Sometimes More, Sometimes Less: Prudence and the Diversification of Risky Insurance Coverage

Lukas Reichel, Hato Schmeiser and Florian Schreiber

Institute of Insurance Economics
University of St.Gallen

ARIA Meeting 2018
Chicago, August 06, 2018
Mossin’s Demand Model as Starting Point


\[
\begin{align*}
\text{probability } 1 - p &: w - c_i(\alpha_i), \\
\text{probability } p &: w - c_i(\alpha_i) + (-1 + \alpha_i)l
\end{align*}
\]

→ Full coverage is optimal iff premium is actuarially fair.

Over- and under-insurance can be optimal under an actuarial fair premium.
Insurance Pools are permanent risk-sharing arrangements among several insurers.

Examples of pools:
- German Pharma Pool
- Austrian Insurance Pool for Terror Risks
- Nuclear Insurance Association of Canada
Motivation

**Single-Insurer Policy**

- PH
- IC

Bilateral contracting

**Insurance Pools**

- PH
- PH
- PH

Pool Organization

IC 1  IC 2  IC 3  IC n

Examples of pools

- **German Pharma Pool**
- **Austrian Insurance Pool for Terror Risks**
- **Nuclear Insurance Association of Canada**

**Co-Insurance Policy**

- PH

Co-Insurance Policies are temporary risk-sharing arrangements among several insurers

- IC 1
- IC 2
- IC 3
- IC n

Contract
**Motivation**

Insurance Pools are permanent risk-sharing arrangements among several insurers.

**Examples of pools**
- German Pharma Pool
- Austrian Insurance Pool for Terror Risks
- Nuclear Insurance Association of Canada

---

**Single-Insurer Policy**

**Co-Insurance Policy**

Co-Insurance Policies are temporary risk-sharing arrangements among several insurers.
Co-Insurance Policy Model

- $n$ co-insurers: each co-insurer holds $\frac{1}{n}$ in premium and losses
- $d_{k,n}$ is the probability of $k$ insolvent insurers in the loss state: $\mathbb{P}[F = k]$
- $F \sim BB \left( q \frac{1-\theta}{\theta}, (1 - q) \frac{1-\theta}{\theta} \right)$, $\theta$ is the joint default correlation factor
- As $\theta \to 0$, $F$ converges to a binomial distribution (independent defaults)
Co-Insurance Policy Model

Assumed premium principle: *Expected Indemnification* x *Proportional Cost Loading*

\[ c_I(\alpha_I, n, \theta) = E[\text{Default Adjusted Indemnification}](1 + \lambda_I) = \alpha_I L p (1 - q \tau) (1 + \lambda_I) = c_I(\alpha_I) \]
Let $u$ be any concave utility function. Then it holds true for the policyholder’s expected utility

$$U_{n,\theta} = (1 - p)u(W_{\text{no loss}}) + p \sum_{k=0}^{n} d_{k,n} u(W_{k,n})$$

that

$$U_{n,\theta} \leq (<) U_{n+1,\theta}, \text{for all } n \geq 1 \text{ and } \theta < 1,$$

i.e. the diversification of the co-insurance policy causes a mean-preserving contraction.
Let \( u \) be any concave utility function. Then it holds true for the policyholder’s expected utility

\[
U_{n, \theta} = (1 - p)u(W_{no \ loss}) + p \sum_{k=0}^{n} d_{k,n} u(W_{k,n})
\]

that

\[
U_{n, \theta_1} \leq (<) U_{n, \theta_2}, \text{ for all } n \geq 1 \text{ and } \theta_1 > \theta_2,
\]

i.e. an increasing default correlation in the co-insurance policy causes a mean-preserving spread.
Effect of Diversification on Optimal Level of Coverage

- Assumed the number of co-insurers increases from $n$ to $n+1$ → Natural question: Is it optimal to increase or to decrease insurance coverage?

- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility. **But:**
Effect of Diversification on Optimal Level of Coverage

- Assumed the number of co-insurers increases from $n$ to $n+1 \rightarrow$ Natural question: Is it optimal to increase or to decrease insurance coverage?

- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility. **But:**

```
Numeric Example:
\[ u(x) = -\exp(-\beta x) \]
```

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial wealth</td>
<td>1.5</td>
</tr>
<tr>
<td>Loss prob. ( p )</td>
<td>5.0 %</td>
</tr>
<tr>
<td>Loss size ( L )</td>
<td>1.0</td>
</tr>
<tr>
<td>Default prob. ( q )</td>
<td>1.0 %</td>
</tr>
<tr>
<td>Correlation ( \theta )</td>
<td>15 %</td>
</tr>
<tr>
<td>Cost loading ( \lambda )</td>
<td>0.0</td>
</tr>
</tbody>
</table>

LGD rate \( \tau = 50\% \)

![Optimal Level of Coverage](image)
Monotonicity Criterion

Let $\alpha^*_{I,n}$ be the optimal insurance demand for $n$ co-insurers and set

$$w^*_n(x) = w - l - c(\alpha^*_I,n) + \alpha^*_I,nL - \alpha^*_I,nxL.$$  

Then, $\alpha^*_{I,n+1} \geq (\leq) \alpha^*_{I,n}$ holds true, if

$$(1 - \frac{w-L}{w^*_n(x)}) \eta(w^*_n(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1],$$  

where $\eta(x) := -xu'''(x)/u''(x)$ is the policyholder’s relative prudence.
Monotonicity Criterion

Let \( \alpha_{i,n}^* \) be the optimal insurance demand for \( n \) co-insurers and set
\[
w_n^*(x) = w - l - c(\alpha_{i,n}^*) + \alpha_{i,n}^* L - \alpha_{i,n}^* x L.
\]
Then, \( \alpha_{i,n+1}^* \geq (\leq) \alpha_{i,n}^* \) holds true, if
\[
\left( 1 - \frac{w-L}{w_n^*(x)} \right) \eta(w_n^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1],
\]
where \( \eta(x) := -xu'''(x)/u''(x) \) is the policyholder’s relative prudence.


Assuming a **MPC** shift of the risky asset’s rate of return, it holds true that:

The investment in the risky asset increases \( \Rightarrow \eta(w) \leq 2 \)
Conclusion

• Insurance policy under default risk can be interpreted as a risky asset
  → Presumably, allows to adapt results from the classic allocation problem

• Diversified co-insurance policies does not necessarily stimulate the insurance demand in the EU-model

• Might also be the case for other risk-mitigating instruments, such as CDS, Letters of Credit, Collateralization, Solvency Regulation, Guarantee Schemes

• Unambiguous results (demand stimulation) can be obtained by restricting on a bounded range of relative prudence (≤ 2)
Thank You
Implication for the single-insurer policy ($n = 1$):

\[
\left(1 - \frac{w-1}{w_1(x)}\right) \eta(w_1^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1], \Rightarrow \alpha_{i,1}^* \leq (\geq) \frac{\text{Optimal demand without default risk}}{1-q(1-r)}
\]


Assuming a zero-mean background risk in the loss state, it holds true that:

*Policyholder is imprudent (prudent) $\Rightarrow \alpha^* \leq (\geq) \text{Optimal demand without background risk}*$