ITALY IN THE EUROZONE:
Technical Appendix

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Abstract

We present a three region monetary model of the world economy. Italy is modeled in detail, including investment and nominal wage rigidity, the rest of the Eurozone is stylized, and the rest of the world is largely exogenous. This Technical Appendix accompanies the paper 'Italy in the Eurozone' by Keuschnigg, Kirschner, Kogler and Winterberg (2020).

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1 Model of a EZ Country

This Technical Appendix documents a three region monetary model. Italy is modeled in detail, the rest of the EZ is modeled rudimentary, with an exogenous income process, and separate monetary policy. The model can accommodate two separate regimes, a monetary union between Italy and the rest of the Eurozone with centralized monetary policy and a fixed internal exchange rate, and independent monetary policies with flexible exchange rates. The model of Italy features nominal wage rigidity, similar to Walsh (2010, chapter 8) and Gali (2015, mostly chapter 6). The model includes fiscal policy and a banking banking sector, taking deposits and equity capital, and lending to firms.

1.1 Production

Final and Intermediate Goods: Demand for final goods is Y_t. The production process involves two stages. Input producers use capital and labor to produce components y_{jt}. Final goods producers (headquarters, HQ) use technology $Y_t^g = \left[\int_0^1 y_{jt}^{\frac{\sigma_v - 1}{\sigma_v}} dj\right]^{\frac{\sigma_v}{\sigma_v - 1}}$, $\sigma_v > 1$, to assemble final goods from components. Given aggregate demand Y_t^g, HQs minimize costs which results in demand for varieties and a price index,

$$y_{jt} = (P_t/p_{jt})^{\sigma_v} Y_t^g, \quad P_t = \left[\int_0^1 p_{jt}^{1-\sigma_v} dj\right]^{1/(1-\sigma_v)}, \quad P_t Y_t^g = \int_0^1 p_{jt} y_{jt} dj. \tag{1}$$

Multiply demand y_{jt} by p_{jt}, integrate and use the price index to get spending $P_t Y_t^g$.

HQ also accumulate capital and hire labor, and rent factor services to input producers on competitive internal markets at prices w_t^L and w_t^K. Input firms are small, taking factor prices and final goods price index P_t as given, but enjoy local market power since their input is differentiated. By (1), the perceived price elasticity of demand is, thus,

$$-\frac{p_{jt}}{y_{jt}} \frac{\partial y_{jt}}{\partial p_{jt}} = \sigma_v > 1. \tag{2}$$

In a first stage, monopolistic input producers minimize costs for any given level of output $y_{j,t}$. Given minimum unit cost, they set a profit maximizing output price subject to a demand function for specialized brands.

Cost Minimization: Firms need labor which is a CES composite of differentiated services. A total amount of labor L_t requires specialized services $L_{j,t}$:

$$L_t = \left[\int_0^1 L_{j,t}^{(\sigma_l - 1)/\sigma_l} dj \right]^{\sigma_l/(\sigma_l - 1)}, \quad \sigma_l > 1. \tag{3}$$

Firms minimize wage costs, $\min_{L_{j,t}} \int_0^1 w_{j,t} L_{j,t} dj$, subject to technology L_t and wages $w_{j,t}$ set by households. By standard steps, demand functions for labor services are

$$L_{j,t} = \left(w_t^L / w_{j,t} \right)^{\sigma_l} L_t, \quad w_t^L = \left[\int_0^1 w_{j,t}^{1-\sigma_l} dj \right]^{1/(1-\sigma_l)}, \quad \int_0^1 w_{j,t} L_{j,t} dj = w_t^L L_t. \tag{4}$$

Total costs are $w_t^L L_t$ and w_t^L is a nominal wage index.

The technology for producing intermediate inputs is

$$y_{jt} = z_t k_{jt}^{\alpha} l_{jt}^{1-\alpha}, \tag{5}$$

where z_t is an autoregressive process affected by infrastructure, see (45) below. Taking (nominal) factor prices w_t^L and w_t^K as given, firms minimize costs per unit of output,

$$m^c \left(w_t^L, w_t^K \right) = \min_{k^u, l^u} w_t^L l_t^u + w_t^K k_t^u + \lambda_t^L \left[1 - z_t \left(k_t^u \right)^{\alpha} \left(l_t^u \right)^{1-\alpha} \right]. \tag{6}$$

Optimality requires $m_t^c = \lambda_t^L$, $w_t^L = m_t^c (1-\alpha) z_t \tilde{k}_t^{\alpha}$ and $w_t^K = m_t^c \alpha z_t / \tilde{k}_t^{1-\alpha}$ where the capital labor ratio is $\tilde{k}_t \equiv k_t^u / l_t^u = \frac{\alpha}{1-\alpha} \frac{w_t^L}{w_t^K}$. Since all firms face the same prices, the capital labor ratio, unit demands and unit cost are symmetric. Unit factor demands are $k_t^u = \alpha m_t^c / w_t^K$ and $l_t^u = (1-\alpha) m_t^c / w_t^L$, which yields marginal cost m_t^c. To sum up,

$$m_t^c = \left(w_t^K / \alpha \right)^{\alpha} \left(w_t^L / (1-\alpha) \right)^{1-\alpha} / z_t, \quad w_t^L = m_t^c (1-\alpha) z_t \tilde{k}_t^{\alpha}, \quad w_t^K = m_t^c \frac{\alpha z_t}{\tilde{k}_t^{1-\alpha}}. \tag{7}$$

Mark-up: The price elasticity of variety demand (1) $-\left(\partial y_{jt} / y_{jt} \right) \left(\partial p_{jt} / p_{jt} \right) = \sigma_v$ is a constant. A specialized producer, supplying the entire market niche, sets an optimal price p_{jt}, taking the price index P_t and final demand Y_t^g as given. Profits are

$$\chi_{jt}^m = \max_{p_{j,t}} \left(p_{jt} - m_t^c \right) y_{jt} \quad s.t. \quad y_{jt} = \left(P_t / p_{jt} \right)^{\sigma_v} Y_t^g. \tag{8}$$

Price setting requires $y_{jt} + \left(p_{jt} - m_t^c \right) \frac{\partial y_{jt}}{\partial p_{jt}} = 0$, which gives a markup

$$p_t = \frac{\sigma_v}{\sigma_v - 1} \cdot m_t^c, \quad \chi_t^m = \left(p_t - m_t^c \right) Y_t^g. \tag{9}$$

Due to symmetry, $p_t = P_t$ and $y_t = Y_t^g$, and aggregate profit is χ_t^m.

1.2 Aggregate Income

Income: Given linear homogeneity of technology, we multiply the unit isoquant in (6), $1 = z_t (k_t^u)^\alpha (l_t^u)^{1-\alpha}$, by aggregate output, use total factor demands $K_{t-1} = k_t^u Y_t^g$ and $L_t = l_t^u Y_t^g$, and write aggregate output as

$$Y_t^g = z_t K_{t-1}^\alpha L_t^{1-\alpha} = z_t \tilde{k}_t^\alpha L_t. \tag{10}$$

Head quarters (HQ) manage employment L_t and capital K_{t-1} and rent out services to production units. They earn nominal factor income $m_t^c Y_t^g = w_t^L L_t + w_t^K K_{t-1}$ on competitive internal markets and collect monopoly profits χ_t^m. Adding up, total earnings of HQ amount to $P_t Y_t^g = \chi_t^m + m_t^c Y_t^g$ or

$$P_t Y_t^g = \chi_t^m + w_t^L L_t + w_t^K K_{t-1}, \quad m_t^c Y_t^g = w_t^L L_t + w_t^K K_{t-1}. \tag{11}$$

Goods Demand: Income finances investment and consumption. Consumers buy a composite quantity $\bar{C}_t$ of domestic and foreign final goods and must spend $\bar{P}_t \bar{C}_t$. The price index $\bar{P}_t$ is the minimum cost per unit of the composite good. Firms invest and must acquire capital goods $\bar{Z}_t$, and spend $\bar{P}_t \bar{Z}_t$. We assume that households and firms use the same composite good and thus face the same price index $\bar{P}_t$.

Households of region i consume a basket of final goods C_t^{ij} from different regions. The index j refers the origin country (production). We think of Italy i (home), the rest of the Eurozone e, and other countries o (RoW). In most cases, we suppress the index i so that $C = C^{ii}$ denotes demand for the home good, and C^{ie} and C^{io} are imports, giving utility

$$\bar{C}_t = \left[\sum_j \left(s^j\right)^{1/\sigma_r} \left(C_t^{ij}\right)^{(\sigma_r - 1)/\sigma_r} \right]^{\sigma_r/(\sigma_r - 1)}, \quad \sum_j s^j = 1, \quad j \in \{i, e, o\}. \tag{12}$$

Expenditure minimization yields a price index $\bar{P}_t$ and a budget $\sum_j P_t^{ij} C_t^{ij}$ where P_t^{ij} are demand prices paid by domestic consumers for home and foreign produced goods. Consumption thus leads to demand for final goods

$$C_t^{ij} = s^j \left(\bar{P}_t / P_t^{ij}\right)^{\sigma_r} \bar{C}_t, \quad \bar{P}_t = \left[\sum_j s^j \left(P_t^{ij}\right)^{1-\sigma_r} \right]^{1/(1-\sigma_r)}, \quad \sum_j P_t^{ij} C_t^{ij} = \bar{P}_t \bar{C}_t. \tag{13}$$

Exchange rates relate import prices in domestic currency to the foreign producer prices $\tilde{P}^e$ and $\tilde{P}^o$ in foreign currency,

$$P_t^{ie} = e_t^{ie} \cdot P_t^e, \quad P_t^{io} = e_t^{io} \cdot P^o, \quad e_t^{eo} \equiv e_t^{io}/e_t^{ie}. \tag{14}$$

Suppose i (Italy) uses Lire, e uses Euros and o Dollars. The exchange rate converts 1 Euro and 1 Dollar into e_t^{ie} and e_t^{io} Lire. Lira prices for imports are P_t^{ie} and P_t^{io} where foreign producer prices P_t^e and P^o are in foreign currency. The inverse rate converts 1 Lira into $1/e_t^{ie}$ Euros and $1/e_t^{io}$ Dollars. If the Eurozone e imports from Italy, it pays a Euro price P_t/e_t^{ie} when the producer price in Lire is P_t. RoW pays a Dollar price P_t/e_t^{io} for imports from Italy. By transitivity, the Euro Dollar exchange rate is $e_t^{eo} \equiv e_t^{io}/e_t^{ie}$, i.e., one Dollar buys e_t^{io} Lire, and one Lira gives $1/e_t^{ie}$ Euros, so that one Dollar in the end buys e_t^{eo} Euros. When Italy is part of the Euro Area and shares the same currency with the rest of the Eurozone, it faces a fixed exchange rate $e_t^{ie} = 1$.

1.3 Consumption and Money Demand

The household is an extended family with individuals $j \in [0, 1]$, each offering specialized labor services $L_{j,t} = N_{j,t}H$, where H is household size and $N_{j,t}$ is labor supply per capita. A household of type j is a monopolist over her specialized labor services. Once labor earnings of each type is optimally determined, the family pools all income and chooses consumption and money holdings.

Preferences for consumption $\bar{C}_t$, labor supply and *real* money balances $\bar{M}_t$ are

$$V_t^h = E_t \sum_{s=0}^{\infty} \beta^s u\left(\bar{C}_{t+s}, \bar{M}_{t+s}, \{N_{j,t+s}\} H\right). \tag{15}$$

Household size is H, and $N_{j,t}$ is labor supply per capita of type j, giving a total of $N_{j,t}H$. The family pools income and, in total, enjoys consumption and money balances of $\bar{C}_t$ and $\bar{M}_t$. Consumption is a composite of domestic and imported final goods, see (12-13). Preferences are homothetic and separable, $u_t = u\left(X\left(\bar{C}_t, \bar{M}_t\right), \{N_{j,t}\} H\right)$, where

$$u_t = \frac{X\left(\bar{C}_t, \bar{M}_t\right)^{1-\sigma_c}}{1-\sigma_c} - \phi_t \cdot \frac{\int_0^1 N_{j,t}^{1+\eta} H dj}{1+\eta}, \quad X_t = \left[s_c \bar{C}_t^{1-\sigma_m} + (1-s_c)\bar{M}_t^{1-\sigma_m}\right]^{\frac{1}{1-\sigma_m}}. \tag{16}$$

We include an autoregressive preference shock $\phi_t = (1-\rho)\bar{\phi} + \rho\phi_{t-1} + \varepsilon_t^{\phi}$ to induce economic fluctuations by labor supply shocks.

Note linear homogeneity and use the money-consumption ratio m_t,

$$X_t = x_t \cdot \bar{C}_t, \quad x_t = \left[s_c + (1-s_c)\, m_t^{1-\sigma_m}\right]^{1/(1-\sigma_m)}, \quad m_t \equiv \bar{M}_t/\bar{C}_t.$$

Get the marginal utilities $u_{C,t} = s_c \bar{C}_t^{-\sigma_m} X_t^{\sigma_m-\sigma_c}$ and $u_{M,t} = (1-s_c)\, \bar{M}_t^{-\sigma_m} X_t^{\sigma_m-\sigma_c}$. Note $u_{CM,t} = (\sigma_m - \sigma_c)\, s_c \bar{C}_t^{-\sigma_m} (1-s_c)\, \bar{M}_t^{-\sigma_m} X_t^{2\sigma_m-\sigma_c-1} \gtrless 0 \Leftrightarrow \sigma_m \gtrless \sigma_c$, Walsh, 66-67,

$$u_{C,t} = \frac{s_c x_t^{\sigma_m-\sigma_c}}{\bar{C}_t^{\sigma_c}}, \quad u_{M,t} = \frac{(1-s_c)\, x_t^{\sigma_m-\sigma_c}}{m_t^{\sigma_m}\bar{C}_t^{\sigma_c}}, \quad MRS_{j,t} = -\frac{u_{N_{j,t}}}{u_{C,t}} = \frac{\phi_t \cdot N_{j,t}^{\eta}\bar{C}_t^{\sigma_c}}{s_c x_t^{\sigma_m-\sigma_c}}.$$

Budget Constraint: The Fisher equation for interest rates ($r_t \approx i_t - \pi_t$) is[1]

$$1+\pi_t = \frac{\left(1+\tau_{t+1}^c\right)\bar{P}_{t+1}}{\left(1+\tau_t^c\right)\bar{P}_t}, \quad 1+r_t = (1+i_t)\,/\,(1+\pi_t) = (1+i_t)\,\frac{\left(1+\tau_t^c\right)\bar{P}_t}{\left(1+\tau_{t+1}^c\right)\bar{P}_{t+1}}. \tag{17}$$

Labor earnings derive from differentiated services $N_{j,t}$ at wages $w_{j,t}$. Insurance within the family perfectly smooths income risk. The family cares only about total earnings. Households pay a wage income tax at rate τ_t and a consumption tax at rate τ_t^c, and are able to reduce tax liability by T_t^l on account of tax evasion. They collect dividends χ_t and χ_t^b from firms and banks, respectively, and receive transfers from social spending E_t and seignorage T_t^M. Income from bank deposits S_t^d includes interest plus repayment of deposits, net of any new savings. Net earnings on government debt holdings, net of any windfall loss due to default, are $\left(1-\tilde{s}^b\right) S_t^G$. The nominal budget is

$$\begin{aligned} \frac{A_t}{1+i_t} &= A_{t-1} + \int_0^1 (1-\tau_t)\, w_{j,t} N_{j,t} H dj + E_t + T_t^l + \chi_t + \chi_t^b \\ &: \; +S_t^d + \left(1-\tilde{s}^b\right) S_t^G + (M_{t-1} - M_t) + T_t^M - \left(1+\tau_t^c\right)\bar{P}_t\bar{C}_t. \end{aligned} \tag{18}$$

The dating convention is that all variables are measured at the beginning of period, except for M_t and A_t which are measured at the end, so that M_{t-1} and A_{t-1} are beginning of t, or end of $t-1$. Nominal money holdings M_{t-1} and real money balances are thus related by

[1] Prices are measured at the beginning of period, so that the inflation rate in t reflects the change in prices from the beginning to the end (beginning of next) period.

