The combined risk of liquidity and lapse in life insurance

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Abstract

Surrender options in endowment life insurance contracts result in a lapse risk due to two major uncertainties: 1) the disparity between the ex-ante fixed surrender amount and the contract’s market value and 2) the inability to predict the exact number of policyholders that will surrender their contracts in a given year. Traditionally, only the first uncertainty is taken into account when assessing the lapse risk. However, the insurer’s asset side influenced by the second uncertainty should not be ignored. If many policyholders simultaneously surrender their contracts, the insurer will lose liquidity. This study models the lapse behavior influenced by the stochastic development of the insurers assets and interest rate fluctuations. We measure the effects of the combined lapse and liquidity risk on the insurer’s financial stability and develop risk management strategies for risk reduction.

Keywords: Endowment life insurance · Liquidity risk · Lapse risk · Life insurance
1 Introduction

Life insurance contracts are offered with embedded options, including surrender options, which are generally required by law. The surrender payout amount is fixed at the beginning of the contract and may be higher or lower than the contract’s market value at the time of surrender. This disparity between the amount fixed in the insurance contract and the market value, combined with the uncertainty of when policyholders surrender their contracts, creates a lapse risk. In the Solvency II framework, the lapse risk is the most important part of the underwriting risk for life insurance companies. After market risk, underwriting risk is the second most serious risk (cf. EIOPA (2011)), and liquidity risk, as an important part of the market risk, is closely related to lapse risk.

Liquidity is the ability to meet an immediate cash demand by ongoing cash inflow or the prompt sale of an asset (cf. Claire et al. (2000)). A liquidity problem arises when an entity does not have sufficient cash or liquid assets to meet its immediate obligations. A liquidity risk is especially important for financial institutions as they generally hold substantial liability. Once financial institutions face liquidity problems, their clients typically lose confidence in them, which worsens the situation (the so-called bank-run problem). When this situation spreads, the whole financial market can fall into a severe economic depression.

Insurance companies arguably face lower liquidity risk than banks. Generally, there is no insurance-run risk in the non-life sector, since payments are only made to policyholders in the event of claims. For life insurers, the liabilities undertaken are usually long-term. In market microstructure theory, Vishwanath and Krishnamurti (2009) claim that investors with different investment holding periods treat liquidity risk differently. To match their long-term liabilities, insurers have a greater tolerance of liquidity risk and enjoy the liquidity premium, which partially explains insurers' reaching-for-yield propensity recorded in Becker and Ivashina (2015).

However, insurers are not at all free from liquidity risk. The AIG case in 2008 clearly shows that the liquidity risk could be substantial for insurance companies. Back in 2000, Claire et al. (2000) addressed liquidity issues in response to the National Association of Insurance Commissioners (NAIC), following the concern of a downgraded put on Guaranteed Investment Contracts (GIC) and the unexpected default of General American in 1999. According to this report, a well-managed company faces little liquidity risk when dealing with a large but expected cash outflow (e.g. the maturity of a contract). Nevertheless, if it offers surrender options, the company incurs a great liquidity risk because of an unpredictable cash outflow. Normally, life insurers have longer-term investments to match their long-term liability. However, the liquidity problem arises if many policyholders surrender their contracts at the same time, reducing the life insurer's financial stability and ultimately infringing upon the policyholders' rights.

As the largest institutional investors in the financial market (cf. European Commission (2014)), insurers are the major capital source for less liquid investments central to society such as infrastructure. Regulators must, therefore, monitor liquidity risk without discouraging long-term and less liquid investments. Liquidity risk, though decisive, is hard to quantify. This risk is therefore not considered when calculating either the Solvency Capital Requirement (SCR) for Solvency II or the Risk-Bearing Capital (RBC) for the Swiss Solvency Test (SST). Instead, these two regimes require insurers to perform a qualitative evaluation of their liquidity risk and their risk management procedure.

In the classical asset-pricing framework, two assets with identical expected cash flows and the
same systematic risk are traded at the same price; otherwise, an arbitrage opportunity arises. Hibbert et al. (2009) review the theoretical and empirical literature and confirm the existence of liquidity risk premium. Two features influence liquidity: exogenous factors (e.g. transaction costs) and endogenous factors (e.g. supply/demand pressure and inventory risk). The latter, also known as a market impact, occurs when a large-trading volume enters the market and moves the equilibrium price.

To incorporate the liquidity risk, Cetin et al. (2004) extend the traditional asset pricing model with a supply curve. Jarrow and Protter (2005) apply this model and derive a liquidity risk measure. These two papers assume that investors are price takers, trading assets at a price lying on the asset’s supply curve, a function of the trading volume. Hence, two assets with identical expected cash flows may be traded at different prices according to their corresponding supply curves and trading volumes. Cetin et al. (2004) believe that the liquidity risk can be avoided only if continuous trading is possible. By continuous trading, a large-volume trade can be divided into infinitely small transactions, each having a negligible impact on its market price. Based on this supply curve, Acerbi and Scandolo (2008) form a Marginal Supply Demand Curve (MSDC) and value an investment portfolio with liquidity concerns by solving an optimization problem with a cash constraint.

Abundant literature about surrender options concentrates on the option valuation. Courtadon (1982) and Grosen and Jørgensen (1997) first value surrender options as Bermuda or American options in a life contract with one single premium payment paid upfront. In a theoretical model, policyholders are usually assumed to follow a certain exercise strategy: the optimal stopping strategy introduced by Andersen (1999) and discussed in Kling et al. (2006), Andersen (1999), and Schmeiser and Wagner (2011), the recursive binomial-tree approach discussed in Cox et al. (1979) applied by Bacinello (2003a), Bacinello (2003b), and Bacinello (2005), and the Least Squares Monte Carlo (LSMC) method suggested by Longstaff and Schwartz (2001) and applied by Bacinello (2008), Bacinello et al. (2009), and Chang and Schmeiser (2017). These strategy assumptions suggest policyholders to be rational in a neoclassical finance sense and exercise the surrender option at its optimal value.

The empirical studies examine the factors influencing the lapse rate. The evidence supports the emergency fund hypothesis, in that policyholders surrender their contracts to acquire emergency funds when facing personal/household shocks (e.g. the unemployment in the USA market (cf. Kuo et al. (2003)) and the Korean market (cf. Kim (2005)) and childbirth and divorce in the German market in Gemmo and Götz (2016)). The interest rate hypothesis argues that the lapse rate rises with the increase in the market interest rate. For traditional life endowment policies, the evidence provided by Kuo et al. (2003), Russell et al. (2013), and Kiesenbauer (2012) are hardly conclusive. However, as the present low and even negative interest rate that has not happened before in history, when the interest rebounds, we cannot exclude the interest rate hypothesis. In addition, Barsotti et al. (2016) model the copycat behavior and confirm the correlation and contagion effects among policyholders. This effect results in a mass surrender in which many policyholders surrender their policies after learning that a large number of policyholders have already exercised their surrender options. The first group surrenders for a certain reason (e.g. personal financial difficulties or rising interest rates), whereas the second group follows the first group. This mass surrender gives rise to the "insurance-run" scenario, causing liquidity problems for insurers, which may lead to a systemic crisis.

