Optimal Reinsurance Programs and Hedging Strategies under Default Risk

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Optimal Coverage Design w/o Default Risk


→ Continuous loss distribution
→ Full coverage is optimal iff premium is actuarially fair
→ With positive cost loading, full coverage above a positive deductible is optimal
→ Thus, XL-reinsurance is optimal

- Introduce insolvency state which occurs with probability $q$
- In insolvency state: contractual payoff is reduced by LGD rate $\tau$
- XL-reinsurance is still optimal
- Deductible is always strictly positive (also for a zero cost loading)
Our research project
→ Introduce measures to mitigate the default risk
→ Optimal design of these measures?
→ Effect on the optimal coverage?
→ Presumed strategies: Explicit hedging and diversification in the reinsurance portfolio
→ Optimal mix of measures?
Hedging of Default Risk: Letter of Credit

**Letter of Credit:** A third party issues a guarantee to resume the primary insurer’s claims deficit, if the reinsurer goes bankrupt.
Diversification of Default Risk

Retention of Primary Insurer

Layer 1

Layer 2

Loss Size X

Reinsurer 1

Reinsurer 2

Reinsurer 3

Reinsurer 4

«Vertical» Diversification

«Horizontal» Diversification

Literature on Multi-Layering (without consideration of default risk)


The Model: One Reinsurer, Default Risk and Hedging

Basic Setting (cf. Arrow, 1963)

- $a_0$: non-stochastic initial wealth, e.g., the primary insurer’s assets
- $X$: loss size, modeled as a non-negative random variable with density $f(x)$ on the support $[0, \bar{x}]$
- $r(X)$: reinsurance reimbursement as contracted given loss size $X$, constraints: $r(0) = 0$ and $r(X) \geq 0$

Default Risk of the Reinsurer (cf. Mahul/Wright, 2004)

- $r(X, D) = (1 - D)r(X)$: actual reimbursement with default risk $D$
- $D$ is an indicator variable with $\mathbb{P}(D = \tau) = q$ and $\mathbb{P}(D = 0) = 1 - q$
- $\tau$ is the loss-given-default (non-stochastic, $1 - \tau$ is the recovery rate); $X$ and $D$ are stoch. independent
- $\pi_r$ is the premium: $\pi_r = (1 + \lambda_r)\mathbb{E}[r(X, D)] = (1 + \lambda_r)(1 - q\tau) \int_0^{\bar{x}} r(x)dx$, with cost loading $\lambda_r \geq 0$

Extension: Hedging Instrument

- Provides payoff $\mathcal{h}(X, D)$, with $\mathcal{h}(X, 0) = 0$ and $\mathcal{h}(X, \tau) = h(X) \geq 0$ (no default risk!)
- $\pi_h$ is the hedging fee: $\pi_h = (1 + \lambda_h)\mathbb{E}[\mathcal{h}(X, D)] = (1 + \lambda_h)q \int_0^{\bar{x}} h(x)dx$, with cost loading $\lambda_h \geq 0$
**The Model: Two Reinsurers, Default Risk and Diversification**

**Default Risks of Two Reinsurers under Diversification**

Two reinsurers: \( r_i(X, D) = (1 - D_i)r_i(X), i = 1,2 \)

Total payoff from the reinsurance portfolio: \( r_1(X, D) + r_2(X, D) = (1 - D_1)r_1(X) + (1 - D_2)r_2(X) \)

\( D_i \) indicates the insolvency of reinsurer \( i \), with \( \mathbb{P}(D_i = \tau_i) = q_i \) and \( \mathbb{P}(D_i = 0) = 1 - q_i \)

\( q_i \) is the marginal default probability of reinsurer \( i \); the joint probabilities are defined as:

\[
Q_i := \mathbb{P}(D_i = \tau_i, D_j = 0), i \neq j; \quad Q_\emptyset := \mathbb{P}(D_1 = 0, D_2 = 0); \quad Q_{12} := \mathbb{P}(D_1 = \tau_1, D_2 = \tau_2)
\]

\( \pi_i \) is the premium or reinsurer \( i \): \( \pi_i = (1 + \lambda_i)(1 - q_i \tau_i) \int_0^x r_i(X)dx \), with cost loading \( \lambda_i \geq 0 \)