$M_{t-1} \equiv \bar{M}_{t-1}\bar{P}_t$ and $M_t \equiv \bar{M}_t\bar{P}_{t+1}$. To capture windfall gains and losses of sudden price changes (unexpected inflation), we must use nominal money holdings as a state variable.

Wage Setting: Firms demand specialized labor services of type j. Individual j faces demand $L_{j,t}$ for her labor type. Being a monopolist, $N_{j,t}H = L_{j,t}$, she sets a wage to exploit market power. Being one among many close substitutes, she takes the wage index w_t^L and aggregate labor demand L_t as given. The demand schedule in (4), expressed per capita, exhibits a constant wage elasticity,

$$N_{j,t} = \left(w_t^L / w_{j,t}\right)^{\sigma_l} L_t / H, \quad \sigma_l = -\frac{w_{j,t}}{N_{j,t}} \frac{\partial N_{j,t}}{\partial w_{j,t}} > 1. \tag{19}$$

For $\sigma_l \to \infty$, demand is infinitely elastic, leading to standard labor supply choice. Instead of wage setting, individuals would be forced to accept the market wage.

At any date, a *random selection* of workers, a fraction $1-\omega$, can set wages. The remaining fraction ω is stuck with a wage set in the past. In period t, an opportunity for new wage setting arrives with probability $f_{t,t} = 1-\omega$. The household will be stuck with that wage with probability ω^i for another i periods. With probability $f_{t+i,t} = (1-\omega)\,\omega^i$, she will still have to charge the same wage at date $t+i$. These probabilities must add up to one, $\sum_{i\geq 0} f_{t,t+i} = \sum_{i\geq 0} (1-\omega)\,\omega^i = 1$.

Looking into the past, $f_{t-i,t} = (1-\omega)\,\omega^i$ is the probability that a wage was set in period $t-i$ and is still binding today since it was never changed since then and remained in place over i periods. Again, these probabilities add up to one,

$$f_{t-i,t} = (1-\omega)\,\omega^i, \quad \sum_{i\geq 0} f_{t-i,t} = \sum_{i\geq 0} (1-\omega)\,\omega^i = 1. \tag{20}$$

Chances are *identically and independently distributed.* By the law of large numbers, the probability of wage setting is equal to the fraction of workers that can adjust. In a cross-section, employees (or their union) of type j have set wages either today or at some earlier date. In period $t-i$, a fraction $1-\omega$ has set a new wage, and of these, a fraction $f_{t-i,t} = (1-\omega)\,\omega^i$ has adjusted never since then and is stuck today with that same wage.

Given symmetry within vintages, a person of type j in period t earns a wage

$$w_{j,t} \in \left\{ \begin{array}{ccccc} w_{t,t} = w_t^*, & w_{t-1,t} = w_{t-1}^*, & w_{t-2,t} = w_{t-2}^*, & \ldots & w_{t-i,t} = w_{t-i}^*, \\ 1-\omega & (1-\omega)\,\omega & (1-\omega)\,\omega^2 & & (1-\omega)\,\omega^i. \end{array} \right\}$$

Each vintage $t-i$ includes many agents j. The control w_t^* becomes a state for the next period and all future ones until the next opportunity for wage setting arrives. To find the value of this state, we introduce a state variable w_{t-1}^*, see below. When a person of type j sets an optimal wage in period t, she will earn a wage next period equal to

$$w_{t,t+1} = \left\{ \begin{array}{ll} w_t^* & \text{with prob. } \omega, \\ w_{t+1}^* & \text{with prob. } 1-\omega, \end{array} \right. \qquad w_{t-1,t} = \left\{ \begin{array}{ll} w_{t-1}^* & \text{with prob. } \omega, \\ w_t^* & \text{with prob. } 1-\omega. \end{array} \right. \tag{21}$$

Similarly, a person of vintage $t-1$ earns w_{t-1}^* when she cannot set a new wage (with probability ω), or she earns w_t^* when she is given a chance to revise (with probability $1-\omega$). The control variable w_t^* becomes a state next period with probability ω, and the control w_{t-1}^* is a state this period.

All agents j of the same vintage $t-i$ are identical which gives the following identity. Only the members of the newest vintage can set their optimal wage w_t^*:

$$\Omega_t \equiv \int_0^1 N_{j,t}^{1+\eta} dj = \textstyle\sum_{i \geq 0} (1-\omega)\,\omega^i N_{t-i,t}^{1+\eta}. \tag{22}$$

Optimization: The Bellman problem is (note $V_{t+1}^h = V^h (A_t, M_t, w_t^* H)$)

$$V_t^h = V^h \left(A_{t-1}, M_{t-1}, w_{t-1}^* H\right) = \max_{\bar{C}_t, \bar{M}_t, w_t^*} u\left(\bar{C}_t, \bar{M}_t, \{N_{j,t}\} H\right) + \beta E_t V_{t+1}^h, \tag{23}$$

subject to (18), (19), (21) and $M_t = \bar{M}_t \bar{P}_{t+1}$. The control w_t^* refers only to types j in vintage t, $N_{j,t} \in N_{t,t}$ which receive a new opportunity for wage setting. Define shadow prices $\lambda_{t+1} \equiv dV_{t+1}/dA_t$, $\lambda_t^M \equiv dV_t/dM_{t-1}$ and $dV_t/dw_{t-1}^* = \mu_t^w$. Using this, optimality conditions for $\bar{C}_t, \bar{M}_t, w_t^*$ and envelope conditions for $A_{t-1}, M_{t-1}, w_{t-1}^*$ are

$$\begin{aligned}
\bar{C}_t &: u_{C,t} = \beta E_t \lambda_{t+1} (1+i_t) (1+\tau_t^c) \bar{P}_t, \\
\bar{M}_t &: u_{M,t} = \beta E_t \left[\lambda_{t+1} (1+i_t) - \lambda_{t+1}^M \right] \bar{P}_{t+1}, \\
w_t^* &: 0 = u_{N_{t,t}} \frac{\partial N_{t,t}}{\partial w_t^*} + \beta \lambda_{t+1} (1+i_t)(1-\tau_t) \frac{\partial (w_t^* N_{t,t})}{\partial w_t^*} + \omega \cdot \beta \mu_{t+1}^w, \\
A_{t-1} &: \lambda_t = \beta E_t \lambda_{t+1} (1+i_t), \\
M_{t-1} &: \lambda_t^M = \beta E_t \lambda_{t+1} (1+i_t), \\
w_{t-1}^* &: \mu_t^w = u_{N_{t-1,t}} \frac{\partial N_{t-1,t}}{\partial w_{t-1}^*} + \beta \lambda_{t+1} (1+i_t)(1-\tau_t) \frac{\partial \left(w_{t-1}^* N_{t-1,t} \right)}{\partial w_{t-1}^*} + \omega \cdot \beta \mu_{t+1}^w.
\end{aligned} \tag{24}$$

Shift forward (24.iv) by one period, multiply by $\beta E_t (1+i_t)(1+\tau_t^c) \bar{P}_t$, rearrange terms to use (i) on both sides, note the Fisher equation and get the Euler condition

$$u_{C,t} = \beta E_t (1+r_t) \cdot u_{C,t+1}, \quad u_{C,t} = \frac{s_c x_t^{\sigma_m - \sigma_c}}{\bar{C}_t^{\sigma_c}}, \quad x_t = \left[s_c + (1-s_c) m_t^{1-\sigma_m} \right]^{\frac{1}{1-\sigma_m}}. \tag{25}$$

A higher real interest tilts consumption to the future, implying larger savings today.

By (iv-v), $\lambda_t^M = \lambda_t$. Using this together with the Fisher equation in (ii) and combining with (i) gives the tangency condition for money demand,

$$\frac{u_{M,t}}{u_{C,t}} = \frac{1-s_c}{s_c m_t^{\sigma_m}} = \frac{i_t}{\left(1+\tau_{t+1}^c\right)(1+r_t)}, \quad m_t = \left(\frac{1-s_c}{s_c} \frac{\left(1+\tau_{t+1}^c\right)(1+r_t)}{i_t} \right)^{\frac{1}{\sigma_m}}, \tag{26}$$

where $m_t \equiv \bar{M}_t / \bar{C}_t$ is the desired money consumption ratio. Money demand depends on the opportunity cost, the return that could have been obtained if it were invested in the market at a rate i_t, or $i_t / (1+r_t)$ in present value. A higher expected consumption tax leads households to shift from consumption to money holdings. Numerically, we solve for household decisions in terms of expected values $u_{C,t+1}$ and π_t, giving $u_{C,t}$ and i_t (in terms of r_t and π_t) and, in turn, m_t, x_t, $\bar{C}_t$ and $\bar{M}_t = m_t \bar{C}_t$, all per capita.

1.4 Wage Setting

Turning to labor supply, we use the demand elasticity σ_l and substitute for $\frac{\partial N_{t,t}}{\partial w_t^*} = -\frac{N_{t,t}}{w_t^*} \sigma_l$ and $\frac{\partial (w_t^* N_{t,t})}{\partial w_t^*} = N_{t,t} + w_t^* \frac{\partial (N_{t,t})}{\partial w_t^*} = -(\sigma_l - 1) N_{t,t}$. Multiply (24.iii) by $\frac{w_t^*}{N_{t,t}}$, use (i), divide by $(\sigma_l - 1)\, u_{C,t}$, substitute $MRS_{t,t} \equiv -\frac{u_{N_{t,t}}}{u_{C,t}}$, multiply by $\frac{w_t^*}{N_{t,t}}$ and rearrange

$$\begin{aligned}
\frac{(1-\tau_t)\, w_t^*}{(1+\tau_t^c)\, \bar{P}_t} &= \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t} + \frac{w_t^*}{N_{t,t}} \omega \frac{\beta u_{C,t+1}}{u_{C,t}} \frac{\mu_{t+1}^w}{(\sigma_l - 1)\, u_{C,t+1}}, \\
\frac{\mu_t^w}{(\sigma_l - 1)\, u_{C,t}} &= \left[\frac{\sigma_l}{\sigma_l - 1} MRS_{t-1,t} - \frac{(1-\tau_t)\, w_{t-1}^*}{(1+\tau_t^c)\, \bar{P}_t} \right] \frac{N_{t-1,t}}{w_{t-1}^*} + \omega \frac{\beta u_{C,t+1}}{u_{C,t}} \frac{\mu_{t+1}^w}{(\sigma_l - 1)\, u_{C,t+1}}.
\end{aligned}$$

Do the same steps to get the second equation from the envelope condition (24.vi). The Euler equation is $\beta \frac{u_{C,t+1}}{u_{C,t}} = \frac{1}{1+r_t}$. Define $\mu_t \equiv \frac{\mu_t^w}{(\sigma_l - 1) u_{C,t}}$ and get the final solution

$$\begin{aligned}
(i) \quad &: \quad \frac{(1-\tau_t)\, w_t^*}{(1+\tau_t^c)\, \bar{P}_t} = \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t} + \frac{w_t^*}{N_{t,t}} \omega \frac{\mu_{t+1}}{1+r_t}, \\
(ii) \quad &: \quad \mu_t = -\left[\frac{(1-\tau_t)\, w_{t-1}^*}{(1+\tau_t^c)\, \bar{P}_t} - \frac{\sigma_l}{\sigma_l - 1} MRS_{t-1,t} \right] \frac{N_{t-1,t}}{w_{t-1}^*} + \omega \frac{\mu_{t+1}}{1+r_t}.
\end{aligned} \tag{27}$$

Computations require $N_{t,t} = \left(w_t^L / w_t^*\right)^{\sigma_l} L_t / H$ and $N_{t-1,t} = \left(w_t^L / w_{t-1}^*\right)^{\sigma_l} L_t / H$ as well as

$$MRS_{t,t} = -\frac{u_{N_{t,t}}}{u_{C,t}} = \frac{\phi_t N_{t,t}^{\eta} \bar{C}_t^{\sigma_c}}{s_c x_t^{\sigma_m - \sigma_c}}, \quad MRS_{t-1,t} = \frac{\phi_t N_{t-1,t}^{\eta} \bar{C}_t^{\sigma_c}}{s_c x_t^{\sigma_m - \sigma_c}} = \left(w_t^* / w_{t-1}^*\right)^{\eta \sigma_l} \cdot MRS_{t,t}.$$

Stationary Solution: In a SS, wages are constant, giving $w_t^* = w_{t-1}^* = w_{t-i}^*$, which also implies that $MRS_{t-i,t}$ and $N_{t-i,t}$ are the same for all vintages. Intuitively, wage rigidity disappears in a SS. Substituting (i) into (ii) implies a static mark-up

$$\frac{(1-\tau)\, w^*}{(1+\tau^c)\, \bar{P}} = \frac{\sigma_l}{\sigma_l - 1} \cdot MRS, \quad \mu = 0. \tag{28}$$

Similarly, if wages were flexible in all periods ($\omega = 0$), they would all be optimally set, i.e., $w_{t-i,t} = w_t^*$. Given $\omega = 0$, (27.i) gives a static wage mark-up in all periods, $\frac{w_t^*}{P_t} = \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t}$, which gives $\mu_{t+1} = \mu_t = 0$. Although derived rather differently as part of the same Bellman problem, the solution exactly corresponds to Gali (2015), see the Appendix for details.

