Optimal Size and Intensity of Job Search Assistance Programs

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Abstract
This paper derives the welfare optimal size and intensity of job search assistance programs in a general equilibrium model where the labor market is affected by search frictions. Both instruments have a priori ambiguous fiscal implications: their direct employment stimulating effects broaden the base of the labor income tax and increase revenues, while also incurring direct costs. At optimal levels, the policy instruments trade off the positive effects on the participants against a marginal increase in taxes, which distorts employment decisions and potentially labor market tightness. We find that the higher unemployment insurance benefits, the lower is the optimal program intensity. Further, the introduction of a job search assistance program is more likely to raise welfare if it is highly effective at improving participants’ job search skills, direct program costs are low and if the general level of taxation in the economy and thus the labor market participation tax are high.

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1 Introduction

Over the last decades, active labor market programs have been part of most industrialized countries’ policies to bring the unemployed back into work. A considerable share of the unemployed are assigned to participate in these programs, and total public expenditures amount to more than 1% of GDP in some OECD countries (OECD, 2006). The most commonly used policies are job search assistance programs, which shall improve the job search skills of the unemployed rather than their productivity in a given occupation. Examples of such measures include training of how to apply for a job, practicing job interviews, but also counselling and direct referrals to potentially suitable positions by the public employment service. These activities typically require no long-term instruction and are therefore relatively cheap compared to other activation measures. However, due to their high prevalence, their costs for taxpayers nevertheless reach up to about 0.3% of GDP in several OECD countries (OECD, 2006).

Microeconometric evaluations of existing programs indicate that, in contrast to other activation measures, job search assistance appears to be effective for a broad range of participants (see the surveys in Fay, 1996; Heckman, LaLonde, and Smith, 1999; Martin and Grubb, 2001; Kluve and Schmidt, 2002). This means that participants have a higher transition rate to employment than if they had not attended the program. However, due to their considerable size, job search assistance programs must be expected to affect also the labor market situation of non-participants. These effects are captured in macroeconomic evaluation studies, which typically analyze the impact of activation measures on the aggregate unemployment or employment rates. Although the evidence is still scarce, most existing studies suggest that the macroeconomic implications are often significant (see for example Calmfors and Skedinger, 1995; Blundell, Costa Dias, Meghir, and Van Reenen, 2004; Boone and Van Ours, 2004, and the references therein).

A number of theoretical papers more systematically characterize how job search assistance programs influence the macroeconomic equilibrium. Calmfors and Lang (1995) study the implications on wages and employment when the program is targeted at the long-term versus the short-term unemployed. Distinguishing between high- and low-skilled workers, Van der Linden (2005) analyzes the effects of a program expansion among the low-skilled by step-wise endogenization of variables. Both studies stress that general equilibrium reactions substantially influence a program’s implications on aggregate employment. Repercussions can even be so severe that the employment rate falls in consequence of a job search assistance program. This possible outcome is also explicitly taken up
by Saint-Paul (1998), who identifies the conditions under which unskilled workers will vote for a labor market program that actually raises unemployment. In a nutshell, by characterizing the implications of job search assistance programs in a general equilibrium setting, these papers highlight the different channels through which workers' employment prospects are affected by these policies.

However, when it comes to judging the overall effects and desirability of a policy, the evaluation criterion that should ultimately be considered is social welfare rather than employment. The above studies provide only to a reduced extent information about this measure: Saint-Paul (1998) discusses utility effects for employed low-skilled workers, but does not capture the welfare consequences for other groups in the economy. Van der Linden (2005), on the other hand, shows a simulation of aggregate welfare as a function of program size, but does not, in his theoretical analysis, integrate the different effects to a social welfare measure. This issue is taken up in this paper.

The aim of this paper is to characterize the effects of job search assistance programs on social welfare and to perform a normative analysis of such a policy. We concentrate on the two most important characteristics: the size of a program, i.e. the number of jobseekers attending, and its intensity, which is a measure of the total services provided to each participant. The optimality criteria for both characteristics are derived in a general equilibrium framework where the labor market is affected by search frictions and wages are bargained between firms and workers. Workers are ex ante homogeneous, and assignment to the job search assistance program is undertaken by the government. We find that both instruments should optimally trade off their direct beneficial impact on program participants (or marginal participants in the case of program size) against the fiscal implications for taxpayers and ensuing distortions of employment and labor market tightness. The fiscal implications consist of two parts: first, program enlargement or intensification of course has direct costs, as more instruction and counselling have to be financed. Secondly, the direct employment enhancing effect on participants also changes the government budget. On the one hand, it widens the base of the labor income tax, thus providing more revenues. On the other hand, it cuts down to the same extent the number of individuals living on unemployment insurance compensation and thus saves benefit expenditures. This sum of taxes and benefits is generally denoted as the participation tax of the unemployment insurance system and can reach considerable levels, especially in European countries with traditionally generous welfare states (see Immervoll, Kleven, Kreiner, and Saez, 2007). Our analysis shows that at the optimal levels of program size and intensity, their net fiscal impact is negative, requiring a marginal increase in the
labor income tax. This not only reduces consumption possibilities of the taxpayers, but also distorts labor market participation. Depending on the relationship between worker’s bargaining power and the elasticity of the matching function with respect to jobseekers, the reaction of labor market tightness provoked by a tax rise might also be distortionary.

