Performance Measurement in the Life Insurance Industry: An Asset-Liability Perspective

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Abstract

Established risk-adjusted investment performance measures such as the Sharpe, the Sortino or the Calmar Ratio have been developed with an exclusive focus on the mutual and hedge fund industries. Consequently, they are less suited for liability-driven investors such as life insurance companies, whose portfolio choice is materially affected by the substantial interest rate sensitivity of their long-term contractual obligations. In order to tackle this limitation, we introduce the Asset-Liability Sharpe Ratio, which is theoretically motivated, computable based on publicly-available data, incentive compatible, and relevant. Hence, it should be a valuable new tool for performance assessment in the life insurance industry.

Keywords: Asset-Liability Management, Life Insurance, Risk-Adjusted Performance Measurement, Rank Correlation

JEL classification: G11; G22

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1 Introduction

A life insurer’s investment performance is, amongst other factors such as financial strength, of great importance to both policyholders and shareholders as it influences their decisions to purchase insurance contracts or invest in the firm’s stock. However, established risk-adjusted financial performance measures such as the Sharpe, the Sortino, or the Calmar Ratio have been developed with an exclusive focus on the mutual and hedge fund industries. Consequently, they are less suited for liability-driven investors such as life insurance companies and pension plans, whose portfolio choice is materially affected by the substantial interest rate sensitivity of their long-term contractual obligations. It is quite astonishing that a specific financial performance measure for these types of asset managers has not evolved yet. After all, we are looking at two of the top three institutional investors worldwide, which, with combined assets under management of more than USD 40 trillion, range way ahead of hedge funds or private equity firms (see, e.g., Swiss Re, 2010; OECD, 2018). In recent years, digital transformation has begun to increase pressure on the industry (see, e.g., Braun and Schreiber, 2017). Hence, to stay relevant for their customers, going forward life insurers will more than ever need to properly evaluate and document their investment performance.

The extant literature mirrors this gap. Pedersen and Rudholm-Alfvin (2003) compare several classical and modern performance measures based on a set of objective criteria. In addition, they run an empirical analysis which illustrates the suitability of the Sharpe Ratio for the financial services sector, while documenting its weaknesses in the context of alternative investments. Furthermore, Eling and Schuhmacher (2007) examine the impact of the performance measure on the evaluation of hedge fund managers. Drawing on a data set of monthly returns for 2’763 hedge funds in the time period between 1985 and 2004, they find virtually identical rankings for a total of 13 ratios. Eling (2008) extends this work by a comprehensive sample of mutual funds that invest in all major asset classes. He confirms his earlier results and concludes that the choice of measure is not critical in practical applications. In contrast to that, Farinelli et al. (2008) raise doubts about the suitability of the Sharpe Ratio for portfolio optimization purposes once the return distributions deviate from normality. Their analysis underlines several studies focus on the optimal asset allocation in the presence of risky liabilities. Sharpe (1987), for instance, suggests a generalized framework aiming to help investors navigating their asset allocation decision process, while Sundaresan and Zapatero (1997) analyze the optimal asset allocation of defined benefit pension plans. Hoevenaars et al. (2008), on the other hand, reveal differences in strategic asset portfolios between asset-only and asset-liability investors. In this regard, they highlight that the latter are not only required to deal with reinvestment risks of government bonds, but also with the asset-liability duration mismatch on their balance sheet.
that more recently introduced asymmetrical parameter-dependent measures such as the ratio by Farinelli and Tibiletti (2008) exhibit a considerably higher robustness. Zakamouline (2011), on the other hand, disproves the results of Eling and Schuhmacher (2007) as well as Eling (2008). Employing the same hedge fund data set and running a supplemental simulation analysis, he reveals shortcomings in these two studies and concludes that the choice of measure in fact does have a major impact on the ranking of fund managers. This view is supported by Ornelas et al. (2012) who also find that the ranking and selection of mutual funds depends on the chosen performance measure.

We complement the existing line-up of performance measures by the Asset-Liability Sharpe Ratio (ALSR), which is tailored to the characteristics of the life insurance industry. Instead of evaluating the asset side of the balance sheet in isolation, the ALSR focuses on the distribution of a purely market risk-driven return on equity (ROE). The latter explicitly takes in to account the interaction between assets and liabilities. Our work comprises four major contributions. First, we develop the ALSR based on theoretical considerations, which underline that it ideally fits the business model of life insurance firms. Second, we show how the ALSR can be easily estimated from publicly-available data, thus being well suited for shareholders and policyholders, who do not have access to company-internal information. Third, we draw on a rigorous model framework to demonstrate that the ALSR favors portfolios that hedge the market-value balance sheet by capturing the stochastic dependence between the assets and the liabilities. Fourth, we run a comprehensive simulation study to illustrate the relevance of the ALSR. To this end, we create a large sample of 10'000 hypothetical life insurance companies. Each company is assumed to hold five major asset classes for which we empirically estimate means, variances, and correlations. The corresponding portfolio weights, capital structures and liability durations are drawn from realistically parametrized distributions. After the sample of firms has been generated, we apply the ALSR together with several existing performance measures, form rankings, and compute the corresponding Spearman and Kendall correlation coefficients. Due to the varying asset-liability hedging properties of the simulated investment portfolios, the ALSR leads to completely different results than conventional performance ratios. To sum up, our new measure is theoretically motivated, easy to estimate, incentive compatible, and conveys information that is not included in existing measures. Hence, it should be a valuable new tool for performance assessment in the life insurance industry.
The remainder of this manuscript is organized as follows. In the next section, we introduce the theoretical motivation of the ALSR, present a model framework for life insurance companies based on which it can be estimated from publicly-available data, and illustrate its incentive effects. Furthermore, in the third section, we determine the empirical risk-return profiles of the portfolio constituents held by the hypothetical life insurers, draw their asset allocations, capital structures, and liability durations and evaluate their investment performance. Finally, the last section contains our conclusion.

