New Frontiers in the Validation of Simulation Models—Structural Dominance Analysis

Markus Schwaninger and Stefan Groesser

University of St. Gallen, Institute of Management,
Dufourstrasse 40a, 9000 St. Gallen, Switzerland
markus.schwaninger@unisg.ch

Abstract. Building better models is crucial for coping with complexity in general, and for the management of organizations in particular. This paper discusses the epistemological aspects of model validation for the achievement of high-quality models. Then it provides an overview of validation methods. The logic of validation is demonstrated by introducing the Structural Dominance Test as a means for testing the correspondence of the structural dominance between model and reality.

Keywords: Modeling, Simulation, Validation, Validity, Model Quality, System Dynamics, Structural Dominance Analysis.

1 Introduction

Validation is the process by which the correspondence between model system and real system is systematically enhanced. It consists in gradually building confidence in the usefulness of a model by applying validation tests. In principle, validation pervades all phases of the modeling process, and, in addition, reaches into the phases of model use and implementation.

In this contribution, the issue of validation is addressed with respect to dynamic models of social systems. Special reference will be made to System Dynamics models. System Dynamics is a methodology for the modeling and simulation of complex, dynamic systems [3]. It is particularly adequate for modeling socio-technical systems, such as private and public enterprises, communities, etc. They are structured as meshes of interconnected feedback loops. Causal relationships, delays, and closed-loop structures are characteristic of System Dynamics models.

Simulation is a way of experimenting with mathematical models to gain insights and then employ them to improve the real system under study. Validity in this context consists of a stringent correspondence between an abstract model system and a concrete "real" system. We will concentrate on the crucial philosophical underpinnings and a canon of methods for model validation.

2 Philosophy of Model Validation

One of the frequent convictions about science is the obsessive idea that proofs are the touchstone of the validity of both theories and models. To orientate model validation,
we follow a different rationale. We argue for the adoption of the philosophical position of critical rationalism, a philosophical position founded by Karl R. Popper.

Critical rationalism posits that, in the social domain, theories can never be definitely proved, but can only reach greater or lesser levels of truth. Scientific proofs are confined to the realm of the formal sciences, namely logic and mathematics [12, 13]. As Popper demonstrates, all theories are provisional. As a consequence, the main criterion for the assessment of a theory or model's truth status is falsification (see also: [17]).

Popper's refutationist concept (as opposed to a verificationist concept) of theory-testing implies both an evolutionist perspective and an empiricist stance. The evolutionist perspective is primary because it welcomes the challenges posed to a theory, since the attempts at falsification lead to an evolutionary process: Successful falsification efforts result in revisions and improvements of the theory. Correspondingly, empiricism is paramount in the social sciences, because the main source for the refutation of a theory is empirical evidence. However, falsification can also be grounded in logical arguments where empirical evidence cannot be obtained.

3 Validation Methods

For the enhancement of model validity, a considerable set of qualitative and quantitative tests has been developed. We give an overview of the types of tests developed for System Dynamics models. Three domains of tests are expounded: the model-related context, model structure, and model behavior.¹

3.1 Tests about Model-Related Context

These tests deal with aspects related to the situation in which the model is to be developed and embedded. They imply meta-level decisions which have to be taken in the first place, before engaging in model-building. Applied ex-post-facto, i.e., after modeling, they allow for assessing the utility of the modeling endeavor as such.

Examples: Issue Identification Test, Adequacy of Methodology Test, System Configuration Test, System Improvement Test.

3.2 Tests of Model Structure

Tests of model structure refer to the "nuts and bolts" of System Dynamics modeling, i.e., to the concepts and interrelationships which represent the real system. Model structure tests - direct and indirect - aim to increase confidence in the structure of the theory created to assess the behavior mode of interest. The model structure can be assessed by means of either direct or indirect inspection. Tests of model structure assess if the logic of the model is attuned to the corresponding structure in the real world.

Examples of Direct Structure Tests: Structure Examination Test, Parameter Examination Test, Direct Extreme Condition Test, Boundary Adequacy Structure Test, Dimensional Consistency Test.

¹ For a detailed description of these tests, see: [18] as well as literature quoted therein.
Examples of Indirect Structure Tests: Indirect Extreme Condition Test, Behavior Sensitivity Test, Integration Error Test, Boundary Adequacy Behavior Test / Boundary Adequacy Policy Test, Loop Dominance Test.