The design of the job search assistance program also responds importantly to the unemployment insurance system. As individuals are risk averse, it is optimal to pay out positive unemployment insurance benefits to those who are out of work, although the distortions generated by the need to finance said expenditures prevent full insurance. The generosity of unemployment compensation affects the optimal intensity of the job search assistance program in opposing ways: on the one hand, it raises the participation tax and inflates the fiscal savings of a more effective program, thus reducing its marginal cost. Secondly, higher benefits diminish job search incentives in general, and therefore also the gain in employment prospects that can be achieved by higher program intensity, leading to a reduction in the marginal benefit of an intensification. We show that if the Hosios condition is met, i.e. the wage bargaining power of workers is equal to the elasticity of the matching function with respect to jobseekers, the second effect is always stronger than the first: the optimal intensity of the job search assistance program is lower the higher unemployment insurance benefits. This result connects well with the analysis of Coe and Snower (1997), who show that active labor market policies are more effective at reducing unemployment when the unemployment insurance system is not too generous. Our analysis complements this result by showing that also from a welfare perspective, having a highly intensive job search assistance program is only optimal if unemployment compensation is low.

Our analysis also provides insights on the question whether a job search assistance program should be introduced in the first place. Program introduction is more likely to improve social welfare if participation significantly increases a worker’s effectiveness in job search activity and if the costs incurred by the first participant are not too high. Further, we show that if the general level of labor income taxation in the economy is high, implying that the fiscal savings in the form of the participation tax compensate the direct program cost, program introduction is also more beneficial. This finding might explain why active labor market policies are especially prevalent in countries with large social welfare systems and high levels of taxation, like Belgium, Denmark, Finland, Germany, the Netherlands or Sweden.

Our paper connects to several strands of the literature. A few recent papers study the optimal sequence of different active and passive labor market policies for the unemployed
(see Pavoni and Violante, 2007; Wunsch, 2007; Spinnewijn, 2008). However, by focusing on individual jobseekers only, these papers do not consider the feedback effects on other agents in the economy, which should be taken into account when designing potentially large programs. As discussed above, the general equilibrium studies of Calmfors and Lang (1995) and Van der Linden (2005) focus mainly on a positive analysis of program effects on employment and thus stop short of deducing normative implications. A notable exception, albeit focusing on a different measure of active labor market policy, is Fredriksson (1999), who studies the optimal number of participants in public employment programs.

The influence of the participation tax on both program introduction decisions and optimal program design also highlights the importance of interactions between different elements of active and passive labor market policies, which has already received some attention in the literature. For instance, in Keuschnigg and Ribi (2009) the participation tax of the unemployment insurance system is shown to play also a significant role in the determination of optimal wage subsidies (as a means for redistribution). Other studies emphasizing the interactions between active and passive policies include Van der Linden (2006), Cardullo and Van der Linden (2006), Coe and Snower (1997) in a general equilibrium context, and the individual-based contributions of Pavoni and Violante (2007), Wunsch (2007) and Spinnewijn (2008).

The paper proceeds as follows. Section 2 introduces the model, and Section 3 discusses the comparative static effects of changes in the government instruments. Section 4 then derives optimal program size and intensity, and Section 5 concludes. The Appendix provides some more technical calculations.

2 A Simple Model

The analytical framework is based on a one-period model of a labor market affected by frictions. This set-up provides a situation where individuals’ job search efforts are only partially successful and there might be a role for a policy that addresses this issue. To focus on the main mechanisms at work, the model is kept simple in other respects.

The economy contains a mass one of ex ante homogeneous workers. In the beginning, all individuals are unemployed and have to exert positive search effort to be able to find a job. Given their search effort, they can then with a certain probability secure a suitable job, paying a net wage \( w - t \). Otherwise, individuals end up unemployed and receive unemployment insurance benefits \( b \) from the state.

To enhance matching in the labor market and stimulate employment, the government
runs an active labor market program that provides job search assistance to the unemployed.\textsuperscript{1} The assignment of unemployed workers to the program is undertaken by the government, and for the designated individuals participation is mandatory. This is also very common in practice, where it is mostly at the discretion of the public employment service to place the unemployed into different programs. The share of program participants in the whole population, i.e. the size of the program, is denoted by $\phi$. For participants, the labor market program leads to an increase in their search effectiveness. Given a level of search effort $s_P$, their actual search effectiveness then amounts to $\delta s_P$ with $\delta > 1$.\textsuperscript{2} The factor $\delta$ is thus interpreted as a measure of the intensity of the program. The two defining characteristics $\phi$ and $\delta$ of the job search assistance program are both policy instruments of the government.