2 The Asset-Liability Sharpe Ratio

2.1 Theoretical Considerations

Financial performance ratios relate an excess return figure achieved by an asset manager to some risk measure (see, e.g., Eling and Schuhmacher, 2007):

\[
\text{Performance Ratio} = \frac{\text{Excess Return}}{\text{Risk Measure}}.
\] (1)

For all types of collective investment vehicles, the returns are commonly determined based on a net asset value (NAV) time series. The NAV, which is defined as the value of a fund’s assets minus that of its liabilities, is the equivalent of a firm’s equity capital and therefore incorporates the effect of leverage. Except from specific debt funds, mutual funds do usually not use debt financing and thus, the return on the NAV exactly equals their return on assets (ROA). In the case of long-short and market-neutral hedge funds, however, which tend to be heavily geared, one needs to differentiate between the ROA and the returns derived from their NAV. This distinction becomes even more important for life insurers in the form of a stock company and defined-benefit pension funds. The reason is that, in contrast to the short-term loans taken out by leveraged funds, participating life insurance contracts can have maturities of more than 30 years (see, e.g., Braun et al., 2019). Therefore the technical reserves of a life insurer exhibit a substantial interest rate sensitivity, which introduces a nonnegligible degree of volatility into the return on liabilities (ROL) and generates a positive stochastic dependence with the ROA. For this reason,

2 This does not hold for mutual insurance companies, for which there is no distinction between the equityholder and policyholder stake (see, e.g., Braun et al., 2015a)

3 See, e.g., Braun et al. (2011) for a discussion of the anatomy of pension funds.
life insurers pursue a liability-driven investment approach. This means that their portfolios overweigh long-term investment-grade fixed-income securities to better match both sides of the market-value balance sheet. The latter is at the center of modern risk-based capital requirements, such as those computed with the Solvency II standard formula (see, e.g., Braun et al., 2015b, 2017, 2018). The loans taken out by hedge funds, in contrast, are mostly short term (see, e.g., Ang et al., 2011). Hence, they exhibit a low duration, resulting in little volatility and correlation with the asset side. Figure 1 is a stylized illustration of these differences between mutual funds, hedge funds and life insurance companies.

Figure 1: Stylized Market-Value Balance Sheets
This figure shows simplified versions of the market-value balance sheets of mutual funds, hedge funds, and life insurers. $A$: market value of the assets, $L$: market value of the liabilities, $NAV$: net asset value, $\sigma_l$: volatility of the return on liabilities, $\rho$: correlation of the asset and liability returns. The stochastic dependence of life insurance assets and liabilities implies that a meaningful investment performance evaluation needs to rely on the ROE instead of the ROA.

In light of the aforementioned considerations, it becomes clear that a meaningful investment performance evaluation for life insurers cannot rely on the ROA. Consequently, we suggest the ALSR, which builds upon mean and standard deviation of a purely market risk-driven ROE, $\bar{r}^e$, excluding the underwriting result of the insurer. It relates $\bar{r}^e$ in excess of the risk-free interest rate $r_f$ in the numerator to the standard deviation of $\bar{r}^e$, $\sigma_e$, in the denominator:

$$\text{ALSR} = \frac{E(\bar{r}^e_t) - r_f}{\sigma_e}.$$  \hspace{1cm} (2)

\[4\] Note that, in contrast to insurance companies, mutual and hedge funds face redemptions. The latter may also give rise to asset-liability considerations, since the asset manager needs to hold cash or highly liquid short-term securities to meet them. However, the associated mechanics are beyond the scope of this paper.

\[5\] In the following, the term performance will exclusively refer to investment performance, which can be influenced by the insurer’s asset management division. Our considerations deliberately exclude operational performance, i.e., the fact that some insurance companies may be much more efficient than others in the business of selling policies to clients.
Just as the classical Sharpe Ratio, the ALSR is rooted in mean-variance preferences (see Sharpe, 1966). Due to the stochastic dependence between the ROA and the ROL as well as the need to exclude the biometric insurance risk on the liability side, the calculation of $\mathbb{E}(\tilde{r}_t^a)$ and $\sigma_r$ is not as straightforward as it may seem. Biometric risks comprise, e.g., mortality, longevity and disability scenarios. Since these cannot be influenced by the firm’s asset managers, they should not affect the ratio used to measure their performance. Thus, one can neither resort to the book value of equity nor the share price of the firm to estimate the underlying return distribution. The reason is that the former does not incorporate shifts in market value, whereas the latter also contains the insurance risk on the balance sheet and is distorted by general stock market sentiment. In the following section, we address this issue by presenting an intuitive model framework that can be used to estimate the first two moments of the required ROE distribution based on publicly-available data. Moreover, we demonstrate that the ALSR provides adequate incentives for asset managers, since a less volatile ROE can be achieved through a better asset-liability hedge. That being said, however, it should be emphasized that our focus is not normative. Put differently, we assume that life insurers generally pursue a liability-driven investment approach. If one accepts this premise, the ALSR can shed light on the asset managers’ relative success in generating investment returns while, at the same time, trying to achieve the best possible matching of assets and liabilities.

2.2 Model Framework for the Moments of the ROE

Asset Side

The market value of the asset portfolio of a life insurance company at time $t$, $\tilde{A}_t$, is obtained by compounding the deterministic initial value, $A_0$, with the stochastic ROA between times 0 and $t$ ($\tilde{r}_t^a$):

$$
\tilde{A}_t = A_0 \cdot (1 + \tilde{r}_t^a),
$$

(3)

The ROA equals the weighted average of the returns on the individual assets in the portfolio:

$$
\tilde{r}_t^a = \sum_{j=1}^{n} w_j \tilde{r}_{jt},
$$

(4)

6Due to modern regulatory frameworks, such as Solvency II, insurers must maintain a sufficient level of risk-bearing capital. This tightly constrains their portfolio choice and naturally incentivizes asset-liability matching (see, e.g., Braun et al., 2015b, 2017, 2018).
with \( n \) representing the number of assets, \( w_j \) being the portfolio weight of asset \( j \), and \( \tilde{r}_{jt} \) denoting the stochastic return on asset \( j \) between times \( 0 \) and \( t \). In the following, we assume \( \tilde{r}_{jt} \sim N(\mu_{\tilde{r}_{jt}}, \sigma_j) \), i.e. all asset returns adhere to a normal distribution. Consequently, the expected value, variance, and standard deviation of \( \tilde{r}_t^a \) can be expressed as:

\[
E(\tilde{r}_t^a) = E\left( \sum_{j=1}^{n} w_j \tilde{r}_{jt} \right) = \sum_{j=1}^{n} w_j E(\tilde{r}_{jt}),
\]

\[
\text{var}(\tilde{r}_t^a) = \text{var}\left( \sum_{j=1}^{n} w_j \tilde{r}_{jt} \right) = \sum_{j=1}^{n} \sum_{k=1}^{n} w_j w_k \rho_{j,k} \sigma_j \sigma_k,
\]

\[
\sigma_a = \sqrt{\text{var}(\tilde{r}_t^a)} = \sqrt{\sum_{j=1}^{n} \sum_{k=1}^{n} w_j w_k \rho_{j,k} \sigma_j \sigma_k},
\]

where \( E(\tilde{r}_{jt}) \) is the expected return of asset \( j \), \( \sigma_j \) the corresponding volatility, and \( \rho_{j,k} \) the correlation coefficient between the returns of assets \( j \) and \( k \).

