On the Measurement of the Elasticity of Labour*

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Abstract

We use a simple macroeconomic model to illustrate the difference between the Frisch elasticity, understood as a structural behavioural parameter, and the elasticity of labour measured through a tax holiday; a setup used in the micro-applied literature to measure the responsiveness of labour to changes in the returns to work. We show that a high Frisch elasticity is compatible with a measured elasticity of the same order of magnitude as the micro-applied estimates.

Keywords— Frisch Elasticity, Labour Supply, Aggregate Hours, Tax Holidays.

JEL Codes— E24, H21, J20.

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1 Introduction

One of the most prominent questions in Economics is how the amount of labour in an economy responds to changes in marginal taxes. A crucial ingredient of the discussion is the elasticity of labour supply to the returns of work, understood as a structural behavioural parameter. One strand of the literature poses that the elasticity of labour is rather low which implies that the efficiency cost of income taxation and, hence, redistribution is small. Another strand of the literature, argues that increasing marginal taxes is costly because it generates a sizeable reduction in aggregate labour supply. The low-elasticity view is predominately based on micro-applied studies that rely on arguably exogenous changes in the returns to work which are used to measure the empirical response of observed hours worked or labour income. The high-elasticity perspective is grounded in structural models which are parametrised to replicate aggregate indicators.

We combine key elements from the two camps to illustrate the complementarity between these two views. In particular, we use the model in Prescott (2004) to analyse the effect of an income tax holiday, a natural experiment that has been used to measure the elasticity of labour in the micro-applied literature (Bianchi, Gudmundsson, and Zoega, 2001; Martínez, Saez, and Siegenthaler, 2018).

We present two main results. First, we show analytically that the elasticity measured through an income tax holiday is not equivalent to the Frisch elasticity. The Frisch elasticity captures how preferences drive the labour supply response of the representative household to a change in the returns of work holding wealth constant. Instead, the measured elasticity reflects changes in the equilibrium quantity of labour, which results from the interaction between labour demand and supply. A tax holiday changes the returns to work by reducing the marginal tax on labour, i.e., the returns to work. The direct response of the representative household is to supply more labour. However, this increase in supply is met with a decrease in the price of labour due to the structure of the demand. The net effect of these two countervailing forces is reflected in the equilibrium level of labour. Secondly, we show quantitatively that, although the Frisch elasticity of the model is high, the measured elasticity through the tax-holiday experiment is of the same order of magnitude as the estimates of the micro-applied literature.

2 Framework

The model in Prescott (2004) summarises the key ingredients of a long-standing and widely-used approach in macroeconomics to model the determination of aggregate labour. There is a forward-looking representative household that faces labour-leisure and consumption-savings decisions. The representative firm combines labour and capital in a Cobb-Douglas production function which produces a numeraire good that is either consumed, invested, or used by the public sector. The government levies taxes on income, consumption, investment, and savings. All markets are perfectly competitive.

\(^1\) See Keane (2011) and Saez, Slemrod, and Giertz (2012).

\(^2\) Keane and Rogerson (2012) and Chetty, Guren, Manoli, and Weber (2013) review in detail some of the mechanisms that bring about the differences between the Micro and Macro elasticities.
The representative household has additive-separable preferences over consumption (c) and leisure (100 − h). The maximisation problem of the representative household is given by:

$$E \sum_{t=0}^{\infty} \beta^t [\log(c) + \theta \log(100 - h)],$$

subject to

$$(1 + \tau^c)c_t + (1 + \tau^x)x_t = (1 - \tau^{inc} - \tau^{ss})w_t h_t + (1 - \tau^k)(r_t - \delta)k_t + \delta k_t + T_t,$$

where $x_t$ is investment, $\tau^x$ is the investment tax, $w_t$ is the real wage rate, $k_t$ is capital, $\tau^k$ is the capital income tax, $r_t$ is the rental price of capital, $\delta$ is the depreciation rate of capital, and $T_t$ are transfers. Note that, from the perspective of the representative household, the returns to work are given by $(1 - \tau^{inc} - \tau^{ss})w_t$.