The number of suitable job matches $M$ that are formed in the economy depends on the number of vacancies $V$ set up by firms and on the effective number of jobseekers

$$S = \phi \cdot \delta s_P + (1 - \phi) \cdot s_N,$$

(1)

where effective search intensities of the two groups (participants and non-participants) are multiplied with the relative weight of the respective group in the population. In accordance with the literature, we assume the matching function to be increasing and linear homogeneous in the arguments $S$ and $V$, or specifically, $M(S, V) = m_0 S^\alpha V^{1-\alpha}$. In what follows, it will be convenient to use the concept of labor market tightness $\theta$, reflecting the ratio of vacancies to the effective number of jobseekers:

$$\theta \equiv \frac{V}{S}.$$

In this static model, the employment rate $e$ in the economy is given by the ratio of successful matches that are formed relative to initial jobseekers, who have mass one:

$$e = m_0 S \theta^{1-\alpha}.$$  

From the point of view of a single individual, the probability $p$ of finding a job depends on whether he has participated in the job search assistance program. If he has not taken part in the program, his search effectiveness $s_N$ implies a probability of finding a job of

$$p_N = \frac{s_N M}{S} = s_N m_0 \theta^{1-\alpha}.$$  

(2)

\textsuperscript{1}We assume that there are no privately run labor market programs in this economy, which is a common assumption in the theoretical literature. In reality, the policy discussion clearly centers on publicly funded programs, and many unemployed workers might also be cash constrained to attend programs for which they would have to pay themselves. Further, some measures of job search assistance programs also contain monitoring elements, and can therefore be provided only by the public authority.

\textsuperscript{2}The index $P$ stands for program participants, the index $N$ denotes non-participants.
Analogously, a person who has participated in the program has an effective search intensity of $\delta s_P$, leading to a job finding probability of 

$$p_P = \frac{\delta s_PM}{S} = \delta s_Pm_0\theta^{1-\alpha}. \quad (3)$$

Job search probabilities for the two groups and the aggregate employment rate thus increase in labor market tightness. Finally, a firm can fill a vacancy with probability $q = m_0\theta^{-\alpha}$, which is decreasing in market tightness.

### 2.1 Job Search Decision

Individuals determine their job search effort to maximize expected utility. Job search incurs effort cost $\varphi(s)$, which is an increasing and convex function of $s$. It is assumed that both groups of jobseekers have the same effort cost function. Individuals who have not participated in the labor market program know that they have a probability $p_N$ of finding a job. In this case, they earn a gross wage $w$, but have to pay a labor income tax of $t$. If they end up unemployed (with probability $1 - p_N$), they receive unemployment insurance benefits of $b$. Their indirect expected utility is thus

$$EU_N = \max_{s_N} p_Nu(w - t) + (1 - p_N)u(b) - \varphi(s_N), \quad (4)$$

where $u$ is a standard concave utility function. Optimal job search effort $s_N$ is determined by the condition

$$m_0\theta^{1-\alpha} \left[ u(w - t) - u(b) \right] = \varphi'(s_N). \quad (5)$$

The left-hand side shows the marginal benefit of increased search effort: as a higher search effort raises the probability of finding a job, see (2), it becomes more likely that the individual can move out of unemployment and thus realize the utility difference $u(w - t) - u(b)$. It is clear that to uphold positive search incentives, this difference must be positive. This will be ensured by wage bargaining. The right-hand side shows the marginal effort cost associated with higher search effort.

For a jobseeker who has participated in the job search assistance program, the probability of finding a job is given by $p_P$ in (3), and expected utility is

$$EU_P = \max_{s_P} p_Pu(w - t) + (1 - p_P)u(b) - \varphi(s_P). \quad (6)$$

We thus assume that the time spent in the program does not directly affect a person’s effort cost per unit of job search activity. This seems reasonable given that job search assistance does not require a very high time input by participants. Therefore a lock-in
effect cannot occur either in our model, which is often found to be important for more intensive program types like training (cf. Lechner, Miquel, and Wunsch, 2006). Optimal job search effort $s_P$ follows from the condition

$$\delta m_0 \theta^{1-\alpha} [u(w - t) - u(b)] = \varphi'(s_P).$$

(7)

Due to program participation, the marginal increase in the job-finding probability is $\delta m_0 \theta^{1-\alpha}$, which is higher than in the case of a non-participant if the program is effective ($\delta > 1$). The convexity of the search cost function then implies that program participants exert higher search effort than non-participants. Comparing (2) and (3), it follows that the probability of finding a job is higher for participants for two reasons: first, they exert higher search effort, and second, their search effort is more effective due to the multiplier $\delta$. We can also show that despite higher effort costs, participants end up with higher expected utility than non-participants, $EU_P > EU_N$ (see Appendix A1). The labor market program thus creates inequality between participants and non-participants.

2.2 Firms

All firms in the economy produce the same numeraire good. Each firm can only create one vacancy, which costs $k$ units of the numeraire. With probability $q$, it then finds a suitable worker to fill the post and produce $y$ units of output, and pays the worker a gross wage of $w$. If it fails to find a worker, its output is zero. A firm’s expected profits are therefore $E(\pi) = q(y - w) - k$. With free entry, firms enter the economy until expected profits are driven down to zero:

$$q(y - w) = k.$$

(8)

The wage is determined by Nash bargaining. Once a successful worker-firm match has been created, both actors know that they can share a rent. Breaking up the relationship would leave both with their outside option, which is zero for the firm and $u(b)$ for the worker as we have assumed one shot matching. With $\gamma \in (0, 1)$ denoting the worker’s bargaining power, the wage is determined by

$$w = \arg\max [u(w - t) - u(b)]^\gamma [y - w]^{1-\gamma},$$

or implicitly by the first order condition

$$\gamma u'(w - t)[y - w] = (1 - \gamma)[u(w - t) - u(b)].$$

(9)
2.3 Equilibrium

The labor market and the government’s budget constraint jointly determine the equilibrium in the economy. There are $V$ vacancies posted by all firms together, and workers’ search behavior implies an effective number of jobseekers $S$. Labor market equilibrium requires that both labor supply and labor demand are equal to the number of matches formed with the given vacancies and jobseekers:

$$e = M(S, V) = qV. \quad (10)$$

Aggregate employment is given by the total mass of program participants and non-participants that were able to secure a suitable job, and is thus a weighted sum of the respective job-finding probabilities: $e = \phi p_P + (1 - \phi) p_N$.

