Limited Information and its Impact on a Policyholder’s Optimal Choice on Deductibles

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Research Questions

Policyholders typically face two sources of uncertainty:
- Uncertainty arising from the randomness of future losses
- Limited information about the functional forms of the loss distribution and the utility function

How can we incorporate limited information about the functional forms in traditional models for the optimal deductible choice?

Which information sources can the policyholder leverage to approximate the functional forms?

Given limited information about the functional forms: Which characteristics should good heuristics for the optimal deductible choice have?
Motivation

Wealth of policyholder

$$Y_D(L) = \begin{cases} 
  W - R(D) & \text{if } L = 0 \\
  W - R(D) - L & \text{if } 0 < L \leq D \\
  W - R(D) - D & \text{if } L > D
\end{cases}$$

Optimal deductible choice

$$D^*_K = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E}[u(Y_D(L))]$$
### Related Literature

#### Deductible policies
- Optimality of deductible policies: Arrow (1963), Raviv (1979), Gollier (2013)

#### Decision making under limited information
- Decision making under uncertainty: Kofler and Menges (2013)
- Decision rules under uncertainty: Bamberg et al. (2019)
Contribution of the Paper

Modeling limited information

- Modeling sources of information in an expected utility framework
- Deriving approximations of the loss distribution
- Deriving approximations of the utility function

Deriving suitable heuristics for choosing an optimal deductible

- Defining desirable characteristics for heuristics
- Analyzing the performance of different heuristics under limited information
Model

- Loss $L$ with cumulative distribution function

$$F(x) = \mathbb{P}(L \leq x) = \begin{cases} 
0 & \text{if } x < 0 \\
p & \text{if } x = 0 \\
p + \int_0^x f(z) \, dz & \text{if } 0 < x < N \\
1 & \text{if } x \geq N 
\end{cases}$$

- Premium $R(D) = (1 + \lambda) C(D)$ with $C(D) = \int_D^N (z - D) f(z) \, dz$

- Utility function $u$ which is everywhere twice differentiable with $u' > 0$ and $u'' \leq 0$

- Optimal Deductible: $D_K^* = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E}[u(Y_D(L))]$
Model

- Cost function $c : \{D_1, \ldots, D_K, N\} \rightarrow \mathbb{R}$ for which $c(D)$ is defined as the value $c \in \mathbb{R}$ which solves

$$\mathbb{E}[u(Y(D^K) - c)] = \mathbb{E}[u(Y(D))]$$

- Expected costs for a heuristic $\hat{\theta}^{(l)}_{m,n}$:

$$\mathbb{E}[c(\hat{\theta}^{(l)}_{m,n})] = \sum_{i=1}^{K} \mathbb{P}(\hat{\theta}^{(l)}_{m,n} = D_i) c(D_i) + \mathbb{P}(\hat{\theta}^{(l)}_{m,n} = N) c(N)$$

- Sources of information:
  - Loss observations $L_1, \ldots, L_n$
  - Functional values of utility function $u(x_1), \ldots, u(x_m)$
  - Deductible policies with deductibles $D_K = (D_1, \ldots, D_K)$ and prices $R_K = (R(D_1), \ldots, R(D_K))$
Approximating the Loss Distribution

**Empirical distribution function**

\[ \hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{(-\infty,x]}(L_i) \]

**Approximation derived from the available deductible policies**

\[ \hat{F}_{K}^{\text{ded}}(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\hat{F}_{K}^{\text{ded}}(0) & \text{if } x = 0 \\
\hat{F}_{K}^{\text{ded}}(0) + \int_{0}^{x} \hat{f}_{K}^{\text{ded}}(y) dy & \text{if } x > 0 
\end{cases} \]

where

\[ \hat{f}_{K}^{\text{ded}}(x) = \mathbb{1}_{(\hat{D}_1,\hat{D}_2]}(x) \hat{\zeta}_{K,1} + \ldots + \mathbb{1}_{(\hat{D}_K,\mathcal{N}]}(x) \hat{\zeta}_{K,K} \]
Approximating the Loss Distribution

- Weighting of the two approximations $\hat{F}_n$ and $\hat{F}_{K}^{\text{ded}}$ with weight $0 \leq \kappa_{n,K} \leq 1$:

$$\hat{F}_{n,K}^* = \kappa_{n,K} \hat{F}_{K}^{\text{ded}} + (1 - \kappa_{n,K}) \hat{F}_n$$

- Deriving the weight $\kappa_{n,K}^*$ which minimizes the distance measure

$$d_{n,K}(\kappa_{n,K}) = (\kappa_{n,K} \mu_{K}^{\text{ded}} + (1 - \kappa_{n,K}) \bar{L} - \mu)^2 + \beta (\kappa_{n,K} \sigma_{n,K}^2 + (1 - \kappa_{n,K}) \sigma_n^2 - \sigma^2)^2$$