Facing the current low-interest challenge, insurers attempt to adjust their products towards those
with lower equity capital requirements, such as unit-linked products. However, due to the competitive market and policyholder needs and expectations, traditional insurance products still play a major role in several European markets (cf. Dany (2018)). Table (1) shows that the traditional life products dominate four out of five (except for the U.K.) major life insurance markets in Europe. Current life insurance policies are offered with a guaranteed return slightly above or equal to zero. Once the interest rate rebounds, policyholders may decide to (or be advised to) surrender and obtain new policies with a higher guaranteed rate. This liquidity issue, caused by surrender options and augmented by the contagion effects among policyholders, may lead to systematic risk and thus has attracted close attention from the regulatory authorities (cf. European Systemic Risk Board (2015), p.16, and International Monetary Fund (2016), p.91). Supervisors in some countries (e.g. France, Japan) are even granted powers to suspend surrender payments (cf. Haefeli and Ruprecht (2012)).

<table>
<thead>
<tr>
<th>Country</th>
<th>Traditional Contract(*)</th>
<th>Unit-Linked Contract(**)</th>
<th>Traditional Contract Portion</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>2,802.65</td>
<td>5,494.92</td>
<td>33.8%</td>
</tr>
<tr>
<td>France</td>
<td>6,376.82</td>
<td>1,270.43</td>
<td>83.4%</td>
</tr>
<tr>
<td>Germany</td>
<td>3,598.03</td>
<td>462.39</td>
<td>88.6%</td>
</tr>
<tr>
<td>Italy</td>
<td>2,121.14</td>
<td>564.56</td>
<td>79.0%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1,004.54</td>
<td>404.42</td>
<td>71.3%</td>
</tr>
</tbody>
</table>

All numbers are in billion Euro; * Technical provisions for life products, excluding health and index-linked & unit-linked; ** Technical provisions for index-linked & unit linked products (cf. EIOPA Insurance Statistics (2017))

Table 1: Top five life insurance markets in Europe in terms of technical provisions.

Consequently, this paper combines the lapse risk and the liquidity risk. This combined risk increases as insurers run into difficulties meeting their obligation to pay out the surrender amount due to the increase in lapse rate or in the case of a mass surrender. We model the lapse rate using the LSMC concept. The lapse probability is based on the difference between the surrender option value and the continuous value, thus incorporating the interest rate impact on the lapse behavior. When the lapse rate is large, insurers may face an immediate default threat even though they are solvent in a long-term perspective. Hence, the combined risk can be measured first by the insurer’s immediate default probability. To avoid default cases, insurers are required to hold certain equity capital. This required capital in Solvency II standard formula is defined as the lapse impact on the insurer’s liability only. We adjust the standard formula by measuring the changes on both the asset and the liability of a life insurance company. Insurer’s assets or portfolio with liquidity concerns can be valued by solving a convex optimization problem with an immediate cash constraint. Moreover, we discuss possible liquidity management methods, maximizing the investment return rate without increase in the combined risk (cf. Claire et al. (2000)). The benefit brought by these methods is reflected in our measure and described in this paper.

The rest of the paper is structured as follows. The second section discusses the liquidity risk and how portfolio value evolves when the liquidity is considered. A life contract with a surrender option is constructed and the lapse rate is modeled in the third section. The fourth section analyzes the numerical example and suggests several possible liquidity measurement methods. The final section concludes this paper and provides recommendations for future research.
Modeling Liquidity Risk

The literature discusses trading liquidity and funding liquidity. Trading liquidity is the ease of trading for certain assets and funding liquidity describes the access of the fund to traders or firms (cf. Hibbert et al. (2009)). The liquidity risk in this paper refers to the first definition. The asset’s liquidity risk relates to the asset’s market characteristics (e.g. tightness, depth, and resilience). In a classic asset pricing model, the market is presumably frictionless and competitive.\(^1\) Relaxing both assumptions, Acerbi and Scandolo (2008) value assets with their respective Marginal Supply Demand Curve (MSDC) and an investment portfolio by solving an optimization problem with a certain cash constraint. Tian et al. (2013) apply the MSDC to the real world and propose an exponential function for the MSDC approximations.

The MSDC is a downward curve, combining the traditional asset pricing model with trading volume impact. This curve demonstrates the realizable price discounted by two sources: exogenous transaction costs and endogenous search frictions. Exogenous transaction costs include brokerage fees and other processing costs. Endogenous search frictions occur when investors have difficulties finding counterparties and thus, are forced to make price concessions (cf. Hibbert et al. (2009)). This search friction depends on the size of the trading asset and the intensity of the market. The realizable value is close to the fair market price when the trading size is negligible compared with the entire market’s size. In contrast, investors face difficulties in liquidating their assets when the market has no capacity to absorb the relatively large trading size. This situation results in a "fire sale," as investors are forced to accept a realizable discounted value that is far below the fair market price.

We base our model on the MSDC, an investment portfolio valuation with a liquidity concern discussed in Acerbi and Scandolo (2008) and Tian et al. (2013). Here, an investment portfolio consists of two asset types: 1) buy and hold (BAH) and 2) mark to market (MTM). While BAH is priced as the fair value under the traditional asset pricing model, the MTM’s value is the realizable value lying on its respective MSDC.

The MSDC is a decreasing function: \( m : \mathbb{R}_+ \rightarrow \mathbb{R} \), which satisfies two conditions:

1. \( m(x) \) is non-increasing, (i.e. \( m(x_1) \geq m(x_2) \) if \( x_1 < x_2 \)) and

2. \( m(x) \) is left-continuous with right limits for \( x > 0 \).\(^2\)

\( v(q_i)_{\text{MTM}} \), the value of the MTM asset \( i \) with \( q_i \) units can be expressed as:

\[
v(q_i)_{\text{MTM}} = \int_0^{q_i} m_i(x)dx.
\]

\( v(q_i)_{\text{BAH}} \), the value of the BAH asset \( i \), is its quantity \( q_i \) times its fair price \( m_i(0) \) (trading size has no influence on price):

\[
v(q_i)_{\text{BAH}} = q_i \cdot m_i(0).
\]

\(^1\)In a frictionless market, no transaction costs are faced. In a competitive market, buying or selling any amount of a security can be conducted without any restrictions and without influencing its market price (cf. e.g. Jarrow and Protter (2005)).

\(^2\)In our model, only the long position is considered.
The relationship between two asset types can be formulated as:

\[ v(q_i)^{MTM} \leq v(q_i)^{BAH} = m_i(0) \cdot q_i. \] (1)

Moreover, for the trading currency asset, \( i = 0, \) \( q_0 \) units of the currency values exactly \( q_0 \) \((m_0(x) = m_0(0) = 1)\). In formal terms, we receive:

\[ m_0(0) = 1 \text{ and } v(q_0)^{MTM} = v(q_0)^{BAH} = q_0. \] (2)

A portfolio contains \( I + 1 \) asset classes with each asset size \( q_i \) \((i = 0...I)\). A vector \( q \in \mathcal{P} \), represents this portfolio with \( \mathcal{P} \in \mathbb{R}^{I+1} \), a vector space, denoting the portfolio space. Every asset class \( i \) thus possesses its own MSDC curve, \( m_i \). As \( m_i \) is a function of its respective asset size \( q_i \), we assume the trading size only influences its own trading asset category. More specifically, \( q_i \) does not affect the MTM price of asset \( j \).