The government has two categories of expenditures: first, it pays out unemployment insurance benefits to the unemployed, which requires outlays of $(1 - e)b$. Second, it bears the cost of the job search assistance program. This cost is denoted by $G(\delta, \phi)$ and increases both with program intensity and program size, $G_\delta > 0$ and $G_\phi > 0$. The government’s sole source of revenues is the labor income tax, leading to income $et$. A balanced budget requires

$$(1 - e)b + G(\delta, \phi) = et. \quad (11)$$

The variables $b$, $\phi$ and $\delta$ are the government’s policy instruments. Via unemployment insurance benefits, it provides social insurance for those who are not successful on the job market. As the labor market is affected by frictions, investing in labor market programs increases employment probabilities of the share of participants $\phi$ by raising their search effectiveness via $\delta$.\(^3\)

3 Policy Changes and Employment Effects

In this section, we analyze how changes in the size and intensity of the job search assistance program affect the employment probabilities of the two groups of workers and aggregate employment in the economy. To isolate these effects, we first derive the general comparative statics of the model.

3.1 Comparative Statics

Starting out from an equilibrium in the economy, this section determines how changes in the government’s policy instruments $b$, $\phi$ and $\delta$ affect equilibrium values of the endogenous variables.

\(^3\)Market clearing for the numeraire good is shown in Appendix A2.
variables. Unless otherwise indicated, the hat notation designates changes in variables relative to their pre-change equilibrium values.

The gross wage $w$ is determined by the bargaining condition (9). In log-linearizing this equation, we apply the approximations

$$u_B \approx u_E - (w - t - b) u'_E,$$

where $\rho \equiv -cu''(c)/u'(c)$ is the coefficient of relative risk aversion of workers and $\chi \equiv \frac{w - t - b}{w - t}$ captures the relative income difference between the employed and the unemployed state.

Indexed utilities stand for consumption utility in the employed ($u_E \equiv u(w - t)$) and the unemployed ($u_B \equiv u(b)$) states. The change in the wage is then given by

$$\hat{w} = \omega \left( \hat{b} + \hat{t} \right), \quad \omega \equiv \frac{(1 - \gamma)(1 + \rho \chi)}{1 + (1 - \gamma)\rho \chi}, \quad 0 < \omega < 1,$$

(12)

where $\hat{b} \equiv db/w$ and $\hat{t} \equiv dt/w$. A rise in the unemployment benefit $b$ improves the outside option of workers. For a given wage level, this reduces the income difference between the two employment states. Via wage bargaining, a part of this reduction is shifted to firms, leading to a higher gross wage. Analogously, an increase in the tax $t$ reduces the net wage and is also partially shifted to firms. Log-linearizing the optimality condition for job search effort (5) (use again the approximations for $u_B$ and $u'_B$) yields the change in search effort of non-participants,

$$\hat{s}_N = \sigma (1 - \alpha) \hat{\theta} + \frac{\sigma}{1 - t^*} \left[ \hat{w} - \hat{t} - (1 + \rho \chi) \hat{b} \right],$$

(13)

with $\sigma \equiv \varphi'(s)/\left(\varphi''(s)s\right) > 0$ determining the magnitude of the response of search effort to a change in the marginal return to searching. The term $t^* \equiv \frac{t + b}{w}$ captures the participation tax. This consists of the total fiscal transfers a worker has to give up when moving from joblessness into employment, i.e. the unemployment insurance benefit he loses plus the tax he additionally has to pay when earning a wage. Equation (13) shows that as a higher labor market tightness and a greater income difference in the two employment states (expression in brackets) increase the return to job search, they stimulate the search effort of non-participants. For program participants, the change in search effort follows from differentiating (7):

$$\hat{s}_P = \sigma \hat{\delta} + \sigma (1 - \alpha) \hat{\theta} + \frac{\sigma}{1 - t^*} \left[ \hat{w} - \hat{t} - (1 + \rho \chi) \hat{b} \right].$$

(14)

In addition to the general equilibrium effects that also affect job search of non-participants, workers who attend the labor market program also raise their search effort in a direct reaction to an increase in program intensity $\delta$. As a higher $\delta$ makes a given level of job search more effective, thus translating into a higher employment probability, it raises the
return to searching and consequently stimulates this activity. The effective number of
jobseekers $S$, defined in (1), finally changes by

$$S\dot{S} = (\delta s_p - s_N) (1 - \phi) \dot{\phi} + \phi s_p \delta \dot{\phi} + \phi \dot{s_p} \dot{s_p} + (1 - \phi) s_N \dot{s_N}, \quad (15)$$

where the relative change in program size is defined as $\dot{\phi} = d\phi/(1 - \phi)$. Increases in
program size and intensity directly raise $S$ as they expand the number of workers who
can benefit from the program and make search effort more effective, respectively. Indirect
effects come about because of the changes in search efforts within the two groups, as
indicated in (13) and (14).