Aggregation: Wages and employment are heterogeneous. They differ by their vintage but are symmetric within each cohort. To get the effect of new and old wages on the wage index $w_t^L = \left[\int_0^1 w_{j,t}^{1-\sigma_l} dj\right]^{1/(1-\sigma_l)}$, we use the distribution of types j in (20) and arrange wages according to vintages, $\left(w_t^L\right)^{1-\sigma_l} = \sum_{i \geq 0} (1-\omega)\, \omega^i w_{t-i,t}^{1-\sigma_l}$, giving

$$\left(w_t^L\right)^{1-\sigma_l} = (1-\omega)\, w_{t,t}^{1-\sigma_l} + \omega \cdot (1-\omega) \left[w_{t-1,t-1}^{1-\sigma_l} + \omega^2 w_{t-2,t-1}^{1-\sigma_l} + \ldots \right].$$

A wage set at date $t-i$ is constant thereafter until the next wage setting. The second line thus uses $w_{t-i,t}=w_{t-i,t-1}$, and the definition of the wage index in $t-1$,

$$\left(w_t^L\right)^{1-\sigma_l}=(1-\omega)\cdot w_{t,t}^{1-\sigma_l}+\omega\cdot\left(w_{t-1}^L\right)^{1-\sigma_l}. \tag{29}$$

The state w_{t-1}^L and current wages $w_{t,t}$ give the evolution of the wage index w_t^L. Total wage earnings are then $w_t^L L_t=\int_0^1 w_{j,t}N_{j,t}H dj$.

The disutility of work is $\phi_t\int_0^1\frac{N_{j,t}^{1+\eta}H}{1+\eta}dj=\phi_t H\Omega_t/\left(1+\eta\right)$ per capita. Substitute the demand function $N_{j,t}$ (19) into (22), $\Omega_t=\left(\left(w_t^L\right)^{\sigma_l}L_t/H\right)^{1+\eta}\cdot\sum_{i\geq 0}\left(1-\omega\right)\omega^i\left(w_{t-i,t}\right)^{-(1+\eta)\sigma_l}$. By the same steps as above, we get $\Omega_t=\left(\left(w_t^L/\bar{w}_t^L\right)^{\sigma_l}L_t/H\right)^{1+\eta}$, where $\bar{w}_t^L$ is defined as

$$\left(\bar{w}_t^L\right)^{-(1+\eta)\sigma_l}=\left(1-\omega\right)\left(w_{t,t}\right)^{-(1+\eta)\sigma_l}+\omega\left(\bar{w}_{t-1}^L\right)^{-(1+\eta)\sigma_l}. \tag{30}$$

Numerical Procedure: To compute (27), we need $MRS_{t-i,t}\equiv-\frac{u_{N_{j,t}}}{u_{C,t}}=\frac{\phi_t\bar{C}_t^{\sigma_c}N_{t-i,t}^{\eta}}{s_c x_t^{\sigma_m-\sigma_c}}$ for vintages $j=t$ and $j=t-1$. Employment of a wage setter is $N_{t,t}=\left(w_t^L/w_t^*\right)^{\sigma_l}L_t/H$, resulting in $MRS_{t,t}$. Noting $w_{t-1,t}=w_{t-1}^*$ similarly gives $N_{t-1,t}=\left(w_t^L/w_{t-1}^*\right)^{\sigma_l}L_t/H$ and $MRS_{t-1,t}$. The expected variable μ_{t+1} thus gives w_t^* from (27.i). Given the predetermined variable w_{t-1}^*, we compute an update μ_t for the expected variable.

1.5 Investment and Debt Financing

Investment is subject to installation costs. To install new capital I_t, HQs spend $\bar{P}_t\bar{Z}_t$ on a capital good consisting of domestic final goods and imports (as in 12-13),

$$\bar{Z}_t=I_t+\frac{\psi}{2}K_{t-1}\left(I_t/K_{t-1}-\delta\right)^2. \tag{31}$$

The installation technology $\bar{Z}_t=\bar{Z}\left(I_t,K_{t-1}\right)$ is linear homogeneous. Define derivatives by $\bar{Z}_{K,t}\equiv d\bar{Z}\left(I_t,K_{t-1}\right)/dK_{t-1}$ and $\bar{Z}_{I,t}$. By linear homogeneity, $I_t\bar{Z}_{I,t}+K_{t-1}\bar{Z}_{K,t}=\bar{Z}_t$ where $\bar{Z}_{I,t}=1+\psi\left(I_t/K_{t-1}-\delta\right)$ and $\bar{Z}_{K,t}=-\frac{\psi}{2}\left[\left(I_t/K_{t-1}+\delta\right)\left(I_t/K_{t-1}-\delta\right)\right]$.

Investment is financed with retained earnings and bank credit. External debt grows by $B_t^l/\left(1+i_t^l\right)=B_{t-1}^l-S_t^l$. After net loan repayment S_t^l, outstanding debt is $B_{t-1}^l-S_t^l$.

Adding interest spending gives new debt $B_t^l = B_{t-1}^l - S_t^l + i_t^l \left(B_{t-1}^l - S_t^l\right)$. If $S_t^l < 0$, the firm takes new loans and accumulates debt. In a SS, $S^l = i^l B^l / \left(1 + i^l\right)$. Dividends are reduced by stationary interest payments. Subtracting wages, investment, debt service and taxes from total earnings leaves dividends equal to

$$\begin{aligned} \chi_t &= P_t Y_t^g - w_t^L L_t - \bar{P}_t \bar{Z}_t - S_t^l - \tau_t T_t^k, \\ T_t^k &= P_t Y_t^g - w_t^L L_t - t^z \bar{P}_t \bar{Z}_t - i_t^l \left(B_{t-1}^l - S_t^l\right) / \left(1 + i_t^k\right). \end{aligned} \tag{32}$$

For simplicity, we lump together corporate and personal taxes on capital income which gets taxed with the overall income tax rate τ_t. The tax base T_t^k allows for deduction of a share t^z of investment spending. Furthermore, Tax savings due to interest expensing must be discounted to the beginning of period, using the firm's discount rate i_t^k. Substituting $P_t Y_t^g = \chi_t^m + w_t^L L_t + w_t^K K_{t-1}$ from (11) gives

$$\begin{aligned} \chi_t &= (1-\tau_t)\left(\chi_t^m + w_t^K K_{t-1}\right) - (1 - t^z \tau_t) \bar{P}_t \bar{Z}_t + \frac{\tau_t i_t^l}{1 + i_t^k}\left(B_{t-1}^l - S_t^l\right) - S_t^l, \\ &: \; B_t^l / \left(1 + i_t^l\right) = b^l \cdot \left(1 + i_t^k\right)(1 - t^z \tau_t) \bar{P}_t K_{t-1}, \\ &: \; S_t^l = B_{t-1}^l - b^l \cdot \left(1 + i_t^k\right)(1 - t^z \tau_t) \bar{P}_t K_{t-1}, \\ &: \; K_t = I_t + (1-\delta) K_{t-1}. \end{aligned} \tag{33}$$

The debt capacity is limited and constrains the use of debt. We keep the debt equity choice exogenous. The firm's debt level $B_{t-1}^l - S_t^l$ is restricted to a fixed fraction b^l of the replacement cost of preexisting capital. Once the feasible debt level B_t^l is set, the flow S_t^l is residually determined. For optimization, we note

$$\begin{aligned} \frac{dS_t^l}{dB_{t-1}^l} &= 1, \quad \frac{dS_t^l}{dK_{t-1}} = -b^l \cdot \left(1 + i_t^k\right)(1 - t^z \tau_t) \bar{P}_t, \\ \frac{dB_t^l}{dB_{t-1}^l} &= 0, \quad \frac{dB_t^l}{dK_{t-1}} = \left(1 + i_t^l\right) b^l \cdot \left(1 + i_t^k\right)(1 - t^z \tau_t) \bar{P}_t. \end{aligned} \tag{34}$$

Firm value V_t (in nominal terms) is dated at the beginning of period. We also allow for an exogenous equity premium $\bar{\theta}^k \geq 1$ which gives

$$V_t = \chi_t + V_{t+1} / \left(1 + i_t^k\right), \quad i_t^k = \theta_t^k \cdot i_t, \quad \theta_t^k = \left(1 - \rho^\theta\right) \bar{\theta}^k + \rho^\theta \theta_{t-1}^k + \varepsilon_t^k. \tag{35}$$

Use the value function, $V_t = V\left(K_{t-1}, B_{t-1}^l\right)$ and define shadow prices $\lambda_t^K \equiv \partial V_t / \partial K_{t-1}$ and $\lambda_t^B \equiv \partial V_t / \partial B_{t-1}^l$. Value maximization $V\left(K_{t-1}, B_{t-1}^l\right) = \max_{I_t} \chi_t + V_{t+1} / \left(1 + i_t^k\right)$ s.t. (33-34) gives optimal investment and new debt. Note that S_t^l is not a choice variable but is determined along with capital accumlation and a binding debt constraint:

$$\begin{aligned}
I_t &: \left(1 - t^z \tau_t\right) \bar{P}_t \bar{Z}_{I,t} = \lambda_{t+1}^K / \left(1 + i_t^k\right), \\
K_{t-1} &: \lambda_t^K = \left(1 - \tau_t\right) w_t^K - \left(1 - t^z \tau_t\right) \bar{P}_t \bar{Z}_{K,t} \\
&: \qquad + \left[1 + i_t^k + \tau_t i_t^l + \left(1 + i_t^l\right) \lambda_{t+1}^B\right] \left(1 - t^z \tau_t\right) b^l \bar{P}_t + (1 - \delta) \frac{\lambda_{t+1}^K}{1 + i_t^k}, \\
B_{t-1}^l &: \lambda_t^B = -1.
\end{aligned} \tag{36}$$

Use the *net* investment rate $x_t^I \equiv I_t / K_{t-1} - \delta = \left(K_t - K_{t-1}\right) / K_{t-1}$ and note marginal cost $\bar{Z}_{I,t} = 1 + \psi x_t^I$ to get optimal investment as in Tobin's Q-theory,

$$x_t^I = \left(Q_t - 1\right) / \psi, \quad Q_t \equiv \frac{\lambda_{t+1}^K / \left(1 + i_t^k\right)}{\left(1 - t^z \tau_t\right) \bar{P}_t}. \tag{37}$$

Tobin's Q_t is the market value $E_t \lambda_{t+1}^K / \left(1 + i_t^k\right)$ per unit of capital, divided by the acquisition cost equal to $\left(1 - t^z \tau_t\right) \bar{P}_t$.

In a SS with $\bar{Z}_I = 1$, $\bar{Z}_K = 0$ and $Q_t = 1$, we can give an intuitive expression for the user cost of capital. By (i), the shadow price is $\lambda^K = \left(1 - t^z \tau_t\right) \left(1 + i^k\right) \bar{P}$. Using this together with $\lambda^B = -\left(1 + \tau_t i_t^l / \left(1 + i_t^k\right)\right)$, (ii) eventually results in

$$w^K = \left[\frac{\delta}{1 - \tau} + \frac{i^k}{1 - \tau} \cdot \left(1 - b^l\right) + i^l \cdot b^l\right] \left(1 - t^z \tau\right) \bar{P}. \tag{38}$$

The term $\left(1 - \tau^z \tau\right) \bar{P}$ is the acquisition cost of a unit of capital, to be financed with debt and equity. The tax inflates the cost of equity $i^k / (1 - \tau)$, but not the cost of debt i^l, since interest on debt is tax deductible, while the (notional) interest on equity is not. The user cost is an average, using the debt ratio b^l as a weight. Finally, replacement investment is fully equity financed by retained earnings, and hence bears a tax adjusted cost of depreciation equal to $\delta^K / (1 - \tau)$.

Lemma 1 ***(Hayashi):*** *Firm value is equal to* $V_t = \lambda_t^K K_{t-1} - B_{t-1}^l + V_t^m$.

Proof. Multiply (36.ii) by K_{t-1}, use $(1-\delta)K_{t-1} = K_t - I_t$, note investment optimality in (36.i), and use linear homogeneity $\bar{Z}_t = I_t \bar{Z}_{I,t} + K_{t-1}\bar{Z}_{K,t}$,

$$\begin{aligned} \lambda_t^K K_{t-1} &= (1-\tau_t) w_t^K K_{t-1} - (1-t^z\tau_t)\bar{P}_t\bar{Z}_t \qquad \text{(i)} \\ &: + \left[1+i_t^k+\tau_t i_t^l + \left(1+i_t^l\right)\lambda_{t+1}^B\right](1-t^z\tau_t) b^l \bar{P}_t K_{t-1} + \lambda_{t+1}^K K_t / \left(1+i_t^k\right), \end{aligned}$$

Substitute $\lambda_{t+1}^B = -1$ and use the debt constraint (33),

$$\begin{aligned} \lambda_t^K K_{t-1} - B_{t-1}^l &= \left[(1-\tau_t) w_t^K K_{t-1} - (1-t^z\tau_t)\bar{P}_t\bar{Z}_t + \frac{\tau_t i_t^l}{1+i_t^k}\left(B_{t-1}^l - S_t^l\right) - S_t^l\right] \\ &: + \left(\lambda_{t+1}^K K_t - B_t^l\right) / \left(1+i_t^k\right), \qquad \text{(ii)} \end{aligned}$$

By the dividend definition (33), the square bracket is equal to $\chi_t - (1-\tau_t)\chi_t^m$. Defining the value of monopoly profits by $V_t^m = (1-\tau_t)\chi_t^m + V_{t+1}^m / \left(1+i_t^k\right)$, we finally get

$$\lambda_t^K K_{t-1} - B_{t-1}^l + V_t^m = \chi_t + \left(K_t \lambda_{t+1}^K - B_t^l + V_{t+1}^m\right) / \left(1+i_t^k\right). \qquad \text{(iii)}$$

Noting that the forward solutions of (35) and (iii) are identical, establishes the result. ■

1.6 Fiscal Policy

The government spends on productive services[2] $P_t G_t$, engages in social spending E_t, potentially pays subsidies $T_t^b = t_t^b d_t^l B_{t-1}^l$ to stabilize banks, and raises tax revenue T_t. It inherits B_{t-1}^G. At the beginning of period, it taxes, spends and possibly defaults on debt at a rate d_t^g (a haircut), thus reducing future liabilities by $d_t^g B_{t-1}^G$.[3] Finally, the government issues new gross debt B_t^G at a price $1/(1+i_t^g)$. The fiscal budget constraint is

$$B_t^G / (1+i_t^g) = B_{t-1}^G - S_t^G, \quad S_t^G = T_t - P_t G_t - E_t - T_t^b + d_t^g B_{t-1}^G. \qquad (39)$$

Default is unexpected. Expected risk is reflected in an interest premium

$$i_t^g = \theta_t^g \cdot i_t, \quad \theta_t^g = 1 - \rho^\theta + \rho^\theta \theta_{t-1}^g + \varepsilon_t^g, \quad d_t^g = \rho^{dg} d_{t-1}^g + \varepsilon_t^{dg}. \qquad (40)$$

[2]Producing public services uses labor and capital. Assuming the same factor intensity as in private production, we can model this as demand for domestic output only, without imports.

