Agent and Principal
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Summary:
In most general terms, agency theory focuses on co-operation in the presence of external effects as well as asymmetric information. To have a look on external effects first, consider two individuals. One of them, the agent, is decision making. He is thus affecting his own welfare and, in addition, that of the other individual called principal. These external effects of the agent's decisions or actions are negative: modifications of the agent's action which are preferred by the principal yield disutilities to the agent. A common example is a situation where the principal is assisted by the agent and the agent is deciding on level and kind of his effort. The principal is thus ready to pay some kind of reward to the agent in return for a certain decision/action/effort. Unfortunately, and this is the second characteristic of situations in agency theory, the principal cannot observe the agent's actions in full detail. The asymmetric information with respect to the agent's decision excludes simple agreements concerning pairs of action and payment.

External effects and asymmetric information prevail in very widespread situations of economic co-operation. The variety of examples include such important relations as those between employer and employee, stockholder and manager, or patient and physician.

From a methodological point of view, the principal-agent relation is closely related to risk sharing, hidden effort, monitoring, hidden characteristics, screening, and self selection. The purpose of this essay is to model and analyse these different features of agency theory in one unified approach. This formal approach is based on linear reward schemes, exponential utility functions, and normal distributions, and it will therefore be called LEN-Model. The LEN-Model allows for explicit presentation of endogenous parameters which determine the agent's decision on effort, the (p. 4) chosen reward.
scheme, and the incorporation of monitoring signals. Hence several insights into how the pattern and design of co-operation depends on exogenous parameters such as the agent’s risk aversion and the variance of environmental risk can be provided.

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1. A General View

1.1 Co-operation

Economics may be viewed as the science of co-operation with regard to the utilization of resources. The basic pattern of co-operation is the exchange of goods, services, information, risk, or rights. If two or more individuals agree to co-operate, each of them will and has to contribute something and is going to receive something in return. Because of this pattern of exchange, the market is a very important organization or set of rules according to which cooperation takes place. Though the market mechanism is not the only design to organize cooperation, markets are efficient if the commodities exchanged have no external effects and if all relevant information is public.

More complex arrangements, however, are required in the presence of external effects or imperfect information. External effects prevail in such cases as that of non-separable labor inputs and that of public goods. Likewise imperfect information, in the sense of uncertainty about the quality of the commodities (skill and effort of labor input, reliability in financial contracting), require a more sophisticated design of the rules of cooperation.
Both external effects and imperfect information are predominating in many situations of economic cooperation. Usually these effects will be mutual. Each of the cooperating individuals affects by her/his decisions the welfare of the others directly, and each individual has some limits to observe the actions of others in full detail. Reciprocally given externalities and common limits to observe explain why cooperation is so complex in real life and why so many different types of arrangements, forms of contracts, institutions, and organizational designs have evolved.


1.2 External Effects

For analytical purposes one has to restrict the view on a simple, single-directed case of external effects and asymmetric information. So, instead of many, consider two individuals only. One of them, the agent, makes his decision \( x \in X \). This decision, in some sense, is made on the quantity/quality of what the agent is going to contribute to what could be called the team. By this decision making the agent does not only influence his own welfare (more effort in team work is connected to individual disutility, for example) but also that of the other individual called principal. (The principal participates in the result of team work which is a consequence of the agent’s effort). Agent and principal have different values associated with the agent’s actions. In other words, the external effects of the agent’s decision making are negative: those modifications of his action which are preferred by the principal yield disutilities to the agent.

Under such conditions, the principal is likely to start negotiations with the agent and offer some compensation, perhaps in form of a payment, if the agent refrains from choosing an action the principal dislikes. This way, both individuals could reach an agreement \( (x, p) \) that commits the agent to a certain decision \( x \in X \) in exchange for a
certain pay $p$ to be made by the principal. It will be easy for them to arrive at an efficient agreement, which therefore could be termed \textit{first-best design of cooperation}. Note that the agent's welfare or utility $U(x, p)$ depends on pairs of action $x$ and pay $p$ (he prefers both lower levels of effort and higher payments). Likewise, the principal's welfare $V(x, p)$ depends on pairs of action $x$ and pay $p$ (she prefers more effort of her partner as well as to give a lower pay). The situation of bargaining on pairs $(x, p)$ can best be illustrated in an Edgeworth-Box.

1.3 Asymmetric Information

Externalities alone cause no deviation from first-best designs of cooperation. Simple bargaining on pairs of actions $x$ and payments $p$ are excluded, however, if external effects occur in combination with asymmetric information. Assume that, for some reason or the other (one reason is presented in Section 2.1) the principal is unable to observe and to verify exactly which action $x$ the agent is or was realizing. Information is asymmetric because the agent, of course, knows which decision he is going to make. But now, if there is no unlimited trust, it does not make sense for the principal to negotiate on pairs $(x, p)$. The agent could make any promise with respect to his action and depart from it later on just because the principal is unable to control or to monitor the agent's decision making.

\textit{(p. 7)} Although there is asymmetric information with respect to the agent's decision $x$ by assumption, there might exist some variables which are correlated to $x$ and the values of which can costlessly be observed by both agent and principal. Such variables provide some or partial information on the agent's decision $x$. Denote variables that partially inform on action $x$ by $y, z, \ldots$. Depending on the particularities of the situation, examples for such variables are firstly the resulting output $y$ of team work and secondly the monitoring signals $z$ resulting from some control devices. Since the values of $y$ and $z$ can be observed by agent and principal without disagreement, reward schemes $p(\ldots)$ can be defined that make the amount of pay $p(y, z)$ a function of these variables $y, z$. More details are presented in Section 2.5.

Now suppose the principal, unable to observe the agent's decision $x$ in an exact and direct way, offers a certain reward scheme $p(\ldots) \in P$, taken from a set $P$ of feasible functions of variables $y, z$. The princi-
pal makes this offer without expecting any pretense or promise of the agent with respect to decision $x$. The principal just invites the agent to accept the scheme $p(\ldots)$ and to make, then, a decision $x$ in his own interest. Consequently, there will be no shirking. The agent, realizing that the actual pay $p(y, z)$ depends on the values of the variables $y, z$ which are related to his action $x$, will make his decision as a response to the scheme $p$. Formally, the agent is now choosing an action $x = \phi(p)$ that depends on the reward scheme $p$. The agent’s response is described by the function $\phi : P \rightarrow X$. In other words, the reward scheme sets an incentive, or, the agent’s decision $x$ is induced by the reward scheme $p$.

1.4 Induced Decision Making

One consequence of information asymmetry is that only designs of cooperation are possible where the action $x = \phi(p)$ is induced by payment $p$. This is a fundamental difference between the first-best situation discussed in Section 1.2, where agent and principal could negotiate on pairs $(x, p)$ of action and payment without further restriction. Under imperfect or asymmetric information, there is the additional constraint that the agent’s action must be induced by payment.