3.3 Tests of Model Behavior

Tests of model behavior are empirical and compare simulation outcomes with data from the real system under study. On that basis, inferences about the adequacy of the model can be made. The empirical data can either be historical or refer to reasonable expectations about possible future developments.

Examples: Behavior Reproduction Tests, Behavior Anticipation Tests, Family Member Test, Surprise Behavior Test, Turing Test.

4 Structural Dominance Test

As an example, we will now demonstrate a new test about the model structure, called the Structural Dominance Test (SDT). In principle, SDT evaluates the relative partial influence of individual feedback loops on the behavior of chosen variables, and compares the result to the real system. Structural dominance signifies which particular piece of model structure is dominant, i.e., most influential for the behavior over a certain period of time. SDT is based on Structural Dominance Analysis [6]. Structural Dominance Analysis is a field of System Dynamics with a three-decade history. Only today are the approaches mature enough to sustain frequent use. Groesser conceptualizes the use of SDA for model validation purposes, namely the SDT [4].

4.1 Axiom of the Structural Dominance Test: Feedback Structure Creates a System's Behaviour

The dominant structure of a feedback system is the cause for the behaviour of the system [3]. The system's behaviour over time can, in principle, be partitioned into such time intervals that for each interval one of three behaviour patterns can be recognized: linear growth, exponential growth, or logarithmic growth [2]. These behaviours are created by the dominant structures. Complex phenomena are generated by the interaction of a multitude of structural feedback loops. This kind of analysis of dynamic complex feedback models is a daily challenge for modellers — even for advanced system dynamicists. The analysis of structural dominance supports modellers in determining which part of the model structure is mainly responsible for creating the system behaviour. On this basis rests the topic of model validation by means of the Structural Dominance Test.

In order to transfer the general notion of the structural dominance analysis to a validation test, the basic elements for the structural analysis of dynamic systems have to be elaborated. These are the concepts of dominant feedback loop, the polarity of a feedback loop and the change of dominant structure.
4.2 Basic Concepts for Structural Dominance Analysis

4.2.1 Dominant Structure

The approaches available for the analysis of a model’s dominant structure use the concept of a dominant feedback loop as their basic element. According to Richardson and Pugh, “a dominant loop is a loop that is primarily responsible for the model behaviour over some time interval” [14: 231]. A feedback loop is a chain of causal effects which includes as a constitutive property at least one state variable. Richardson’s and Pugh’s definition of a dominant feedback loop is valid for relatively small structures (=first or second order models, see [15]). Higher order systems result in more complex interactions among a greater number of feedback loops. In such models, it is likely that multiple feedback loops are simultaneously responsible for causing the system’s behaviour; we term this the dominance of a feedback loop cluster.

4.2.2 Polarity of a Feedback Loop

The polarity of a feedback loop indicates the direction of the resulting behaviour of a feedback loop as the consequence of a change in any variable of the respective loop. Given an increase in a variable’s value by one unit (\(dx_{i0} = 1\)), a feedback loop with a positive (self-reinforcing) polarity further amplifies the initial increase, i.e., \(dx_{i1} > 1\), \((dx/dt)/dx > 0\). A negative (balancing) feedback loop reduces an initial increase accordingly, i.e., \(dx_{i1} < 1\), \((dx/dt)/dx < 0\). Expressed in mathematical terms: the polarity of a feedback loop is the sign of the loop (Equation 1). The sign is the multiplication of the partial derivatives of all bi-variate causal relationships within that feedback loop. The dominant polarity of the feedback structure can be inferred by comparing the polarity of feedback loops with the model’s dominant pattern in each time interval [2].

\[
\text{sgn} \left( \frac{\partial x_1}{\partial x_1} \right) \times \left( \frac{\partial x_1}{\partial x_1} \right) \times \left( \frac{\partial x_1}{\partial x_1} \right) \times \cdots \left( \frac{\partial x_n}{\partial t} \right)
\]

Equation 1. Definition of the polarity of a feedback loop

The notation of dominant polarity is bi-unique in systems with only one feedback loop (=simple system). In a system with two or more feedback loops, all feedback loops which contain the target variable as an element must be taken into account to calculate the dominant polarity.\(^2\)

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\(^2\) State variables are system variables, which represent accumulations over time.