The number of firms in the economy and thus, for a given $S$, also labor market tightness
are determined by the zero profit condition (8). Using the matching function to express
the probability of filling a vacancy, $q = m_0 \theta^{-\alpha}$, implies

$$\hat{\theta} = -\frac{w \hat{\theta} \alpha}{\alpha(y - w)}. \quad (16)$$

A higher gross wage reduces the firms’ rent of a successful job match. To rebalance the
zero profit condition, the probability of filling a vacancy must therefore rise, implying a
reduction in labor market tightness.

By the definition of the matching function, an equilibrium on the labor market is
ensured, and changes in employment, $\dot{e} = \dot{S} + (1 - \alpha) \dot{\theta}$, equate changes in labor demand,
$\dot{q} + \dot{V}$. Last, the tax rate $t$ is endogenously determined to balance the government budget
constraint (11), and differentiating yields

$$\dot{t} = \frac{1 - e}{e} \dot{b} - t^* \dot{e} + \frac{G \delta}{ew} \dot{\delta} + \frac{G \delta (1 - \phi)}{ew} \dot{\phi}. \quad (17)$$

For a given unemployment rate, higher benefit payments $b$ raise expenditures, and must
be financed by higher taxes on labor income. Similarly, when increased size or intensity
make the labor market program more costly, this must also be covered by higher taxes.
A higher employment rate, on the other hand, reduces the number of benefit recipients
and, at the same time, increases the number of taxpayers. Thus, for each additionally
employed, revenues in proportion to the participation tax $t^*$ are added to the state’s
budget, allowing for a corresponding reduction in the labor income tax. Inserting for the
change in employment, and using equations (12)-(16) lets us write the change in the tax
as a function of changes in the policy parameters only:

\[
\hat{t} = \frac{\kappa}{\Psi} \hat{b} + \frac{\xi}{\Psi} (1 - \phi) \hat{\phi} + \frac{\lambda}{\Psi} \hat{\delta}, \\
\kappa \equiv \frac{1 - e}{e} + t^* \psi + \frac{t^* \sigma \rho \chi}{1 - t^*}, \\
\xi \equiv \frac{G_{\phi}}{e w} - t^* \frac{\delta s_P - s_N}{S}, \\
\lambda \equiv \frac{G_{\delta}}{e w} - t^* (1 + \sigma) \frac{\phi s_P}{S}, \\
\Psi \equiv 1 - t^* \psi, \\
\psi \equiv (\sigma + 1) \frac{(1 - \alpha) w \omega}{\alpha (y - w)} + \frac{\sigma (1 - \omega)}{1 - t^*} > 0.
\]

For stability reasons, it is required that \(\Psi > 0\). This term captures the behavioral responses of jobseekers and firms to an increase in the tax that lead to a reduction in employment. The ensuing erosion of the tax base implies that the tax must be raised by a greater amount to generate a certain level of revenues than would be required in the absence of any endogenous behavioral response.

An increase in the unemployment insurance benefit \(b\) has an unambiguously positive effect on the tax \(t\). In addition to the direct effect that higher expenditures per unemployed require more financing already identified in \((17)\), a higher \(b\) raises the gross wage but reduces the net wage and labor market tightness, which dampens search efforts of both groups of jobseekers. The ensuing reduction in employment then raises the fiscal burden for the remaining workers.

An increase in the size \(\phi\) of the job search assistance program has two counteracting effects on the tax: on the one hand, program expansion has a direct marginal cost \(G_{\phi} > 0\), which must be covered by higher taxes. On the other hand, as more workers benefit from higher search effectiveness, substituting \(\delta s_P\) for \(s_N\) in their probability to find a job, this has a direct positive impact on the employment rate. As discussed above, this leads to fiscal savings in proportion to the participation tax \(t^*\), which implies the labor income tax can be reduced. The total effect of a change in \(\phi\) on \(t\) is ambiguous.

A rise in program intensity \(\delta\) has analogous effects on the tax as a change in \(\phi\). The increase in program costs \(G_{\delta} > 0\) puts an additional burden on the public budget. A more intensive program, however, raises search effectiveness of participants both directly and indirectly by stimulating search effort. As a result, program participants face a higher probability of finding a suitable job, which boosts overall employment. This has again the positive implications for the fiscal budget discussed above. The aggregate effect of an increase in \(\delta\) on the public finances and thus on the tax rate that must balance the budget is again ambiguous.
3.2 Employment Effects of Changes in Size and Intensity

Having fully determined the comparative statics of the model, we can now isolate the effects of changes in program size and intensity on the employment probabilities for the different groups and aggregate employment. This lets us relate our results more clearly to existing studies of macroeconomic effects of job search assistance programs, in particular to Van der Linden (2005). In this section, we keep the level of unemployment insurance benefits constant.