[3]Default reduces future primary surpluses and, thereby, future income of households and banks which instantaneously reduces current market value of debt.

To prevent unstable debt, the government must run a consolidation policy. We specify a policy rule for the 'structural' component $\tilde{S}_t^G$ of the primary surplus which excludes any surprise expenditures or windfall gains. Indeed, the Maastricht rules impose restrictions on the structural rather than the actual deficit, and also specify a long-run debt to GDP ratio $\bar{b}^g = B^G/(PY)$. The parameter γ^g determines how fast debt is reduced (or increased) to reach the long-run target. The consolidation rule thus specifies a structural surplus

$$\tilde{S}_t^G = \left(1 - \frac{\gamma^g}{1+i_t^g}\right) B_{t-1}^G - \frac{1-\gamma^g}{1+i_t^g} \bar{b}^g P_t Y_t. \tag{41}$$

In the absence of fiscal shocks, debt would exclusively depend on the target surplus,

$$\frac{B_t^G}{1+i_t^g} = B_{t-1}^G - \tilde{S}_t^G \quad \Rightarrow \quad B_t^G = \gamma^g \cdot B_{t-1}^G + (1-\gamma^g) \cdot \bar{b}^g P_t Y_t, \quad 0 < \gamma^g < 1. \tag{42}$$

The stabilization rule (41) makes debt converge to $B^G = \bar{b}^g PY$, equal to $\bar{b}^g\%$ of GDP.

The actual surplus deviates from the structural surplus when there are unexpected shocks or windfall gains. Spending policies and required tax revenues T_t are

$$\begin{aligned} P_t G_t &= \bar{g} \cdot P_t Y_t - \xi^g \cdot \tilde{S}_t^G + \varepsilon_t^G, \\ E_t &= \bar{e} \cdot w_t^L L_t - \xi^e \cdot \tilde{S}_t^G + \varepsilon_t^E, \\ T_t &= \bar{g} \cdot P_t Y_t + \bar{e} \cdot w_t^L L_t + (1 - \xi^g - \xi^e) \cdot \tilde{S}_t^G. \end{aligned} \tag{43}$$

Productive spending consists of a normal level $\bar{g} P_t Y_t$, reduced by spending cuts to finance a share ξ^g of the required primary surplus. Social spending reflects a normal replacement rate $\bar{e}$ of wage earnings. Spending cuts must contribute a share ξ^e to budget consolidation. Spending shocks ε_t^G and ε_t^E as well as unexpected subsidies to banks T_t^b are not immediately financed with taxes but raise next period's debt and are consolidated later on. Similarly, windfall gains $d_t^g B_{t-1}^G$ due to sovereign default reduce future debt. In consequence, the required tax revenue T_t finances only the structural part of government spending, $\bar{g} P_t Y_t + \bar{e} w_t^L L_t$, plus additional tax increases $(1 - \xi^g - \xi^e)\, \tilde{S}_t^G$ needed to reduce public debt. The parameters ξ^e and ξ^g determine whether consolidation policy is tax or expenditure based. If ξ^e and ξ^g are low, most of budget consolidation is tax based while high values indicate budget consolidation with spending cuts.

To see how unconsolidated fiscal shocks affect fiscal debt dynamics, substitute the tax and spending rules (43) into the primary surplus (39),

$$S_t^G = \tilde{S}_t^G - \varepsilon_t^G - \varepsilon_t^E - T_t^b + d_t^g B_{t-1}^G. \tag{44}$$

In the absence of shocks, $d_t^g = 0$ and $S_t^G = \tilde{S}_t^G$, which leads to stable debt as in (42). Any unexpected spending first raises public debt before is gets consolidated in future periods. Similarly, sovereign default is essentially a windfall gain, reducing debt and creating more fiscal space, at the expense of private investors and banks.[4]

When raising ξ^g and ξ^e, the government shifts budget consolidation from tax increases to spending cuts which links to the research by Alesina et al. (2015) on the effectiveness of tax versus spending based budget consolidation. In the spirit of Barro (1990), we assume that a higher stock of productive infrastructure K_t^G (broadly defined) boosts factor productivity by z_t. The public capital stock accumulates by

$$K_t^G = G_t + (1-\delta^g) K_{t-1}^G, \quad z_t = (1-\rho)\,\bar{z}\left(K_t^G/\bar{K}^G\right)^{\sigma^z} + \rho z_{t-1} + \varepsilon_t^z. \tag{45}$$

Higher tax rates discourage labor supply and investment, thereby reducing growth. Public revenues stem from taxing wages and profits at rate τ_t and consumption spending at rate τ_t^c. Given the profit tax base by T_t^k, tax revenue is

$$T_t = \tau_t \cdot w_t^L L_t + \tau_t \cdot T_t^k + \tau_t^c \cdot \bar{P}_t \bar{C}_t - T_t^l, \quad T_t^l = \left(1-\rho^T\right) \bar{t}^l P_t Y_t + \rho^T T_{t-1}^l + \varepsilon_t^T. \tag{46}$$

Tax revenue shrinks on account of tax base erosion, leading to tax losses T_t^l. We assume that the tax yield is reduced by $\bar{t}^l$% of GDP in the long-run. The larger the tax losses, the higher tax rates must be to generate the required revenue, thereby magnifying tax

[4]Use (39) and substitute the policy rules (43): $B_t^G/(1+i_t^g) = B_{t-1}^G - \tilde{S}_t^G + \varepsilon_t^G + \varepsilon_t^E + T_t^b - d_t^g B_{t-1}^G$. Also noting the consolidation policy in (41) finally results in actual debt dynamics

$$B_t^G = \gamma^g B_{t-1}^G + (1-\gamma^g)\,\bar{b}^g P_t Y_t + (1+i_t^g)\left[\varepsilon_t^G + \varepsilon_t^E + T_t^b - d_t^g B_{t-1}^G\right],$$

which eventually follows (42) when the transitory parts in the last term vanish.

distortions and slowing growth. Finally, we introduce a common scaler t_t^s to implement tax based budget consolidation,

$$\tau_t = t_t^s \cdot \tau_0, \quad \tau_t^c = t_t^s \cdot \tau_0^c. \tag{47}$$

In the initial equilibrium, $t^s = 1$. When the government needs more revenue, it may scale up all tax rates by a common factor t_t^s. A change in the tax structure, e.g., a shift from income to consumption taxes, could also be implemented by first reducing the parameter τ_0 and then computing the budget balancing increase in the tax rate scaler t_t^s.

1.7 Banking Sector

The government issues debt B_t^G in total, of which B_t^g is held by banks, and $B_t \equiv B_t^G - B_t^g$ by investors (private households). Banks acquire a fixed share $\tilde{s}^b$ of new bonds which adds to the stock B_{t-1}^g. They also receive a part $\tilde{s}^b$ of interest and repayment S_t^G. Unexpected sovereign default is a windfall loss to banks, reducing next period's bond value by the haircut $d_t^g B_{t-1}^g$. Sovereign bond holdings evolve as

$$\begin{aligned} B_t^g / (1 + i_t^g) &= (1 - d_t^g) B_{t-1}^g - \tilde{s}^b S_t^G, & B_t^g &= \tilde{s}^b B_t^G, \\ B_t / (1 + i_t^g) &= (1 - d_t^g) B_{t-1} - \left(1 - \tilde{s}^b\right) S_t^G, & B_t &= \left(1 - \tilde{s}^b\right) B_t^G. \end{aligned} \tag{48}$$

Banks provide credit B_t^l to firms and B_t^g to the government. After net loan repayment S_t^l, debt at the end of period is $B_t^l = B_{t-1}^l - S_t^l + i_t^l \left(B_{t-1}^l - S_t^l\right)$, equal to outstanding debt $B_{t-1}^l - S_t^l$ plus interest. Similarly, after repayment S_t^d, deposits D_t are

$$B_t^l / \left(1 + i_t^l\right) = B_{t-1}^l - S_t^l, \quad D_t / \left(1 + i_t^d\right) = D_{t-1} - S_t^d. \tag{49}$$

Bank runs lead to prohibitive costs of deposit funding. We thus allow deposit interest to include a 'risk-premium', reflecting panic driven taste shocks,

$$\begin{aligned} i_t^d &= \theta_t^d \cdot i_t, \quad \theta_t^d = 1 - \rho^\theta + \rho^\theta \theta_{t-1}^d + \varepsilon_t^d, \\ i_t^b &= \theta_t^b \cdot i_t, \quad \theta_t^b = \left(1 - \rho^\theta\right) \bar{\theta}^b + \rho^\theta \theta_{t-1}^b + \varepsilon_t^b. \end{aligned} \tag{50}$$

Banks effectively get repaid only $\left(1-d_t^l\right)B_{t-1}^l$ since liquadion of bad debt burns a value $d_t^l B_{t-1}^l$ in the liquidation process. Credit losses reflect real costs and are proportional to the share of bad loans,

$$d_t^l = \ell \cdot s_t^l, \quad T_t^b = t_t^b d_t^l B_{t-1}. \tag{51}$$

To keep up lending in a crisis, the government may subsidize bank losses at a rate t_t^b. The effective *net* revenue on loans, consisting of net repayment minus outflows of new credit, amounts to $\left(1-d_t^l\right)B_{t-1}^l - B_t^l/\left(1+i_t^l\right) = S_t^l - d_t^l B_{t-1}^l$. With net deposit inflow S_t^d, the bank's budget constraint relates inflows to outflows,

$$\left(1-d_t^l\right)B_{t-1}^l + \left(1-d_t^g\right)B_{t-1}^g + \frac{D_t}{1+i_t^d} + T_t^b = \frac{B_t^l}{1+i_t^l} + \frac{B_t^g}{1+i_t^g} + D_{t-1} + \chi_t^b. \tag{52}$$

Using net revenues from loans and from deposits in (48-49) gives dividends

$$\chi_t^b = S_t^l + \tilde{s}^b S_t^G - S_t^d - \left(1-t_t^b\right)d_t^l B_{t-1}^l. \tag{53}$$

To refinance lending, banks collect deposits D_t and raise equity E_t^b,

$$E_t^b/\left(1+i_t^b\right) = E_{t-1}^b - \chi_t^b, \tag{54}$$

where E_{t-1}^b is equity inherited from the past, and $E_{t-1}^b - \chi_t^b$ is equity ex dividend. The balance sheet restricts total assets to equity plus deposits,

$$\frac{B_t^l}{1+i_t^l} + \frac{B_t^g}{1+i_t^g} = \frac{D_t}{1+i_t^d} + \frac{E_t^b}{1+i_t^b}. \tag{55}$$

Substitute the budget (52). Equity (ex dividend and net of credit losses) is thus equal to net worth at $t-1$, $E_t^b/\left(1+i_t^b\right) = \left(1-\left(1-t_t^b\right)d_t^l\right)B_{t-1}^l + \left(1-d_t^g\right)B_{t-1}^g - D_{t-1} - \chi_t^b$. Using equity issues in (54) gives the balance sheet at the end of $t-1$,

$$E_{t-1}^b + D_{t-1} = \left(1-\left(1-t_t^b\right)d_t^l\right)B_{t-1}^l + \left(1-d_t^g\right)B_{t-1}^g. \tag{56}$$

Net assets (after depreciation and default) are refinanced with deposits and bank equity.

Basel regulations define regulatory capital as a fraction of risk weighted assets where $\tilde{\kappa}^B$ is the risk-weight on business loans and $\tilde{\kappa}^G$ on government debt. For example, $\tilde{\kappa}^B$ is

8% while $\tilde{\kappa}^G$ is 0% since, according to Basel regulations, government debt is deemed to be safe. Adding a voluntary buffer of κ percent of total assets, banks must raise total equity equal to $\kappa^B = \tilde{\kappa}^B + \kappa$ percent of business loans and $\kappa^G = \kappa$ percent of government loans. The balance sheet then determines the volume of required deposits,

$$\frac{E_t^b}{1+i_t^b} = \kappa^B \frac{B_t^l}{1+i_t^l} + \kappa^G \frac{B_t^g}{1+i_t^g} \quad \Rightarrow \quad \frac{D_t}{1+i_t^d} = \left(1-\kappa^B\right)\frac{B_t^l}{1+i_t^l} + \left(1-\kappa^G\right)\frac{B_t^g}{1+i_t^g}. \tag{57}$$

Equity value is $V_t^b = \chi_t^b + V_{t+1}^b / \left(1+i_t^b\right)$. We solve the Bellman problem s.t. (49,57),

$$\begin{aligned} V_t^b\left(D_{t-1}, B_{t-1}^l\right) &= \max_{S_t^l, S_t^d} \; \chi_t^b + \frac{V_{t+1}^b\left(D_t, B_t^l\right)}{1+i_t^b} \\ &: \qquad +\mu_t \cdot \left[\left(1-\kappa^B\right)\frac{B_t^l}{1+i_t^l} + \left(1-\kappa^G\right)\frac{B_t^g}{1+i_t^g} - \frac{D_t}{1+i_t^d}\right]. \end{aligned} \tag{58}$$

Note that banks do not optimize sovereign debt holdings. Given the required return i_t^g, they rather passively absorb what is offered to them. Using $\lambda_{t+1}^D \equiv dV_{t+1}^b/dD_t$ and $\lambda_t^B \equiv dV_t^b/dB_{t-1}^l$, optimality and envelope conditions are

$$\begin{aligned} &i. && S_t^l: && 1 = \left(1-\kappa^B\right)\mu_t + \left(1+i_t^l\right)\lambda_{t+1}^B / \left(1+i_t^b\right), \\ &ii. && S_t^d: && 1 = \mu_t - \left(1+i_t^d\right)\lambda_{t+1}^D / \left(1+i_t^b\right), \\ &iii. && B_{t-1}^l: && \lambda_t^B = -\left(1-t_t^b\right)d_t^l + \left(1-\kappa^B\right)\mu_t + \left(1+i_t^l\right)\lambda_{t+1}^B / \left(1+i_t^b\right), \\ &iv. && D_{t-1}: && \lambda_t^D = -\mu_t + \left(1+i_t^d\right)\lambda_{t+1}^D / \left(1+i_t^b\right). \end{aligned} \tag{59}$$

Combine (i,iii) and (ii,iv) to get $\lambda_t^B = 1 - \left(1-t_t^b\right)d_t^l$ and $\lambda_t^D = -1$. Solve (ii) for the multiplier μ_t and substitute into (i) to obtain loan pricing,

$$1+i_t^l = \frac{\kappa^B \cdot \left(1+i_t^b\right) + \left(1-\kappa^B\right)\cdot\left(1+i_t^d\right)}{1-\left(1-t_{t+1}^b\right)d_{t+1}^l}, \qquad \mu_t = \frac{i_t^b - i_t^d}{1+i_t^b} > 0. \tag{60}$$

The loan rate i_t^l is a mark-up over the cost of capital which is a weighted average of deposit interest and the cost of equity. The mark-up factor reflects default risk and expected depreciation of NPLs.