(p. 8) Denote by $E$ the set of pairs $(x, p)$ that are efficient with respect to the welfare $U$ of agent and the welfare $V$ of principal. Thus $E$ is the set of first-best designs of cooperation. Further, let $I$ be the set of pairs $(\phi(p), p)$ of action and payment, where the action is induced by payment. The set $I$ contains all designs that are feasible under information asymmetry. The information asymmetry would cause no problem at all if both sets $E$ and $I$ were identical. Any first-best design of cooperation could then be realized through induced decision making. One could already be satisfied in some weaker sense if the sets $E$ and $I$ had one or some elements in common. In such cases, at least one or some first-best designs of cooperation could be reached through induced decision making. Situations where $E$ and $I$ coincide or have some common elements are usually referred to as incentive compatibility.

In all other cases, the fact that some of the relevant information is not public causes a deviation from first-best and efficient designs (set $E$). Then all designs in $I$ are dominated by designs in $E$ and, for that reason, are second best only.
Few attempts have been made to measure the disadvantage between first-best and second-best designs in terms of a real number. Such measures are called *agency costs* in the tradition of M. C. Jensen and W. H. Meckling (1976). In figurative terms, agency costs measure the distance between the set $E$ of first-best designs, which are an utopian fiction in the presence of asymmetric information, and the set $I$ of designs where the agent's decision is induced by a payment scheme. The distance between two sets, however, can be measured in many different ways such that a particular definition of agency costs can easily be criticized with regard to appropriateness. In particular, one has to be very careful when using agency costs to compare and evaluate alternative second-best arrangements.

Another and presumably less ambiguous way is to define agency costs as the decision-theoretic *value of perfect information*: How much would the principal at most be willing to pay for becoming able to observe the agent’s decision correctly? Agency costs as value of perfect information provide an upper bound for monitoring costs. If there were the possibility to introduce a perfectly working monitoring device it would be rejected if the costs of the device surmount the information value, see Section 2.4.

1.5 Hierarchy and Delegation

Note that no hierarchy was assumed so far. Neither was the principal assumed to be the boss nor the agent to be her subordinate as one might associate from the designations of the two cooperating partners. Consequently, the expression of a team seems to be much more appropriate. Agent is simply that member of the team who can vary his action/effort/behavior/input. Principal is that member of the team who cannot costlessly observe the agent’s action/effort/behavior/input. Therefore, team members are bounded to schemes that set incentives. If person $A$ buys insurance from company $P$, company $P$ can hardly observe the care person $A$ shows to avoid the accident, and nevertheless there is no hierarchical cooperation between $A$ and $P$, see M. Spence and R. Zeckhauser (1971).

The relations between employer and employee as well as between stockholder and manager are very important examples for an agent-principal relation. Although most approaches are based on the identifications of principal and employer or principal and stockholder, resp., some aspects of these relations require to see the subordinate
as principal and the superior as agent, see P. Swoboda (1987). In fact, the reward systems of hierarchical organizations sometimes provide more incentives for bosses than for subordinates.

Further, no formal contract was supposed to legalize the relation between agent and principal. Moreover, not necessarily it is the case that "the principal delegates some decision making to the agent", though the delegation of decision making provides a reasonable explanation of why the principal cannot observe the agent's doing in full detail. But there are many other situations different from the "delegation of decision making" where it is easy to see that the principal has some difficulties in controlling the agent's action/effort/behavior/input. One example is the situation of insurance mentioned above.

1.6 Hidden Effort, Hidden Characteristics

The elaboration of Agency Theory requires a closer look to a number of different issues. One major task is to present a variety of different situations where a principal cannot completely observe an agent. In addition, reasonable argumentations have to be given for this information asymmetry. One should distinguish two situations which were termed by K. J. Arrow (1986): hidden efforts and hidden characteristics.

In many cases agent and principal cooperate within an organization and they know each other quite well. Each of them might provide some inputs to the team, but the principal's inputs are not under discussion here. The input provided by the agent are labor or management services and what can hardly be observed by others is the agent's effort. Effort is not only diligence and sweat but could also refer to the agent's renunciation of consumption on the job. Hidden effort and managerial discretion thus refer to the same situation.

The total team output and hence the principal's welfare depend on the agent's effort, but additionally also on some exogenous risk (state of nature). Although the principal knows the probability distribution of this risk, she might be unable to come to know which state nature was actually realizing. Consequently, she is unable to separate low effort from bad luck. If results turned out to be poor, the principal cannot conclude that the agent's effort must have been low. So it is the environmental uncertainty that explains why the principal is unable to deduce the agent's effort from the resulting team output.
As stated, the team members know each other. In particular, the principal knows the characteristics of her agent such as his skill and his attitude toward risk. Although the principal is unable to observe her agent’s effort, she can predict the way in which the agent will behave under certain conditions. She can calculate the agent’s response (function $\phi: P \rightarrow X$) to a certain reward scheme. The principal can thus study the impact of reward schemes on her own wealth, and, determine a reward scheme that is best with respect to her own interest and subject to the constraint that the agent’s effort is induced by the reward scheme.

In the basic situation of hidden effort the reward will be a function of team output $y$. This can be generalized if there is a monitoring signal $z$, i.e., a statistic that is correlated to the agent’s effort. The issue of monitoring is thus related to the situation of hidden effort.

A situation quite different from hidden effort is that of hidden characteristics. Here cooperation occurs across markets and the principal is unable to observe the agent’s decision in time. A principal on the one side of the market gets into contact with many individuals, potential agents, on the other side. The principal has to make an offer in the moment of getting into contact with one of these agents. The agents, however, differ in their characteristics. Although the principal might know the distribution of characteristics, she usually will be uncertain about the particular type of agent. How to make an offer that is appropriate without knowing the individual characteristic?

In such cases of hidden characteristics the principal will look for sorting devices or install additional instruments that partially reveal hidden characteristics through screening. An important screening device consists of a set of payment schemes which allow for self selection through agents. Self selection schemes should be designed such that each agent has an incentive to reveal his type and his characteristics through choice. Such a scheme is presented in Section 2.6.
2. A Closer Look

2.1 Risk Sharing

A common situation of hidden effort is one in which the principal seeks help from the agent because her wealth depends on services the agent can provide. The agent can offer these services in various quantities and qualities upon which he alone decides. Formally, the agent chooses an element \( x \) from a set \( X \) of feasible actions. This decision, in its manifold aspects, is called effort. So far the external effects are outlined. On the other hand, the principal’s wealth is not only affected by the agent’s effort. Another factor is some kind of exogenous risk the probability distribution of which neither principal nor agent can control. Describe this state of nature by the random variable \( \tilde{\theta} \). Thus the principal’s gross wealth, denoted by \( \tilde{y} \), can be viewed as a function of effort \( x \) and risk \( \tilde{\theta} \),

\[
\tilde{y} = f(x, \tilde{\theta}).
\]

It might be indicated to visualize this situation as one of production although sometimes this notion must be interpreted in a broad sense. Anyway, the principal’s gross wealth \( \tilde{y} \) will be called output or result. The only input upon which a decision can be made is the agent’s effort \( x \). If there were any other inputs, their quantities and qualities will be supposed to be either fixed or settled beforehand.