\(^3\) Even though the identification and calculation of the dominant polarity according to the procedure established by Richardson [15] are of limited convenience for multi-loop systems, they are helpful to illustrate the basic logic of the approach. The simultaneous consideration of multiple feedback loops becomes increasingly impossible for the analyst. The authors estimate that it is impossible to mentally cope with the dynamic complexity of more than 3-5 feedback loops.
4.2.3 Change in Dominant Structure

A change in dominant structure marks the point in time at which the dominance of one feedback loop is neutralized by the dominance of another. This change is identifiable by an alteration of the signum of the feedback structure [15]. The example in Equation 2 shows that the state variable (x) and the modelled system context (plainly represented by the intercept a and the slope b) determine the dominant polarity and the position in time \[x=\frac{a}{b}\] of the change in the dominant polarity. Taking the example from Equation 2, the reinforcing structure (positive signum) dominates for relatively small values of x; correspondingly, this changes the dominance to a balancing structure (negative signum) for relatively large values of x.

\[
\text{Dominant Polarity} = \text{sgn} (a - bx) = \begin{cases} + & \text{if } x < \frac{a}{b} \\ - & \text{if } x > \frac{a}{b} \end{cases} \quad ; \text{with } a, b > 0;
\]

**Equation 2.** The change of the signum (sgn) indicates the change of the dominant polarity. The example shows a linear function with the intercept a and the slope b.

The interpretation of the change of the dominant structure depends on the degree of complexity of the considered structure. In simple systems with two feedback loops with opposing polarities, a change of the polarity displays a change in the dominance of the feedback loops. In more complex systems, this interpretation is no longer valid. A change in the dominant structure can no longer be directly inferred from individual feedback loops. In a system containing more than three feedback loops only the polarity of a cluster of loops can be calculated. If a change of polarity occurs, the whole feedback structure then has the opposing polarity; individual feedback loops, which might be responsible for the dominant behaviour, cannot be detected. It might be, for instance, that within a given time interval, during which no change of the signum takes place, two positive feedback loops dominate each other in sequence. Obviously, the dominance of the individual loops shifts over time. This sequential dominance, however, cannot be recognized by the indicator of the dominant polarity. Diagnosing changes in structural dominance within a time interval that is continuously dominated by either positive or negative loops requires mathematical Eigenvalue methods for the exact analytical determination of the structural dominance over time [5]. Several mathematical approaches of structural dominance analyses have emerged over the past decade ([6], [7], [8], [10], [11]). An experimental approach to the analysis of dominant structure has emerged early in the field of System Dynamics [2]. However, this approach lacks the mathematical rigour of the Eigenvalue-analytical methods.

4.3 Validation Logic

The validation logic of the structural dominance test corresponds to the general logic of any other validation effort: the analyst – or a group of participants in a group modelling project – compares the model assumptions, model inputs, as well as model outcomes with the perceived counterparts of the modelled section of reality (Figure 1). The evaluation of the correspondence of the model and the modelled part of reality is (always) based on the available empirical material. This fundamental validation logic will now be applied to the analysis of the structural dominance of a model. Thereby, the unique contribution of the structural dominance test will become more vivid.
Modelling with System Dynamics tries to elicit the basic system structure with a special focus on the following properties: feedback structure of the system, dynamic behaviour of the system, and change of the dominant system structure over time. System Dynamics analysts explore the reality based on these three criteria. As a result, they manifest their insights in their mental models about dynamic systems [1] (Figure 1). The major difference between the informal presentation of the reality in the mental model and the reality itself is that the perception of reality is imperfect given the perceptual apparatus of the observer. Subject-related distortions can be reduced by group-based modelling practices and validation methods [19].

![Diagram](image)

**Fig. 1.** Association of the modelling of the reality, the mental model of the observer, and the formal simulation model as well as the validation process. Validation by means of Structural Dominance Analysis supports the modeller in identifying the dominant feedback structure of the formal model, and enables evaluation of the correspondence between the formal model and the modelled part of reality.