### Liability Side

The market value of the life insurer’s liabilities at time \( t \), \( \tilde{L}_t \), is driven by the stochastic ROL between times \( 0 \) and \( t \) (\( \tilde{r}_t^l \)):

\[
\tilde{L}_t = L_0 \cdot (1 + \tilde{r}_t^l).
\]

As discussed above, we want to isolate the effect of interest rate changes on the liabilities, while ignoring insurance risk. To this end, we assume a flat term structure and approximate Equation (8) as follows:

\[
\tilde{L}_t \approx L_0 + \frac{\delta L_0}{\delta y} \Delta y_t
\]

\[
= L_0 \cdot (1 + \frac{\delta L_0}{\delta y} \frac{\Delta y_t}{L_0}),
\]

\( \Delta y_t \) represents the change in the yield curve at time \( t \).

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The suitability of this assumption will be tested by means of distribution fitting in the fourth section.
where $\Delta \tilde{y}_t = \tilde{y}_t - y_0$ is the absolute change in the yield between times 0 and $t$, and $\delta L_0/\delta y$ is the partial derivative (sensitivity) of the present value of the liabilities with regard to the current yield level $y$. The latter can be computed based on the expected cash flows to the policyholders $E(\tilde{C}_t^l)$ and their times of occurrence $\tau$:

$$\frac{\delta L_0}{\delta y} = -\frac{1}{1 + y} \sum_{\tau=1}^{T} \tau \cdot E(\tilde{C}_t^l) \cdot (1 + y)^{-\tau} < 0. \tag{10}$$

Comparing Equations (8) and (9) and additionally employing the definition of the modified duration of the life insurer’s liabilities, $D_l = \frac{\delta L_0}{\delta y}$, we obtain the following approximation for the ROL:

$$\tilde{r}_l^t = \tilde{L}_t - L_0 \approx \frac{\delta L_0}{\delta y} \frac{\Delta \tilde{y}_t}{L_0} = -D_l \Delta \tilde{y}_t. \tag{11}$$

Suppose that shifts in the yield are normally distributed: $\Delta \tilde{y}_t \sim N(E(\Delta \tilde{y}_t), \sigma_y)$. It is now straightforward to derive the expected value, variance, and standard deviation of $\tilde{r}_l^t$:

$$E(\tilde{r}_l^t) \approx E(-D_l \Delta \tilde{y}_t) = -D_l E(\Delta \tilde{y}_t), \tag{12}$$

$$\text{var}(\tilde{r}_l^t) \approx \text{var}(-D_l \Delta \tilde{y}_t) = D_l^2 \cdot \text{var}(\Delta \tilde{y}_t), \tag{13}$$

$$\sigma_l \approx \sqrt{\text{var}(\tilde{r}_l^t)} = \sqrt{D_l^2 \sigma_y^2} = D_l \sigma_y. \tag{14}$$

**Equity Capital**

Having fully described both the asset and the liability side of the market-value balance sheet, we combine Equations (3) and (8) to arrive at the time-$t$ market value of the life insurer’s equity capital, $\tilde{E}_t$:

$$\tilde{E}_t = \tilde{A}_t - \tilde{L}_t = A_0(1 + \tilde{r}_a^t) - L_0(1 + \tilde{r}_l^t). \tag{15}$$

The market risk-driven stochastic ROE ($\tilde{r}_e^t$) is therefore given by the following expression:

$$\tilde{r}_e^t = \frac{\tilde{E}_t - E_0}{E_0} = \frac{A_0}{E_0} \cdot (1 + \tilde{r}_a^t) - \frac{L_0}{E_0} \cdot (1 + \tilde{r}_l^t) - 1. \tag{16}$$
Since both $\tilde{r}_a^t$ and $\tilde{r}_l^t$ are normally distributed, so is $\tilde{r}_e^t$. Consequently, the following equations hold for the expected value, variance, and standard deviation of $\tilde{r}_e^t$:

$$E(\tilde{r}_e^t) = \frac{A_0}{E_0} \cdot (1 + E(\tilde{r}_a^t)) - \frac{L_0}{E_0} \cdot (1 + E(\tilde{r}_l^t)) - 1, \quad (17)$$

$$\text{var}(\tilde{r}_e^t) = \frac{A_0^2}{E_0^2} \cdot \text{var}(\tilde{r}_a^t) + \frac{L_0^2}{E_0^2} \cdot \text{var}(\tilde{r}_l^t) - 2 \frac{A_0 L_0}{E_0^2} \cdot \text{cov}(\tilde{r}_a^t, \tilde{r}_l^t), \quad (18)$$

$$\sigma_e = \sqrt{\text{var}(\tilde{r}_e^t)} = \frac{1}{E_0} \sqrt{\frac{A_0^2 \cdot \sigma_a^2 + L_0^2 \cdot \sigma_l^2 - 2 A_0 L_0 \cdot \sigma_a \sigma_l \cdot \rho(\tilde{r}_a^t, \tilde{r}_l^t)}{\text{var}(\tilde{r}_a^t) \cdot \text{var}(\tilde{r}_l^t)}}. \quad (19)$$

The normality assumptions can be easily relaxed to account for higher moments of the ROA, ROL, and ROE distributions. In the absence of sufficiently detailed historical ROL time series, however, this would remain a rather theoretical exercise, since it is virtually impossible to obtain reliable parameter estimates for more complex representations of the liability side.

**Asset-Liability Correlation**

A non-trivial element in calculating the ROE volatility $\sigma_e$ via Equation (19) is the correlation between $\tilde{r}_a^t$ and $\tilde{r}_l^t$. Since time series data for $\tilde{r}_l^t$ is typically difficult to obtain without access to the life insurance company’s internal database, we take advantage of the approximations in Equations (11) and (14) to derive the following relationship:

$$\rho(\tilde{r}_a^t, \tilde{r}_l^t) = \frac{\text{cov}(\tilde{r}_a^t, \tilde{r}_l^t)}{\sigma_a \sigma_l} = \frac{\text{cov}(\tilde{r}_a^t, -D_l \Delta \tilde{y}_t)}{\sigma_a D_l \sigma_y} \quad (20)$$

$$= - \frac{D_l \cdot \text{cov}(\tilde{r}_a^t, \Delta \tilde{y}_t)}{\sigma_a D_l \sigma_y} = - \frac{\text{cov}(\tilde{r}_a^t, \Delta \tilde{y}_t)}{\sigma_a \sigma_y} = - \rho(\tilde{r}_a^t, \Delta \tilde{y}_t).$$

Equation (20) implies that $\rho(\tilde{r}_a^t, \tilde{r}_l^t)$ can be estimated through $\rho(\tilde{r}_a^t, \Delta \tilde{y}_t)$, i.e. the correlation between the ROA and the absolute change in the yield. As will be shown below, this correlation rises for a higher fraction of fixed-income securities in the investment portfolio and therefore accounts for ROE immunization strategies by the asset management.
Impact of Asset Allocation and Bond Immunization