\( V(q) \) defines the value of the portfolio bounded by the maximum value \( V^{\text{max}}(q) \) and the minimum value \( V^{\text{min}}(q) \). For the maximum value, all assets in this portfolio are classified as BAH, valued at \( v(q_i)^{BAH} \). The minimum value occurs when there is a tremendous cash demand, and all the assets should be liquidated immediately. Hence, all the assets are classified as MTM with discount value \( v(q_i)^{MTM} \).

For the maximum value of an investment portfolio, we have:

\[ V^{\text{max}}(q) = \sum_{i=0}^{I} v_i(q_i)^{BAH} = \sum_{i=0}^{I} m_i(0) \cdot q_i. \] (3)

For the minimum value of an investment portfolio, we have:

\[ V^{\text{min}}(q) = \sum_{i=0}^{I} v_i(q_i)^{MTM} = \sum_{i=0}^{I} \int_{0}^{q_i} m_i(x)dx. \] (4)

The portfolio value \( V(q) \) boundaries are given by:

\[ V^{\text{max}}(q) \geq V(q) \geq V^{\text{min}}(q). \] (5)

Institutional investment portfolios are usually subject to a cash-position constraint \( a \) proposed by their asset management department or regulatory requirement. \( L(a) \), a closed and convex subset of \( \mathcal{P} \), represents a liquidation strategy to satisfy this constraint. The formal definition can be written as:

\[ L(a) := \{ q \in \mathcal{P} | q_0 \geq a \geq 0 \}. \]

\( V^{L(a)} \) defines the maximum value attainable in this constraint. If \( L(a) = \emptyset \), \( V^{L(a)} \) is defined as \(-\infty\). In such a case, it is not possible to achieve this cash constraint with the existing portfolio \( q \). Ignoring the case of \( V^{L(a)} = -\infty \), \( V^{L(a)} \) can be determined by \( s^* \), where \( s^* \in \mathcal{P} \) is the solution of
an optimization problem. With \( s^* \), 
\[ V^L(a) = V^{\max}(q - s^*) + V^{\min}(s^*) \]
This optimization problem solution, \( s^* \), is unique and given as follows:\(^3\)
\[ s^*_i = \begin{cases} 
\min(\max(0, m^{-1}_i(q_0)), q_0) & q_0 < a \\
0 & q_0 > a, 
\end{cases} \]

where \( m^{-1}_i \) denotes the inverse of the MSDC function \( m_i \). The Lagrange multiplier \( \lambda \) can be seen as the marginal cost of liquidation per trading currency, determined by \( L(s^*) = a - q_0 \).

Each investment asset \( i \) possesses its own MSDC, an exponential function: \( m_i(x) = m^+_i e^{-k_i \sqrt{x}} \).

Figure (1) illustrates the impact of the liquidity index \( k_i \) on an asset's realizable value \( v(q_i) \). With a unit price \( m^+_i = 1 \), \( v(q_i)^{BAH} \) lies on the diagonal line where \( k_i = 0 \), as no liquidity is considered and \( m_i(x) = m^+_i \). When \( k_i \) increases, the realizable value decreases. The distances between the diagonal line and the other curves reflect the liquidity risk, which grows as \( q_i \) increases. Moreover, the larger the \( k_i \), the faster the risk grows. To simplify this model, we set \( k_i \) to a constant. However, in real cases, \( k_i \) can be stochastic and correlated, not only among different \( k_j \), \( i \neq j \) but also with the asset prices \( m^+_i \) and \( m^+_j \).

\[ v^B_A H, k=0 \]
\[ v^B_A H, k=10^{-3} \]
\[ v^B_A H, k=10^{-2} \]
\[ v^B_A H, k=10^{-1} \]
\[ v^B_A H, k=5 \times 10^{-3} \]

Combining the exponential function \( m_i(x) = m^+_i e^{-k_i \sqrt{x}} \) with Equation (3) to (6), the investment

\(^3\)A detailed proof can be found in Acerbi and Scandolo (2008).
portfolio value with liquidity concerns, $V^L(a)$ can be derived as follows:

$$m_i(x) = m_i^+ e^{-k_i \sqrt{x}},$$

$$v(q_i)^{MTM} = \int_0^{q_i} m_i(x) dx = \frac{2m_i^+}{k_i^2} (1 - (k_i \sqrt{q_i})(e^{-k_i \sqrt{q_i}})),$$

(7)

$$V^\text{min}(q) = \sum_{i=1}^{I} \frac{2m_i^+}{k_i^2} (1 - (1 + (k_i \sqrt{s_i^*})(e^{-k_i \sqrt{s_i^*}}))).$$

The optimization solution derived in Equation (6) leads to:

$$s_i^* = \left(\frac{\log(1 + \lambda)}{k_i}\right)^2,$$

with $\lambda = e^x - 1$, $(1 - a/ \sum_{i=1}^{I} \frac{2m_i^+}{k_i^2})e^x - x - 1 = 0$ for $x > 0$.

Therefore, $V^L(a)$ can be derived as:

$$V^L(a) = V^{\text{max}}(q - s^*) + V^{\text{min}}(s^*) = \sum_{i=0}^{I} m_i(0) (q_i - s_i^*) + \sum_{i=1}^{I} \frac{2m_i^+}{k_i^2} (1 - (k_i \sqrt{s_i^*})(e^{-k_i \sqrt{s_i^*}})).$$

(8)

### 3 Model Framework

#### 3.1 Participating Life Insurance Contracts with Surrender Option

We consider a life insurance endowment contract as discussed, for example, in Schmeiser and Wagner (2011) and Chang and Schmeiser (2017). This contract has a guaranteed-surplus feature with a surrender option. The contract duration is $T$ with the time index $t = 1 \ldots T$. $p_x$ denotes the probability that a $x$-aged policyholder survives the next $t$ year. $q_x = 1 - p_x$, on the contrary, refers to the possibility that this policyholder dies in the coming year. We assume that the mortality risk is of pure unsystematic nature. In the beginning of year $t$, policyholders pay an annual fixed premium $B$ condition that at the end of year $t - 1$, their policies are in force (i.e. policyholders are alive and the surrender options have not been exercised yet). With the technical discount rate $r$, the present value (PV) of the premium payment can be written as $B \sum_{t=0}^{T-1} q_x (1 + r)^{-t}$.