Of course, the principal wants to buy some input from the agent but, unfortunately, she cannot observe how much the agent is providing and how good he is performing. In other words, the principal is assumed to be unable to observe the agent’s effort decision \( x \in X \). One implication of the exogenous risk \( \tilde{\theta} \) is that it gives a reason for the assumed information asymmetry. If the principal is not completely ignorant, she will usually know the production function \( f \) (how her gross wealth is affected by her agent’s effort and the exogenous risk), and she will know the probability distribution of \( \tilde{\theta} \). Later she will also observe the realization \( y \) of her gross wealth \( \tilde{y} \). But, to speak in figurative terms, she might be too distant from the location of production in order to see which state \( \theta \) nature realized. Consequently, the principal cannot infer the agent’s effort from the knowledge of both technology \( f \) and result \( y \). The information asymmetry rules out negotiations with the aim to close with an agreement on effort.
Assume that the realization $y$ of the output can be observed by both agent and principal correctly and without costs. Hence the principal can offer a \textit{payment scheme} $p(.,.)$ where the actual payment $p(y)$ to be made to the agent depends on the realization $y$ of output. Clearly, the principal will then keep the residuum $y - p(y)$ as her net wealth. Denote by $P$ the set of such schemes $p(.)$ from which the principal is choosing one in order to offer it to her agent.

So far the agent need not make any committing declaration or contract in any legal sense. He will just realize the principal's offer, consider it in his decision-making calculations, and accept the money later when the realization $y$ becomes known. Note, however, that for some reward scheme it could happen under a particular realization of output that the actual payment is negative. In such a case, the agent were to pay the corresponding amount to the principal. In order not to exclude such schemes from further consideration, the right will be assigned to the agent to decide whether or not to accept a payment scheme. If the agent accepts a payment scheme $p(.)$ he declares himself willing to make an eventual transfer in the case $p(y)$ is negative. But the agent is never supposed to make any promise with regard to his effort decision which could not be checked by the principal anyway.

Let $c(x)$ be the agent's \textit{disutility of effort} in terms of a money equivalent. So to speak, $c(x)$ is the cost the agent has to pay by himself for the services he is going to provide as input. If the agent was offered and had accepted the payment scheme $p \in P$ and is now going to decide upon his effort $x \in X$, he is confronted with net wealth

\begin{equation}
\tilde{w}(x, p) = p(f(x, \tilde{\theta})) - c(x).
\end{equation}

Since the result (1) is uncertain at that moment of decision making, the wealth $w$ will be uncertain, too. In the particular case the scheme $p(.)$ is constant in $y$ such that the agent receives a fixed wage rather than sharing the result, his wealth is free of risk. The welfare derived from wealth $w$ can be formalized by the expected utility $E[u(\tilde{w})]$, or, what is done here, the agent's welfare $U$ is expressed in terms of the certainty equivalent

\begin{equation}
U(x, p) := u^{-1}\left(E[u(\tilde{w})]\right).
\end{equation}
Thereby, $u$ denotes the Neumann-Morgenstern utility function of the agent. He is supposed to be risk averse ($u$ is concave), and hence the certainty equivalent $U$ of wealth is below the expected value $E[\bar{w}]$. The difference between the two entities was called risk premium by J. W. Pratt (1964).

(p. 14) A second implication of the exogenous uncertainty $\bar{\theta}$ introduced in (1) is that it raises the issue of risk sharing. The more a payment scheme lets the agent share the uncertain result $\bar{y}$, the more risky becomes his wealth (2). Suppose the principal wants to set an incentive to her agent by offering a considerable result sharing. The agent is not only requiring a compensation for his disutility of effort $c(x)$. Because of his risk aversion, the agent needs also a higher risk premium in order to maintain a certain level of welfare.

That risk premium may turn out to be inefficient from a risk-sharing point of view. Suppose the principal is risk neutral so she could bear all the risk without requiring a premium. The principal keeps all the risk with her residuum $\bar{y} - p(\bar{y})$ if the scheme $p(.)$ is constant such that the agent receives a fixed wage independent of the uncertain result. Such a fixed-wage payment, however, will set no incentives.

2.2 Induced Effort

How will the agent respond to a payment scheme $p(.)$? He will choose his effort such that his welfare (3) is maximized. Let $x^* \in X$ denote an optimal decision,

\begin{equation}
U(x^*, p) = \max \{U(x, p) \mid x \in X\}.
\end{equation}

The effort chosen depends, among other things, on the payment scheme and hence we write $x^* = \phi(p)$. Omit questions of existence and uniqueness (for some of the problems involved see S. J. Grossman and O. D. Hart (1983)), and solve (4) for each $p \in P$. This yields the response function $\phi: P \rightarrow X$ that describes the way in which the agent responds to reward schemes. In other words, $\phi$ describes how effort is induced. Note that under scheme $p$ the agent can and will attain the welfare $U(\phi(p), p)$.

The decision on effort is not the only choice to be made. Distinguish four consequential choices. The first choice is made by the principal.
who selects a payment scheme \( p \in P \) and suggests it to the agent. The second decision is made by the agent when he either accepts or refuses the scheme suggested. The agent makes his decision on acceptance in view of some other opportunities he might have and the best of which guarantees a certain reservation welfare \( m \). Evidently, the agent is accepting a payment scheme \( p \) only if the welfare attained is not below the reservation level,

\[
(5) \quad U(\phi(p), p) \geq m.
\]

For that reason, the inequality (5) is called reservation constraint. If the agent refuses, the principal will presumably suggest another payment scheme. So there might be some bargaining and the first two decisions turn out to be interrelated. To make here a clear statement, we proceed on the assumption that the agent accepts a scheme \( p \) if and only if the reservation constraint (5) is satisfied. The reservation level \( m \) is thereby either belonging to the data or is resulting from negotiations. In short, \( m \) is considered as an exogenous parameter.

The third decision: If the agent accepted a reward scheme \( p \) he is going to choose his effort \( x^* = \phi(p) \). The fourth and final step of that sequence is the realization of the state of nature, more precisely, the realization \( y \) of \( \tilde{y} \) becomes known to both principal and agent. Only now the actual payment \( p(y) \) can be made. This ends the cooperation.

Nothing was said hitherto about the first decision in that chain of four choices. How will the principal choose a scheme \( p \) from set \( P \)? The principal’s wealth is the residuum \( \tilde{y} - p(\tilde{y}) \), and her welfare (again expressed in terms of a certainty equivalent) is

\[
(6) \quad V(x, p) = v^{-1}\left(E[v(\tilde{y} - p(\tilde{y}))]\right).
\]

Where \( v \) denotes the Neumann-Morgenstern utility function of the principal. The welfare (6) depends on the agent’s effort \( x \) since the result \( \tilde{y} \) depends on \( x \).