Due to their interest rate sensitivity as represented by the modified duration, mainly the returns of fixed income instruments covary with $\Delta \tilde{y}_t$. Hence, the value of $\rho(\tilde{r}_t^a, \Delta \tilde{y}_t)$ and, thus, $\rho(\tilde{r}_t^a, \tilde{r}_t^b)$ depends on the firm’s asset allocation and bond immunization strategies. Analytically, this can be shown by decomposing $\tilde{r}_t^a$ into a part $\tilde{r}_t^b$ that comes from the bond portfolio as well as a part $\tilde{r}_t^s$ that comprises instruments with a negligible duration such as stocks:

$$\tilde{r}_t^a = \gamma \cdot \tilde{r}_t^b + (1 - \gamma) \cdot \tilde{r}_t^s. \quad (21)$$

Here, $\gamma$ represents the allocation to the fixed income subportfolio (including government and corporate bonds). Analogously to the interest rate sensitive liabilities (see Equation (11)), $\tilde{r}_t^b$ can be estimated as follows:

$$\tilde{r}_t^b = \frac{\tilde{B}_t - B_0}{B_0} = \frac{\delta B_0}{\delta y} \frac{\Delta \tilde{y}_t}{B_0} = -D_b \Delta \tilde{y}_t, \quad (22)$$

where $B_0$ stands for the present value of the bonds, $D_b = -\frac{\delta B_0}{\delta y} / B_0$ is their modified duration, and $\delta B_0 / \delta y$ derives from their expected cash flows $E(\tilde{C}_t^b)$ as well as the corresponding occurrence dates $\tau$:

$$\frac{\delta B_0}{\delta y} = -\frac{1}{1 + y} \sum_{\tau=1}^{T} \tau \cdot E(\tilde{C}_t^b) \cdot (1 + y)^{-\tau} < 0. \quad (23)$$

Hence, we obtain the expected value, variance, and standard deviation of $\tilde{r}_t^b$ as shown below:

$$E(\tilde{r}_t^b) = E(-D_b \Delta \tilde{y}_t) = -D_b E(\Delta \tilde{y}_t), \quad (24)$$

$$\text{var}(\tilde{r}_t^b) = \text{var}(-D_b \Delta \tilde{y}_t) = D_b^2 \cdot \text{var}(\Delta \tilde{y}_t), \quad (25)$$

$$\sigma_r = \sqrt{\text{var}(\tilde{r}_t^b)} = D_b \sigma_y. \quad (26)$$

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8 Several authors such as Boquist et al. (1975), Livingston (1978), Lanstein and Sharpe (1978), and Leibowitz (1986), among others, discuss the duration of common stocks and the relationship with an equity beta. In our model, however, we assume that mainly fixed income instruments covary with changes in the yield $\Delta \tilde{y}_t$. 

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10
It is now possible to express the variance of \( \tilde{r}_t^a \) as well as its covariance with \( \tilde{r}_t^l \) in terms of the contributions of \( \tilde{r}_t^b \) and \( \tilde{r}_t^t \):

\[
\text{var}(\tilde{r}_t^a) = \gamma^2 \cdot \text{var}(\tilde{r}_t^b) + (1 - \gamma)^2 \cdot \text{var}(\tilde{r}_t^t) + 2\gamma(1 - \gamma) \cdot \text{cov}(\tilde{r}_t^b, \tilde{r}_t^t) \tag{27}
\]

\[
= \gamma^2 \sigma_b^2 + (1 - \gamma)^2 \sigma_t^2 + 2\gamma(1 - \gamma) \cdot \text{cov}(\Delta \tilde{y}_l, \tilde{r}_t) \]

\[
= \gamma^2 \sigma_b^2 \sigma_t^2 + (1 - \gamma)^2 \sigma_t^2 - 2\gamma(1 - \gamma) \cdot \text{cov}(\Delta \tilde{y}_l, \tilde{r}_t)
\]

\[
\text{cov}(\tilde{r}_t^a, \tilde{r}_t^l) = \text{cov}(\gamma \cdot \tilde{r}_t^b + (1 - \gamma) \cdot \tilde{r}_t^t, -D_l \Delta \tilde{y}_l) \tag{28}
\]

\[
= -\gamma D_l \cdot \text{cov}(\tilde{r}_t^b, \Delta \tilde{y}_l) - (1 - \gamma) D_l \cdot \text{cov}(\tilde{r}_t^t, \Delta \tilde{y}_l)
\]

\[
= -\gamma D_l \cdot \text{cov}(\Delta \tilde{y}_l, \Delta \tilde{y}_l) - (1 - \gamma) D_l \cdot \text{cov}(\tilde{r}_t^t, \Delta \tilde{y}_l)
\]

\[
= \gamma D_b D_l \sigma_y^2 - (1 - \gamma) D_l \cdot \rho(\tilde{r}_t^t, \Delta \tilde{y}_l) \sigma_x \sigma_y.
\]

Finally, using Equations (14), (27), and (28) and realizing that the modified duration of the firm’s overall assets is defined as \( D_a = \gamma D_b \), we may break down the correlation \( \rho(\tilde{r}_t^a, \tilde{r}_t^l) \) as follows:

\[
\rho(\tilde{r}_t^a, \tilde{r}_t^l) = \frac{\text{cov}(\tilde{r}_t^a, \tilde{r}_t^l)}{\sigma_a \sigma_l} \tag{29}
\]

\[
= \frac{D_a D_l \sigma_y^2 - (1 - \gamma) D_l \cdot \rho(\tilde{r}_t^t, \Delta \tilde{y}_l) \sigma_x \sigma_y}{\sqrt{D_a^2 \sigma_y^2 + (1 - \gamma)^2 \sigma_x^2}}
\]

\[
= \frac{D_a - (1 - \gamma) \cdot \rho(\tilde{r}_t^t, \Delta \tilde{y}_l) \frac{\sigma_x \sigma_y}{\sigma_y}}{\sqrt{D_a^2 + (1 - \gamma)^2 \sigma_x^2}}
\]

\[
= \frac{D_a - (1 - \gamma) \cdot \rho(\tilde{r}_t^t, \Delta \tilde{y}_l) \frac{\sigma_x \sigma_y}{\sigma_y}}{\sqrt{D_a^2 - 2D_a(1 - \gamma) \cdot \rho(\tilde{r}_t^t, \Delta \tilde{y}_l) \frac{\sigma_x \sigma_y}{\sigma_y}}}
\]

\[A \text{ derivation of the equality } D_a = \gamma D_b \text{ can be found in the Appendix, which additionally contains mathematical expressions of } \rho(\tilde{r}_t^a, \tilde{r}_t^l) \text{ for special values of } \gamma \text{ and } \rho(\tilde{r}_t^t, \Delta \tilde{y}_l).\]
\[
\frac{\delta p(\tilde{r}_t^a, \tilde{r}_t^l)}{\delta \gamma} > 0 \quad \frac{\delta p(\tilde{r}_t^a, \tilde{r}_t^l)}{\delta \Delta \tilde{y}_t} > 0
\]