The policyholders are entitled to either death or survival benefits. Constant death benefits $\gamma$ are paid to the beneficiaries at the end of year $t$ if the policyholders become deceased during time period $t - 1$ to $t$. Otherwise, the policyholders survive the contract period and receive survival benefits. These survival benefits are guaranteed with the minimum equal to the death benefits $\gamma$ plus a surplus participation. To calculate the premium from the safe side, we assume that the discounting rate is the guaranteed rate $g$ (cf. Linnemann (2003)), and the surplus of the
survival benefits is zero. Hence, the survival benefits are equal to the minimum amount $\gamma$. The relationship can be summarized in the following equation:

$$B \sum_{t=0}^{T-1} t p_x (1 + g)^{-t} = \gamma \left( \sum_{t=0}^{T-1} t p_x q_{x+t} (1 + g)^{-(t+1)} + r p_x (1 + g)^{-T} \right).$$  \tag{9}

With Equation (9), the death benefits $\gamma$ can be determined by:

$$\gamma = \frac{B \sum_{t=0}^{T-1} t p_x (1 + g)^{-t}}{\left( \sum_{t=0}^{T-1} t p_x q_{x+t} (1 + g)^{-(t+1)} + r p_x (1 + g)^{-T} \right)}.$$ \tag{10}

The actual survival benefits include the guaranteed and the surplus equal to the policy accumulated asset at maturity $A_T$. To calculate $A_T$, we first separate the premium payment $B$ into two parts: $B^R_t$ and $B^A_t$. $B^R_t$ is the term life premium calculated as the death probability $(q_{x+t})$ times the difference between the death benefits $\gamma$ and the accumulated asset at $t-1$ or $A_{t-1}$. The remaining $B^A_t$ as the saving premiums becomes part of the accumulated asset at the beginning of $t$. In formal terms, we have:

$$B = B^R_t + B^A_t,$$ \tag{11}

where $B^R_t = q_{x+t-1} \max(\gamma - A_{t-1}, 0)$, and thus $B^A_t = B - B^R_t$.

At the beginning of year $t$, the sum of $A_{t-1}$ and $t-1 p_x B^A_t$ evolves with a return rate, containing both the guaranteed rate $g$ and the surplus with participation rate $\alpha$ via the following equation:

$$A_t = (A_{t-1} + t-1 p_x B^A_t) \cdot (\max(g, \alpha \cdot r_t) + 1);$$ \tag{12}

thereby, $A_0 = 0$, and $r_t$ denotes the insurer’s investment portfolio return at year $t$.

This portfolio contains $\pi$ ($0 \leq \pi \leq 1$) portion of the risky asset with a return rate of $r^A_t$ and $1 - \pi$ portion of the government bond with a return rate of $r^f_t$. While $r^f_t$ is subject to the spot interest rate risk only, $r^A_t$ is related to the investment risk, including the spot rate risk and the asset risk. As such, $r_t$ can be written in the following way:

$$r_t = (1 - \pi) r^f_t + (\pi) r^A_t.$$ \tag{13}

The spot rate risk element is assumed to evolve according to the one-factor Vasicek Model (cf. Vasicek (1977)):

$$(dr^f_t)^P = \kappa (\theta - r^f_t) dt + \sigma_f dZ^P,$$

where $Z^P$ is a Wiener process on a probability space $(\Omega, \phi, \mathbb{P})$. $\sigma_f$ determines how much randomness of $Z^P$ is acquired while $\kappa$ and $\theta$ are two positive constants, representing the speed of the reversion and the long-term mean, respectively. For the risk-neutral measure $\mathbb{Q}$, a constant market price of risk $\lambda$ is introduced, and the interest spot rate under the risk-neutral measure $\mathbb{Q}$ is changed to:

$$(dr^f_t)^Q = \kappa (\theta - \frac{\sigma_f \lambda}{\kappa} r^f_t) dt + \sigma_f dZ^Q,$$
where $Z^Q$ denotes a Wiener process under the risk-neutral measure $Q$.

A one-period spot rate under the Vasicek model for the neutral measure $Q$ and the real world measure $P$ can be derived as follows:

$$(r^f_t)^P = r_0 \cdot e^{(-\kappa \Delta t)} + \theta (1 - e^{-\kappa \Delta t}) + \frac{\sigma_f}{\sqrt{2\kappa}} \sqrt{1 - e^{-2\kappa \Delta t}} Z^P_t,$$

and

$$(r^f_t)^Q = r_0 \cdot e^{(-\kappa \Delta t)} + \left(\theta - \frac{\sigma_f \lambda}{\kappa}\right) (1 - e^{-\kappa \Delta t}) + \frac{\sigma_f}{\sqrt{2\kappa}} \sqrt{1 - e^{-2\kappa \Delta t}} Z^Q_t.$$ (14)

The investment risk denotes the risk the insurer faces when investing in the capital market. This risk source includes the spot rate risk and the asset risk. We assume that the asset risk follows a geometric Brownian motion (with a deterministic asset drift $\mu$ and volatility $\sigma_A$). For the real-world measure $P$, $r^A_t$ can be formally described as:

$$(r^A_t)^P = \mu - \frac{\sigma_A}{2} + \sigma_A \left(\rho Z^P_t + \sqrt{1 - \rho^2} W^P_t\right),$$ (15)

where $\rho$ indicates the correlation coefficient between the spot rate risk and the asset risk.

Under the risk-neutral measure $Q$, the deterministic drift for the asset risk changes into the stochastic interest rate $r^f_t$ derived in Equation (14):

$$(r^A_t)^Q = r^f_t - \frac{\sigma_A}{2} + \sigma_A \left(\rho Z^Q_t + \sqrt{1 - \rho^2} W^Q_t\right).$$ (16)

The contract’s net present value (NPV) $\Pi$ is defined as the PV difference between the two cash flows, the premium payments paid by the insured to the insurer, and the benefits payments paid by the insurer to the insured. Hence, the NPV of a basic contract under the $Q$ measure is:

$$\Pi = E^Q(\gamma \sum_{t=0}^T t p_x q_x t \delta_{t+1} + A_T \delta_T - B \sum_{t=0}^{T-1} t p_x \delta_t),$$ (17)

where $\delta_t = \prod_{i=1}^t (1 + r_i)^{-1}$.

With the predetermined parameters, we can derive the participation rate $\alpha$ (0 ≤ $\alpha$ ≤ 1) such that the basic contract is "fair" for policy and equity holders (i.e. $\Pi = 0$).4

3.2 Modeling Lapse Rate with Least Square Monte Carlo Method

Every year, besides the death benefit, insurers encounter extra cash outflow due to the surrender option offered. In contrast to the death benefit, which is rather unsystematic and can be well

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4As mortality risk is assumed to be unsystematic, the death and survival benefits are expected cash outflow and do not cause liquidity issues. Hence, the fair contract condition for $\Pi$ is not violated even when incorporating liquidity risk.
estimated, lapse rates are harder to predict and can be subject to systematic risk. The policyholders’ surrender behavior, or the lapse rate, relates to individuals’ financial status as well as to the financial market conditions. The emergency fund hypothesis assumes that individuals surrender their contract to acquire the surrender amount when an emergency occurs. According to the interest hypothesis, policyholders find the existing insurance policies less attractive and tend to surrender their contracts when the interest rate increases. Figure (2) shows the lapse rates in the German market from 2005 to 2014. This rate spikes in 2008 during the financial crisis, a difficult financial status or emergency situation for a large number of policyholders.