The conceptualization and creation of a formal model is fully grounded only in the mental models of the analysts. The formal model should incorporate the structure of the feedback system, the system’s dynamic behaviour, and the change of its dominant structures over time. Validation efforts in all possible validation domains [18] attempt to ensure the correspondence of the formal model and its counterparts in reality. The dominance analysis focuses on the comparison of the dominant feedback structures in the simulation model and reality. Thereby, it becomes possible to test their correspondence. The following decision logic is applied:

\[
\text{Structural Dominance}_{\text{formal}} = \begin{cases} 
\text{true} & \rightarrow \text{Abort Validation} \\
\text{false} & \rightarrow \text{Reformulation}
\end{cases}
\]

If the dominant structure of the formal model corresponds to the dominant structure of the perceived part of reality over time, no further validation is required – the potential of the structural dominance test is then exhausted. However, if the structures do not
correspond, it must be that either the formal model does not reflect reality adequately or that the mental model is not an adequate representation of reality. In either case, it is appropriate to reconsider both the formal and the mental models. Thereby, immense possibilities for substantial learning effects open up. The tracing of the dominant structures in the formal model supports locating the discrepancy between model and reality.

In the following, the principle of the structural dominance test is briefly applied. A full application and analysis is provided in [4]. The example is based on a simplified version of Meadow's commodity cycle theory [9]. At its core, it consists of two interacting feedback mechanisms (B1 and B2) which are mutually regulated by the commodity price (Figure 2). The price stimulates capacity expansion which is introduced into the market with a medium-term time lag. Demand-side reactions to price changes are short-term oriented. The interaction of both goal-seeking feedback loops (B1 and B2) creates an oscillatory behaviour of the indicator variable 'price' (see Figure 3, dotted line 'price').

Figure 3 plots the Behaviour Pattern Index (BPI; solid line; for a detailed definition of the BPI; see [2]). The BPI is an operationalization of both the concept of dominant polarity (see 4.2.) and the pattern of growth (linear, exponential, and logarithmic) of the variable of interest. For positive values of the BPI, the commodity price (dotted line) exhibits an exponential-reinforcing growth pattern (e.g., for the time intervals \( t_{p1} = [4, 6] \), \( t_{p2} = [15, 18] \)); for negative values of the BPI, the price follows a logarithmic, i.e., goal-seeking, behaviour (for example, \( t_{n1} = [0, 4] \), \( t_{n2} = [6, 15] \)). An additional demand step of 30% of the regular demand (for 10 months) disturbs the model initial equilibrium at \( P_{commodity} = 50 \) [USD].

For the excess demand (+36% for \( t > 10 \) months), the equilibrium price is now at \( P_{commodity} = 55 \) [USD]; the equilibrium demand at 675 [units]. In the short-term, the higher demand meets a relatively fixed production capacity which almost immediately results in higher commodity prices. Consequently, the demand per capita is reduced towards the equilibrium demand; this is shown by the logarithmic growth pattern.

\(^4 \rho \) stands for positive, \( n \) for negative values of BPI.
[0<\textless{}t<4] of the BPI. Due to the time delay in the per-capita demand adjustment, the system undershoots the equilibrium demand leading to a brief reinforced correction process in the time interval from 4<\textless{}t<6. After a delay for building, additional production capacity enters the market, leading to further decline in the commodity price. In terms of the model structure, capacity expansion (B2) now dominates the development of the price; B1 is still active. For t>10, the excess demand is reduced; a different equilibrium price and demand is now established. Both loops try to achieve their implicit goals; hence, the main dynamics of the BPI show logarithmic growth. Several spikes of exponential growth tendencies occur (e.g., 15<\textless{}t<18), which indicate a reinforcing correction process of the demand as explained above.

The example chosen is a well-tested theory about commodity prices. At first, it might be counter-intuitive that a balancing feedback loop can generate a reinforcing growth behaviour for short time periods. However, a balancing loop which shows lagged action can exhibit transitional reinforcing growth dynamics [16].

The application of the SDT does not indicate a falsification of the model; the model's change in structural dominance seems to correspond to reality. In this case, the SDT cannot invalidate the model.

5 Conclusion

Validation is a rich and well-defined process by which the confidence in a model is gradually enhanced, with the help of the application of a battery of tests. Validity is always a matter of degree, never an absolute property.

By this demonstration of the Structural Dominance Test, we have made a case for a new approach which extends the frontiers of model validation. More generally, we have used the description of SDT as an exemplar of the logic underlying the validation process.
Simulation based on formal dynamic models is likely to become ever more important for research and practice. It will continue to support scholars and managers at all levels in research, decision-making, and policy design. The more that models are relied upon in all these areas, the greater the importance of their quality. Therefore, model validation is one of the major challenges that we face in advancing modeling and simulation.

References