(30)

2.3 Estimation Based on Publicly-Available Data

Equations (17) and (19) illustrate that the two essential input parameters for the ALSR, i.e. the mean and the standard deviation of the market risk-driven ROE (\(\tilde{r}_t^a\)) depend on a total of seven parameters: \(A_0\), \(L_0\), \(E(\tilde{r}_t^a)\), \(\sigma_a\), \(\sigma_l\) and \(\rho(\tilde{r}_t^a, \tilde{r}_t^l)\). The most recent market values of assets and liabilities (\(A_0\) and \(L_0\)) or the corresponding equity capital (\(E_0\)) are included in the life insurer’s regulatory report.\(^{10}\) The other five parameters can be estimated from time series data. Individual asset-level figures for the calculation of \(\tilde{r}_t^a\) via Equation (4) will be hard to obtain for company outsiders. However, annual reports contain the portfolio weights \(w_j\) for the strategic asset allocation. In combination with index return time series that benchmark different asset classes (e.g., government bonds, stocks etc.), it is possible to generate a well-informed proxy for \(\tilde{r}_t^a\).\(^{11}\) Calculating the first two moments is then straightforward. The better the indices match the actual portfolio compositions of the life insurers, the more accurate the results. Moreover, as visible in Equations (12) and (14), the estimation of \(E(\tilde{r}_t^l)\) and \(\sigma_l\) merely requires the modified duration from the annual or solvency report of the insurer as well as the mean and standard deviation of the yield change \(\Delta \tilde{y}_t\). The latter can be estimated from time series data available on Datastream or Bloomberg, which also enters the approximation of \(\rho(\tilde{r}_t^a, \tilde{r}_t^l)\) by means of Equation (20). Given these considerations, it should be straightforward to compute the ALSR in practice. To underline this notion, we will present an exemplary calibration of the model framework with in the fourth section.

2.4 Incentive effects

Figure 2 relies on our model framework and illustrates how \(e\) reacts to its two main drivers \(\sigma_a\) and \(\rho(\tilde{r}_t^a, \tilde{r}_t^l)\). It is based on the following input parameter values: \(L_0/A_0 = 0.75\) (i.e. \(A_0/E_0 = 4\) and \(L_0/E_0 = 3\)) and \(\sigma_l = 0.1\). If \(\rho(\tilde{r}_t^a, \tilde{r}_t^l) = 0\), \(e\) rises monotonically in \(\sigma_a\), since Equation (19) turns into \(e = 1/E_0 \sqrt{A_0^2 \sigma_a + L_0^2 \sigma_l}\). In contrast, if \(\rho(\tilde{r}_t^a, \tilde{r}_t^l) = 1\), then \(e\) initially falls in \(\sigma_a\), until it reaches its

\(^{10}\)Under Pillar 3 of Solvency II, e.g., insurance companies need to disclose information on their market value balance sheet to the public in the form of a detailed Solvency and Financial Condition Report (SFCR) (see, e.g., EIOPA, 2015).

\(^{11}\)In contrast to other types of investors, life insurance companies typically pursue buy and hold investment strategies with a particular focus on long-duration assets (see, e.g., Gründl et al., 2016, Fitch Ratings, 2019), which allows us to draw on benchmark indices for deriving their asset allocations.
global minimum of zero at \( \sigma_a = L_0/A_0 \cdot \sigma_l = 0.075 \). Subsequently, it rises monotonically in \( \sigma_a \). Furthermore, if \( \sigma_a = 0 \), Equation (19) turns into \( \sigma_e = L_0/E_0 \cdot \sigma_l = 0.3 \). In this case, the \( \sigma_e \) is independent of the asset-liability correlation. For all other (positive) values of \( \sigma_a \), however, \( \sigma_e \) declines when the asset-liability correlation \( \rho(\tilde{r}_t, \tilde{r}_l^a) \) increases.

From the perspective of a life insurer’s asset management department, the liability side must be treated as given and represents a crucial driver of investment decisions. In contrast, both \( \sigma_a \) and \( \rho(\tilde{r}_t, \tilde{r}_l^a) \) can be determined through the portfolio choice. Since the ALSR is governed by \( \sigma_e \) as the risk measure in the denominator, it consistently promotes asset allocations that are associated with a higher asset-liability correlation and, in turn, a less volatile ROE. In other words, by incentivizing asset managers to

\[ \text{Figure 2: Sensitivities of the ROE Volatility} \]

This figure illustrates the sensitivities of the ROE volatility \( \sigma_e \) with regard to the ROA volatility \( \sigma_a \) and the asset-liability correlation \( \rho(\tilde{r}_t, \tilde{r}_l^a) \) in the adopted model framework for life insurance companies. It is based on the following parameter values: \( L_0/A_0 = 0.75 \) (i.e. \( A_0/E_0 = 4 \) and \( L_0/E_0 = 3 \)) and \( \sigma_l = 0.1 \). The global minimum of \( \sigma_e \) equals zero and is reached for \( \rho(\tilde{r}_t, \tilde{r}_l^a) = 1 \) and \( \sigma_a = L_0/A_0 \cdot \sigma_l = 0.075 \). In addition, if \( \sigma_a = 0 \), we have \( \sigma_e = L_0/E_0 \cdot \sigma_l = 0.3 \).

\[^{12}\text{To see this, set } \rho(\tilde{r}_t, \tilde{r}_l^a) \text{ to one and then solve the first order condition of Equation (19) with regard to } \sigma_a.\]
improve the risk-return profile of the asset side and the asset-liability match at the same time, the ALSR discourages isolated investment decisions.

