find – as function of $\alpha_1$ and the liability regime – values for $E_0^1$ and $E_0^2$ s.t.

$$(P_0, E_0^1, E_0^2) = \left(PV(P_1), PV(E_1^1), PV(E_1^2)\right)$$
### Numerical Example – Setting (2/2)

- Pool claims are modelled as jump-diffusion process (GBM & Poisson Process, *Merton (1976)*)
- Assets returns are modelled as ordinary GBM
- The claims’ face value at time $t = 0$ is fixed at $PV(C_1) = 100 (A1)$
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- $\alpha_1 \in \{0, 0, 0, 1, \ldots, 1\}$; for ensuring comparability of different shares and liability regimes we focus on a reference case for which the contract safety level is throughout $PV(D_1) = 0.5 (A2)$
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<table>
<thead>
<tr>
<th>Risk-free Rate</th>
<th>Asset Process</th>
<th>Claims Process</th>
<th>Case IV</th>
<th>Case V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uniformity</td>
<td>Pos. Correlation</td>
<td>Neg. Correlation</td>
<td>Incr. Asset Risk (w/o taxation)</td>
</tr>
<tr>
<td>Uniformity</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>Pos. Correlation</td>
<td>3%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Neg. Correlation</td>
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<td>20%</td>
<td>20%</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.5</td>
<td>- 0.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

We revoke the assumption of a frictionless market and introduce corporate income taxation. Policyholders are burdened with present value of tax payment at time $t = 0$. Some degree of risk-aversion and inability to replicate payoffs is assumed for policyholders. Applied tax rate $\tau = 35\%$
Observations

- Fair equity required for pre-given safety level increases for both regimes in risk/premium share.
- For all allocations: \( E_0^i (Several\ Liability) \geq E_0^j (Joint\ Liability) \)
- For the marginal cases: \( E_0^i (Several\ Liability) = E_0^j (Joint\ Liability) \)
- The aggregated equity reaches for joint liability a minimum at \((\alpha_1, \alpha_2) = (0.5, 0.5)\)
- For several liability, the aggregated equity is constant
- \(\rightarrow\) For several liability no effects from increasingly diversified risk-sharing; also no effect from increasing \(n\)
### Numerical Example – Results Case I, II and III

<table>
<thead>
<tr>
<th></th>
<th>Case I Uniformity</th>
<th>Case II Pos. Correlation</th>
<th>Case III Neg. Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free Rate</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>Asset Process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility IC1</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Volatility IC2</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Correlation</td>
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<td>0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>Claims Process</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Volatility Diffusion</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Jump Freq.</td>
<td>10 yr.</td>
<td>10 yr.</td>
<td>10 yr.</td>
</tr>
<tr>
<td>Jump Factor</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Corr. Asset-Claims</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Observations**

- Plot shows fair premium-equity-combinations (aggregated) for the allocation \((\alpha_1, \alpha_2) = (0.5, 0.5)\)
- In a regime of several liability, the fair combinations do not depend on the ICs’ asset correlation
- In a regime of joint liability, negative correlation reduces the required equity for keeping the safety level
- In contrast, positive correlation in a regime of joint liability increases the equity requirement
- Several liability appears as limit case of joint liability as correlation goes towards 1
Numerical Example – Results Case IV

Observations

- In general, tax loading in a regime of joint liability is less than in a regime of several liability
- Tax benefit amplifies when asset correlation is negative
- Tax benefit shrinks as correlation goes towards 1

Applied tax rate $\tau = 35\%$
Observations

- Risk-shift of IC 2 solely affects policyholder in a regime of several liability: severe wealth transfer from PH to EH of IC 2

- In a regime of joint liability risk-shift affects policyholder as well as equity holders of IC 1: wealth transfer from PH and EH of IC 1 to EH of IC 2
Summarized Conclusions

Regime of Several Liability
- No diversification effects from risk sharing, just an allocation between several parties
- No impact of asset correlation on fair premium and equity values
- Risk-shifting problems only matter for policyholders

Regime of Joint Liability
- Increased diversification (i.e. $\alpha_i \to 1/n$) leads to reduced required equity to achieve the pre-given safety level
- Regime of joint liability will pass into several liability if risk allocation goes towards marginal allocation (i.e. only one insurer bears the business)
- Analogously, the regime will pass into several liability if asset correlation goes towards 1
- For increased negative correlation, the distinction between both regimes becomes more material
- Both equity holders as well as policyholders must be concerned about risk-shifting problems

→ From perspective of policyholder the regime of joint liability is preferable due to lower frictional costs and the utility to share the risk-shifting problem with other parties jointly
Thank You

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