Limited Information and its Impact on a Policyholder's Optimal Choice on Deductibles

Jan–Christian Fey*[1] Hato Schmeiser[1] Florian Schreiber[2]

[1]University of St. Gallen, Switzerland

[2]Lucerne University of Applied Science and Arts, Switzerland

World Risk and Insurance Economics Congress (WRIEC), 2020

*Corresponding author: jan-christian.fey@unisg.ch

Research Questions

- Policyholders typically face two sources of uncertainty:
  - Uncertainty arising from the randomness of future losses
  - Limited information about the functional forms of the loss distribution and the utility function
- How can we incorporate limited information about the functional forms in traditional models for the optimal deductible choice?
- Which information sources can the policyholder leverage to approximate the functional forms?
- Given limited information about the functional forms: Which characteristics should good heuristics for the optimal deductible choice have?

Motivation

Wealth of policyholder

$$Y_D(L) = \begin{cases} W - R(D) & \text{if } L = 0 \\ W - R(D) - L & \text{if } 0 < L \leq D \\ W - R(D) - D & \text{if } L > D \end{cases}.$$

Optimal deductible choice

$$D_K^* = \underset{D \in \{D_1, \ldots, D_K, N\}}{\arg\max} \; \mathbb{E}\left[u\left(Y_D(L)\right)\right]$$

Related Literature

Deductible policies

- Optimality of deductible policies: Arrow (1963), Raviv (1979), Gollier (2013)
- Optimal deductible choice: Schlesinger (1981), Schlesinger (2013), Moffet (1977), Mossin (1968)

Decision making under limited information

- Decision making under uncertainty: Kofler and Menges (2013)
- Decision rules under uncertainty: Bamberg et al. (2019)

Contribution of the Paper

Modeling limited information

- Modeling sources of information in an expected utility framework
- Deriving approximations of the loss distribution
- Deriving approximations of the utility function

Deriving suitable heuristics for choosing an optimal deductible

- Defining desirable characteristics for heuristics
- Analyzing the performance of different heuristics under limited information

Model

- Loss L with cumulative distribution function

$$F(x) = \mathbb{P}(L \leq x) = \begin{cases} 0 & \text{if } x < 0 \\ p & \text{if } x = 0 \\ p + \int_0^x f(z)\, dz & \text{if } 0 < x < N \\ 1 & \text{if } x \geq N \end{cases}$$

- Premium $R(D) = (1+\lambda)\, C(D)$ with $C(D) = \int_D^N (z - D)\, f(z)\, dz$
- Utility function u which is everywhere twice differentiable with $u' > 0$ and $u'' \leq 0$
- Optimal Deductible: $D_K^* = \arg\max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E}\left[u\left(Y_D(L)\right)\right]$

Model

- Cost function $c : \{D_1, \ldots, D_K, N\} \to \mathbb{R}$ for which $c(D)$ is defined as the value $c \in \mathbb{R}$ which solves

$$\mathbb{E}[u(Y(D_K^*) - c)] = \mathbb{E}[u(Y(D))]$$

- Expected costs for a heuristic $\hat{\theta}_{m,n}^{(l)}$:

$$\mathbb{E}[c(\hat{\theta}_{m,n}^{(l)})] = \sum_{i=1}^{K} \mathbb{P}(\hat{\theta}_{m,n}^{(l)} = D_i)c(D_i) + \mathbb{P}(\hat{\theta}_{m,n}^{(l)} = N)c(N)$$

- Sources of information:
  - Loss observations $L_1, \ldots, L_n$
  - Functional values of utility function $u(x_1), \ldots, u(x_m)$
  - Deductible policies with deductibles $\mathbf{D}_K = (D_1, \ldots, D_K)$ and prices $\mathbf{R}_K = (R(D_1), \ldots, R(D_K))$

Approximating the Loss Distribution

Empirical distribution function

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{(-\infty,x]}(L_i)$$

Approximation derived from the available deductible policies

$$\hat{F}_K^{ded}(x) = \begin{cases} 0 & \text{if } x < 0 \\ \hat{\hat{F}}_K^{ded}(0) & \text{if } x = 0 \\ \hat{\hat{F}}_K^{ded}(0) + \int_0^x \hat{f}_k^{ded}(y)dy & \text{if } x > 0 \end{cases},$$

where

$$\hat{f}_K^{ded}(x) = \mathbb{1}_{(\hat{D}_1,\hat{D}_2]}(x)\hat{z}_{K,1} + ... + \mathbb{1}_{(\hat{D}_K,N]}(x)\hat{z}_{K,K}$$