3 Simulation Study

Having established the motivation behind the ALSR and illustrated its sensitivities as well as associated incentive effects, we now want to analyze its practical relevance. Eling and Schuhmacher (2007) found high rank correlations between most established ratios. Hence, another metric is only warranted if, in addition to being designed based on a solid theoretical reasoning, it conveys additional information and therefore leads to a notably different performance assessment. To see whether the ALSR fulfills this criterion, we now compare it to a broad range of established measures, including the Sharpe Ratio (see Sharpe, 1966), Omega (see Shadwick and Keating, 2002), the Sortino Ratio (see Sortino and van der Meer, 1991), Kappa₃ (see Kaplan and Knowles, 2004), the Calmar Ratio (see Young, 1991), the Sterling Ratio (see Kestner, 1996), the Burke Ratio (see Burke, 1994), the Excess Return on VaR (see Dowd, 2000), the Conditional Sharpe Ratio (see Agarwal, 2004, and the Modified Sharpe Ratio (see Gregoriou and Gueyie, 2003). To this end, we resort to a comprehensive simulation study. It should be emphasized that, in doing so, we do not contradict our claim from the previous section regarding the possibility to estimate the measure based on publicly available data. Instead, the simulation is considered superior to a pure empirical comparison of the aforementioned financial performance measures, since life insurers are much fewer in numbers than mutual funds or hedge funds. Hence, the simulation allows us to analyze the properties of the ALSR on a much larger sample than would be empirically available.

3.1 Empirical Risk-Return Profiles

Prior to drawing the asset allocations of our sample firms, we need to determine their feasible investment space. In this regard, we focus on five asset classes commonly held by life insurers, i.e. government bonds, corporate bonds, stocks, real estate, and hedge funds. Each asset class will be represented by a benchmark index. In the last two decades three major crises shook the capital markets worldwide: the collapse of the new economy in 2001, the global financial crisis from 2007 to 2009, as well as the European debt crisis starting at the end of 2009. Consequently, some indices exhibit negative mean returns if the

\footnote{For detailed information on these measures, the reader is referred to the cited literature.}
period under consideration is too short. We therefore draw on longer time series and estimate expected values, standard deviations, and covariances for the period from January, 1990 to December, 2014. By doing so, we ensure that our data spans various interest rate environments as well as business cycles. Furthermore, it is consistent with the long-term investment horizon of life insurers, relating to their liabilities with 25+ years to maturity.

The Barclays US Treasury Index and the Barclays US Corporate Index serve as benchmarks for the life insurer’s government and corporate bond holdings, respectively. The equity subportfolio, on the other hand, is proxied by the S&P 500. We further assume that the S&P/Case-Shiller National Home Price Index captures the risk-return characteristics of the real estate asset class. Since insurers may also invest in alternative asset classes such as hedge funds, we additionally draw on the HFRI Fund Weighted Composite Index that comprises more than 2,000 single funds. Each of them either has a minimum of USD 50 mn assets under management or a track record of at least twelve months. Finally, we take the mean of the three-month US Treasury Bill rate as risk-free interest rate, while the absolute change in the yield ($\Delta y_t$) is calculated from the 10-year US Treasury Bond (zero coupon). Table 1 contains a set of descriptive statistics for the aforementioned indices.

<table>
<thead>
<tr>
<th>No.</th>
<th>Asset Class</th>
<th>Index Representing the Subportfolio</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Government Bonds</td>
<td>Barclays US Treasury Index (TR)</td>
<td>6.94%</td>
<td>5.01%</td>
<td>5.53</td>
</tr>
<tr>
<td>2</td>
<td>Corporate Bonds</td>
<td>Barclays US Corporate Index (TR)</td>
<td>8.00%</td>
<td>5.98%</td>
<td>7.21</td>
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<tr>
<td>3</td>
<td>Stocks</td>
<td>S&amp;P 500 Index (TR)</td>
<td>6.99%</td>
<td>10.41%</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>Real Estate</td>
<td>S&amp;P/Case-Shiller National Home Price Index (TR)</td>
<td>3.16%</td>
<td>2.44%</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>Hedge Funds</td>
<td>HFRI Fund Weighted Index (TR)</td>
<td>10.33%</td>
<td>6.80%</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1: Descriptive Statistics (Annualized) for Monthly Return Time Series (01/01/1990 – 12/31/2014)

This table shows the mean ($\mu$) and standard deviation ($\sigma$) of the monthly return time series. Each index measures the total return (TR), i.e. includes both coupons and dividends. For the bond subportfolios, the modified duration as of 12/31/2014 is presented. All figures are shown on an annual basis.

3.2 Simulation Input

In order to compare the ALSR with selected classical performance measures, we construct a total of 10,000 hypothetical life insurance companies. More specifically, we draw the asset and liability sides of their balance sheets from appropriate distributions. Regarding the portfolio weights, we resort to uniform

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14 All figures have been obtained from Datastream.
15 For further information please refer to www.hedgefundresearch.com.
distributions with a lower limit of zero and an upper bound equal to typical legal investment limits that insurance companies have to obey for the asset class under consideration. This proceeding is reasonable given the fact that the strategic asset allocations of insurance companies are relatively stable and their tactical choices reflect a buy and hold approach. In line with Braun et al. (2017), we limit corporate bonds holdings to 10 percent, stock investments to 20 percent, real estate holdings to 25 percent, and hedge funds to 5 percent. Once the weights for these four subportfolios have been determined, the fraction of government bonds is chosen such that the accumulated shares of all five asset classes sum to 100 percent. Regarding the liability side of the balance sheet, the leverage ratios \( L_0/A_0 \) are drawn from a beta distribution with shape parameters \( \alpha = 200 \) and \( \beta = 50 \). Subsequently, it is straightforward to derive the remaining ratios for Equations (17) and (19) as follows: \( A_0/E_0 = 1/(1 - L_0/A_0) \) and \( L_0/E_0 = L_0/A_0/(1 - L_0/A_0) \). The durations of the long-term life insurance liabilities \( D_l \), on the other hand, are obtained from a beta distribution with shape parameters \( \alpha = 200 \) and \( \beta = 1200 \). Due to its analytical properties, the beta distribution is highly flexible and therefore frequently used to model random variables that are defined on the finite interval \([0, 1]\), such as default probabilities or recovery rates (see, e.g., Renault and Scaillet, 2004; Jankowitsch et al., 2007). Our parametrizations ensure that the draws for \( L_0/A_0 \) and \( D_l \) are consistent with empirically-observed ranges for these variables.

### 3.3 Simulation Output

Table 2 shows some descriptive statistics for our sample of life insurance companies. It contains the mean, median, standard deviation (S.D.), minimum, and maximum for the central variables that shape a life insurer’s asset and liability structure. Additionally, Figure 3 displays mean-variance representations of the resulting ROAs (subfigure 3a) and ROEs (subfigure 3b), whereas Figure 4 highlights further sample characteristics regarding the fixed income portfolio weight \( \gamma \) (subfigures 4a and b) as well as the ROE volatility (subfigures 4c and d). Overall, these results demonstrate that our simulation has generated a realistic set of 10,000 life insurers which vary widely regarding their asset allocation and liability characteristics.\(^{16}\) Hence, it forms a solid basis for an evaluation of the impact of the ALSR compared to conventional performance measures.