![Figure 2: Lapse rate in German market (Sources: BaFin (2014))](image)

We adjust the Least Squares Monte Carlo strategy (LSMC) to determine the lapse rate at each year $t$. LSMC was first introduced in Longstaff and Schwartz (2001) for American option pricing. Since then, it has been applied in surrender option valuations in life insurance contracts (cf. e.g. Andreatta and Corradin (2003), Bacinello (2008), and Chang and Schmeiser (2017)) as the optimal exercise strategy.

For the following, if not stated otherwise, rational policyholders’ behavior is defined in a neoclassical finance term: assuming policyholders have no liquidity constraints, rational policyholders will exercise the surrender option at its optimal value (if the option is in the money throughout the contract period). With surrender options ($\vartheta$), policyholders can choose to terminate their contracts at the end of year $t$ ($t < T$) and receive a surrender amount. Assuming that no fee is applied when surrendering a contract at $t = \theta$, the surrender amount equals the policy’s accumulated assets at $\theta$ ($A_\theta$). The PV of this option $\vartheta_\theta$ can be determined as the PV difference of a contract with and without the surrender option as follows:

$$
\vartheta_t = \Pi - E^Q[\gamma \sum_{t=1}^{\theta-1} t p_x q_{x+t} \delta_{t+1} + A_\theta \delta_\theta - B \sum_{t=0}^{\theta-1} t p_x \delta_t]
$$

$$
= E^Q[-\gamma \sum_{t=0}^{T} t p_x q_{x+t} \delta_{t+1} + A_\theta \delta_\theta - A_T \delta_T + B \sum_{t=0}^{T} t p_x \delta_t].
$$

(18)
following the LSMC strategy, by the end of every contract year, the exercise value and the
continuous value are compared. Based on the adjusted LSMC described in Chang and
Schmeiser (2017), the continuous value \( \tilde{C}(\vartheta) \) and the exercise value \( \vartheta_t \) are estimated as \( \hat{C}(\vartheta) \) and \( \hat{\vartheta}_t \) at the end of each year \( t \). These estimated values are approximated with the coefficients \( \alpha^k \) and \( \alpha'^k \), together with the basis functions, \( v^k \) \( v'^k \), and parameters \( x^k_t \), representing all relevant information accessible at \( t \). In our model, \( x^k_t \) includes the discount rate \( r^k_t \) and the surrender amount at \( t \), or \( A_t \). These approximations are given by the following equations:

\[
\hat{C}(\vartheta) = E^Q[\tilde{C}(\vartheta)|F] \approx \sum_{k=0}^{K} \alpha^k v^k(x^k_t,...,x^k_T),
\]

\[
\hat{\vartheta} = E^Q[\tilde{\vartheta}|F] \approx \sum_{k=0}^{K} \alpha'^k v'^k(x'^k_t,...,x'^k_T).
\]

The second approximation estimates \( \alpha^k \) and \( \alpha'^k \) as \( \hat{\alpha}^k \) and \( \hat{\alpha}'^k \) as follows:

\[
\hat{\alpha}^k_t = \arg \min \left\{ \sum_{n=1}^{N} \left[ \tilde{C}(\vartheta) - \sum_{k=0}^{K} \alpha^k v^k(x^k_t,...,x^k_T) \right] \right\},
\]

\[
\hat{\alpha}'^k_t = \arg \min \left\{ \sum_{n=1}^{N} \left[ \hat{\vartheta}_t - \sum_{k=0}^{K} \alpha'^k v'^k(x'^k_t,...,x'^k_T) \right] \right\}.
\]

Rational policyholders decide whether to surrender their contracts by comparing the option’s estimated exercise value with the continuous value. The continuous value is dependent on its optimal stopping point \( \vartheta^* \), derived in the following steps.

In the following equations, \( n \) denotes the \( n \)th simulation path \( (n = 1,...N) \).

1. Set all optimal exercise points at \( T \), i.e. \( ^n\vartheta_T = T \); thereby, the options expire with a value of zero, \( ^n\vartheta_T = 0 \).

2. One year backwards at \( T - 1 \): for a continuous value, we have \( ^n\tilde{C}(\vartheta) = ^n\tilde{\vartheta}_{T-1} = 0 \). The exercise value \( ^n\tilde{\vartheta}_{T-1} \) can be estimated with the previous formulas. Exercise the option if \( ^n\tilde{\vartheta}_{T-1} > ^n\tilde{C}(\vartheta) \), and set \( ^n\vartheta^* = T - 1 \). Otherwise, \( ^n\vartheta^* \) remains unchanged.

3. Backwards for \( t = T - 2,...1 \): with estimated \( ^n\tilde{\vartheta}_t \) and \( ^n\tilde{C}(\vartheta) \), exercise the option if \( ^n\tilde{\vartheta}_t > ^n\tilde{C}(\vartheta) \), and set \( ^n\vartheta^* = t \). Otherwise, \( ^n\vartheta^* \) remains unchanged.

The LSMC strategy incorporates up-to-date information \( (x^k_t) \). These results hence simulate the rational policyholder’s behavior explained by the interest-rate hypothesis. This hypothesis, however, only partially explains the lapse behavior. The lapse rate should be modeled within a probabilistic function as follows:

\[
\varphi_t = f(\dot{\tilde{C}}(\vartheta), \dot{\vartheta}_t),
\]

where \( 0 \leq f(\dot{\tilde{C}}(\vartheta), \dot{\vartheta}_t) \leq 1 \), and the lapse rate \( (\varphi_t) \) equals the likelihood of exercising the surrender due to the option value. Here we set \( f \), a logistic function:

\[
f(\dot{\tilde{C}}(\vartheta), \dot{\vartheta}_t) = 1/(1 + \exp(-\phi_t(\dot{\tilde{C}}(\vartheta) - \dot{\vartheta}_t))/\psi
\]

5More details are given in Chang and Schmeiser (2017)
suggested in Luce et al. (1963).

φₜ and ψ are two constants. φₜ controls the noise or the individual lapse behaviors that the interest rate hypothesis does not explain. ψ and φₜ are determined so that the 99.5% quantile of ϱₜ in the simulations is 30%. If no φₜ > 0 exists for the 99.5% quantile of ϱₜ equal to 30%, the surrender option is deep out of the money, and the interest-rate hypothesis explains no lapse behavior. In such a case, only policyholders in need of emergency funding surrender their contract.