Lemma 2 ***(Hayashi)****: Bank value is $V_t^b = \left(1-\left(1-t_{t+1}^b\right)d_{t+1}^l\right)B_{t-1}^l - D_{t-1} + V^g$, where V^g is the bank's valuation of government debt.*

Proof. Multiply the envelope conditions by stocks and use (49) and optimality,

$$\lambda_t^B B_{t-1}^l = -\left(1-t_t^b\right) d_t^l B_{t-1}^l + \left(1-\kappa^B\right) \mu_t \left(B_{t-1}^l - S_t^l\right) + S_t^l + \frac{\lambda_{t+1}^B B_t^l}{1+i_t^b}, \tag{i}$$

$$\lambda_t^D D_{t-1} = -\mu_t \left(D_{t-1} - S_t^d\right) - S_t^d + \frac{\lambda_{t+1}^D D_t}{1+i_t^b}. \tag{ii}$$

Adding up and using (53) together with (48-49) gives

$$\lambda_t^B B_{t-1}^l + \lambda_t^D D_{t-1} = \chi_t^b - \tilde{s}^b S_t^G + \mu_t \cdot \left[\left(1-\kappa^B\right) \frac{B_t^l}{1+i_t^l} - \frac{D_t}{1+i_t^d}\right] + \frac{\lambda_{t+1}^B B_t^l + \lambda_{t+1}^D D_t}{1+i_t^b}. \tag{iii}$$

The multiplier μ_t is positive by (60). Hence, the constraint is binding which we use to rewrite the square bracket,

$$\lambda_t^B B_{t-1}^l + \lambda_t^D D_{t-1} = \chi_t^b - \left[\tilde{s}^b S_t^G + \mu_t \left(1-\kappa^G\right) \frac{B_t^g}{1+i_t^g}\right] + \frac{\lambda_{t+1}^B B_t^l + \lambda_{t+1}^D D_t}{1+i_t^b}. \tag{iv}$$

Using the square bracket, we define the value of government debt holdings as

$$V_t^g = \tilde{s}^b S_t^G + \mu_t \left(1-\kappa^G\right) \frac{B_t^g}{1+i_t^g} + \frac{V_{t+1}^g}{1+i_t^b}. \tag{v}$$

Using this, we rewrite (iv) as

$$\lambda_t^B B_{t-1}^l + \lambda_t^D D_{t-1} + V_t^g = \chi_t^b + \frac{\lambda_{t+1}^B B_t^l + \lambda_{t+1}^D D_t + V_{t+1}^g}{1+i_t^b}. \tag{vi}$$

Comparing (vi) with the valuation equation $V_t^b = \chi_t^b + V_{t+1}^b / \left(1+i_t^b\right)$ gives

$$V_t^b = \lambda_t^B B_{t-1}^l + \lambda_t^D D_{t-1} + V_t^g \quad \Leftrightarrow \quad V_t^b = \left(1 - \left(1-t_t^b\right) d_t^l\right) B_{t-1}^l - D_{t-1} + V_t^g. \tag{vii}$$

The second equality uses shadow prices. Since $V_t^g \neq (1-d_t^g) B_{t-1}^g$, we find that the market value of the bank differs from the book value of equity E_t, as noted in (56). ■

Government support reflects asset purchases similar to the troubled asset relief program (TARP) of the U.S. during the financial crisis. The government purchases a fraction t_t^b of the loan portfolio and pays the face value of one. Total asset purchases equal $t_t^b B_{t-1}^l$. After absorbing credit losses $T_t^b = t_t^b d_t^l B_{t-1}$, the government sells back the 'cleaned' assets at the depreciated value $\left(1-d_t^l\right) t_t^b B_{t-1}^l$. The net transfer to the banking sector equals T_t^b. The subsidy t_t^b reduces the markup factor, leading banks to set smaller loan rates.

The program aims to keep up private lending at low rates, thereby reducing the severity of a crisis and speeding up the recovery.

The share of bad loans follows an autoregressive process

$$s_t^l = \left(1-\rho^{sl}\right)\bar{s}^l \cdot \left(\bar{Y}_t/Y_t\right)^{\sigma^{sl}} + \rho^{sl} s_{t-1}^l + \varepsilon_t^{sl}, \quad t_t^b = \rho^{sl} t_{t-1}^b + \varepsilon_t^{tb}. \tag{61}$$

When actual output Y_t falls short of potential output $\bar{Y}_t$, the share of bad loans shifts up with elasticity σ^{sl}. In a SS, $\bar{Y}_t = Y_t$ and $\varepsilon_t^{sl} = 0$, giving a stationary value $s^l = \bar{s}^l$. The subsidy rate on bad loans follows the policy process in the second equation. While banks take the subsidy rate t_t^b as given, it is a choice variable of the government. The program is activated only transitorily if the share of non-performing loans s_t^l is very high. When the program is terminated, the subsidy rate vanishes with speed ρ^{sl}.

1.8 General Equilibrium

Gross output Y_t^g is reduced by resource costs of credit losses, equal to $d_t^l B_t^l / P_t$ since B_t^l is in nominal terms. Noting (10), we define net output or GDP equal to

$$Y_t = Y_t^g - d_t^l B_{t-1}^l / P_t, \quad Y_t^g = z_t K_{t-1}^{\alpha} L_t^{1-\alpha}. \tag{62}$$

Monetary Policy: We analyze fluctuations around a steady state with constant money supply and zero inflation. We specify a policy rule as in Ascari and Ropele (2013) and Sargent and Surico (2011),

$$M_t^s = (1-\rho^m)\,\phi^m \bar{Y}_{t-1} \cdot \frac{\left(\bar{Y}_{t-1}/Y_t\right)^{\psi_y}}{(1+\pi_t)^{\psi_\pi}} + \rho^m M_{t-1}^s + \varepsilon_t^m, \quad T_t^M = M_t^s - M_{t-1}^s. \tag{63}$$

Trend output $\bar{Y}_{t-1}$ is smoothed over the business cycle according to

$$\bar{Y}_t = \delta^m Y_t + (1-\delta^m)\,\bar{Y}_{t-1} = \sum_{s=0}^{\infty} \delta^m (1-\delta^m)^s\, Y_{t-s}. \tag{64}$$

Weights add up to $\sum_s \delta^m (1-\delta^m)^s = 1$. With a smaller rate δ^m, the 'memory' of past output realizations is longer, and current output realizations weigh less heavily in trend output. The trend component of money supply $\phi^m \bar{Y}_{t-1}$ accommodates a permanent increase in output so that the price level can stay roughly constant.

Money supply consists of a trend and a cyclical component. In a SS, actual output is at its trend level, $Y = \bar{Y}$. Depending on parameters ψ_y and ψ_π, the cyclical part is meant to dampen fluctuations. If current output is below trend output, $Y_t < \bar{Y}_{t-1}$, money supply scales up by a factor $\left(\bar{Y}_{t-1}/Y_t\right)^{\psi_y} > 1$, while the oppposite happens in a boom. Similarly, if actual inflation exceeds the trend rate ($\pi_t > 0$), money supply is scaled down by $1/\left(1+\pi_t\right)^{\psi_\pi} < 0$. The opposite happens in a recession where $\pi_t < 0$.

Suppose B_t^f is foreign debt, denominated in domestic currency and growing by

$$B_t^f/\left(1+i_t\right) = B_{t-1}^f - TB_t, \quad TB_t = P_t E_t^x - P_t^{ie}\left(C_t^{ie} + Z_t^{ie}\right) - P_t^{io}\left(C_t^{io} + Z_t^{io}\right). \tag{65}$$

The trade balance TB_t, in domestic currency, is a surplus equal to the value of exports minus imports. Exports reflect import demand of the other regions and will be specified below. In a SS, trade surpluses pay for interest on foreign debt, $iB^f/\left(1+i\right) = TB$.

In (65), foreign debt is denominated in domestic currency (Lira), and is exclusively held by Eurozone investors.[5] There is exchange rate risk. If an EZ saver invests 1 Euro at home, she earns gross interest $1 + i_t^e$. If she invests 1 Euro in the Italian bond, she gets e_t^{ie} Lire at the beginning of period which grow by $1 + i_t$ and are converted back at a rate $1/e_{t+1}^{ie}$, giving end of period wealth equal to $\left(1+i_t\right) e_t^{ie}/e_{t+1}^{ie}$. Standard interest rate parity prevents arbitrage. However, when countries are subject to default risk, investors request a premium $\theta_t \geq 1$. This premium is assumed to rise in line with debt to GDP ratios rising beyond some benchmark, $b_t^f \geq \bar{b}^f$. Modified interest rate parity requires

$$\left(1+i_t\right) e_t^{ie}/e_{t+1}^{ie} = \left(1+i_t^e\right)\theta_t. \tag{66}$$

The return of the Italian bond in Euros must exceed the domestic return $1+i_t^e$ by a factor of θ_t. Assets are perfect substitutes up to the premium θ_t. When the country's debt ratio rises, investors start to worry about solvency and ask for a higher premium. In addition,

[5] Our focus is on Italy and the Eurozone. We thus do not explain capital flows with RoW.

we introduce autoregressive shocks to capture sudden stop phenomena in foreign funding,

$$\theta_t = \left(1-\rho^f\right)\left[1+\gamma\left(e^{b_t^f-\bar{b}^f}-1\right)\right]+\rho^f\theta_{t-1}+\varepsilon_t^\theta, \quad b_t^f \equiv B_t^f/\left(P_tY_t\right). \tag{67}$$

In a SS, exchange rates are constant and $i = i^e = 1/\beta$, to support stationary consumption. The country premium must disappear, $\theta \to 1$ which requires $b_t^f \to \bar{b}^f$. The model thus explains fluctuations around a stationary foreign debt to GDP ratio. The debt sensitivity of the country premium assures stability of savings in an open economy.

Market Clearing: Equilibrium requires market clearing

$$Y_t = C_t + G_t + Z_t + E_t^x, \quad A_t = -B_t^f, \quad \bar{M}_t = M_t^s/\bar{P}_{t+1}. \tag{68}$$

Demand for home goods stem from consumption, public spending and investment, C_t, G_t and Z_t. Households own firms and banks. They receive dividends in (18), reflecting a return on equity values. Therefore, A_t is residual savings which is equal to net foreign assets, the negative of foreign debt. There is no separate condition for labor market clearing since each household type j is a 'local' monopolist and serves the entire market, $N_{j,t}H = L_{j,t}$. The private sector chooses real money balances which must be equal to money supply, $\bar{M}_t = M_t^s/\bar{P}_{t+1}$. Market clearing determines the price level $\bar{P}_{t+1} = M_t^s/\bar{M}_t$, where M_t^s is nominal money supply and $\bar{M}_t = m_t \cdot \bar{C}_t$ is real money demand in (26). Money market clearing reflects our dating convention that measures money holdings and money supply M_t^s at the end of period.

Lemma 3 ***(Walras' Law)*** *Excess demands are related by*

$$\left(A_t + B_t^f\right)/\left(1+i_t\right) + \left(M_t - M_t^s\right) + P_t\left(C_t + G_t + Z_t + E_t^x - Y_t\right) = 0. \tag{69}$$

Proof. Wage income is $\int_0^1 w_{j,t}N_{j,t}Hdj = w_t^L L_t$. Use also $T_t^M = M_t^s - M_{t-1}^s$ in the household budget (18). Market clearing holds identically in $t-1$, i.e. $M_{t-1} = M_{t-1}^s$. Use dividends χ_t and χ_t^b from (32,53), and the fiscal budget in (39,46). Using also the definition of GDP in (62) gives

$$\begin{aligned} \frac{A_t}{1+i_t} - A_{t-1} + \left(M_t - M_t^s\right) &= P_tY_t - P_tG_t - \bar{P}_t\left(\bar{C}_t + \bar{Z}_t\right) \qquad (70) \\ &= P_t\left(Y_t - G_t - C_t - Z_t - E_t^x\right) + TB_t. \end{aligned}$$

Expand by export values $P_t E_t^x$, substitute the budget $\bar{P}_t \bar{C}_t = P_t C_t + P_t^{ie} C_t^{ie} + P_t^{io} C_t^{io}$, and similarly for investment spending $\bar{P}_t \bar{Z}_t$, and finally use the definition of the trade balance to get the last equality. With output market clearing, the trade surplus is thus equal to national income minus domestic absorption. By the income expenditure identity in the first equation, national income is equal to domestic absorption plus aggregate savings, $P_t Y_t = P_t G_t + \bar{P}_t \left(\bar{C}_t + \bar{Z}_t\right) + [A_t / (1 + i_t) - A_{t-1} + (M_t - M_t^s)]$. Replace the trade balance by the current account in (65). Noting that $A_{t-1} + B_{t-1}^f = 0$ holds identically by last period's capital market clearing finally proves the result in (69). By Walras' Law, one of the market clearing conditions is redundant. ■

2 The World Economy

2.1 Rest of Eurozone

Households: Eurozone modeling is extremely simple. We postulate an autoregressive income process for Eurozone GDP,

$$Y_t^e = (1 - \rho) Y_0^e + \rho Y_{t-1}^e + \varepsilon_t^{Y,e}. \tag{71}$$

Households collect income, consume and save. Financial wealth in Euros grows by

$$A_t^e / (1 + i_t^e) = A_{t-1}^e + \bar{\imath}_t^e + P_t^e Y_t^e + \left(M_{t-1}^e - M_t^e\right) + T_t^{M,e} - \bar{P}_t^e \bar{C}_t^e, \tag{72}$$

where $\bar{\imath}_t^e$ is the differential return on holding Italian bonds which are part of financial wealth. Given the risk premium θ_t, Italian bonds pay higher interest than domestic assets which earn i_t^e only, see the discussion in (80) below.