**Assumption on Heterogeneity**

\[
q_1 \geq q_2 \quad \tau_1 \geq \tau_2 \quad \lambda_1 \leq \lambda_2
\]

- Reinsurer 1 is assumed to have an equal or **higher default risk**
- Reinsurer 1 is assumed to have an equal or **lower cost loading**
Expected-Utility-Optimization

1. The primary insurer’s equity after reinsurance and hedging (diversification) is:

\[ E(X, D; r, h) = a_0 - \pi_r - \pi_h - X + r(X, D) + h(X, D) \]

\[ E(X, D; r_1, r_2) = a_0 - \pi_r - \pi_h - X + r_1(X, D) + r_2(X, D) \]

2. It is assumed that the primary insurer (or its risk-managers) behaves like a risk-averse decision-maker with concave utility function \( u \) (cf. Froot/Stein, 1998).

3. It is aimed at finding the reinsurance-hedging combination \( (r^*(x), h^*(x)) \), and the reinsurance portfolio strategy \( (r_1^*(x), r_2^*(x)) \), respectively, that maximize the expected utility:

\[ \max_{r,h,\pi_r,\pi_h} \mathbb{E}[u(E(X, D; r, h))] \text{ subject to } \]

\[ r(x) \geq 0, \text{ for } x \in [0, \bar{x}], \]

\[ h(x) \geq 0, \text{ for } x \in [0, \bar{x}], \]

\[ \pi_r = (1 + \lambda_r)(1 - q_r \tau_r) \int_0^{\bar{x}} r(x) dx, \]

\[ \pi_h = (1 + \lambda_h)q \int_0^{\bar{x}} h(x) dx. \]

\[ \max_{r_1, r_2, \pi} \mathbb{E}[u(E(X, D; r_1, r_2))] \text{ subject to } \]

\[ r_i(x) \geq 0, \text{ for } x \in [0, \bar{x}], i = 1,2 \]

\[ \pi = \sum_{i=1}^{2} (1 + \lambda_i)(1 - q_i \tau_i) \int_0^{\bar{x}} \eta_i(X) dx. \]
Optimal Reinsurance-Hedging Combination

Spread $\Delta_\lambda := \lambda_h - \lambda_r = ?$

- Retention of Primary Insurer
- Non-proportional Reinsurance
- XL Cover
- Loss Size X
- Reinsurance Strategy
- Hedging Strategy
Optimal Reinsurance-Hedging Combination (1/3)

Spread $\Delta \lambda := \lambda_h - \lambda_r = 0 \rightarrow$ Full hedging is optimal

- XL cover is the optimal reinsurance policy
- Full hedging of default risk from XL cover is optimal
- Deductible is the same as for the default-free demand model
Optimal Reinsurance-Hedging Combination (2/3)

Spread $\Delta_\lambda := \lambda_h - \lambda_r > 0 \Rightarrow$ Under-hedging is optimal

- Coverage is split in two layers: first layer remains unhedged; second layer is fully hedged
- Deductible is slightly higher than in the default-free model
- More-than-full coverage for layer 1 is optimal, if $0 < \tau < 1$; for $\tau = 1$: full coverage
Optimal Reinsurance-Hedging Combination (3/3)

Spread $\Delta \lambda := \lambda_h - \lambda_r < 0 \rightarrow$ Over-hedging is optimal

- XL cover is optimal reinsurance policy
- More-than full hedging is optimal (speculating on reinsurer’s default)
- Deductible is slightly smaller than in the default-free case
**Numeric Example**

### Parameter Setting
- Initial assets $a_0 = 15$
- Reinsurance cost loading: $\lambda_r = 0.2$
- Default risk of reinsurer: $q = 0.05$, $\tau = 1.0$ (total default)
- Loss size $X$ has a truncated exponential distribution with $\bar{x} = 10$
- Primary insurer has exponential utility function with risk-aversion parameter $\beta$
  - High Risk Aversion: $\beta = 10/a_0$
  - Medium Risk Aversion: $\beta = 5/a_0$
  - Low Risk Aversion: $\beta = 2/a_0$
Optimal Diversification Strategies