- Properties of $\kappa_{n,K}^*$:
  - $\lim_{n \to \infty} \kappa_{n,K}^* = 0 \ \mathbb{P}$-almost surely
  - $\lim_{K \to \infty} \kappa_{n,K}^* = 1$
Approximating the Loss Distribution

**Figure 1**: Distribution of $\kappa_{n,K}^*$ for different $n$ and $D_K$

(a) $D_K = (500, 1000, \ldots, 2500)$
(b) $D_K = (500, 1000, \ldots, 10000)$
Preference Uncertainty

Figure 2: Illustration of the functions $u_m$ and $\tilde{u}_m$ for $x_2 = (x_1, x_2)$
Heuristics for the Optimal Deductible Choice

Definition

A heuristic $\hat{\theta}^{(I)}_{m,n}$ under the information vector $I = (D_K, R_K)$ is an estimator of the optimal deductible level $D^*_K$. It is called

(i) consistent if $\lim_{m,n \to \infty} \hat{\theta}^{(I)}_{m,n} = D^*_K$ $\mathbb{P}$-almost surely.

(ii) unbiased if $\lim_{m \to \infty} \mathbb{E}[\hat{\theta}^{(I)}_{m,n}] = D^*_K$ holds for all $n \in \mathbb{N}$.

(iii) preference independent if $\hat{\theta}^{(I)}_{0,n} = \hat{\theta}^{(I)}_{m,n}$ $\mathbb{P}$-almost surely for all $m \in \mathbb{N}$ for a fixed but arbitrary $n \in \mathbb{N}$.

If $u'' < 0$ and $D^*_K \neq N$, the heuristic $\hat{\theta}^{(I)}_{m,n}$ is called

(iv) preference consistent if

$$\lim_{n \to \infty} \mathbb{E}[c(\hat{\theta}^{(I)}_{0,n})] - \lim_{m,n \to \infty} \mathbb{E}[c(\hat{\theta}^{(I)}_{m,n})] > 0.$$
### Examples of Heuristics

#### Preference independent heuristics

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\theta}_{m,n}^{(I),1})</td>
<td>(\hat{\theta}<em>{m,n}^{(I),1} = \arg \min</em>{D \in {D_1, \ldots, D_K, N}} \min \left( \max_{i \in {1, \ldots, n}} L_i, D \right) + R(D))</td>
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<tr>
<td>(\hat{\theta}_{m,n}^{(I),2})</td>
<td>(\hat{\theta}<em>{m,n}^{(I),2} = \arg \min</em>{D \in {D_1, \ldots, D_K, N}} \left( \frac{1}{n} \sum_{i=1}^{n} \min(L_i, D) \right) + R(D))</td>
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<tr>
<td>(\hat{\theta}_{m,n}^{(I),3})</td>
<td>(\hat{\theta}<em>{m,n}^{(I),3} = \arg \min</em>{D \in {D_1, \ldots, D_K, N}} \int_0^N \min(I, D) d\hat{F}_n^*(l) + R(D))</td>
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#### Preference dependent heuristics

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<td>(\hat{\theta}_{m,n}^{(I),4})</td>
<td>(\hat{\theta}<em>{m,n}^{(I),4} = \arg \max</em>{D \in {D_1, \ldots, D_K, N}} \frac{1}{n} \sum_{i=1}^{n} \tilde{u}_m(Y_D(L_i)))</td>
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<tr>
<td>(\hat{\theta}_{m,n}^{(I),5})</td>
<td>(\hat{\theta}<em>{m,n}^{(I),5} = \arg \max</em>{D \in {D_1, \ldots, D_K, N}} \int_0^N \tilde{u}_m(Y_D(I)) d\hat{F}_n^*(l))</td>
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### Preference Independent Heuristics

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(a) Mean costs for $\hat{\theta}_{m,n}^{(1)}$

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(b) Mean costs for $\hat{\theta}_{m,n}^{(3)}$
Preference Dependent Heuristics

(a) Mean costs for $\hat{\theta}^{(l)}_{m,n}$ for $m = 5$

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(b) Mean costs for $\hat{\theta}^{(l)}_{m,n}$ for $m = 20$

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(b) Mean costs for $\hat{\theta}_{m,n}^{(l),5}$ for $m = 20$
Conclusion

- Including limited information in traditional models for the optimal deductible choice
- Derivation of approximation for loss distribution from different sources of information
- Discussion of approximations for the utility function
- Identification of suitable heuristics for the optimal deductible choice
References