Agents spend $\bar{P}_t^e \bar{C}_t^e = P_t^e C_t^e + P_t^{ei} C_t^{ei} + P_t^{eo} C_t^{eo}$ where $P_t^{ei} = P_t^i / e_t^{ie}$ and $P_t^{eo} = P^o \cdot e_t^{eo}$ are local demand prices in Euros. One Dollar is worth e_t^{eo} Euros. If a RoW good costs P^o Dollars, the price in Euros is $P_t^{eo} = P^o \cdot e_t^{eo}$. EZ consumers demand three goods, subject to $\bar{C}_t^e = \left[\sum_r \left(s_r^{er}\right)^{1/\sigma_r} \left(C_t^{er}\right)^{(\sigma_r - 1)/\sigma_r}\right]^{\sigma_r/(\sigma_r - 1)}$. Minimizing expenditure yields a price index

$\bar{P}_t^e$, demand C_t^{er} for goods of region r, and total spending $\bar{P}_t^e \bar{C}_t^e$,

$$C_t^{er} = s_r^{er} \left(\bar{P}_t^e / P_t^{er}\right)^{\sigma_r} \bar{C}_t^e, \quad \bar{P}_t^e = \left[\sum_r s_r^{er} \left(P_t^{er}\right)^{1-\sigma_r}\right]^{1/(1-\sigma_r)}. \tag{73}$$

Preferences for consumption $\bar{C}_t^e$ and real money balances $\bar{M}_t^e$ are

$$V_t^e = E_t \sum_{s=0}^{\infty} \beta^s \frac{X\left(\bar{C}_t^e, \bar{M}_t^e\right)^{1-\sigma_c}}{1-\sigma_c}, \quad X_t^e = \left[s_c \left(\bar{C}_t^e\right)^{1-\sigma_m} + (1-s_c)\left(\bar{M}_t^e\right)^{1-\sigma_m}\right]^{\frac{1}{1-\sigma_m}}. \tag{74}$$

By linear homogeneity, and using the money-consumption ratio m_t^e,

$$X_t^e = x_t^e \cdot \bar{C}_t^e, \quad x_t^e = \left[s_c + (1-s_c)\left(m_t^e\right)^{1-\sigma_m}\right]^{1/(1-\sigma_m)}, \quad m_t^e \equiv \bar{M}_t^e / \bar{C}_t^e.$$

Using $u\left(X_t^e\right) = \left(X_t^e\right)^{1-\sigma_c} / (1-\sigma_c)$ gives marginal utilities $u_{C,t}^e = s_c \left(\bar{C}_t^e\right)^{-\sigma_m} \left(X_t^e\right)^{\sigma_m - \sigma_c}$ and $u_{M,t}^e = (1-s_c)\left(\bar{M}_t^e\right)^{-\sigma_m} \left(X_t^e\right)^{\sigma_m-\sigma_c}$. As in Walsh (2010), pp. 66-67, we eventually get $u_{CM,t}^e = (\sigma_m - \sigma_c) s_c \left(\bar{C}_t^e\right)^{-\sigma_m} (1-s_c)\left(\bar{M}_t^e\right)^{-\sigma_m} \left(X_t^e\right)^{2\sigma_m - \sigma_c - 1} \gtrless 0 \Leftrightarrow \sigma_m \gtrless \sigma_c$. Note

$$u_{C,t}^e = \frac{s_c \left(x_t^e\right)^{\sigma_m - \sigma_c}}{\left(\bar{C}_t^e\right)^{\sigma_c}}, \quad u_{M,t} = \frac{(1-s_c)\left(x_t^e\right)^{\sigma_m-\sigma_c}}{\left(m_t^e\right)^{\sigma_m}\left(\bar{C}_t^e\right)^{\sigma_c}}.$$

The Fisher equation relating real and nominal interest rates is

$$1 + \pi_t^e = \bar{P}_{t+1}^e / \bar{P}_t^e, \quad 1 + r_t^e = \left(1 + i_t^e\right) / \left(1 + \pi_t^e\right) = \left(1 + i_t^e\right) \bar{P}_t^e / \bar{P}_{t+1}^e. \tag{75}$$

Using $V_t^e = u_t^e + \beta V_{t+1}^e$, the Bellman problem

$$V^e\left(A_{t-1}^e, M_{t-1}^e\right) = \max_{\bar{C}_t^e, \bar{M}_t^e} \ u\left(\bar{C}_t^e, \bar{M}_t^e\right) + \beta E_t V^e\left(A_t^e, M_t^e\right),$$

subject to (72) and $M_t^e = \bar{M}_t^e \bar{P}_{t+1}^e$. Define $\lambda_{t+1}^e \equiv dV_{t+1}^e / dA_t^e$ and $\lambda_t^{M,e} \equiv dV_t^e / dM_{t-1}^e$. Optimality conditions for $\bar{C}_t^e, \bar{M}_t^e$ and envelope conditions for A_{t-1}^e, M_{t-1}^e are

$$\begin{aligned}
\bar{C}_t^e &: \ u_{C,t}^e = \beta E_t \lambda_{t+1}^e \left(1 + i_t^e\right) \bar{P}_t^e, \\
\bar{M}_t^e &: \ u_{M,t}^e = \beta E_t \left[\lambda_{t+1}^e \left(1 + i_t^e\right) - \lambda_{t+1}^{M,e}\right] \bar{P}_{t+1}^e, \\
A_{t-1}^e &: \ \lambda_t^e = \beta E_t \lambda_{t+1}^e \left(1 + i_t^e\right), \\
M_{t-1}^e &: \ \lambda_t^{M,e} = \beta E_t \lambda_{t+1}^e \left(1 + i_t^e\right).
\end{aligned} \tag{76}$$

Shift forward (76.iii) by one period, multiply by $\beta E_t \left(1+i_t^e\right) \bar{P}_t^e$, rearrange terms to use (i) on both sides, note the Fisher equation and get the Euler condition

$$u_{C,t}^e = \beta E_t \left(1+r_t^e\right) \cdot u_{C,t+1}^e, \quad u_{C,t}^e = s_c \left(x_t^e\right)^{\sigma_m - \sigma_c} / \left(\bar{C}_t^e\right)^{\sigma_c}, \tag{77}$$

where $x_t^e = \left[s_c + (1-s_c)\left(m_t^e\right)^{1-\sigma_m}\right]^{1/(1-\sigma_m)}$.

By (iii-iv), $\lambda_t^{M,e} = \lambda_t^e$. Using this in (ii), combining with (i) and noting the Fisher equation gives the tangency condition for money demand,

$$\frac{u_{M,t}}{u_{C,t}} = \frac{1-s_c}{s_c} / \left(m_t^e\right)^{\sigma_m} = \frac{i_t^e}{1+r_t^e} \quad \Rightarrow \quad m_t^e \equiv \frac{\bar{M}_t^e}{\bar{C}_t^e} = \left(\frac{1-s_c}{s_c} \frac{1+r_t^e}{i_t^e}\right)^{1/\sigma_m}. \tag{78}$$

The policy rule for money supply is

$$\begin{aligned} M_t^{s,e} &= (1-\rho^m)\, \phi^{m,e} \bar{Y}_{t-1}^e \cdot \frac{\left(\bar{Y}_{t-1}^e / Y_t^e\right)^{\psi_y}}{\left(1+\pi_t^e\right)^{\psi_\pi}} + \rho^m M_{t-1}^{s,e} + \varepsilon_t^{m,e}, \\ \bar{Y}_t^e &= \delta^m Y_t^e + (1-\delta^m)\, \bar{Y}_{t-1}^e, \quad T_t^{M,e} = M_t^{s,e} - M_{t-1}^{s,e}. \end{aligned} \tag{79}$$

Trend output $\bar{Y}_{t-1}^e$ is smoothed over the business cycle. The trend component of money supply thus expands by $\phi^m \bar{Y}_{t-1}$ and accommodates a permanent increase in output so that the price level can stay roughly constant.

Equilibrium: Net foreign debt B_t^e, denominated in Euros, grows by

$$B_t^e / \left(1+i_t^e\right) = B_{t-1}^e - TB_t^e - \bar{\imath}_t^e, \quad TB_t^e = P_t^e E_t^{x,e} - P_t^{ei} C_t^{ei} - P_t^{eo} C_t^{eo}, \tag{80}$$

where $\bar{\imath}_t^e \equiv (\theta_t - 1)\left(TB_t^e - B_{t-1}^e\right)$. Since EZ holds Italian bonds and is, thus, a creditor country, net foreign debt is negative, i.e., it is net wealth. Since Italian bonds pay a premium θ_t, interest exceeds the domestic rate i_t^e, which generates differential income $\bar{\imath}_t^e$. The trade surplus plus differential interest earnings both add to next periods foreign assets. Using $\bar{\imath}_t^e$, bond holdings thus accumulate by $B_t^e = \left(1+i_t^e\right)\theta_t \cdot \left(B_{t-1}^e - TB_t^e\right)$.

Equilibrium requires market clearing

$$Y_t^e = C_t^e + E_t^{x,e}, \quad A_t^e = -B_t^e, \quad \bar{M}_t^e = M_t^{s,e} / \bar{P}_{t+1}^e. \tag{81}$$

Savings is equal to net foreign assets, the negative of foreign debt.

Lemma 4 *(**Walras' Law**) Excess demands are related by*

$$\left(A_t^e + B_t^e\right) / \left(1 + i_t^e\right) + \left(M_t^e - M_t^{s,e}\right) + P_t^e \left(C_t^e + E_t^{x,e} - Y_t^e\right) = 0. \tag{82}$$

Proof. Use $T_t^{M,e} = M_t^{s,e} - M_{t-1}^{s,e}$ in the household budget (72), and note that market clearing holds as an identity in period $t-1$, i.e. $\bar{P}_t^e \bar{M}_{t-1}^e = M_{t-1}^e = M_{t-1}^{s,e}$,

$$\frac{A_t^e}{1+i_t^e} - A_{t-1}^e + \left(M_t^e - M_t^{s,e}\right) = P_t^e Y_t^e - \bar{P}_t^e \bar{C}_t^e + \bar{\imath}_t^e = P_t^e \left(Y_t^e - C_t^e - E_t^{x,e}\right) + TB_t^e + \bar{\imath}_t^e. \tag{83}$$

To get the last equality, expand by exports $P_t^e E_t^{x,e}$, substitute the budget $\bar{P}_t^e \bar{C}_t^e$, and use the trade balance. Next, replace the trade balance by the current account in (80), note that $A_{t-1}^e + B_{t-1}^e = 0$ holds identically by last period's market clearing, and get the result in (82). By Walras' Law, one of the market clearing conditions is redundant. ■

2.2 Rest of the World

RoW consists of other countries (indexed by o). Since our focus is on Italy and the Eurozone, we allow only for internal capital flows. Hence, net foreign assets of RoW are zero and trade is balanced. The foreign final good serves as *numeraire*, the local price is $P^o = 1$. We postulate export demand functions for Italian and EZ exports to RoW,

$$C_t^{oi} = s^{oi} \cdot \left(e_t^{io}/P_t\right)^{\sigma_r}, \quad C_t^{oe} = s^{oe} \cdot \left(e_t^{eo}/P_t^e\right)^{\sigma_r}. \tag{84}$$

Now all export demands are specified, giving

$$E_t^x = C_t^{ei} + C_t^{oi}, \quad E_t^{x,e} = C_t^{ie} + Z_t^{ie} + C_t^{oe}, \quad E_t^{x,o} = C_t^{io} + Z_t^{io} + C_t^{eo}. \tag{85}$$

With zero net foreign assets, trade must be balanced

$$TB_t^o = P^o E_t^{x,o} - P_t^{oe} C_t^{oe} - P_t^{oi} C_t^{oi} = 0. \tag{86}$$

Lemma 5 *(**Walras' Law**) In the total world economy, the sum of trade balances, converting them into the same currency (e.g. Lire), must add up to zero,*

$$TB_t + e_t^{ie} TB_t^e + e_t^{io} TB_t^o = 0. \tag{87}$$

Since RoW is closed to external capital flows, global capital market clearing, expressed in the same currency, requires

$$A_t + e^{ie}_{t+1} A^e_t = -\left(B^f_t + e^{ie}_{t+1} B^e_t\right) = 0. \tag{88}$$

Proof. Substitute the definitions of trade balances and of exports in (85) and use $P^{ie}_t = e^{ie}_t P^e_t$, $P^{io}_t = e^{io}_t P^o$ and $P^{ei}_t = P_t / e^{ie}_t$, $P^{oi}_t = P_t / e^{io}_t$, and finally $P^{oe}_t = e^{oe}_t P^e_t$, $P^{eo}_t = e^{eo}_t P^o$, $e^{io}_t e^{oe}_t = e^{ie}_t$ and $e^{ie}_t e^{eo}_t = e^{io}_t$ to get the result in (87). Imports of one country are the exports of another, and the trade surplus of one is a deficit of another.

National capital market clearing requires $A^j_t = -B^j_t$. Substitute the current accounts and use $B^e_t = (1 + i^e_t)\,\theta_t \left(B^e_{t-1} - TB^e_t\right)$ for the Eurozone as noted after (80),

$$\begin{aligned}
\left(B^f_t + e^{ie}_{t+1} B^e_t\right) &= (1 + i_t)\left(B^f_{t-1} - TB_t\right) + (1 + i^e_t)\,\theta_t \frac{e^{ie}_{t+1}}{e^{ie}_t} \cdot \left(e^{ie}_t B^e_{t-1} - e^{ie}_t TB^e_t\right) \quad \text{(i)} \\
&= (1 + i_t)\left[\left(B^f_{t-1} + e^{ie}_t B^e_{t-1}\right) - \left(TB_t + e^{ie}_t TB^e_t\right)\right] = 0.
\end{aligned}$$

The second line uses modified interest parity in (66). Capital market clearing in the last period implies $B^f_{t-1} + e^{ie}_t B^e_{t-1} = 0$. We also have $TB_t + e^{ie}_t TB^e_t = 0$ by (87) since trade in RoW is balanced, $TB^o_t = 0$, which gives global capital market clearing. ■

2.3 Currency Union

Common Monetary Policy: In a *currency union*, there is only 1 monetary policy subject to 1 money market clearing, and the exchange rate is fixed at $e^{ie} = 1$. Money supply is based on the state of the whole union which is a weighted average of the two regions. We use weights $s^Y = PY / (PY + P^e Y^e)$ and $1 - s^Y$ equal to the calibrated shares in total Eurozone GDP of Italy and the rest. We define a 'price index' $\bar{P}^u_t$ and get

$$Y^u_t \equiv \left(P_t Y_t + P^e_t Y^e_t\right) / \bar{P}^u_t, \quad \bar{P}^u_t \equiv s^Y \bar{P}_t + \left(1 - s^Y\right) \bar{P}^e_t, \quad 1 + \pi^u_t \equiv \bar{P}^u_{t+1} / \bar{P}^u_t. \tag{89}$$

Total Eurozone trend output is $\bar{Y}^u_t = \delta^m Y^u_t + (1 - \delta^m)\,\bar{Y}^u_{t-1}$.