The steady-state equilibrium implies that the quantity of hours worked in the economy (h) is determined by:

$$h = \frac{1 - \alpha}{\tau - \frac{\theta c}{1 - \tau}} + (1 - \alpha),$$

where $\frac{c}{y}$ is the consumption-to-income ratio, $\theta$ is a preference parameter that determines the weight of leisure in the utility function, $1 - \alpha$ is the labour share of income in the production function, and $\tau$ is the effective marginal tax on labour income. The effective marginal tax is a function of the consumption ($\tau^c$), labour income ($\tau^{inc}$), and social security marginal taxes ($\tau^{ss}$):

$$\tau = \frac{\tau^{inc} + \tau^{ss} + \tau^c}{1 + \tau^c}.$$

The utility function determines the structural Frisch elasticity of labour supply:

$$\epsilon_u = \frac{100 - h}{h}.$$

A labour income tax holiday corresponds to setting the labour income tax to zero ($\tau^{inc} = 0$). The literature that uses this tax holidays to identify the Frisch elasticity, interprets this change in income taxes as a change in the returns to work. However, this change in the tax rate also affects the equilibrium level of hours through two countervailing forces: first the change in the amount of labour supplied by the household and second the change in the equilibrium wage rate induced by the labour demand.

Let us denote $h$ the equilibrium level of hours when the marginal labour income tax is positive ($\tau^{inc} > 0$) and $h'$ the equilibrium level of hours under a tax holiday ($\tau^{inc} = 0$). The elasticity of a tax holiday experiment is defined as:

$$\epsilon = \frac{\Delta h}{\Delta (1 - \tau^{inc} - \tau^{ss})},$$

where $\Delta h$ is the relative change in hours from an equilibrium with positive income taxes to the tax holiday equilibrium. $\Delta (1 - \tau^{inc} - \tau^{ss})$ is the relative change in the returns to work generated

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3It is assumed that the household is endowed with 100 hours per week which can be allocated to leisure or work.
by the tax holiday:

$$\Delta(1 - \tau^{inc} - \tau^{ss}) = \frac{(1 - \tau^{ss}) - (1 - \tau^{inc} - \tau^{ss})}{1 - \tau^{inc} - \tau^{ss}} = \frac{\tau^{inc}}{1 - \tau^{ss} - \tau^{inc}}.$$ 

Given the expression for the equilibrium level of hours in Equation 1, the change in hours is:

$$\Delta h = h' - h = \frac{\theta \xi (\tau - \tau')}{{(1 - \tau)(\theta \xi y + (1 - \tau')(1 - \alpha))}},$$

where $$\tau' = \frac{\tau^{ss} - \tau^{inc}}{1 + \tau^c}$$. Hence the measured elasticity from a tax holiday is:

$$\epsilon = \frac{(1 - \tau^{ss} - \tau^{inc})\theta \xi (\tau - \tau')}{\tau^{inc}(1 - \tau)(\theta \xi y + (1 - \tau')(1 - \alpha))}.$$ (2)

That is, the measured elasticity ($$\epsilon$$) is not equivalent to the structural Frisch elasticity ($$\epsilon^u$$). In what follows, we evaluate the difference between these two elasticities and discuss the implications for inference.

### 3 Parametrisation

The procedure to parametrise the model involves two steps. First, we require data on the consumption-to-income ratio ($$\xi/y$$), the effective tax rate ($$\tau$$), average weekly hours worked, and the capital share ($$\alpha$$). Second, we calibrate the weight of leisure in the utility function of households ($$\theta$$) by minimising the sum of squared distances between hours worked as predicted by the model ($$h$$) and hours worked observed in the data.