Cost loading spread:

\[ \Delta_r = \lambda_2 - \lambda_1 \geq 0 \]

Exchange rate to substitute one unit of coverage from reinsurer \( j \) by one unit of coverage from reinsurer \( i \):

\[ \gamma_{ij} = \frac{(1 + \lambda_i)(1 - q_i \tau_i)}{(1 + \lambda_j)(1 - q_j \tau_j)} \]

Payoff ratio of one unit of coverage from reinsurer \( i \) against one unit of coverage from reinsurer \( j \) under a default of reinsurer \( i \):

\[ \phi_{ij}^i = \frac{(1 - \tau_i)Q_i + (1 - \tau_i)Q_{12}}{Q_i + (1 - \tau_j)Q_{12}} \]
Identical Cost Loadings: $q_1 \geq q_2 \tau_1 \geq \tau_2, \Delta_r = 0$ (1/2)

$\gamma_{21} \geq \phi_{21}^2 \rightarrow$ Vertically diversified XL cover is optimal

Total Indemnification (w/o default)

Loss Size X

Share of

- Reinsurer 1
- Reinsurer 2

Retention of Primary Insurer

Non-proportional Reinsurance
Identical Cost Loadings: $q_1 \geq q_2 \tau_1 \geq \tau_2, \Delta_r = 0$ (2/2)

$\gamma_{21} < \phi_{21}^2 \Rightarrow$ Non-diversification is optimal; saver reinsurer only
Different Cost Loadings: $q_1 \geq q_2 \tau_1 \geq \tau_2, \Delta_r > 0$ (1/3)

$\gamma_{12} \leq \phi_{12}^1 \rightarrow$ Non-diversification is optimal; riskier reinsurer only

Total Indemnification (w/o default)

Loss Size $X$

Retention of Primary Insurer

Non-proportional Reinsurance

Share of

- Reinsurer 1
- Reinsurer 2
Different Cost Loadings: $q_1 \geq q_2 \quad \tau_1 \geq \tau_2 \quad \Delta_r > 0 \quad (2/3)$

$\gamma_{12} > \phi_{12}^1 \text{ and } \gamma_{21} \geq \phi_{21}^2 \rightarrow \text{Horizontal and vertical diversification is optimal}$

Total Indemnification (w/o default)

Loss Size $X$

Retention of Primary Insurer

Non-proportional Reinsurance

Share of

Reinsurer 1

Reinsurer 2
Optimal Reinsurance-Hedging Combination (2/3)

Spread $\Delta_\lambda := \lambda_h - \lambda_r > 0 \Rightarrow$ Under-hedging is optimal

- Coverage is split in two layers: first layer remains unhedged; second layer is fully hedged
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Different Cost Loadings: \( q_1 \geq q_2, \tau_1 \geq \tau_2, \Delta r > 0 \) (2/3)

\[ \gamma_{12} > \phi_{12}^1 \text{ and } \gamma_{21} \geq \phi_{21}^2 \rightarrow \text{Horizontal and vertical diversification is optimal} \]

Total Indemnification (w/o default)

Loss Size X

Layer 1

Layer 2

Share of

Reinsurer 1

Reinsurer 2

Retention of Primary Insurer

Non-proportional Reinsurance
Different Cost Loadings: \( q_1 \geq q_2, \tau_1 \geq \tau_2, \Delta_r > 0 \) (3/3)

\[ \gamma_{12} > \phi_{12}^1 \text{ and } \gamma_{21} < \phi_{21}^2 \rightarrow \text{Horizontal and vertical diversification is optimal} \]
Conclusion

So far

• Costly management of default risk should be concentrated on higher layers
• Basically, preference for reinsurers with low default risk, but only if corresponding costs do not outweigh the utility from a lower default risk
• Combination of horizontal and vertical diversification may be optimal for placing saver, but expensive reinsurer in higher layers.

Still to be researched

• Optimal shares of riskier and saver reinsurer, respectively, under horizontal diversification
• Better understanding of cost and payoff rates $\gamma_{ij}$ and $\phi_{ij}$
• Length of the different layers / position of their attachment points
• Combination of hedging and diversification: substitution and optimal risk-management mix.
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