Insurers encounter an additional potential cash outflow due to the surrender payout at the end of each year. To calculate the asset portfolio value in Equation (8), aₜ is the cash flow constraint for the optimization problem solution s*. We determine aₜ as the surrender payout. This payout equals the exercise portion ϱₜ times the surrender amount Aₜ:

\[ aₜ^n = Aₜ^n \cdot ϱₜ^n. \]  

4 Numerical Example

4.1 Parameter Choices

We consider a life insurance endowment contract with the following parameters: annual premium B equals 12,000 currency units and the duration is T = 10. For Equation (14), (15), (16), and the guaranteed interest rate g, we take the parameters used in Braun et al. (2015), calibrated based on observations of the German market data. With Equation (10), the death benefit yields 127,921.5 currency units. The insurance company sells this life contract to 5 \cdot 10^4 policyholders with the same mortality probability.\(^7\)

The investment strategies only allow long positions, and the insurer invests all the premiums received: q₀ = 0 and qᵢ ≥ 0, for i = 1...I. The insurer’s investment portfolio contains I = 1 + I₄ asset categories: one government-bond category with q₁ units and I₄ risky assets with qᵢ units, i = 2...I. I₄ is fixed to be 1 in the original setup.

In Equation (7), the liquidity indices kᵢ, kᵢ > 0 for i > 0, as all the holding assets, government bonds included, contain liquidity risk. k₁ is set to be ln((100 − 0.11)/100)/10,000 ≃ −1.1 \cdot 10⁻⁵, or 11 basis points with a trade value of $10,000 per transaction. We set kᵢ = ln((100 − 0.5)/100)/100ᵢ⁺ ≃ −5 \cdot 10⁻⁵ for i > 1, or 50 basis points per $10,000 trade-value transaction. The results are generated by Monte Carlo simulations with the same set of 10⁵ iterations for all simulations.

Table (2) lists the relevant parameters for the base case.

\(^6\)As discussed in the next section, in the Solvency II standard formula, the extreme case for mass lapse is set to 30%.

\(^7\)Mortality probabilities are for a 30-year-old US woman in 1994 based on the data from the HMD (Human Mortality Database).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>annual premium per contract (in currency units)</td>
<td>12,000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>death benefits (in currency units)</td>
<td>127,921.5</td>
</tr>
<tr>
<td>$g$</td>
<td>guaranteed interest rate</td>
<td>1.25%</td>
</tr>
<tr>
<td>$x$</td>
<td>initial age</td>
<td>30</td>
</tr>
<tr>
<td>$T$</td>
<td>time to maturity (in years)</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>interest rate volatility</td>
<td>0.6%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>interest rate reversion speed</td>
<td>8%</td>
</tr>
<tr>
<td>$r_0$</td>
<td>initial interest rate</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>long-term interest rate mean</td>
<td>2.4%</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>market price of risk</td>
<td>$-0.18$</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>asset volatility</td>
<td>19.1%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>deterministic drift for the investment return under the empirical measure $\mathbb{P}$</td>
<td>8.2%</td>
</tr>
</tbody>
</table>

$i$ | asset class for $i = 0, 1, \ldots, I$ with $I = I^A + 1$, $i = 0$ for cash position, $i = 1$ for the government-bond class, and $i = 2 \ldots I$ for a risky asset with $I^A$ classes. If not stated otherwise, $I^A = 1$. |

$q_i$ | asset unit for asset class $i$: $q_0 = 0$ for the cash position, and $q_i \geq 0$ for $i = 1, \ldots, I$ for other asset classes. |

$k_i$ | liquidity risk index: $k_1 = -\ln((100 - 0.11)/100)/10,000^{0.5} \approx -1.10 \cdot 10^{-5},$ and $k_i = -\ln((100 - 0.5)/100)/10,000^{0.5} \approx 5 \cdot 10^{-5},$ for $i = 2 \ldots I$. |

Table 2: Parameters used for the base case scenario

### 4.2 General Setup

Figure (3) demonstrates the participation rate, $\alpha$, under the fairness condition for different $\pi$ (asset allocation) with different $\theta$ (long-term interests). On the one hand, for the same guaranteed rate, policyholders demand a higher $\alpha$ for higher $\theta$. On the other hand, with the same $\theta$, higher $\pi$ leads to lower $\alpha$. When investing more in risky assets, insurers bear a higher risk and thus offer less of a participation rate to reach a risk-adequate return for the shareholders. Figure (4) presents the distribution of the contract return, $\max(g, \alpha \cdot r_t)$ described in Equation (12). Although larger $\pi$ leads to a higher discount rate, the slopes are relatively flat especially when $\pi > 20\%$, as the participation rate $\alpha$ decreases with the increasing $\pi$.

Figure (5) shows the insurer’s ultimate profit per annual premium when the contract matures. This figure does not consider the costs of holding capital. The average profit increases with
larger $\pi$ while the impact of different $\theta$ on the profit is rather limited. In addition, the profit ranges expand as $\pi$ increases with relatively limited downside development. Without any capital requirement, insurers are, therefore, motivated to invest 100\% of their capital in risky assets to reach the maximum profit with a bounded negative risk. Regulators should, therefore, design an adequate capital requirement scheme in which holding a larger risky asset requires insurers to hold a higher equity capital amount. Insurers then face trade-offs between reaching higher expected returns and holding less equity capital.

4.3 Results with Modeled Lapse Rate

Figure (6) shows the lapse behavior distribution, with the 99.5\% quantile set at 30\%. At the later stage, the option value can be predicted more accurately. The lapse behavior is thus more
concentrated. On the contrary, at the early age, the lapse rate is around zero except for the extreme cases. Due to this concentration, from Figure (7), the average lapse rate is higher at the later stage. At this later stage, policyholders are more certain about the surrender value and can use the surrender option more precisely. With \( \pi = 0 \), as the return rate is more stable, the option value can be predicted relatively easier. Thus, the lapse rate is higher as the policyholders are more likely to surrender their options once the option is in the money. When \( \pi > 0 \), the asset allocation has trivial influence on the modeled lapse rate.

![Figure 6: Lapse rate distribution with different contract year](image)

![Figure 7: Average lapse rate for different asset allocation](image)

Insurers go bankrupt immediately when the cash payment \( (a_t) \) cannot be achieved even if they liquidate all of their asset holdings (i.e. \( L(a) = 0 \) and \( V_{L(a)} = -\infty \)). With the lapse rate above, Figure (8(a)) shows that the bankruptcy threat becomes more serious as life contracts develop. From Figure (8(b)), insurers face a much larger bankruptcy probability when \( \pi > 80\% \) with the modeled lapse rate.

This growing probability does not result from the increases in the lapse amount. From Figure (8(c)), the surrender amount per annual premium (black line) is independent from the asset allocation except for \( \pi = 0 \) because of the little impact of \( \pi \) on the lapse rate. In contrast, the red line shows the portfolio value reduction caused by the lapse amount \((V^0 - V_{L(a)})\) with \( V \) calculated in Equation (8) per annual premium. Portfolio value decreases substantially when \( \pi \) reaches around 80%. Subsequently, the bankruptcy probability soars as \( \pi > 80\% \).
Figure 9 presents the results with the following four scenarios considered:

1. An unexpected spike in the interest rate at $t = 0$: The long-term interest rate mean ($\theta$) increases to 3.4% as soon as the contract is issued.