We parametrise the model using data for the G7 countries, as in Prescott (2004), and data for the 26 cantons (member states) of Switzerland. For the G7 countries, we follow the methodology outlined by Prescott (2004), and compute $$c/y$$, hours worked, and the tax rates $$\tau$$ using national accounts data. The data is taken from the United Nations System of National Accounts (SNA), the Organisation for Economic Co-operation and Development (OCDE), and covers the years 2014-2017.

For Switzerland, we use nationally representative household surveys to measure macroeconomic aggregates at the cantonal level, because Swiss cantons do not record national accounts. Studying Switzerland presents two main advantages. First, in contrast to the G7 countries, Swiss cantons share the same labour market institutions and independently set income taxes. For instance, cantonal tax rates in labour income range between 19.5% and 31.8% and average hours worked per week between 19.8 and 29.2. Secondly, Martínez et al. (2018) provide an estimate of the measured elasticity of labour income in Switzerland using a real-life tax holiday event.

We use the Swiss Household Panel (SHP) to compute the consumption-to-income ratios and the effective tax rates for each canton. We define cantonal income as the aggregate of gross

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4Note that the tax holiday does not affect the structural marginal propensity to consume.

5The G7 countries are Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.
household income net of public and private transfers. To measure consumption, we subtract savings, debt payments, and mortgage payments from disposable income, which we define as income net of social security contributions, direct taxes, health insurance payments, and payments to other households. Also, we use the methodology by Guner, Kaygusuz, and Ventura (2014) to estimate the marginal social security and labour income taxes.\textsuperscript{6} The data on hours worked is taken from the Swiss Labour Force Survey (SLFS). Finally, because Swiss cantons cannot set consumption taxes, we follow Prescott (2004) and measure the consumption tax in Switzerland using data on indirect taxes and household consumption expenditure from SNA. That is, all cantons have the same consumption tax.\textsuperscript{7}

4 Measurement

In Table 1, we report our estimates of the Frisch elasticity ($\epsilon^u$) and the measured elasticity ($\epsilon$) for the G7 countries and Switzerland. For all countries, the Frisch elasticity is high and in line with the estimates found in the macroeconomic literature. However, the measured elasticity is four-to-six times smaller than the Frisch elasticity and ranges from 0.68 for the United States to 0.75 for France. To the best of our knowledge, there exist estimates from real-life tax holidays for two countries: Switzerland and Iceland. For Switzerland, the estimates from Martínez et al. (2018) range between 0.03 and 0.29. For Iceland, the range is between 0.07 and 0.42 (Bianchi et al. (2001); Sigurdsson (2019); Stefánsson (2019)). Our results indicate that the parametrised model implies measured elasticities that are of a similar magnitude to the micro-applied estimates.

Table 1: The Frisch elasticity of the model is in-line with the high estimates of the literature while the measured elasticity is close to the low estimates.

<table>
<thead>
<tr>
<th>Country</th>
<th>$h$</th>
<th>$\epsilon^u$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>25.19</td>
<td>2.97</td>
<td>0.71</td>
</tr>
<tr>
<td>Germany</td>
<td>21.44</td>
<td>3.67</td>
<td>0.72</td>
</tr>
<tr>
<td>France</td>
<td>17.82</td>
<td>4.61</td>
<td>0.75</td>
</tr>
<tr>
<td>Italy</td>
<td>18.43</td>
<td>4.43</td>
<td>0.73</td>
</tr>
<tr>
<td>United States</td>
<td>26.23</td>
<td>2.81</td>
<td>0.68</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>22.63</td>
<td>3.42</td>
<td>0.71</td>
</tr>
<tr>
<td>Canada</td>
<td>24.65</td>
<td>3.06</td>
<td>0.69</td>
</tr>
<tr>
<td>Japan</td>
<td>25.92</td>
<td>2.86</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Notes: $h$ is hours worked as predicted by the model. The estimates for the G7 are an average of the estimates obtained using the cross-country data. The estimate for Switzerland is the population-weighted average of cantonal estimates obtained using the cantonal data.

\textsuperscript{6}Our estimates are robust to using different functional forms to estimate marginal taxes.

