Sometimes More, Sometimes Less: Prudence and the Diversification of Risky Insurance Coverage

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Mossin's Demand Model as Starting Point

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. *Journal of Political Economy*. 76(4):533-568.

→ Full coverage is optimal iff premium is actuarially fair.

Doherty & Schlesinger's Nonperformance Model

Doherty, N. & Schlesinger, H. (1990). Rational Insurance Purchasing: Consideration of Contract Nonperformance. *Quarterly Journal of Economics*, 105(1):243-253.

→ Over- and under-insurance can be optimal under an actuarial fair premium.

Motivation

Single-Insurer Policy

Bilateral contracting

Insurance Pools

Insurance Pools are permanent risk-sharing arrangements among several insurers

Examples of pools

German Pharma Pool

Austrian Insurance Pool for Terror Risks

Nuclear Insurance Association of Canada

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Co-Insurance Policy Model

- n co-insurers: each co-insurer holds $\frac{1}{n}$ in premium and losses
- $d_{k,n}$ is the probability of k insolvent insurers in the loss state: $\mathbb{P}[F = k]$
- $F \sim BB\left(q \frac{1-\theta}{\theta}, (1-q) \frac{1-\theta}{\theta}\right)$, θ is the joint default correlation factor
- As $\theta \rightarrow 0$, F converges to a binomial distribution (independent defaults)

Co-Insurance Policy Model

Assumed premium principle: *Expected Indemnification x Proportional Cost Loading*

$$\begin{aligned}
 c_I(\alpha_I, n, \theta) &= E[\text{Default Adjusted Indemnification}](1 + \lambda_I) \\
 &= \alpha_I L p (1 - q\tau)(1 + \lambda_I) = c_I(\alpha_I)
 \end{aligned}$$

Diversified Co-Insurance Policy Causes a MPC

Let u be any concave utility function. Then it holds true for the policyholder's expected utility

$$U_{n,\theta} = (1 - p)u(W_{no\ loss}) + p \sum_{k=0}^n d_{k,n} u(W_{k,n})$$

that

$$U_{n,\theta} \leq (<) U_{n+1,\theta}, \text{ for all } n \geq 1 \text{ and } \theta < 1,$$

i.e. the diversification of the co-insurance policy causes a mean-preserving contraction.

Increased Default Correlation Causes a MPS

Let u be any concave utility function. Then it holds true for the policyholder's expected utility

$$U_{n,\theta} = (1 - p)u(W_{no\ loss}) + p \sum_{k=0}^n d_{k,n} u(W_{k,n})$$

that

$$U_{n,\theta_1} \leq (<) U_{n,\theta_2}, \text{ for all } n \geq 1 \text{ and } \theta_1 > \theta_2,$$

i.e. an increasing default correlation in the co-insurance policy causes a mean-preserving spread.

Effect of Diversification on Optimal Level of Coverage

- Assumed the number of co-insurers increases from n to $n+1$ → Natural question: Is it optimal to **increase** or to **decrease** insurance **coverage**?
- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility. **But:**

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Numeric Example:

$$u(x) = -\exp(-\beta x)$$

Initial wealth w	1.5
Loss prob. p	5.0 %
Loss size L	1.0
Default prob. q	1.0 %
Correlation θ	15 %
Cost loading λ_I	0.0

Monotonicity Criterion

Let $\alpha_{I,n}^*$ be the optimal insurance demand for n co-insurers and set

$$w_n^*(x) = w - l - c(\alpha_{I,n}^*) + \alpha_{I,n}^*L - \alpha_{I,n}^*xL.$$

Then, $\alpha_{I,n+1}^* \geq (\leq) \alpha_{I,n}^*$ holds true, if

$$\left(1 - \frac{w-L}{w_n^*(x)}\right) \eta(w_n^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1],$$

where $\eta(x) := -xu'''(x)/u''(x)$ is the policyholder's relative prudence.

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Choi, G., Kim, I., & Snow, A. (2001). Comparative Statics Predictions for Changes in Uncertainty in the Portfolio and Savings Problems. *Bulletin of Economic Research*, 53(1):61-72:

Assuming a MPC shift of the risky asset's rate of return, it holds true that:

The investment in the risky asset increases $\Leftrightarrow \eta(w) \leq 2$

Conclusion

- Insurance policy under default risk can be interpreted as a risky asset
→ Presumably, allows to adapt results from the classic allocation problem
- Diversified co-insurance policies does not necessarily stimulate the insurance demand in the EU-model
- Might also be the case for other risk-mitigating instruments, such as CDS, Letters of Credit, Collateralization, Solvency Regulation, Guarantee Schemes
- Unambiguous results (demand stimulation) can be obtained by restricting on a bounded range of relative prudence (≤ 2)

Thank You

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Single-Insurer Policy and Prudence

Implication for the single-insurer policy ($n = 1$):

$$\left(1 - \frac{w-l}{w_1^*(x)}\right) \eta(w_1^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1], \Rightarrow \alpha_{I,1}^* \leq (\geq) \frac{\text{Optimal demand without default risk}}{1-q(1-r)}$$

Fei, W. & Schlesinger, H. (2008). Precautionary Insurance Demand With State-Dependent Background Risk. *Journal of Risk and Insurance*, 75(1):1-16:

Assuming a zero-mean background risk in the loss state, it holds true that:

Policyholder is imprudent (prudent) $\Rightarrow \alpha^* \leq (\geq)$ Optimal demand without background risk