Money market clearing requires $M^{s,u} \equiv M^s + M^{s,e} = \bar{P}\bar{M} + \bar{P}^e\bar{M}^e$. The trend component of money supply is $M^{s,u} = \phi^{m,u}\bar{Y}^u$ with a coefficient $\phi^{m,u} \equiv \frac{\bar{P}\bar{M}+\bar{P}^e\bar{M}^e}{\bar{Y}^u}$. The common monetary policy rule includes trend and countercyclical components,

$$M_t^{s,u} = (1-\rho^m)\,\phi^{m,u}\bar{Y}_{t-1}^u \cdot \frac{\left(\bar{Y}_{t-1}^u/Y_t^u\right)^{\psi_y}}{\left(1+\pi_t^u\right)^{\psi_\pi}} + \rho^m M_{t-1}^{s,u} + \varepsilon_t^{m,u}. \tag{90}$$

We allocate money supply $M_t^{s,u} \equiv M_t^s + M_t^{s,e} = \bar{P}_{t+1}\bar{M}_t + \bar{P}_{t+1}^e\bar{M}_t^e$ to accommodate money demand in each region, $M_t^s = \bar{P}_{t+1}\bar{M}_t$ and $M_t^{s,e} = \bar{P}_{t+1}^e\bar{M}_t^e$. In this case, seignorage income cancels from household budgets and Walras' Law holds separately for each country.

If starting from a SS, and if no 'other' shock occurs, a Eurozone exit cannot have any effect! To see this, note that money supply is stationary in a SS without a cyclical component, and is equal to $M^s = \phi^m\bar{Y}$ and $M^{s,e} = \phi^{m,e}\bar{Y}^e$ in the two member states. Adding up, expanding by $\bar{Y}^u = \bar{Y} + \bar{Y}^e$ and using the definitions above gives

$$M^s + M^{s,e} = \phi^m\bar{Y} + \phi^{m,e}\bar{Y}^e = \left[\phi^m\frac{\bar{Y}}{\bar{Y}^u} + \phi^{m,e}\frac{\bar{Y}^e}{\bar{Y}^u}\right]\bar{Y}^u = \phi^{m,u}\bar{Y}^u = M^{s,u}.$$

Nothing changes if total money supply is replaced by its two components which follow exactly the same policy rules. For this to be the case, the coefficients must be related by $\phi^{m,u} = \phi^m\frac{\bar{Y}}{\bar{Y}^u} + \phi^{m,e}\frac{\bar{Y}^e}{\bar{Y}^u}$. Total money supply is the sum of the two components if and only if the coefficient $\phi^{m,u}$ is defined in this way. In addition, if the two countries are in a stationary equilibrium, there is no reason for the exchange rate $e^{ie} = 1$ to adjust, which proves the claim. We use this scenario to check consistency of our model.

Monetary Regimes: The binary variable $EZ \in \{1,0\}$ indicates the monetary regime. Setting $EZ = 1$ implements the currency union with common monetary policy while $EZ = 0$ refers to the exit scenario with autonomous, separate policies:

$$\begin{array}{rcllcl} & & \text{stay in EZ } (EZ=1) & & \text{exit } (EZ=0) \\ \hline M_t^s & = & EZ\cdot\bar{P}_{t+1}\bar{M}_t & + & (1-EZ)\cdot M_t^s\,(63)\,, \\ M_t^{s,e} & = & EZ\cdot\bar{P}_{t+1}^e\bar{M}_t^e & + & (1-EZ)\cdot M_t^{s,e}\,(79)\,, \\ 0 & = & EZ\cdot\left(\bar{P}_{t+1}\bar{M}_t + \bar{P}_{t+1}^e\bar{M}_t^e - M_t^{s,u}\right) & + & (1-EZ)\cdot\left(\bar{P}_{t+1}\bar{M}_t - M_t^s\right), \\ 0 & = & EZ\cdot\left(e_t^{ie}-1\right) & + & (1-EZ)\cdot\left(\bar{P}_{t+1}^e\bar{M}_t^e - M_t^{s,e}\right). \end{array} \tag{91}$$

In both scenarios, we compute total money supply $M_t^{s,u}$ as in (90), although it will only be used in the EMU scenario. The column $(EZ = 1)$ refers to common monetary policy. The exchange rate is fixed at $e_t^{ie} = 1$, and the area wide money market clears. The first two elements in the column $(EZ = 1)$ indicate that money supply flows to each region according to money demand. The second column $(EZ = 0)$ refers to an exit, switching to two independent policies that separately determine M_t^s and $M_t^{s,e}$ by the rules (63) and (79). Each money market clears in isolation and the exchange rate e^{ie} floats freely.

References

[1] Calvo, Guillermo A. (1983), Staggered Prices in a Utility Maximizing Framework, *Journal of Monetary Economics 12*, 383-398.

[2] Gali, Jordi (2015), *Monetary Policy, Inflation, and the Business Cycle*, 2nd ed., Princeton University Press.

[3] Walsh, Carl E. (2010), *Monetary Theory and Policy*, 3rd ed., MIT Press.

[4] Yun, Tack (1996), Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles, *Journal of Monetary Economics 37*, 345-370.

3 Appendix

We show that the solution in (27) exactly corresponds to Gali (2015). To see this, note that w_t^* remains fixed until the next opportunity arrives. Shift forward (27.ii) by one period and rearrange (i),

$$(i) \; : \; 0 = -\left[\frac{(1-\tau_t)\, w_t^*}{(1+\tau_t^c)\, \bar{P}_t} - \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t}\right] \frac{N_{t,t}}{w_t^*} + \omega \frac{\mu_{t+1}}{1+r_t},$$

$$(ii) \; : \; \mu_{t+1} = -\left[\frac{(1-\tau_{t+1})\, w_t^*}{\left(1+\tau_{t+1}^c\right) \bar{P}_{t+1}} - \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t+1}\right] \frac{N_{t,t+1}}{w_t^*} + \omega \cdot \frac{\mu_{t+2}}{1+r_{t+1}}.$$

Substitute the Euler equation $\frac{1}{1+r_t} = \beta \frac{u_{C,t+1}}{u_{C,t}}$, multiply by $u_{C,t}$, and define $v_{t+i} \equiv u_{C,t+i}\mu_{t+i}$,

$$: \; 0 = -\left[\frac{(1-\tau_t)\, w_t^*}{(1+\tau_t^c)\, \bar{P}_t} - \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t}\right] \frac{N_{t,t}}{w_t^*} u_{C,t} + \beta\omega \cdot v_{t+1},$$

$$: \; v_{t+1} = -\left[\frac{(1-\tau_{t+1})\, w_t^*}{\left(1+\tau_{t+1}^c\right) \bar{P}_{t+1}} - \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t+1}\right] \frac{N_{t,t+1}}{w_t^*} u_{C,t+1} + \beta\omega \cdot v_{t+2}.$$

Define $X_{t,t+i} \equiv \left[\frac{(1-\tau_t)w_t^*}{\left(1+\tau_{t+i}^c\right)\bar{P}_{t+i}} - \frac{\sigma_l}{\sigma_l-1} \cdot MRS_{t,t+i}\right] \frac{N_{t,t+i}}{w_t^*} u_{C,t+i}$ and note that w_t^* remains fixed. In general, the second equation is $v_{t+i} = -X_{t,t+i} + \beta\omega \cdot v_{t+i+1}$. Solving forward establishes $0 = -\sum_{i\geq 0} (\beta\omega)^i X_{t,t+i}$. Substitute $X_{t,t+i}$, note that the constant term w_t^* (set today and unchanged thereafter) cancels out, and finally get

$$0 = \sum\nolimits_{i\geq 0} (\beta\omega)^i \left[\frac{(1-\tau_t)\, w_t^*}{\left(1+\tau_{t+i}^c\right) \bar{P}_{t+i}} - \frac{\sigma_l}{\sigma_l - 1} \cdot MRS_{t,t+i}\right] N_{t,t+i} u_{C,t+i}.$$

Setting taxes to zero, this exactly corresponds to equation 11 of Gali (2015, p. 168).

4 Model Representation in Dynare

1. $z_t = (1-\rho)\, \bar{z} \left(K_t^G / \bar{K}^G\right)^{\sigma^z} + \rho z_{t-1} + \varepsilon_t^z$, productivity shocks, **ITALY**
2. $\tau_t = t_t^s \tau_0$, tax rate scaling for budged balance
3. $\tau_t^c = t_t^s \tau_0^c$,
4. $P_t^{ie} = e_t^{ie} P_t^e$, import prices
5. $P_t^{io} = e_t^{io} P^o$,
6. $\bar{P}_t = \left[s^{ii} (P_t)^{1-\sigma_r} + s^{ie} \left(P_t^{ie}\right)^{1-\sigma_r} + s^{io} \left(P_t^{io}\right)^{1-\sigma_r}\right]^{1/(1-\sigma_r)}$, price index
7. $b_t^f = B_t^f / (P_t Y_t)$, foreign debt to GDP ratio
8. $\theta_t = \left(1-\rho^f\right) \left[1 + \gamma \left(e^{b_t^f - \bar{b}^f} - 1\right)\right] + \rho^f \theta_{t-1} + \varepsilon_t^\theta$, country risk premium
9. $1 + i_t = (1 + i_t^e)\, \theta_t e_{t+1}^{ie} / e_t^{ie}$, interest parity
10. $1 + \pi_t = \left(1 + \tau_{t+1}^c\right) \bar{P}_{t+1} / \left((1 + \tau_t^c)\, \bar{P}_t\right)$,
11. $1 + r_t = (1 + i_t) / (1 + \pi_t)$,
12. $\theta_t^g = 1 - \rho^\theta + \rho^\theta \theta_{t-1}^g + \varepsilon_t^g$, government interest shock
13. $\theta_t^d = 1 - \rho^\theta + \rho^\theta \theta_{t-1}^d + \varepsilon_t^d$, deposit shock
14. $\theta_t^b = \left(1 - \rho^\theta\right) \bar{\theta}^b + \rho^\theta \theta_{t-1}^b + \varepsilon_t^b$, equity premium shocks
15. $\theta_t^k = \left(1 - \rho^\theta\right) \bar{\theta}^k + \rho^\theta \theta_{t-1}^k + \varepsilon_t^k$,
16. $i_t^g = \theta_t^g i_t$, interest premium

17. $i_t^d = \theta_t^d i_t$,

18. $i_t^b = \theta_t^b i_t$,

19. $i_t^k = \theta_t^k i_t$,

20. $u_{C,t} = \beta\left(1+r_t\right)\cdot u_{C,t+1}$, Euler equation

21. $m_t = \left[\left(1-s_c\right)\left(1+\tau_{t+1}^c\right)\left(1+r_t\right)/\left(s_c i_t\right)\right]^{1/\sigma_m}$, money consumption ratio

22. $x_t = \left[s_c + \left(1-s_c\right) m_t{}^{1-\sigma_m}\right]^{1/(1-\sigma_m)}$,

23. $\bar{C}_t = \left(s_c x_t^{\sigma_m-\sigma_c}/u_{C,t}\right)^{1/\sigma_c}$,

24. $\bar{M}_t = m_t \cdot \bar{C}_t$, money demand

25. $\left(w_t^L\right)^{1-\sigma_l} = \left(1-\omega\right)\left(w_t^*\right)^{1-\sigma_l} + \omega\left(w_{t-1}^L\right)^{1-\sigma_l}$, wage index

26. $N_{t,t} = \left(w_t^L/w_t^*\right)^{\sigma_l} L_t/H$, per capita

27. $N_{t-1,t} = \left(w_t^L/w_{t-1}^*\right)^{\sigma_l} L_t/H$, vintage $t-1$ at date t

28. $\phi_t = \left(1-\rho\right)\bar{\phi} + \rho\phi_{t-1} + \varepsilon_t^{\phi}$,

29. $MRS_{t,t} = \phi_t N_{t,t}^{\eta} \bar{C}_t^{\sigma_c}/\left(s_c x_t^{\sigma_m-\sigma_c}\right)$,

30. $\frac{1-\tau_t}{1+\tau_t^c}\frac{w_t^*}{\bar{P}_t} = \frac{\sigma_l}{\sigma_l-1} MRS_{t,t} + \left(w_t^*/N_{t,t}\right)\omega\mu_{t+1}^w/\left(1+r_t\right)$, new wage setting

31. $\mu_t^w = -\left[\frac{1-\tau_t}{1+\tau_t^c}\frac{w_{t-1}^*}{\bar{P}_t} - \frac{\sigma_l}{\sigma_l-1}\left(w_t^*/w_{t-1}^*\right)^{\eta\sigma_l} MRS_{t,t}\right] N_{t-1,t}/w_{t-1}^* + \omega\mu_{t+1}^w/\left(1+r_t\right)$,

32. $m_t^c = P_t \cdot \left(\sigma_v - 1\right)/\sigma_v$, markup pricing

33. $\tilde{k}_t = K_{t-1}/L_t$,

34. $w_t^L = \left(1-\alpha\right) m_t^c z_t \tilde{k}_t^{\alpha}$, factor prices

35. $w_t^K = \alpha m_t^c z_t/\tilde{k}_t^{1-\alpha}$,

36. $Q_t = \left(\lambda_{t+1}^K/\left(1+i_t^k\right)\right)/\left(\left(1-t^z\tau_t\right)\bar{P}_t\right)$, INVESTMENT

37. $K_t = \left(1 + \left(Q_t - 1\right)/\psi\right) K_{t-1}$,

38. $I_t = K_t - \left(1-\delta\right) K_{t-1}$,

39. $\bar{Z}_t = I_t + \frac{\psi}{2} K_{t-1}\left(I_t/K_{t-1} - \delta\right)^2$, adjustment costs

40. $\bar{Z}_{K,t} = -\frac{\psi}{2}\left(I_t/K_{t-1} + \delta\right)\left(I_t/K_{t-1} - \delta\right)$,

41. $s_t^l = \left(1-\rho^{sl}\right)\bar{s}^l\left(\bar{Y}_t/Y_t\right)^{\sigma^{sl}} + \rho^{sl} s_{t-1}^l + \varepsilon_t^{sl}$, NPL share

42. $d_t^l = ls_t^l$,

43. $t_t^b = \rho^{sl} t_{t-1}^b + \varepsilon_t^{tb}$, subsidy rate bank rescue

44. $1 + i_t^l = \left(\kappa^B \left(1 + i_t^b\right) + \left(1 - \kappa^B\right) \left(1 + i_t^d\right)\right) / \left(1 - \left(1 - t_{t+1}^b\right) d_{t+1}^l\right)$,

45. $\lambda_t^K = (1 - \tau_t) w_t^K + \left[\left(i_t^k - (1 - \tau_t) i_t^l\right) b^l - \bar{Z}_{K,t}\right] (1 - t^z \tau_t) \bar{P}_t + (1 - \delta) \lambda_{t+1}^K / \left(1 + i_t^k\right)$,

46. $B_t^l / \left(1 + i_t^l\right) = b^l \left(1 + i_t^k\right) (1 - t^z \tau_t) \bar{P}_t K_{t-1}$, debt capacity