Approximating the Loss Distribution

- Weighting of the two approximations $\hat{F}_n$ and $\hat{F}_K^{ded}$ with weight $0 \leq \kappa_{n,K} \leq 1$:

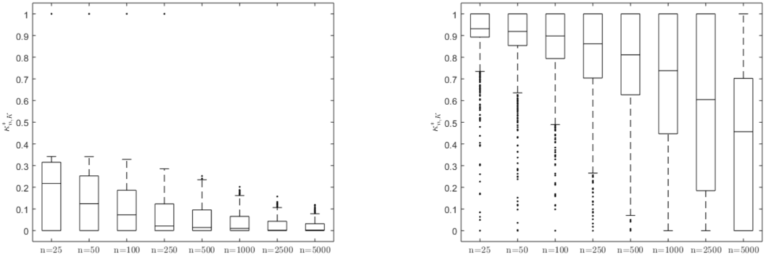
$$\hat{F}^*_{n,K} = \kappa_{n,K} \hat{F}_K^{ded} + (1 - \kappa_{n,K}) \hat{F}_n$$

- Deriving the weight $\kappa^*_{n,K}$ which minimizes the distance measure

$$d_{n,K}\left(\kappa_{n,K}\right) = \left(\kappa_{n,K} \mu_K^{ded} + (1 - \kappa_{n,K}) \bar{L} - \mu\right)^2 + \beta \left(\kappa_{n,K} \sigma^2_{ded,K} + (1 - \kappa_{n,K}) \sigma_n^2 - \sigma^2\right)^2$$

- Properties of $\kappa^*_{n,K}$:
  - $\lim_{n \to \infty} \kappa^*_{n,K} = 0$ $\mathbb{P}$-almost surely
  - $\lim_{K \to \infty} \kappa^*_{n,K} = 1$

Approximating the Loss Distribution



(a) $\mathbf{D}_K = (500, 1000, \ldots, 2500)$ (b) $\mathbf{D}_K = (500, 1000, \ldots, 10000)$

Figure 1: Distribution of $\kappa^*_{n,K}$ for different n and $\mathbf{D}_K$

Preference Uncertainty

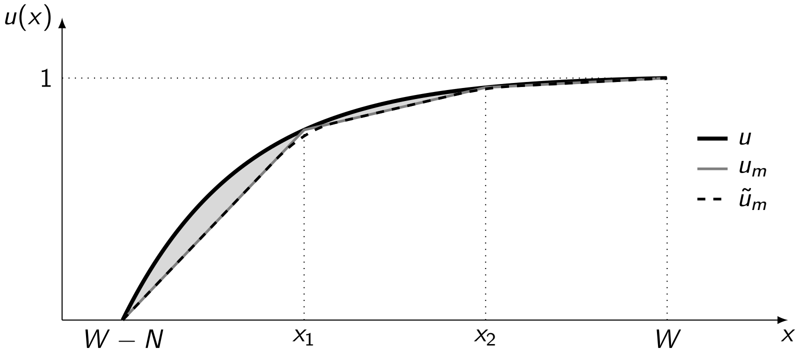


Figure 2: Illustration of the functions u_m and $\tilde{u}_m$ for $\mathbf{x}_2 = (x_1, x_2)$

Heuristics for the Optimal Deductible Choice

Definition

A heuristic $\hat{\theta}^{(I)}_{m,n}$ under the information vector $I = (\mathbf{D}_K, \mathbf{R}_K)$ is an estimator of the optimal deductible level D^*_K. It is called

(i) consistent if $\lim_{m,n\to\infty} \hat{\theta}^{(I)}_{m,n} = D^*_K$ $\mathbb{P}$-almost surely.

(ii) unbiased if $\lim_{m\to\infty} \mathbb{E}[\hat{\theta}^{(I)}_{m,n}] = D^*_K$ holds for all $n \in \mathbb{N}$.

(iii) preference independent if $\hat{\theta}^{(I)}_{0,n} = \hat{\theta}^{(I)}_{m,n}$ $\mathbb{P}$-almost surely for all $m \in \mathbb{N}$ for a fixed but arbitrary $n \in \mathbb{N}$.