One of the stronger assumptions in the hidden-effort situation is that the principal knows all relevant characteristics of the co-operating agent. The relevant characteristics of the agent are: utility function \( u \), disutility \( c(.) \), set of feasible effort decisions \( X \), and the reserva-
tion level $m$. With that knowledge the principal can calculate the way $\phi$ in which the agent will respond (p. 16) $x^* = \phi(p)$ to reward schemes $p \in P$. This assumption simplifies the principal’s decision to

\begin{equation}
(7) \text{ maximize } V(\phi(p), p) \text{ with respect to } p \in P \text{ subject to the reservation constraint (5)}
\end{equation}

A solution of (7) will be denoted by $p_m^*$. As was indicated by the subscript $m$, the reservation level usually has a major impact on the scheme selected. Of course, the optimal scheme also depends on data such as the technology $f$, the agent’s risk aversion $-u''/u'$, and the variance $Var[\tilde{\phi}]$ of the exogenous risk.

A final remark is made on the assumption according which the principal knows the agent’s characteristics and is thus in the position to predict her agent’s decision making although she is, due to the information asymmetry, unable to verify her calculations by observation. What makes then the difference between the ability to predict and the ability to observe? Suppose the principal selects the scheme $p$ and predicts, by herself, that the agent will respond with effort $x^* = \phi(p)$. What the agent will do in fact is to choose exactly that effort $x^*$. The problem is not that there could be any difference between what the principal predicts and what the agent really does. The principal’s prediction is always correct. Rather than that the true problem is: both individuals cannot freely negotiate in order to agree upon any pair $(x, p)$ of effort and payment. Suppose, for a moment, both individuals would agree to realize a particular pair $(\bar{x}, \bar{p})$ where $\bar{x} \neq \phi(\bar{p})$. Then the principal, unable to observe the agent, can predict that the agent will realize the effort $x^* = \phi(\bar{p})$ in disaccord with the agreement. And the selfish agent will, in fact, make his decision $x^*$ as predicted. Consequently, both individuals are restricted in their cooperation to those specific pairs $(x^*, p)$, where effort is induced by the payment $x^* = \phi(p)$. For that reason, there is no need and no sense to discuss on effort at all. Agent and principal just speak on payment schemes $p$ and none of them has doubts about the corresponding effort induced. Since they do not settle effort, there is no shirking.

The discussion between agent and principal on the payment scheme was modeled here in that way: The principal selects, from all (p. 17) payment schemes which guarantee the agent a certain welfare $U \geq m$, that scheme $p_m^*$ which maximizes her own welfare $V$. The resulting
design of cooperation is characterized by the pair \((x_m^*, p_m^*)\) of induced effort \(x_m^* = \phi(p_m^*)\) and payment \(p_m^*\). By variation of the parameter \(m\) one gets the elements of the set \(I\) of second-best designs defined in Section 1.4.

3. The LEN-Model

The hidden-effort situation as outlined in the last section cannot be solved in its general form. In order to study how the induced effort and the selected payment scheme depend on the data and parameters of the model, we further specify functions and variables. The set of specifying assumptions suggested here is called Linear-Exponential-Normal-Model, since

(L) output \(\bar{y}\) is a linear function of risk \(\tilde{\theta}\), and feasible payment schemes \(p(.) \in P\) are linear functions of output,

(E) the utility function \(u\) of the agent is exponential; likewise the principal has constant absolute risk aversion,

(N) the risk \(\tilde{\theta}\) is normally distributed.

Specifications (N), (L) imply that both the agent's wealth and the principal's residuum are normally distributed. That, in conjunction with (E), implies that the certainty equivalents (3), (6) can be expressed as expected value minus half the variance times risk aversion (G. Bamberg and K. Spremann (1981)). A simple version of the Len-Model is:

\[
\bar{y} = f(x, \tilde{\theta}) := x + \tilde{\theta}, \quad x \in X := [0, 1/2],
\]
\[
\tilde{\theta} \text{ normal, } E[\tilde{\theta}] = 0, \text{Var}[\tilde{\theta}] = \sigma^2
\]
\[
p \in P \text{ if and only if } p(y) = r + s \cdot y
\]
\[
u(w) = -\exp(-\alpha \cdot w), \quad \alpha > 0
\]
\(v\) linear (principal is risk neutral)
The agent’s effort has one dimension only and the result $\tilde{y}$ is the sum of effort $x$ and the one-dimensional random variable $\tilde{\theta}$. The agent has constant risk aversion denoted by $\alpha = -u''/u' > 0$ and the principal is risk neutral $-v''/v' = 0$. In order to describe increasing marginal disutility of effort, the function $c(x)$ is supposed to be quadratic.

Two parameters $r,s$ determine feasible payment schemes: $r$ will be called fee and $s$ will be called share. So far there are no restrictions on $r,s \in \mathbb{R}$ although the share may be viewed as constrained to $0 \leq s \leq 1$. In the case $s = 0$ the principal pays a fixed fee $r$ for the services provided by the agent, independent of team profit. In the case $s = 1$ it is the agent who bears all the risk, while the principal’s wealth will be risk free under such an agreement. The fee $r$ can be negative, tao, which could be indicated in particular if the agent receives a positive share $s > 0$ of the result. One could then refer to $r$ as a rent paid to the principal, and we will use the term rent independent of whether $r$ is positive, negative, or equal to zero. From now on, we write the scheme as pair $(r,s)$ of rent and share.

Analysis and results presented in the sequel depend, as always, on the specific assumptions made. In particular, the class of linear payment schemes has a major impact. We just mention that non-linear schemes have been suggested. Quite often arrangements can be found where the agent’s reward is not a linear function of team output. Sometimes, the agent participates in gains but not in losses.

$$p(y) = r + s \cdot \max\{0,y\}$$

such that the risk premium demanded will be reduced. Denote the set of payment schemes (8) by $P^+$. Not only is the question which are the parameters $r,s$ chosen in the situation where $P^+$ is the set of feasible arrangements. Another issue is whether or not the best schemes in $P^+$ are superior to linear profit-sharing arrangements in $P$. In other words: which arrangements would be chosen in the set $P \cup P^+$? Another common arrangement is a bonus-penalty scheme: a fixed fee $r$ is applied as long as the profit is not below a certain critical level $Y_L$ combined with a fine $t \sim 0$ for too poor
For an analysis of penalty schemes see also J. Mirrlees (1975).

Our version of the LEN-Model has three exogenous parameters:
the agent's risk aversion $a > 0$, the reservation level $m$, and the variance $\sigma^2 > 0$ of the environmental risk. There are three endogenous variables: the agent's effort $x$, and rent $r$ and share $s$ which determine the payment scheme. The purpose is to study how the induced effort and how the payment scheme $(r,s)$ depend on the exogenous parameters.