\textsuperscript{7}In Appendix Section A we provide further details on the construction of the data sources and the computation of model inputs. For 8 out of the 26 cantons, there exist data equivalent to national accounts. We provide a comparison between our estimates based on micro-data and these national-accounts equivalents.
5 Conclusion

We provide a parsimonious illustration of the differences between the Frisch elasticity, understood as a structural behavioural parameter, and the labour response that is measured through a tax holiday; a setup used in micro-applied studies to identify the elasticity of labour. We show that the simple model exhibits a high Frisch elasticity while the elasticity measured through the tax holiday is similar to the micro-applied estimates in the literature.
References


A Data

In this section, we provide more details on the construction of the cantonal dataset for the parametrization of Switzerland. The parametrization to Switzerland draws from multiple data sources: the UN System of National Accounts (SNA), the Swiss Household Panel (SHP) administered by FORS and the Swiss Federal Statistical Office’s (BFS) Household Budget Survey (HABE), labour Force Survey (SLFS), and population statistics. In what follows, we provide more details on the construction of the dataset.

Measurement of cantonal consumption, income and tax rates

We compute $c/y$ and the tax rates $\tau^{ss}$ and $\tau^{inc}$ using data from the SHP 2014-2017. It readily delivers information on the primary household income $y$ which is obtained by subtracting public and private transfers from gross household income. We use this data to measure household consumption $c$ as the residual of primary income after savings, debt and mortgage payments: $^8$

$^8$Here the formula with exact variable names from the survey: 

$c = \text{disposable household income} - \text{annual savings} - \text{annual debt payments} - \text{annual mortgage payments}$

We also use the SHP to measure marginal social security and income tax rates. We estimate the parameters of two tax functions as proposed by Benabou (2002) and Guner et al. (2014). To do so, we use data on social security contributions and income tax payments, which we then aggregate to the cantonal level using survey weights, and use to estimate the marginal tax rates of a canton.

Measurement of labour supply

Our empirical measure of labour supply is weekly labour supply $h$. We use data from the Swiss labour Force Survey (SLFS), in particular data on annual hours worked by permanent residents, including foreigners with Swiss resident permits. This is important since labour income is taxed in the canton of residence. We then divide these numbers by 52.14 and by active population of each canton, again obtained from the BFS and including foreign permanent residents, to measure weekly labour supply.

Measurement of value-added consumption tax

In Switzerland, value added consumption tax are set at the federal level. Therefore, we measure $\tau^c$ using UN System of National Accounts (SNA) data on indirect taxes and household consumption expenditure, and set $\tau^c$ equal to this national rate for all cantons.

Micro-data estimates vs. National Accounts Equivalents

Table 2 shows estimates of the consumption-to-income ratio for the eight most populous cantons. For these cantons, the FSO provides estimates of regional GDP and consumption that are equivalent to the data registered in national accounts. The data reveals that our estimates from the SHP are very close the their national accounts counterparts.
Table 2: The estimates for $c/y$ obtained from National Accounts Equivalents and the micro-data are very similar.

<table>
<thead>
<tr>
<th></th>
<th>SHP</th>
<th>HABE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argau</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>Bern</td>
<td>0.66</td>
<td>0.70</td>
</tr>
<tr>
<td>Geneva</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td>Luzern</td>
<td>0.65</td>
<td>0.67</td>
</tr>
<tr>
<td>St. Gallen</td>
<td>0.67</td>
<td>0.66</td>
</tr>
<tr>
<td>Ticino</td>
<td>0.65</td>
<td>0.74</td>
</tr>
<tr>
<td>Vaud</td>
<td>0.63</td>
<td>0.69</td>
</tr>
<tr>
<td>Zurich</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>Population-weighted Average</td>
<td>0.67</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Notes: The National Account Equivalents are obtained from the Household Income and Expenditure Survey (HABE) of the FSO. These estimates are only available for the eight most populous cantons. The table reports an average for years 2014-2017.