The program participants’ probability \( p_P \) of finding suitable employment is defined in (3) and changes according to \( \dot{p}_P = \dot{\delta} + \dot{s}_P + (1 - \alpha)\dot{\theta} \). Inserting from (14), (16), (12) and (18) yields

\[
\dot{p}_P = (\sigma + 1) \hat{\delta} - \psi \lambda \frac{\delta \hat{\delta}}{\Psi} - \psi \xi (1 - \phi) \frac{\hat{\phi}}{\Psi}.
\]  

A higher program intensity directly improves employment prospects as it raises the effectiveness of job search, which also increases the returns to searching and thus stimulates this activity. On the other hand, a change in intensity also has fiscal implications as discussed in (18). When a rise in \( \delta \) requires a higher tax rate, this reduces the net wage and labor market tightness, leading to a fall in the return to job search and thus in the effort put to this activity. A fall in labor market tightness also reduces the probability of being matched to a suitable firm. In aggregate, if \( \lambda < 0 \), the indirect effects following from an increase in program intensity reduce the employment probability of program participants, thus counteracting the direct positive effects.

In the case of program size \( \phi \), search effort of participants only changes in response to the equilibrium feedback of the implied change in the tax on the consumption utility differential and labor market tightness. This is complemented by the direct effect of labor market tightness on the employment probability. If program enlargement leads to higher taxes, the effect on the job-finding probability of those who already participate in the program is unambiguously negative.

For workers not assigned to participate in the job search assistance program, there are no direct effects of changes in \( \delta \) and \( \phi \) on either search effort or the probability to find a job. The respective equilibrium adjustments follow from inserting (13), (16), (12) and (18) into \( \dot{p}_N = \dot{s}_N + (1 - \alpha)\dot{\theta} \) and are given by

\[
\dot{p}_N = -\psi \lambda \frac{\delta \hat{\delta}}{\Psi} - \psi \xi (1 - \phi) \frac{\hat{\phi}}{\Psi}.
\]  

They are thus the same as the equilibrium feedback effects for program participants. Finally, we can derive the impact on aggregate employment by differentiating \( e = \phi p_P + \)
(1 − φ)p_N and substituting from (19) and (20):
\[
\hat{e} = (1 + \sigma) \frac{\phi p_P}{e} \delta + \frac{p_p - p_N}{e} (1 - \phi) \phi - \psi \xi (1 - \phi) \phi.
\]

On the one hand, an increase in program intensity has a positive direct effect on employment within the group of participants. They benefit from higher effectiveness of their job search effort, and raise their effort in response. Both effects translate into a higher employment probability for this group. On the other hand, the fiscal consequences of a rise in δ and their implications for the equilibrium wage and labor market tightness affect all individuals in the same way, leading to an employment change that is proportional to λ. Aggregate employment effects might thus be positive as long as the fiscal consequences do not require too high an increase in the tax rate.

When the number of program participants is raised, this also has a positive direct effect on employment. As we have seen in Section 2.1, individuals who have attended the program always have a higher probability of finding employment than non-participants. Thus, increasing the share of the population entering the program directly increases the employment rate by the respective differential. The effects on the public budget and thus on the tax rate again lead to general equilibrium adjustments of the search efforts and market tightness, which affect all workers in the same way. Depending on whether the fiscal gains of program expansion exceed the fiscal costs or not, the indirect equilibrium effects might be positive or negative. When the necessary increase in the tax rate turns out to be too high, aggregate employment might even be reduced when more jobseekers enter the program.

These results confirm the findings of Van der Linden (2005), who shows in simulations how the positive direct employment effects of an expansion of a job search assistance programs can be more than compensated when all general equilibrium implications are considered. Our discussion provides further insights into the theoretical conditions that must be satisfied for such an outcome to occur, as it relates the change in employment fully to the fundamental changes in program characteristics φ and δ.

4 Optimal Program Size and Intensity

Having seen how changes in the size and the intensity of a job search assistance program affect employment probabilities of both participants and non-participants, we now analyze how these instruments should be set optimally. Social welfare W is defined as aggregate welfare of all individuals, \( W = \phi EU_P + (1 - \phi) EU_N \), and, because population
size is normalized to one, corresponds to the expected utility of a person before program assignment has taken place. Differentiation shows that social welfare is, on the one hand, affected by changes in the expected utility of the different groups of workers, and, on the other hand, by a changing composition of program participants versus non-participants in the population:

\[
dW = \phi dEU_P + (1 - \phi)dEU_N + [EU_P - EU_N](1 - \phi)\hat{\phi}. \tag{22}
\]

In order to derive social welfare effects and the optimality criteria for the program characteristics, it is first necessary to analyze how these instruments affect expected utility of the different groups, which we do in the next subsection. To be able to discuss also the effects of passive labor market policy on welfare, we again allow for changes in unemployment insurance benefits \(b\). In Subsections 4.2 and 4.3, we then turn to the determination of optimal program size and intensity, respectively.