The principal will steps which answer way will the agent

find an optimal payment scheme $(r_m, s_m)$ in three

three questions. The first question is: in which respond $x^*$ to a payment scheme $(r,s)$?

In the LEN-Model the agent's wealth (2) is equal to

\[
(10) \quad w(x; r, s) = 2 - r + (x + e)s - x
\]

and the derived welfare (certainty equivalent (3)) is

\[
U(x; r, s) = -a - \mathbb{E}[w] - 2 \text{Var}[w] = 2 a 2 2
\]
Maximization of $U$ with respect to effort $x$ yields the agent's response

$$x^* = \frac{1}{2} J(r, s)$$

Note that $0 \leq s \leq 1$ implies $x^* \in X$. This proves:

**Theorem 1**: Neither the rent $r$ nor (the result of negotiations on) the reservation level $m$ have an impact on the agent's effort. In particular, a fixed-fee arrangement, $s = 0$, induces the agent to the lowest feasible effort, $x^* = \frac{1}{2} J(r, 0) = 0$, however large the rent $r$ may be.

The second question is: which payment schemes $(r, s)$ will be accepted by the agent in view of the reservation constraint (5)?

Equations (11), (12) imply that the attained welfare is

$$U(x^*; r, s) = r + \frac{1}{2} (1 - 2a \sigma^2)$$

such that the reservation constraint is satisfied if the rent $r$ has the size

$$r \geq \frac{s^2 2 m - \frac{1}{2} (1 - 2a \sigma^2)}{s^2},$$

at least.

A common hypothesis is that the fee or rent $r$ can be reduced in an arrangement if the share $s$ is increased. As (14) indicates, however, that is correct only if both the agent's risk aversion $a$ and the variance $\sigma^2$ are small enough, i.e., if $2a \sigma^2 < 1$. To see the reason, recognize the difference between expected value and certainty equivalent of the agent's wealth (10) as a risk premium. The risk premium is equal to $(a/2) s^2 \sigma^2$. Thus a rising share $s$ has three effects. (i) A
higher bonus simply increases the expected income. (ii) A higher share induces the agent to more effort and
he is participating in a better result. (iii) The agent is demanding
a higher risk premium because he is going to bear more of the risk as
the share is increased. If the third effect outweighs the first and the
second effect, the overall result is that the fee r has to
be increased instead of decreased as a higher share is envisaged.

THEOREM 2: If the agent's risk aversion a and/or the variance 020f
the environmental risk are large (in the sense of 1 < 2a02), an in-
crease of the share s requires an incr.ease of the re nt r.
The rationale of this result is that the agent will not only share in
"profits" y > 0 but in "losses" y < 0, too.

The third question is: which payment scheme (r,s) maximizes the
principal's welfare given the agent's response (12) and subject to the
reservation constraint(in the form (14))?

The principal's wealth is the residuum
(15 )
y - (r + sy)
(1 - s) (x + 8) - r
and her welfare, because of her risk neutrality, is the expected
wealth
(16)
V(x;r,s)
(1 - s) x - r.

Considering induced effort (12), the principal wants to maximize
(17 )
V(x*;r,s)
s
(1 - s) "2 - r
with respect to rent rand share s such that the reservation con-
straint (14) is satisfied. Insert (14) into (17) and see that it
means to maximize

(18)
\[ v = s s_2 2 (1 - s)^2 - m + \text{LI} (1 - 2aa) \]

which turns out to be a function of \( s \) alone. The share that maximizes (18) is easily determined,

(19)
\[ * \quad s^{*} = \frac{1 - 2aa}{1 + 2aa} \]

From (14), (19) follows the rent selected,

(20) \[ r^{*} = m - 4(1 + 2aa)^2 \]

whereas (12) gives the induced effort

(21)
\[ * \quad x^{*} = p(r^{m}, s^{*}) \]
\[ 1 - 2 + 2aa \]

Remember that the agent's welfare is \( U^{*} = m \) whereas the principal attains the welfare

(22) \[ V^{*} \]
The agent's share $s_m$ effort $x_m$' expected output $E[f(x_m, \theta)]$, and the principal's welfare $V^*$ are inversely related to $a_a^2$.

One implication of (20) is that the principal's welfare is inversely related to her agent's risk aversion $a$.

THEOREM 3: If the principal could choose between two agents who differ only with respect to risk aversion, she prefers the agent with the lower risk aversion.

To comment on (19), the principal finds it best to reduce the share $s^*$ as the agent's risk aversion $a$ and/or the variance $a^2$ of exogenous risk increase. This is because the agent will then ask a higher risk premium. However, the agent will never get a fixed-fee salary.

THEOREM 4:

No fixed-fee agreement $(r, O)$ will be made, however large the agent's risk aversion is.

On the other hand, the principal prefers to keep a residuum almost free of risk, $s^* \approx 1$, if the agent's risk aversion $a$ or the variance $a^2$ are small. In such situations it is cheap to motivate through profit sharing since the risk premium required by the agent is small. An extreme situation is that of a risk neutral agent $a = 0$. A risk neutral agent bears all the risk, $s = 1$. The effort $x = 1/2$ induced by that share $s_m = 1$ can be seen as first best as will be shown in the next section. The welfare attained by the principal assumes the largest value ever possible $V^* = 1/4 - m$. One can therefore conclude:

THEOREM 5: It is the connection of unobservability (of the agent's effort) and of risk aversion (of the agent) that excludes first-best arrangements.
2.4
Agency Costs

In the seminal paper by M.C. Jensen and W.H. Meckling (1976) agency costs were proposed to be a key tool in evaluating alternative designs of a principal-agent relation. The authors defined agency costs as the sum of (i) the monitoring expenditure by the principal (no such expenditures are modelled here), (ii) the bonding expenditures by the agent, and (iii) the residual loss, i.e. the monetary equivalent of the reduction in welfare experienced by the principal due to the divergence between the agent's decisions and "those decisions which would maximize the welfare of the principal" (1976, p. 308). The latter formulation, however, is not clear and ambiguous if taken literally, see D. Schneider (1987), R.H. Schmidt (1987).

In Section 1.4 agency costs were defined as an index that measures the distance between the set E of first-best designs and the set I of second-best designs. In order to determine agency costs along this line, we have to specify what measure of distance between E and I should be used. In addition, one has to explore for what purposes that index termed agency costs can serve. Agency costs as a measure of distance can be presumed to give an estimation of how much the given second-best design could be improved if there were a monitoring device informing on the agent’s effort. In fact, the nature of agency costs will be seen as a decision-theoretic value of perfect information. But one should be very careful when agency costs are suggested as a tool to evaluate alternative second-best designs.

Formally, we consider two particular designs, one belonging to the set E, the other to I. Both designs assign the same level m of welfare to the agent. Agency costs, in the sense of a distance measure, are the difference of the principal’s welfare in these two designs. In the LEN-Model it turns out that this difference is independent of the parameter m. This rather abstract definition will now be made more concrete in terms of the information value.