If $u'' < 0$ and $D^*_K \neq N$, the heuristic $\hat{\theta}^{(I)}_{m,n}$ is called

(iv) preference consistent if

$$\lim_{n\to\infty} \mathbb{E}[c(\hat{\theta}^{(I)}_{0,n})] - \lim_{m,n\to\infty} \mathbb{E}[c(\hat{\theta}^{(I)}_{m,n})] > 0.$$

Examples of Heuristics

Preference independent heuristics

- $\hat{\theta}^{(I),1}_{m,n} = \arg\min_{D \in \{D_1,\dots,D_K,N\}} \min\left(\max_{i\in\{1,\dots,n\}} L_i, D\right) + R(D)$
- $\hat{\theta}^{(I),2}_{m,n} = \arg\min_{D \in \{D_1,\dots,D_K,N\}} \left(\frac{1}{n}\sum_{i=1}^{n} \min(L_i, D)\right) + R(D)$
- $\hat{\theta}^{(I),3}_{m,n} = \arg\min_{D \in \{D_1,\dots,D_K,N\}} \int_0^N \min(l, D)\, d\hat{F}^*_{n,K}(l) + R(D)$

Preference dependent heuristics

- $\hat{\theta}^{(I),4}_{m,n} = \arg\max_{D \in \{D_1,\dots,D_K,N\}} \frac{1}{n}\sum_{i=1}^{n} \tilde{u}_m(Y_D(L_i))$
- $\hat{\theta}^{(I),5}_{m,n} = \arg\max_{D \in \{D_1,\dots,D_K,N\}} \int_0^N \tilde{u}_m(Y_D(l))\, d\hat{F}^*_{n,K}(l)$

Preference Independent Heuristics

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	13.35	5.92	15.38	38.36	69.21	315.56
n=10	20.09	7.97	11.17	20.03	33.99	151.7
n=100	26.33	9.87	7.28	3.05	1.39	0

(a) Mean costs for $\hat{\theta}^{(I),1}_{m,n}$

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	5.23	3.45	20.45	60.41	111.58	512.71
n=10	6.20	3.29	18.79	55.33	102.32	511.58
n=100	9.61	3.89	15.62	43.63	80.37	454.15

(b) Mean costs for $\hat{\theta}^{(I),3}_{m,n}$

Preference Dependent Heuristics

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	8.48	4.43	18.42	51.61	94.66	433.98
n=10	9.23	4.20	16.87	47.02	86.37	438.52
n=100	10.17	4.08	15.16	41.03	73.84	439.38

(a) Mean costs for $\hat{\theta}_{m,n}^{(I),4}$ for $m = 5$

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	8.48	4.43	18.41	51.61	94.66	433.98
n=10	9.23	4.20	16.84	47.02	86.37	438.52
n=100	10.19	4.10	13.62	36.92	64.76	420.80

(b) Mean costs for $\hat{\theta}_{m,n}^{(I),4}$ for $m = 20$

Preference Dependent Heuristics

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	5.71	3.48	17.79	47.86	3.52	388.82
n=10	6.46	3.28	16.09	37.61	3.64	411.93
n=100	9.62	3.91	14.93	27.88	8.33	396.28

(a) Mean costs for $\hat{\theta}^{(I),5}_{m,n}$ for $m = 5$

	$\eta = 0.1$	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3$	$\eta = 4$
n=5	5.71	3.49	17.69	47.64	3.51	383.44
n=10	6.49	3.29	15.80	36.59	3.64	403.56
n=100	9.64	3.91	12.93	18.90	3.18	384.56

(b) Mean costs for $\hat{\theta}^{(I),5}_{m,n}$ for $m = 20$

Conclusion

- Including limited information in traditional models for the optimal deductible choice
- Derivation of approximation for loss distribution from different sources of information
- Discussion of approximations for the utility function
- Identification of suitable heuristics for the optimal deductible choice

References

Arrow, K. J. (1963). Uncertainty and the welfare economics of medical care. *The American Economic Review*, pages 941–969.

Bamberg, G., Coenenberg, A. G., and Krapp, M. (2019). *Betriebswirtschaftliche Entscheidungslehre*. Vahlen.

Gollier, C. (2013). The economics of optimal insurance design. In *Handbook of Insurance*, pages 107–122. Springer.

Kofler, E. and Menges, G. (2013). *Entscheidungen bei unvollständiger Information*, volume 136. Springer-Verlag.

Moffet, D. (1977). Optimal deductible and consumption theory. *Journal of Risk and Insurance*, pages 669–682.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy*, 76(4, Part 1):553–568.

Raviv, A. (1979). The design of an optimal insurance policy. *The American Economic Review*, 69(1):84–96.

Schlesinger, H. (1981). The optimal level of deductibility in insurance contracts. *Journal of Risk and Insurance*, pages 465–481.

Schlesinger, H. (2013). The theory of insurance demand. In *Handbook of Insurance*, pages 167–184. Springer.