### 4.1 Welfare Effects of Changes in Policy Instruments

In Appendix A3 we show that the change in program participants’ expected utility \((6)\) can be written as

\[
\frac{dEU_P}{u'_E} = p_P(w - t - b)\hat{\delta} + \Gamma p_P(1 - \alpha)\hat{\theta} + (1 - p_P)w(1 + \rho\chi)\hat{b} - p_Pw\hat{t},
\]

\[
\Gamma \equiv \frac{(y - w)(\gamma - \alpha)}{(1 - \gamma)(1 - \alpha)}. \tag{23}
\]

The division by marginal utility \(u'_E\) implies changes in income equivalent units. The first term on the right captures the direct impact of a higher employment probability due to a more intensive program. As participants become more likely to find a job and realize the income difference between the two employment states, their expected utility increases. The second term relates to efficiency effects of a change in labor market tightness. When workers’ bargaining power \(\gamma\) is high relative to the elasticity of the matching function with respect to jobseekers \(S\), \(\gamma > \alpha\) in \(\Gamma\), the bargained gross wage is too high from an efficiency perspective. Consequently, too few firms enter the economy, resulting in inefficiently high unemployment. Because a tighter labor market raises employment, an increase in \(\theta\) then improves efficiency in the model. As already shown by Hosios (1990), when \(\gamma = \alpha\), bargaining is efficient and a change in \(\theta\) has no direct implications \((\Gamma = 0)\) on expected utility. The third term shows the impact of a change in unemployment insurance benefits on participants’ consumption utility. Due to risk aversion, higher benefits imply a higher than one-to-one gain in income equivalent units. Similarly, when the tax rate
increases, utility in the employed state is correspondingly reduced, as captured in the last term in (23).

Analogously, we can derive the change in non-participants’ expected utility as

\[
\frac{dE U_N}{u_E} = \Gamma p_N (1 - \alpha) \hat{\theta} + (1 - p_N) w (1 + \rho \chi) \hat{b} - p_N w \hat{t}.
\]  

Utility responds in the same manner to changes in \( \theta, t \) and \( b \) as in the case of program participants. The only difference to equation (23) is that program intensity does not directly affect workers’ employment prospects, and thus expected utility, here. Dividing equation (22) by marginal utility \( u_E' \) in the employed state and inserting the results from (23) and (24), using the equality \( \phi p_P + (1 - \phi) p_N = e \) and finally substituting for \( \hat{t} \) from (17) yields the change in social welfare

\[
\frac{dW}{u_E} = \phi p_P (w - t - b) \hat{\delta} + \frac{E U_P - E U_N}{u_E} (1 - \phi) \hat{\delta} - G \phi \hat{\delta} \\
- G \phi (1 - \phi) \hat{\phi} + (1 - e) w \rho \chi \hat{b} + \Gamma e (1 - \alpha) \hat{\theta} + e w t^* \hat{e}.
\]  

In this exposition, we see the direct effects of changes in the policy instruments and their implied impacts on efficiency. Higher program intensity stimulates employment probabilities of participants (as in (23)), and a larger program size lets more individuals attain expected utility \( E U_P \) instead of \( E U_N \). We know from the discussion in Subsection 2.1 that this difference is positive. However, program costs \( G \) rise in both program characteristics, which reduces resources available for individuals. The second term on the second line of (25) shows the gains from insurance that arise when the unemployed receive a higher transfer. This gain increases with the risk aversion parameter \( \rho \) and the income difference in the two employment states, as captured in \( \chi \). The next expression corresponds again to the potential inefficiency of wage bargaining and the ensuing implications that arise from a change in market tightness, as discussed below equation (23). The last term in (25) reflects the excess burden of the welfare state. From the workers’ point of view, the participation tax \( t^* \) constitutes the fiscal cost of the transition from unemployment to employment and thus negatively affects employment decisions. An increase in employment \( e \) reduces this excess burden and raises social welfare.

### 4.2 Program Introduction and Optimal Program Size

Using (25), social welfare changes with the size of the job search assistance program according to (remember that \( d \phi = (1 - \phi) \hat{d} \phi \))

\[
\frac{dW}{u_E \cdot d\phi} = \frac{E U_P - E U_N}{u_E} - G \phi + \Gamma e (1 - \alpha) \frac{\hat{\theta}}{d \phi} + e w t^* \frac{\hat{e}}{d \phi}.
\]
Inserting for the effect on the employment rate from (21) shows that the direct fiscal implications of program enlargement stem from increased revenues in the form of the participation tax from those workers who are additionally employed because of their program attendance, minus the direct marginal program costs. Noting that \( \frac{\delta s_P - s_N}{S} \), these two effects can be summarized again by using \( \xi \): \( ew + ewt^* \frac{\psi}{\Psi} = \frac{ew}{\Psi} \) and using (16) and (18) finally yields

\[
\frac{dW}{u_E} \cdot d\phi = \frac{EU_P - EU_N}{u_E} - \frac{ew}{\Psi} \frac{\xi}{1 - \alpha} - \frac{\Gamma e}{\alpha(y - w)} \frac{(1 - \alpha)w\omega}{\Psi} \xi.
\]

Apart from the fact that a higher number of participants means that more workers can enjoy expected utility \( EU_P \) instead of \( EU_N \), all direct and indirect implications of an increase in \( \phi \) are proportional to the fiscal net effect \( \xi \). The second term on the right contains the total employment effects, net of the marginal program costs, and is a negative function of \( \xi \). The third term captures again the efficiency effect due to the change in labor market tightness, and is in negative proportion to \( \xi \) for \( \gamma > \alpha \) and in positive proportion for \( \gamma < \alpha \). However, inserting for \( \Gamma \) and \( \omega \) shows that the second and third terms taken together are always a negative multiple of \( \xi \).