The rationale of the principal-agent relationship is that the agent’s effort cannot be observed by the principal. A rigorous approach has
thus to define agency costs as a value of information: how much will the principal offer, at most, if he could observe the agent's effort?

If the principal has perfect information on the true effort of the agent, both team members can bargain and agree upon any effort in exchange for any payment. No longer has effort to be induced by a payment scheme. Under perfect information the principal would thus address to

Maximize
V(x,p)
(23)
subject to
U(x,p) ~ m
with respect to
pEP and
X E X
if the agent is willing to enter into cooperation as long as his welfare reaches the level m. Denote a solution of problem (23) by (xo, po). This design (xo, po) is the first-best design chosen to represent E.

* *

From the set I we choose the design (xm' Pm) that solves the problem

Maximize
V(x,p)
subject to
U(x,p) ~ m
(24)
with respect to
pEP
where
\[ x = \langle j|p \rangle \text{ is induced.} \]

Agency costs are now defined as difference

\[
(25) \quad AC = V(x_{m'} P_m) - V(x_{m'} P_m)
\]

For the LEN-Model it is easy to see that

\[
(26) \quad x = 1/2
\]

This is the efficient effort upon agent and principal would agree if effort could be observed. The payment they will agree upon is 0

\[
rm
\]

\[
m ,
\]

\[
(27)
\]

\[
s^2 = 0 .
\]

The results (26), (27) and (22) yield agency costs in the LEN-Model as

\[
(28)
\]

\[
AC = 2
\]

\[
e w
\]

\[
4ew^2 + 2
\]

independent of the agent’s welfare m.

Since AC is increasing with \( a^2 \) we get

THEOREM 6: The unobservability of the agent’s effort becomes as more a drawback the larger the agent’s risk aversion and the larger the variance of the environmental risk are.

Another insight provided by (28) concerns the output variance \( a^2 \). The theory of finance tells that diversification is not an issue for the single firm because all unsystematic risks can be eliminated in well
diversified portfolios. This result, however, remains no longer true in the context of agency theory. Diversification within a single firm implies a lower variance $a_2$ and thus reduced agency costs. Consequently, the dependency of agency cost on the variance $a_2$ may suggest to form teams, where the team output has, because of diversification, lower variance compared with the output variances of separated units. The reward of team members is then made as depending on the output of the whole team, rather than making reward a function of individual output.

The literature on teams, see A.A. Alchian and H. Demsetz (1972), often presents this rationale for the existence of teams: the team output can be observed but not be separated and presented as sum of what each of the team members contributed. The analysis presented here suggests another rationale for the existence of teams: the team output is diversified (lower $a_2$) and hence, taken as a basis to reward team members, reduces the required risk premiums.

A final implication of (28) concerns the question of what happens in cases where the variance $a_2$ is quite large and cannot be reduced through diversification. It may thus happen that agency costs are so high or, equivalently, that the principal's welfare is (22) is such low that she prefers no cooperation with an agent at all. The principal, perhaps, has other opportunities which determine a certain reservation level also for herself. Three ways to overcome such a situation of too high agency costs can be outlined.

Firstly, one could enlarge the set $P$ of feasible payment schemes. Consider nonlinear schemes of the form (8) or bonus-penalty schemes of the form (9). If such schemes were feasible it could be the case that second-best designs come closer to first-best results. In a particular setup J. Mirrlees (1975) demonstrated the superiority of payment schemes that impose heavy penalties on a suitable range of outputs.

Secondly, one could consider monitoring devices that give additional information, though not perfect in every case, on the agent's effort. Costless monitoring signals were introduced by M. Harris and A. Raviv (1979), B. Holmström (1979), S. Shavell (1979), F. Gjesdal (1982), N. Singh (1985). An analysis of monitoring signals within the LEN-Model follows in the next section, and extension of these results with respect to costly monitoring was done by M. Blickle (1987).
Thirdly, society could encourage trust. If nowhere cooperation is starting, society can be supposed to develop and to reward behavior such as honesty, reliability, and altruism. In the literature on organization, such forms of behavior are induced through the process of indoctrination: the member of the organization internalizes cooperative criteria, which replace selfishness even within the reign of managerial discretion.

2.5 Monitoring Signals

Both the general model (Sections 2.1, 2.2) and the special LEN-Model (Section 2.3) on the hidden-effort situation can be extended in order to incorporate organizational instruments which monitor the agent and measure his effort. Generally speaking, there might be a (multidimensional) signal \( z \) which, more or less exactly, reveals the agent’s (multidimensional) effort \( x \). Such a monitoring signal \( z \) must thus be a function of \( x \), though not a function of \( x \) alone. More or less exactly means that some additional uncertainty influences the value \( z \) of the signal \( z \),

\[
(29) \quad z = h(x) + E
\]

Such a monitoring signal may be seen as a sufficient statistic.

Both principal and agent are supposed to know the observation function \( h \) as well as the probability distribution of the observation error \( E \). Nature will realize the random variable \( E \) at the same time when \( S \) is realized. Like \( S \), the principal will not learn the realization \( E \) of \( E \). Nevertheless, the principal can find it better to make the reward not only depending on output \( y \) but also on the monitoring signal \( z \). When cooperation is started, the principal is thus suggesting a payment scheme \( p(y, z) \) as a function of output \( y \) and the monitoring signal \( z \).

In its simplest form, the observation function \( h \) and the parameters of the probability distribution of the observation error \( E \) are given beforehand. The principal has thus to decide whether to utilize the signal \( z \) in the reward scheme or not, and if yes, in which way the payment should depend on \( z \). Such an extension will now be studied within the framework of the LEN-Model.
In more complex cases, the form of the observation function $h$ or distributional parameters of the observation error $E$ might belong

variances of separated units. The reward of team members is then made as depending on the output of the whole team, rather than making reward a function of individual output.

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\[
(29)
\]

\[ z = h(x) + \varepsilon \]

Such a monitoring signal may be seen as a sufficient statistic.

Both principal and agent are supposed to know the observation function \( h \) as well as the probability distribution of the observation error \( \varepsilon \). Nature will realize the random variable \( \varepsilon \) at the same time when \( 6 \) is realized. Like \( 6 \), the principal will not learn the realization \( E \) of \( \varepsilon \). Nevertheless, the principal can find it better to make the reward not only depending on output \( y \) but also on the monitoring signal \( z \). When cooperation is started, the principal is thus suggesting a payment scheme \( p(y,z) \) as a function of output \( y \) and the monitoring signal \( z \).

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In more complex cases, the form of the observation function \( h \) or distributional parameters of the observation error \( \varepsilon \) might belong
to the principal’s decisions. Even more, alternative monitoring devices can imply different monitoring cost. To give an example, let effort $x$ and signal $z$ be one-dimensional variables, $z = x + E$, $E|EJ = 0$, and the variance $\text{Var}(EJ) = a^2$ being a decision variable.