\(^{16}\)Fitch Ratings (2019) provides recent asset allocation figures for U.S. life insurers. The latter show that fixed income investments account for 72 percent, while investments in mortgage loans sum up to approximately 13 percent of the invested assets at the end of 2018. Similar results for German life insurers are provided by BaFin (2018).
### Table 2: Descriptive Statistics of the Life Insurer Sample

This table characterizes our sample of 10,000 hypothetical life insurance companies. The upper part contains the mean, median, standard deviation (S.D.), minimum, and maximum for the five portfolio weights, the expected ROA, and the ROA volatility. The lower part, in turn, shows these descriptive statistics for the leverage ratio, the liability duration, the expected ROL, the ROL volatility, and the asset-liability correlation. The five portfolio weights have been drawn from uniform distributions on the intervals [0,0.1] for corporate bonds, [0,0.2] for stocks, [0,0.25] for real estate, and [0,0.05] for hedge funds. Government bonds represent the residual category and are determined such that the portfolio weights always sum to one. The leverage ratios and modified durations have been drawn from beta distributions with $\alpha = 200$ and $\beta = 50$ as well as $\alpha = 200$ and $\beta = 1200$, respectively. All other variables can be calculated based on these basic parameters according to the model framework introduced in the second section.

#### 3.4 Performance Rankings

**Classical Performance Measures**

Following Eling and Schuhmacher (2007), we set the threshold $\tau$ of the three considered LPM-based performance measures to the average annual risk-free interest rate between January, 1990 and December, 2014 ($r_f = 3.06$ percent). Moreover, the Calmar, Sterling, and Burke ratios are based on the five largest drawdowns, while the three Value at Risk-based performance measures are derived at the significance level of $\alpha = 5$ percent. Finally, for each of the twelve performance measures, we rank the 10,000 hypothetical life insurers according to the measured values and calculate the pairwise rank correlation coefficients of Spearman ($\rho$) and Kendall ($\tau$). Tables 3 and 4 provide an overview of the results. In both tables, the last row displays the respective average rank correlation with the classical performance.
Figure 3: Portfolio Characteristics of the Life Insurer Sample

This figure depicts the sample asset portfolios of the 10,000 life insurance companies in the $\mu - \sigma$ space. Subfigures (a) and (b) focus on mean and standard deviation of the ROA and ROE, respectively. In addition, subfigure (a) includes the efficient frontier for the available asset classes. The figures illustrate that our simulation has generated a realistic set of 10,000 life insurance companies with varying asset and liability characteristics.

measures (1) to (11). Moreover, all rank correlations equal to or greater than 0.70 are highlighted in dark gray.

As indicated by the gray shaded areas in Table 3, some performance measures correlate strongly, while others do not. The rank correlation coefficient of the Sharpe ratio, for instance, lies between 0.03 (Calmar Ratio) and 0.96 (Modified Sharpe Ratio) with the average being 0.62. Regarding the average rank correlation, particularly the Sterling Ratio (0.81), the Burke Ratio (0.80), the Sortino Ratio (0.77), and Kappa$^3$ (0.76) stand out. As was to be expected, we find high rank correlations for performance measures that come from the same family. That is, the Sortino Ratio is highly correlated with Kappa$^3$ (0.96) and Omega (0.85), whereas the Calmar Ratio correlates strongly with the Burke Ratio (0.91) and Sterling Ratio (0.89). We further find high rank correlations between Kappa$^3$ and the Conditional Sharpe Ratio (0.88), as well as the Excess Return on Value at Risk and the Modified Sharpe Ratio (0.87).

\[\text{In order to derive the average Spearman rank correlation, we first transformed the pairwise } \rho \text{-values into } z\text{-values by means of the Fisher transformation (Zar, 2005). We then calculated the respective average } z\text{-value per performance measure and reconverted them into average } \rho\text{-values (Corey et al., 1998). For the average Kendall rank correlation, on the other hand, an additional interim step is needed (Walker, 2003). That is, we first transformed Kendall’s } \tau \text{ into Pearson’s } r \text{ by means of Kendall’s formula (Kendall, 1975). In the second step, we again drew on the aforementioned Fisher transformation, i.e. we derived the average } z\text{-values and reconverted them into average } r\text{-values. Finally, the latter have been retransformed into average } \tau\text{-values.}\]
This figure illustrates how the sample of 10,000 life insurance companies is composed in terms of fixed-income portfolio weight and asset-liability correlation (subfigure 4a), fixed-income portfolio weight and ROA volatility (subfigure 4b), asset-liability correlation and ROE volatility (subfigure 4c) as well as ROA volatility and ROE volatility (subfigure 4d).

Although the results for the pairwise Kendall rank correlations in Table 4 turn out almost identical, they are on a much lower level in absolute terms. More specifically, the average coefficient of the Sharpe Ratio amounts to 0.33 and to 0.51 for the Sortino Ratio. Again, the highest rank correlations are observed between the Burke and Sterling Ratio (0.98), the Sharpe Ratio and the Modified Sharpe Ratio (0.83), as well as the Sortino Ratio and Kappa_3 (0.83). To sum up, our results stand in contrast to those of Eling and Schuhmacher (2007) and Eling (2008), who found average rank correlations greater than 0.95 and 0.96, respectively. Consistent with Zakamouline (2011), we demonstrate that the ten conventional ratios lead to significantly different rank orders.
Table 3: Spearman Rank Correlations of the Performance Measures

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
</tr>
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<tr>
<td>Omega</td>
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<tr>
<td>Sortino</td>
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<td>0.81</td>
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<td>Modified Sharpe</td>
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<td>0.84</td>
<td>0.70</td>
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<td>0.04</td>
<td>−0.11</td>
<td>−0.14</td>
<td>−0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 3: Spearman Rank Correlations of the Performance Measures
This table shows the pairwise Spearman rank correlation coefficients for the twelve performance measures. Additionally, we provide the averages of the rank correlations for the classical performance measures (1) to (10) that have been derived using the Fisher transformation. All rank correlation figures equal to or greater than 0.70 are highlighted in dark gray.