2. An unexpected spike in the interest rate at $t = 5$: The long-term interest rate mean ($\theta$) increases to 3.4% at $t = 5$.

3. An unexpected drop in the interest rate at $t = 0$: The long-term interest rate mean ($\theta$) decreases to 1.4% as soon as the contract is issued.

4. An unexpected drop in the interest rate at $t = 5$: The long-term interest rate mean ($\theta$) decreases to 1.4% at $t = 5$.

From Figure (9(a)), when the interest rate increases, the lapse rate increases accordingly. The earlier the interest rate jumps, the higher the lapse rate inflates. For cases with decreasing interest rates and with $t < 2$, the lapse rate is irrelevant in this model, as in the 99.5% quantile, the continuous value is higher than the exercise value, or the surrender option is deep out of the money. In such a case, the interest-hypothesis suggests no lapse in behavior at all. Insurers benefit from the negative option value if policyholders surrender the contract, while the (partial) rational policyholders do not exercise their surrender option.

The interest risk enlarges the lapse risk. The bankruptcy probability increases right after the interest rate unexpectedly climbs. At the later stage, as there are fewer policyholders left and fewer accumulated assets, the bankruptcy probability decreases compared to the base case in which the interest rate develops as expected. The increase in the bankruptcy probability is caused not only by the larger surrender amount due to the higher lapse rate but also by the severer portfolio value reduction. Figure (9(c)) and Figure (9(d)) present the interest impact.
on lapse amount and the portfolio value reduction respectively. Both values increase when the interest rate increases unexpectedly. If the interest rate falls unexpectedly, the bankruptcy probability drops as policyholders are less likely to surrender their profitable contracts.

![Graphs showing relationship between lapse rate and contract year, bankruptcy probability and contract year, lapse amount and contract year, and portfolio value reduction and contract year.](image)

(a) Relationship between lapse rate and the contract year with $\pi = 1$
(b) Relationship between bankruptcy probability and the contract year with $\pi = 1$
(c) Relationship between lapse amount and the contract year (per annual premium with $\pi = 1$)
(d) Relationship between the portfolio value reduction and the contract year (per annual premium with $\pi = 1$)

Figure 9: Bankruptcy probability, lapse amount, and the portfolio value reduction for different unexpected interest rate developments (with $\pi = 1$)

4.4 Liquidity and Lapse Risk

The lapse rate is typically assumed to be constant, ranging between 5% and 10% depended on expert opinions. For different lapse rates, Figure (10) demonstrates different bankruptcy probabilities that insurance companies face as the contract develops in time. The bankruptcy threat enlarges as the contract continues even with a steady lapse rate due to an accumulation of surrender amounts. Moreover, the bankruptcy risk increases substantially with $\varrho = 10\%$ compared to $\varrho = 5\%$. Bankruptcy risk increases as $\pi$ comes closer to 1. From Figure (11), the bankruptcy rate grows abruptly for $\pi > 90\%$. Under Solvency II, the regulatory authority should demand
extra liquidity to avoid a bankruptcy probability greater than 0.5%.

Figure (12) shows the portfolio value deducted by the surrender amount, $(V^L(a) - a)$ at $T - 1$. In general, larger values of $\pi$ with higher investment returns lead to higher portfolio values. However, when $\pi > 95\%$, this value drops substantially as in some simulation paths, insurance companies default due to liquidity problems.

**Risk measure under Solvency II**

The standard formula (SF) in Solvency II determines the required solvency capital due to lapse risk. This capital is calculated as the adverse change in the insurer’s liability or the surrender amount due to the unexpected lapse behavior. This measurement can be expressed as:

$$Lapse^{SF} : \max(a_t^{extreme} - a_t^{normal}, 0),$$

where $a_t^{extreme}$ and $a_t^{normal}$ denote the expected surrender payouts in the extreme scenario and in the normal case. Three extreme scenarios are considered in Solvency II, and the lapse risk is the highest amount generated among these three scenarios: the increase in the lapse rate, the mass lapse event, and the decrease in the lapse rate. As in our model, the risk increases with an increase in the lapse rate, we here consider the first and the second extreme scenarios only (cf. EIOPA (2014)):

I. The increase in the lapse scenario: a 50% increase with respect to the assumed lapse rate.

II. The mass lapse scenario: a 30% lapse rate is suggested for the retail business.

While SF considers the lapse rate’s influence on the insurer’s liability side, the lapse rate may have an even higher impact on the insurer’s portfolio value as shown in the previous section. The surrender options prevent insurance companies from perfect duration matching. Insurers’ portfolio value, $V^L(a)$, decreases with a fairly high unexpected cash constraint (as the sudden increase in lapse rate ($a$ in Equation (8))).
Figure (13(a)) and (13(b)) show the changes in the insurer’s portfolio value when the extreme scenarios occur. These changes increase substantially when $\pi$ reaches 80% for scenario I and 60% for scenario II. When $\pi$ is close to 1, the curves become flat or even decrease as the insurance company faces bankruptcy threat even in the normal case. Scenario II usually results in a higher lapse risk compared with scenario I, as generally the lapse rate is expected to be less or around 10%. From Figure (13(c)), the lapse risk calculated by SF is independent from the asset allocation. The higher assumed lapse rate generates a lower shock for the extreme scenario II. The lower lapse rates, therefore, causes a slightly higher lapse risk under the SF. From Figure (13(a)) and (13(b)), insurers are more likely to go bankrupt when investing in more risky assets. The risk calculated under SF, however, does not reflect this trend, as SF only considers the risk caused by the lapse rate changes.

Figure 13: Changes in portfolio value and lapse risk calculated under SF

To reflect the combined risk including both the lapse and the liquidity impact, we adjust the SF formula to the adjusted formula (AF) as follows:

$$Lapse^{AF} : \max(Portfolio\ Value^{normal} - Portfolio\ Value^{extreme}, 0)$$

with portfolio value=$V^L(a)$−$a$ when $V^L(a) > -\infty$, or portfolio value=$-a$ when $V^L(a) = -\infty$.

Figure (14) shows $Lapse^{AF}$, the combined risk under this measure, and Figure (15) presents the difference between these two measures $(Lapse^{AF} - Lapse^{SF})$. This difference shows the extra liquidity risk the standard formula fails to calculate. The difference is close to zero until $\pi$ reaches 60%. With more than 60% risky assets in the portfolio, insurers are more likely to face a liquidity threat. Due to this difference, Figure (16) compares the optimal asset allocation, generating the highest NPV, assuming insurers are required to hold extra capital as $Lapse^{AF}$ or $Lapse^{SF}$ respectively. With SF, the extra capital the insurer is required to hold is irrelevant to the asset allocation. Therefore, they will be motivated to always invest 100% in the illiquid asset so to generate the highest liquidity premium. Hence, insurers will be subject to higher liquidity risk that SF fails to detect. The optimal risk allocation is around 60% if the risk capital is calculated under the AF and as long as $R_{cc}$ is positive. The higher the $R_{cc}$, the more expensive the extra capital is. Insurers will therefore tend to invest less in the illiquid asset, giving up some part of the liquidity premium.
4.5 Liquidity Management

The previous section shows that the lapse risk can be greatly underestimated if the liquidity impact on the asset side is excluded. Lapse risk and liquidity risk are closely related and should be estimated simultaneously. In the following, we suggest several ways to manage the liquidity risk to mitigate the combined risk of liquidity and lapse. Contrary to SF, the AF considers the impact on the portfolio value and, therefore, can reflect the benefit generated by these liquidity management methods. Insurers will only be encouraged to tackle the liquidity issues when the AF is in force.