Thereby, monitoring costs increase in some way as a smaller error variance is chosen. The question is then not only how to make the reward depending on $z$ but also: how much wealth should be devoted to make monitoring more precise, i.e., reduce $\sigma^2$.

Now, a costless monitoring signal is considered and introduced into the LEN framework. A straightforward extension of the simple version of the LEN-Model presented in section 2.3 is

output $Y = f(x,e) = x + e'$

Signal $- hex)$

$z = + E = X + E$,

- $2^{-1/2} E \sim N(0,a^2)$ \quad $\text{COV}(e,E) = 0$ ,

$S \sim N(0,a^2)$ , \quad $E$ ,

effort $x \quad X = [0,1/2]$,

payment scheme pEP iff

$p(y,z)$

$r + sy + tz$,

agent’s risk aversion $-u''/u'$

$a$ ,

principal risk neutral, disutility of effort
The analytical solution follows the steps presented in Section 2.3. The principal wants to choose a reward scheme given by the triple $(i;s,t)$ such that her welfare, the expected residual wealth,

\[ y - (r + sy + tz), \]

is maximized. As before, the principal knows the agent's characteristics $a,X,c,m$ and can thus predict the agent's response $x^* = \psi(r,s,t)$ to a reward scheme $(r,s,t)$.

The agent's wealth, similar to (10), is normally distributed,

\begin{equation}
\begin{aligned}
\text{w}(x;r,s,t) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(r + sex + S + tex + E - X)^2}{2}ight) \\
\text{U}(x;r,s,t) &= r + (s + t)x - x^2 - 2(s aS + t aE)
\end{aligned}
\end{equation}

Maximization of (31) with respect to $x$ yields

\begin{equation}
\begin{aligned}
x^* &= \psi(r,s,t) \\
&= s + t
\end{aligned}
\end{equation}

which is the induced effort.

Further, the agent is assumed to accept a reward scheme $(r,s,t)$ if and only if the reservation constraint $\text{U}(x^*,r,s,t) \sim m$ is satisfied. A remark on the reservation level will be made below. The constraint requires a rent $r$ which has the level

\begin{equation}
\begin{aligned}
\text{c}(x) \\
&= x^2
\end{aligned}
\end{equation}
The principal's welfare is

\[ v = \mathbb{E}[y - (r + sy + tz)] \] (34)

This welfare, taken as a function of \( s \) and \( t \), is, for \( \varepsilon \) sufficient small, concave and will be maximized for

\[ s^* = 1 - t^* + 2aa^\varepsilon \] (35)

The linear system (35) has the explicit solution

\[ \begin{align*}
1
\end{align*} \]
Equations (36), together with (33), provide the reward scheme \((r^*, s^*, t^*)\) selected. Some of the properties are noteworthy.

At first \(t^* > 0\), in particular, \(t^* \to 0\). This means that the principal prefers to make the reward depending on the monitoring signal \(z\) however inaccurate it is, i.e., however large the variance \(\sigma_r^2\) of the observation error may be. If that variance becomes larger and larger, ceteris paribus, \(t^*\) is chosen smaller, and the share \(s^*\) selected increases and tends to the value (19).

Another result of (36) concerns the question whether a wage should be paid for labor input (time of presence) or for labor output (result of work). Consider again a varying exactness of the monitoring signal as measured by the variance \(\sigma_r^2\). As \(\sigma_r^2\) becomes smaller,

\[ \frac{2}{1 + 2a_a + as/ae} \]

the share \(s^*\) decreases. If \(\sigma_r^2\) tends to zero, which means that the effort (labor input) can almost accurately be observed, the optimal share vanishes, \(s^* = 0\). At the same time, \(t^*\) tends to 1 where 1 is the marginal value of effort to the principal. If effort were observable, the optimal reward scheme is just a price for units of effort, and each effort unit is rewarded according to its marginal value. The rent \(r\) has a distributional effect only. So understand the share \(s\) as a wage paid for labor output and \(t\) as a wage for labor input. The formula (36) indicates how to mix a payment for output with a payment for input in cases of observation errors.
A further comment is made on the bias to signal effort instead of really working. Extend the LEN-Model such that the agent's effort

\[ x = (x_1, x_2) \]

is now a two-dimensional decision variable. Symmetry is achieved through assumptions

\begin{equation}
\begin{aligned}
\text{output} & \quad Y = f(x, \theta) = x_1 + x_2 + \theta, \\
\text{disutility} & \quad c(x) = 2 \left( 2 \right) = x_1 + x_2
\end{aligned}
\end{equation}

This makes all efforts \((x_1, x_2)\) with \(x_1 + x_2\) inefficient, since the

\((X_1 + x_2, x_1 + X_2)\) yields the same result at reduced dis-effort \(2' 2\)

utility:

\[ x_1 + x_2 > 2 \left( (x_1 + x_2)/2 \right) \]

Now assume there is a monitoring signal which informs on \(x_1\) but not on \(x_2\)

\begin{equation}
\begin{aligned}
\text{signal} & \quad z = x_1 + E \\
\text{salary depends on the monitoring signal,} & \quad t^* f 0, \text{ although the agent will respond with an inefficient action } x_1 > x_2. \text{ So to speak, the principal is aware and predicts that the agent utilizes working time to signal effort rather than to work. Nevertheless, the principal prefers to have the signal be part of the reward.}
\end{aligned}
\end{equation}

A final remark concerns the question whether or not the agent refuses cooperation when the principal is going to introduce an additional monitoring device. As an homo economicus, the agent would be indifferent if his welfare was unchanged. Might be the principal is willing to increase that reservation level \(m\) and is, nevertheless, better off. A principal prepared to modify \(m\) can expect that the agent looks by himself for signals that inform on his effort. Another point is that some kinds of monitoring devices cause additional disutilities to
the agent which need compensation. Consequently, there are three reasons why the introduction of monitoring signals can be costly. One of course is that the technology of monitoring requires resources. The second reason is that the agent asks for new negotiations on the level m of his welfare. The third reason is that a disutility caused by monitoring must be compensated.

2.6 Screening

So far, the principal has been assumed to know all the agent's characteristics such as risk aversion and so forth. The analysis presented in Sections 2.1 through 2.5 focussed on hidden effort and monitoring. As outlined in 1.6, another issue of agency theory is screening. Sorting and screening devices become necessary in situations where some of the agent's characteristics are hidden. Perhaps the most important class of screening devices is that of self selection schemes. This is because everybody prefers free choice to inquisition even if the final outcomes are the same.

This section presents basic ideas in the design of self-selection schemes, see K.J. Arrow (1986). To be designed is a set of contracts such that each individual chooses the contract which is designed to fit his or her type. Thus, the individual's characteristics are revealed through choice.