Asset-Liability Sharpe Ratio

The key finding of our analysis is that the rank orders generated by the ALSR exhibit very low correlations with those of the ten classical performance measures. Consider Table 4, e.g., where all results in the last row are equal to or lower than 0.09 in absolute terms. Consequently, as theoretically envisioned, the ALSR is effective and relevant, since it exhibits a heavy impact on the results of a financial performance analysis in the life insurance industry. The reason is that the conventional measures lack a liability-driven perspective and therefore only evaluate $E(\bar{r}_a^e)$ and $\sigma_a$. The ALSR, in contrast, additionally takes into account the degree of asset-liability matching on the insurer’s market balance sheet. Therefore, given two portfolios offer the same $E(\bar{r}_a^e)$ and $\sigma_a$, it consistently promotes the one that is associated with a lower $\sigma_e$ (see Figure 2). To sum up, the choice of measure is critical for the evaluation of a life insurer’s asset management.
Table 4: Kendall’s Tau of the Performance Measures

This table shows the pairwise Kendall rank correlation coefficients for the twelve performance measures. Additionally, we provide the averages of the rank correlations for the classical performance measures (1) to (11) that have been derived using the Fisher transformation. All rank correlation figures equal to or greater than 0.70 are highlighted in dark gray.

4 Conclusion

We complement the existing line-up of performance measures by the Asset-Liability Sharpe Ratio (ALSR), which is specifically tailored to the characteristics of the life insurance industry. It relates a purely market risk-driven average excess return on equity (ROE) to the associated volatility, thereby capturing the stochastic dependence between the assets and the liabilities on the market-value balance sheet and its impact on the overall risk situation of the firm. We deliver four contributions: i) a theoretical motivation, explaining that the ALSR fits in the given context; ii) an illustration how the ALSR can be estimated from publicly-available data; iii) a model-based derivation of the ALSR’s sensitivities that highlights its incentive effects; iv) a simulation study which proves that the ALSR is relevant, as it leads to completely different rankings than conventional measures. Based on our findings, we conclude that the ALSR should be a valuable new tool for performance assessment in the life insurance industry.

We see at least four directions for future research. First, since detailed long-term data on assets and liabilities of a large number of life insurance companies is very difficult to obtain, we based our work on a hypothetical sample and model-based approximations. Accordingly, it would be desirable to repeat the analysis on real-life time series for all relevant variables. Second, we equipped the denominator of the ALSR with the standard deviation of the ROE, which is well suited as long as returns are normally distributed. Although we were able to confirm this assumption for our empirical calibration, it might
be insightful to consider the impact a modification of our “standard ALSR” with lower partial moment or drawdown-based risk measures would have on the rank orders. Third, as the ALSR essentially favors portfolios that optimize the trade-off between the ROA, the ROA volatility, and the correlation of ROA and ROL, one could formulate and solve a tri-criterion portfolio selection in the sense of Hirschberger et al. (2013). On this basis, adept asset managers would be able to explicitly choose their preferred asset allocation from a nondominated surface of ALSR-maximal alternatives. Finally, further research seems to be needed regarding the relevance of the classical ratios, on which the literature is divided. Our results add to the debate in favor of those authors, who argue that the choice of performance measure truly matters.
5 Appendix

Asset Duration vs. Bond Duration

\[ D_a = \sum_{j=1}^{5} w_j D_j \]  
\[ = (w_1 + w_2) \sum_{j=1}^{2} \left( w_j \frac{D_j}{D_r} \right) + \left( w_3 + w_4 + w_5 \right) \sum_{j=3}^{5} \left( w_j \frac{D_j}{D_s} \right) \]  
\[ = \gamma D_r + (1-\gamma) \frac{D_s}{\gamma} \]  

Asset-Liability Correlation Special Cases

if \( \gamma = 1 \), then:

\[ \rho(\bar{r}_t^a, \bar{r}_t^l) = \frac{D_a}{\sqrt{D_a^2}} = 1 \]  

if \( \rho(\bar{r}_t^a, \Delta \tilde{y}_t) = 1 \), then:

\[ \rho(\bar{r}_t^a, \bar{r}_t^l) = \frac{D_a - (1-\gamma) \frac{D_s}{\sigma_y}}{\sqrt{(D_a - (1-\gamma)^2 \frac{D_s}{\sigma_y})^2}} = 1 \]  

if \( \gamma \neq 1 \), then \( \rho(\bar{r}_t^a, \bar{r}_t^l) < 1 \) if:

\[ \left( D_a - (1-\gamma) \frac{D_a}{\sigma_s} \right)^2 < D_a^2 - 2D_a(1-\gamma) \rho(\bar{r}_t^a, \Delta \tilde{y}_t) \frac{\sigma_s}{\sigma_y} + (1-\gamma)^2 \frac{\sigma_s^2}{\sigma_y^2} \]  
\[ (1-\gamma)^2 \frac{\sigma_s^2}{\sigma_y^2} < (1-\gamma)^2 \frac{\sigma_s^2}{\sigma_y^2} \]  
\[ \rho(\bar{r}_t^a, \Delta \tilde{y}_t) < 1 \]

For instance, if \( \rho(\bar{r}_t^a, \Delta \tilde{y}_t) = 0 \), then:
\[ \rho(\tilde{r}_t^a, \tilde{r}_t^b) = \frac{D_a}{\sqrt{(D_a^2 + (1 - \gamma)^2 \sigma_y^2)}} \]  

(38)

Hence, \( \rho(\tilde{r}_t^a, \tilde{r}_t^b) < 1 \) if:

\[ D_a < \sqrt{(D_a^2 + (1 - \gamma)^2 \sigma_y^2)} \]  

(39)

\[ D_a^2 < D_a^2 + (1 - \gamma)^2 \sigma_y^2 \]  

(40)

\[ \gamma < 1 \]  

(41)

**Implicit Asset Duration**

It is possible to derive an implicit asset side duration from the empirical variances and covariances. From Equation (27), we can infer two conditions:

**Condition 1**

\[ \gamma^2 \text{var}(\tilde{r}_t^b) = D_a^2 \text{var}(\Delta \tilde{y}_t) \]  

(42)

\[ D_a = \gamma \frac{\sigma_b}{\sigma_y}, \]

**Condition 2**

\[ 2\gamma(1 - \gamma) \text{cov}(\tilde{r}_t^b, \tilde{r}_t^b) = -2\gamma D_b(1 - \gamma) \text{cov}(\tilde{r}_t^b, \Delta \tilde{y}_t) \]  

(43)

\[ D_a = \gamma \left( \frac{-\text{cov}(\tilde{r}_t^b, \tilde{r}_t^b)}{\text{cov}(\tilde{r}_t^b, \Delta \tilde{y}_t)} \right). \]

Similarly, the following condition is embedded in Equation (30):
Condition 3

\[-\gamma D_1 \text{cov}(\tilde{r}_t^p, \Delta \tilde{y}_t) = D_a D_1 \sigma_y^2\]  \hfill (44)

\[D_a = \gamma \left( - \frac{\text{cov}(\tilde{r}_t^p, \Delta \tilde{y}_t)}{\sigma_y^2} \right).\]

Note that not all three conditions can be perfectly fulfilled at the same time. Hence, we need to numerically estimate $D_a$ by minimizing the sum of squared deviations.
References