47. $S_t^l = B_{t-1}^l - B_t^l / \left(1 + i_t^l\right)$,

48. $Y_t^g = z_t \tilde{k}_t^\alpha L_t$, gross output

49. $Y_t = Y_t^g - d_t^l B_{t-1}^l / P_t$, GDP

50. $\bar{Y}_t = \delta^m Y_t + (1 - \delta^m) \bar{Y}_{t-1}$, potential output

51. $\chi_t^m = (P_t - m_t^c) Y_t^g$,

52. $T_t^k = P_t Y_t^g - w_t^L L_t - t^z \bar{P}_t \bar{Z}_t - i_t^l \left(B_{t-1}^l - S_t^l\right) / \left(1 + i_t^k\right)$,

53. $\chi_t = P_t Y_t^g - w_t^L L_t - \bar{P}_t \bar{Z}_t - S_t^l - \tau_t T_t^k$,

54. $V_t = \chi_t + V_{t+1} / \left(1 + i_t^k\right)$,

55. $\tilde{S}_t^G = (1 - \gamma^g / (1 + i_t^g)) B_{t-1}^G - ((1 - \gamma^g) / (1 + i_t^g)) \bar{b}^g P_t Y_t$, FISCAL POLICY

56. $P_t G_t = \bar{g} \cdot P_t Y_t - \xi^g \cdot \tilde{S}_t^G + \varepsilon_t^G$,

57. $E_t = \bar{e} \cdot w_t^L L_t - \xi^e \cdot \tilde{S}_t^G + \varepsilon_t^E$,

58. $T_t = \bar{g} \cdot P_t Y_t + \bar{e} \cdot w_t^L L_t + (1 - \xi^g - \xi^e) \tilde{S}_t^G$, required tax revenue

59. $T_t^l = \left(1 - \rho^T\right) \bar{t}^l P_t Y_t + \rho^T T_{t-1}^l + \varepsilon_t^T$, tax base erosion

60. $T_t = \tau_t \cdot w_t^L L_t + \tau_t \cdot T_t^k + \tau_t^c \cdot \bar{P}_t \bar{C}_t - T_t^l$, budget balance

61. $K_t^G = G_t + (1 - \delta^g) K_{t-1}^G$,

62. $d_t^g = \rho^{dg} d_{t-1}^g + \varepsilon_t^{dg}$, unexpected default

63. $S_t^G = \tilde{S}_t^G - \varepsilon_t^G - \varepsilon_t^E - t_t^b d_t^l B_{t-1}^l + d_t^g B_{t-1}^G$,

64. $B_t^G / (1 + i_t^g) = B_{t-1}^G - S_t^G$,

65. $E_t^b / \left(1 + i_t^b\right) = \kappa^B B_t^l / \left(1 + i_t^l\right) + \kappa^G \tilde{s}^b B_t^G / (1 + i_t^g)$, bank equity

66. $D_t / \left(1 + i_t^d\right) = \left(1 - \kappa^B\right) B_t^l / \left(1 + i_t^l\right) + \left(1 - \kappa^G\right) \tilde{s}^b B_t^G / (1 + i_t^g)$, deposits

67. $S_t^d = D_{t-1} - D_t / \left(1 + i_t^d\right)$,

68. $\chi_t^b = S_t^l + \tilde{s}^b S_t^G - S_t^d - \left(1 - t_t^b\right) d_t^l B_{t-1}^l$,

69. $Y_t^e = (1-\rho)\, Y_0^e + \rho Y_{t-1}^e + \varepsilon_t^{Y,e}$, **EUROZONE**

70. $\bar{Y}_t^e = \delta^m Y_t^e + (1-\delta^m)\, \bar{Y}_{t-1}^e$, potential output

71. $e_t^{eo} = e_t^{io}/e_t^{ie}$,

72. $P_t^{ei} = P_t/e_t^{ie}$, import prices

73. $P_t^{eo} = P^o e_t^{eo}$,

74. $\bar{P}_t^e = \left[s^{ee}\left(P_t^e\right)^{1-\sigma_r} + s^{ei}\left(P_t^{ei}\right)^{1-\sigma_r} + s^{eo}\left(P_t^{eo}\right)^{1-\sigma_r}\right]^{1/(1-\sigma_r)}$, price index

75. $1 + \pi_t^e = \bar{P}_{t+1}^e/\bar{P}_t^e$,

76. $1 + r_t^e = (1 + i_t^e)\,/\,(1 + \pi_t^e)$,

77. $u_{C,t}^e = \beta\,(1 + r_t^e) \cdot u_{C,t+1}^e$, Euler equation

78. $m_t^e = \left[(1 - s_c)\,(1 + r_t^e)\,/\,(s_c i_t^e)\right]^{1/\sigma_m}$, money consumption ratio

79. $x_t^e = \left[s_c + (1 - s_c)\,(m_t^e)^{1-\sigma_m}\right]^{1/(1-\sigma_m)}$,

80. $\bar{C}_t^e = \left(s_c\,(x_t^e)^{\sigma_m - \sigma_c}/u_{C,t}^e\right)^{1/\sigma_c}$,

81. $\bar{M}_t^e = m_t^e \cdot \bar{C}_t^e$, money demand

82. $C_t^{oi} = s^{oi}\,(e_t^{io}/P_t)^{\sigma_r}$, **TRADE FLOWS**

83. $C_t^{oe} = s^{oe}\,(e_t^{eo}/P_t^e)^{\sigma_r}$, export demand functions

84. $C_t = s^{ii}\left(\bar{P}_t/P_t\right)^{\sigma_r} \bar{C}_t$, demand structure

85. $C_t^{ie} = s^{ie}\left(\bar{P}_t/P_t^{ie}\right)^{\sigma_r} \bar{C}_t$,

86. $C_t^{io} = s^{io}\left(\bar{P}_t/P_t^{io}\right)^{\sigma_r} \bar{C}_t$,

87. $Z_t = s^{ii}\left(\bar{P}_t/P_t\right)^{\sigma_r} \bar{Z}_t$,

88. $Z_t^{ie} = s^{ie}\left(\bar{P}_t/P_t^{ie}\right)^{\sigma_r} \bar{Z}_t$,

89. $Z_t^{io} = s^{io}\left(\bar{P}_t/P_t^{io}\right)^{\sigma_r} \bar{Z}_t$,

90. $C_t^e = s^{ee}\left(\bar{P}_t^e/P_t^e\right)^{\sigma_r} \bar{C}_t^e$,

91. $C_t^{ei} = s^{ei}\left(\bar{P}_t/P_t^{ei}\right)^{\sigma_r} \bar{C}_t^e$,

92. $C_t^{eo} = s^{eo}\left(\bar{P}_t/P_t^{eo}\right)^{\sigma_r} \bar{C}_t^e$,

93. $E_t^x = C_t^{ei} + C_t^{oi}$, exports

94. $E_t^{x,e} = C_t^{ie} + Z_t^{ie} + C_t^{oe}$,

95. $E_t^{x,o} = C_t^{io} + Z_t^{io} + C_t^{eo}$,

96. $TB_t = P_t E_t^x - P_t^{ie}\left(C_t^{ie} + Z_t^{ie}\right) - P_t^{io}\left(C_t^{io} + Z_t^{io}\right)$, trade balance

97. $TB_t^e = P_t^e E_t^{x,e} - P_t^{ei} C_t^{ei} - P_t^{eo} C_t^{eo}$,

98. $TB_t^o = P^o E_t^{x,o} - C_t^{oi} P_t / e_t^{io} - C_t^{oe} P_t^e / e_t^{eo}$,

99. $Y_t = C_t + Z_t + G_t + E_t^x$, output market clearing

100. $B_t^f = (1 + i_t)\left(B_{t-1}^f - TB_t\right)$,

101. $B_t^e = -B_t^f / e_t^{ie}$,

102. $\zeta_t = (1 - \tau_t)\, w_t^L L_t + E_t + T_t^l + \chi_t + \chi_t^b + S_t^d + \left(1 - \tilde{s}^b\right) S_t^G - (1 + \tau_t^c)\, \bar{P}_t \bar{C}_t - TB_t$,

103. $Y_t^e = C_t^e + E_t^{x,e}$,

104. $\zeta_t^e = P_t^e Y_t^e - \bar{P}_t^e \bar{C}_t^e - TB_t^e$,

105. $TB_t^o = 0$,

106. $\bar{P}_t^u \equiv s^Y \bar{P}_t + \left(1 - s^Y\right) \bar{P}_t^e$, **MONETARY POLICY**

107. $1 + \pi_t^u \equiv \bar{P}_{t+1}^u / \bar{P}_t^u$, EZ inflation

108. $Y_t^u \equiv \left(P_t Y_t + P_t^e Y_t^e\right) / \bar{P}_t^u$, EZ output

109. $\bar{Y}_t^u = \delta^m Y_t^u + (1 - \delta^m)\, \bar{Y}_{t-1}^u$,

110. $M_t^{s,u} = (1 - \rho^m)\, \phi^{m,u} \bar{Y}_{t-1}^u \frac{\left(\bar{Y}_{t-1}^u / Y_t^u\right)^{\psi_y}}{(1+\pi_t^u)^{\psi_\pi}} + \rho^m M_{t-1}^{s,u} + \varepsilon_t^{m,u}$,

111. $M_t^s = EZ \cdot \bar{P}_{t+1} \bar{M}_t + (1 - EZ) \cdot \left((1 - \rho^m)\, \phi^m \bar{Y}_{t-1} \frac{\left(\bar{Y}_{t-1} / Y_t\right)^{\psi_y}}{(1+\pi_t)^{\psi_\pi}} + \rho^m M_{t-1}^s + \varepsilon_t^m \right)$,

112. $M_t^{s,e} = EZ \cdot \bar{P}_{t+1}^e \bar{M}_t^e + (1 - EZ) \cdot \left((1 - \rho^m)\, \phi^{m,e} \bar{Y}_{t-1}^e \cdot \frac{\left(\bar{Y}_{t-1}^e / Y_t^e\right)^{\psi_y}}{(1+\pi_t^e)^{\psi_\pi}} + \rho^m M_{t-1}^{s,e} + \varepsilon_t^{m,e} \right)$,

113. $0 = EZ \cdot \left(\bar{P}_{t+1} \bar{M}_t + \bar{P}_{t+1}^e \bar{M}_t^e - M_t^{s,u}\right) + (1 - EZ) \cdot \left(\bar{P}_{t+1} \bar{M}_t - M_t^s\right)$, market clearing

114. $0 = EZ \cdot (e_t^{ie} - 1) + (1 - EZ) \cdot \left(\bar{P}_{t+1}^e \bar{M}_t^e - M_t^{s,e}\right)$.

Model Statistics: Expected variables are indexed by $t+1$, predetermined ones by $t-1$.

- 114 equations for 114 endogenous variables: z, K^G, t^s, τ, τ^c, e^{ie}, e^{io}, P, P^e, P^{ie},[10] P^{io}, $\bar{P}$, B^f, Y, b^f, θ, i, i^e, π, r,[20] θ^g, θ^d, θ^b, θ^k, i^d, i^g, i^k, i^b, u_C, m,[30] x, $\bar{C}$, $\bar{M}$, w^L, w^*, N, N_1, L, ϕ, MRS,[40] μ^w, m^c, K, $\tilde{k}$, w^K, Q, λ^K, I, $\bar{Z}$, $\bar{Z}_K$,[50] $\bar{Y}$, s^l, d^l, t^b, i^l, B^l, S^l, Y^g, χ^m, T^k,[60] χ, V, $\tilde{S}^G$, B^G, G, E, T, T^l, d^g, S^G,[70] E^b, D, S^d, χ^b, Y^e, $\bar{Y}^e$, e^{eo}, P^{ei}, P^{eo}, $\bar{P}^e$,[80] π^e, r^e, u^e_C, m^e, x^e, $\bar{C}^e$, $\bar{M}^e$, C^{oi}, C^{oe}, C,[90] C^{ie}, C^{io}, Z, Z^{ie}, Z^{io}, C^e, C^{ei}, C^{eo}, E^x, $E^{x,e}$,[100] $E^{x,o}$, TB, TB^e, TB^o, B^e, ζ, ζ^e, $\bar{P}^u$, π^u, Y^u,[110] $\bar{Y}^u$, $M^{s,u}$, M^s, $M^{s,e}$.[114]

- 17 exogenous variables: ε^z, ε^θ, ε^g, ε^d, ε^b, ε^k, ε^ϕ, ε^{sl}, ε^{tb}, ε^G,[10] ε^E, ε^T, ε^{dg}, $\varepsilon^{Y,e}$, ε^m, $\varepsilon^{m,e}$, $\varepsilon^{m,u}$.[17]

- 64 parameters: ρ, $\bar{z}$, $\bar{K}^G$, σ^z, τ_0, τ^c_0, P^o, s^{ii}, s^{ie}, s^{io},[10] σ_r, γ, $\bar{b}^f$, ρ^f, ρ^θ, $\bar{\theta}^k$, $\bar{\theta}^b$, β, s_c, σ_m,[20] σ_c, σ_l, ω, H, $\bar{\phi}$, η, σ_v, α, t^z, ψ,[30] δ, $\bar{s}^l$, ρ^{sl}, σ^{sl}, l, κ^B, κ^G, b^l, δ^m, γ^g,[40] $\bar{b}^g$, $\bar{g}$, $\bar{e}$, ξ^g, ξ^e, ρ^T, $\bar{t}^l$, δ^g, ρ^{dg}, $\bar{s}^b$,[50] Y^e_0, s^{ee}, s^{ei}, s^{eo}, s^{oi}, s^{oe}, s^Y, ρ^m, $\phi^{m,u}$, ψ_y,[60] ψ_π, EZ, ϕ^m, $\phi^{m,e}$.[64]

- 26 predetermined variables: z_{t-1}, θ_{t-1}, θ^g_{t-1}, θ^d_{t-1}, θ^b_{t-1}, θ^b_{t-1}, w^L_{t-1}, w^*_{t-1}, ϕ_{t-1}, K_{t-1},[10] s^l_{t-1}, t^b_{t-1}, B^l_{t-1}, $\bar{Y}_{t-1}$, B^G_{t-1}, T^l_{t-1}, K^G_{t-1}, d^g_{t-1}, D_{t-1}, Y^e_{t-1},[20] $\bar{Y}^e_{t-1}$, B^f_{t-1}, $\bar{Y}^u_{t-1}$, $M^{s,u}_{t-1}$, M^s_{t-1}, $M^{s,e}_{t-1}$.[26]

- 12 expected variables: e^{ie}_{t+1}, $\bar{P}_{t+1}$, τ^c_{t+1}, $u_{C,t+1}$, μ^w_{t+1}, λ^K_{t+1}, t^b_{t+1}, d^l_{t+1}, V_{t+1}, $\bar{P}^e_{t+1}$,[10] $u^e_{C,t+1}$, $\bar{P}^u_{t+1}$.

END