Here, the hidden characteristic is supposed to be the agent's risk aversion. All results are derived within the framework of the LEN-Model. But no monitoring signals are considered in order not to burden the notation.

Consider a labour market with job searchers (possible agents) on the one side and the principal in search of an agent on the other side. This time, the issue is cooperation happening across the market. The principal is ready to offer reward schemes (r,s) with

\[
\text{shares}(a) = \frac{1}{2aa^2},
\]

\[
r(a,m,s) = s^2 2
\]

\[
\text{rent} = m - 4 (1 - 2aa),
\]

see (19), (14). However, the principal does not know a job searcher's risk aversion a this time.
Note that (39) is the optimal share as a function of risk aversion $a$, whereas (40) denotes the smallest rent an agent with risk aversion $a$ and reservation level $m$ would accept, if the share were equal to $s$, independent of whether or not this $s$ is optimal in the sense of (39). Another result that should be recalled is the certainty equivalent of an agent's wealth under contract $(r,s)$, now denoted by $U_a(r,s)$:

$$U_a(r,s) = \frac{s^2}{2a} \frac{r + 4(1 - 2aG)}{r + 4(1 - 2aG)},$$

see (13). Suppose the principal knows there are two types $k = 0,1$ of job searching agents who differ only with respect to their risk aversion $a_0,a_1$. The principal knows further that low-risk-averse job searchers $k = 0$ are, to simplify notation, risk neutral and that type-1 job searchers have risk aversion $a$,

$$a_0 \quad a \quad a_1,$$

Both types of agents may have, so far, the same exogenous reservation level $m$ and identical disutilities of effort. As has already been demonstrated, the principal prefers type 0 job searchers to type 1 agents. According to (22), the principal's difference in welfare of getting a type-1 or type-0 agent is

$$v - V = \frac{v - V}{2}.$$
But, in order not to conclude with a search model, assume the principal's aim is not to refuse type 1 agents. She just wants to offer to each job searcher a contract which she, the principal, finds best.

Everything were easy if the principal could costlessly find out a job searcher's type. He would then offer:

contract 0:
share s(O) = 1
rent r(O,m,sO) = 1
	ro = m - '4

to type 0 agents, and

contract 1:
share
sl :=
s(a)
1
- 2
1 + 20.0-
rent
r1 :=
r(a,m,sl)

according to (39)
to individuals type 1.

What will happen if the principal cannot identify an agent's type and is going to allow all job searchers to choose among contracts 0,1?
The answer is that the set of contracts 0,1 breaks down as a self-selection device: Agents of any type decide for contract 1. The proof is twofold. Firstly agents type 1 understand that

\[(44)\]
\[U_a(0_0's_0) = m - 0.0-2\]
\[< m\]
\[U_a(1_1,sl)\]

and consequently prefer contract 1 to contract 0. Secondly, an individual type 0 realizes that welfare under contract 1 is

\[(45)\]
\[U_0(1_1,sl) = m + 2ao2\]
\[\frac{(Sl/2)^2}{2}\]
\[2\]
\[0.0-\]
\[m + 2 2\]
\[2(1 + 20.0- )\]

which exceeds the welfare under contract 0,

\[(46)\]
\[U_0(0_0's_0)\]
\[m < U_0(1_1,sl)\]

In other words: As long as there are type-1 job searchers in the labour market and type-O agents cannot be excluded from choosing contract 1 (which is designed to type-1 individuals), the reservation utility of type 0 agents is endogenously increased from m to the level (45).

Both comparisons (44), (46) demonstrate that a self-selection device made up of contracts 0,1 will break down.

Fortunately there is a straight-forward revision. Realizing that type 0 agent’s reservation level mO is now endogenously given through (45),
while the reservation of type 1 agents is still at the old level m, the principal could modify contract 0 correspondingly. This modification is called

contract 2 : 
shares2 .- So = 1
rent r2 r(0,mO,s2) 

2
1 aa
m - - + 2 2
4 2(1 + 2aa )

While the share remains unchanged, the rent refers now to mO rather than to m.

Is the set of contracts 1,2 working as a self-selection device? The answer is yes.

To prove this answer one has to consider the choice between contracts 1,2 for each type of individuals. Firstly, contract 2 was constructed in such a way that type-O agents are indifferent between contracts 1,2. So increase the rent of contract 2 by one dollar or so to induce type-O agents definitely decide for contract 2. Secondly, type-1 agents still prefer contract 1 when having the choice among contracts 1,2. This follows from

(48 )

" 

Ua(r2,s2) 

m + aa2 2

aa

2(1 + 2aa2)2 - ~ < m

Ua(r1,sl) .
One should not forget, however, to ask the principal what she thinks about the self-selection device of contracts 1,2. The final question reads: is the principal really better off under this self-selection device where each agent reveals his type through choice? Comparison is made with respect to the situation before, where everybody just got contract 1.

Let us see the answer: If a type-O agent decides for contract 2 instead of contract 1, there are two changes of the principal's welfare. Firstly, there is an increase of welfare $V$ according to (43). Secondly, there is the cost $mO - m$ associated with the endogenous reservation welfare of type-O agents. The net effect turns out to be positive.

(49)

$$ (V_O - V_a) - (mO - m) $$

$$ 2 $$

$$ 2 $$

$$ (aa > 0 . \) $$

$$ \backslash, + 2aa2 $$

Consequently, the principal prefers to offer the self-selection device. Her incentive to replace the uniform contract 1 by the device of self-selection between contracts 1,2 becomes the greater, the more different job searchers are with respect to their hidden characteristic risk aversion.

Finally, the principal can further increase her net gain (49). She realizes that the costs of the self-selection device, $mO - m$, are due to the fact that type-O agents cannot be excluded from choosing contract 1. So the trick is to modify contract 1 such that the induced increase of type-O agent's reservation level will not be as much. This is possible, indeed. Since contract 1 maximizes the principal's welfare, a small variation from $sI$ to $s3:= sI - 0$ and from rent $rI$ to $r3:= r(a,m,s3)$ causes a welfare loss of second order only. On the other hand, the difference $UO(rI,sI) - UO(r3,s3)$ of type-O agent's welfare is of first order in $o$. This means that the principal improves herself when offering self-selection between contracts 2,3 rather than self-selection between contracts 1,2.

As was pointed out by K.J. Arrow (1986) it is typical for problems of hidden characteristics that not all types of searching individauls...
can find exactly that offer they would get if their characteristics were known by the other market side. This result can be cast in those words: Consider, on one side of the market, "weak" as well as "strong" individuals, characteristics that are hidden to the other market side. Usually, weak individuals need help and must be treated with care. In order to induce strong people to renounce on care and to help themselves, they must get an extra bonus: not for justice, but to set incentives. The size of the bonus depends on the weakness of the weak, or to be more precise, the amount of care devoted to the weak. Sometimes, the weak are not treated with the proper care, just to make the strong peoples' bonus a bit smaller.

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