4.5.1 Cash Flow Matching

Cash flow matching is the most common and easiest method to reduce the combined risk. Insurers use the premium received in the beginning of the next term to pay the surrender amount that occurs in the last term. As a result, the insurers do not need to liquidate their illiquid assets and suffer a fire sale discount.

Figure (17(a)) shows that this method reduces the bankruptcy probability. From Figure (17(b)), this method generates a much smaller risk compared with the result without any mitigation method. Note that although the cash flow matching provides a rather efficient solution and reduces the bankruptcy probability, it can lead to even more volatile future cash flows. When lapse rate increases, not only does the surrender amount increase but the expected premium also decreases. Therefore, the lapse risk under this method climbs steeply when $\pi$ reaches 80%. From Figure (17(c)), with cash flow matching, insurers invest less than 80% in the illiquid asset.

4.5.2 Liquidity Reduction in MSDC

Another way to reduce the combined risk is to deal with the MSDC function: By reducing the liquidity index ($k$), insurers suffer less of a price discount when they liquidate their assets. In practice, investing in a more liquid market, though lowers the liquidity index, forgoes the liquidity premium and generates lower return rates. Insurers may also define a maximum delivery period, during which they should make the surrender payment. With this predefined period in contracts, insurers can pay out the surrender amount immediately in the normal case. When facing liquidity...
issues, insurers have freedom to delay the payment and prolong the liquidation period of assets, having a minor impact on the market equilibrium price. With a longer liquidation period, insurers can separate one large-amount transaction into numerous small-scale transactions, each independent of the others. In our example, \( m(x) = m^*e^{-k\sqrt{x}} \). If insurers are able to separate one transaction into \( NT \) (with \( NT > 1 \)) transactions, the new MSDC becomes:

\[
NT \cdot m(x/NT) = NT \cdot m^*e^{-k\sqrt{x}/NT} = NT \cdot m^*e^{-k'/\sqrt{x}},
\]

where \( k' = k/\sqrt{NT} \) (with \( k' < k \) as \( NT > 1 \)).

Liquidity issues can also be tackled by diversifying the investment asset. So far, with \( I^A = 1 \), the insurer is assumed to invest in only one illiquid asset (or all assets’ liquidity is tightly correlated). Liquidity issues decrease if it is possible to find different investment assets with their liquidity independent among one another (\( I^A > 1 \)). Figure (17) shows the result when \( I^A = 5 \).\(^8\)

Figure (17(a)) and Figure (17(b)) demonstrate that bankruptcy probability (overlaps with the blue line) and the combined risk can be significantly reduced if \( NT = 100 \) or if \( I^A = 5 \). Due to the reduced capital required, \( \pi \) in the optimal asset allocation increases especially in the case of \( NT = 100 \).

\[
\text{(a) Bankruptcy probability} \quad \text{(b) Combined risk under AF and lapse risk under SF} \quad \text{(c) Optimal asset allocation with different } r_{cc}
\]

Figure 17: Results under liquidity management for lapse rate equals 5%

5 Conclusions

The interest rate has remained low for a long time in most financial markets, but it may increase at any time in the near future. Many national banks begin to raise their interest rates gradually. The lapse risk in the life insurance sector is especially associated to the interest-rate environment. The LSMC setup suggests a method to model the lapse rate influenced by changes in the interest rate. This impact is described in the interest-rate hypothesis. Under this model, the bankruptcy probability is influenced by both the interest rate development (through lapse risk in the traditional calculation) and the asset allocation (through liquidity risk).

\(^8\)Here, we assume that even though the liquidity is independent, different risky assets’ values are completely positively correlated for an easy comparison.
This paper shows that the lapse risk deteriorates when the liquidity risk is taken into account. This combined risk of liquidity and lapse should be carefully addressed so to derive an appropriate asset allocation strategy. Solvency II considers the lapse risk as the impact of the lapse rate change on the insurer’s liabilities. This measure may generate a wrong incentive for insurers, resulting in a reaching-for-yield investment behavior, as the asset risk, other than the credit risk, does not cause any extra solvency capital requirement. While insurers enjoy a high liquidity premium, the liquidity risk may become a threat to the insurer’s solvency, to policyholders, and to the stability of the economy in general.

Liquidity management methods include cash flow matching and liquidity reduction in MSDC. Within one life insurance sector, cash flow matching without proper asset allocation causes higher future cash flow volatility. Nevertheless, cash flow matching among different units may alleviate this problem. Although regulators typically do not allow subsidization across different legal entities, an insurance conglomerate can provide liquidity across units with ring-fencing provisions or through repurchasing agreements (cf. Haefeli and Ruprecht (2012)). For MSDC, several factors such as market depth determine the liquidity index. The market depth varies with asset categories and macro-market conditions. Another way to prevent a severe realized value discount is to prolong the liquidation period. A longer liquidation period allows investors to liquidize their illiquid assets by conducting numerous independent transactions, each involving negligible transaction units. In practice, the regulatory authority in Switzerland is empowered to prolong the payment duration up to one and half years. The supervisors in France and Japan have the authority to suspend the surrender payments under certain circumstances. These forces have not yet been used. Instead of solving the liquidity problem, they may cause panic, actually bringing forward the issue when they are in force. To avoid this problem, insurers can include the maximum-delivery-period clause for the surrender option in the policy contract. As shown in the paper, a liquidity problem can also be alleviated through asset diversification. Regulators should use adequate risk measures to motivate the insurers to acquire liquidity management methods.

Both the policyholder’s behavior and the liquidity conditions are stochastic. The liquidity condition varies dramatically with the market situation and has a direct impact on the holding of the assets’ realizable value. With a simplified numerical example, this paper shows that liquidity plays an important role in a life insurer’s solvency condition. Different asset classes possess different MSDC functions and liquidity indices, depending on the market conditions. Therefore, insurers and regulators should consider the impact of liquidity and revise the liquidity assumption on a regular basis.

Several issues could be discussed in future research: the combined risk modeled in this paper considers the liquidity impact on the lapse risk, while the liquidity situation is assumed steady. More specifically, we set the liquidity indices constant in the model. However, in reality, the indices depend on financial market conditions and are correlated among asset categories and stochastic asset prices. A framework considering this factor could provide additional important insights in the analysis of liquidity risk in the life insurance sector.
References


BaFin (2014). Risk situation in the German financial system. *Deutsche Bundesbank Eurosystem*.


European Commission.
Human Mortality Database (2016). University of California Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). www.mortality.org.