Assuming that \( W \) is a concave function of program size \( \phi \), a necessary prerequisite for the desirability of introducing a job search assistance program is that the derivative of social welfare with respect to \( \phi \) is positive at \( \phi = 0 \). Several factors make this case more likely. First, if the program is highly effective, making the participants’ chances to find employment significantly higher than nonparticipants’, the difference in expected utilities \( EU_P - EU_N \) and in effective search intensities \( \delta s_P - s_N \) is large. This improves the welfare effects of a program introduction. Secondly, small marginal program costs \( G_{\phi} \) for the first participants also help to justify its implementation.

The third factor is the general level of taxation, which reflects the generosity of the welfare state and the size of other government expenditures. If this level is high it also implies a large participation tax, and a rise in employment due to the direct effect of the labor market program on participants thus generates large fiscal savings. However, these positive welfare implications are counteracted by the negative impact of high taxes on search efforts for both groups of jobseekers. As the job search incentives of participants are reduced to a greater extent, the relative difference in employment prospects and in expected utility shrinks. In Appendix A4, we show for the case of \( \gamma = \alpha \), i.e. when the Hosios condition is met, that the first effect dominates when tax levels are so high that \( \xi < 0 \). Thus, the welfare effects of program introduction are more beneficial if the economy has a high general level of taxation.
This finding can to some extent explain why countries with high levels of labor income taxation like Belgium, Denmark, Finland, Germany, the Netherlands or Sweden run a large number of active labor market programs at the same time (cf. OECD, 2006). The initial fiscal gains that can be generated by employment enhancing policies are then so large that they can justify the additional outlays for these activation measures.

It is also conceivable that all workers in the economy optimally participate in the program. Formally, this requires the derivative of social welfare in (26) to be positive at the maximal size $\phi = 1$. In this case, the effectiveness of the program relative to the marginal costs of program expansion would have to be high even when already many workers attend the program.

If the derivative of social welfare in (26) is positive at $\phi = 0$ and negative at $\phi = 1$, the optimal size of the job search assistance program is determined by the condition

$$\frac{EU_P - EU_N}{w^*_E} = \left( ew + \Gamma e \frac{(1 - \alpha)w\omega}{\alpha(y - w)} \right) \frac{\xi}{\Psi}.$$ 

As the term on the left-hand side is positive (see Subsection 2.1), and inserting for $\Gamma$ shows that the sum in brackets is also greater than zero, the net tax effect $\xi$ of a higher program size must be positive as well. The marginal program costs should thus be higher than expected public savings from the direct increase in the employment probability of the marginal attendant. From a distributional perspective, the search assistance program creates inequality between the groups of participants and non-participants, and the gain in expected utility that the marginal participant can obtain compensates for the consequences of a marginal increase in the required tax level and the ensuing distortionary effects. These include a distortion in the employment decision which is implicit in the wage bargaining process and is proportional to the participation tax $t^*$, and the distortion of labor market tightness if $\Gamma > 0$, i.e. $\gamma > \alpha$. However, if the bargaining power of workers is comparatively low, $\gamma < \alpha \Rightarrow \Gamma < 0$, employment is inefficiently high and its tax-induced reduction even increases efficiency.

This optimality condition can be compared to the recommendations for program assignment made in OECD (2005, Chapter 5). It is argued there that programs should be chosen according to the fiscal savings they generate in the form of the participation tax, which should exceed their costs. Our optimality condition for program size makes clear that when placement officers decide on assigning a jobseeker to a particular program that is already in place, they should make sure to consider the marginal program costs that follow from an additional participant. Depending on the size of the fixed costs of a program, these can be higher or lower than the average costs, which are for instance
normally reported in cost-benefit analyses (cf. Dolton and O’Neill, 2002; Van Reenen, 2004). Further, this decision rule ignores that the job search assistance program has direct benefits for its participants, which improves the expected utility of an individual before program assignment has taken place. In fact, the rule maximizes expected utility of non-participants (see equation (24)). From the point of view of a person who does not yet know if he will be assigned to participate in the job search assistance program, this would lead to a too small program size.

4.3 Optimal Program Intensity

Now turning to the analysis of optimal program intensity, the derivative of social welfare with respect to $\delta$ follows from inserting the changes in the employment rate (21) (note that $p_P/e = \delta s_P/S$ and in market tightness (following from (16), (12) and (18)) into (25) and summarizing as above:

$$\frac{dW}{d\delta} = \phi s_P m_0 \theta^{1-\alpha} (w - t - b) - e w \lambda \frac{\psi}{\Psi} - \Gamma e \frac{(1 - \alpha) w \omega}{\alpha (y - w)} \frac{\lambda}{\Psi}.$$  \hspace{1cm} (27)

Analogous to the case of program size, the second term on the right captures both the direct fiscal consequences of a change in program intensity for taxpayers and the general equilibrium implications that affect employment. Both effects are proportional to $\lambda = \frac{G_s}{e w} - t^* (1 + \sigma) \frac{m_0}{S}$, as is the impact on efficiency due to a change in labor market tightness (third term). Inserting for $\Gamma$ and $\omega$ shows that the two terms taken together are a negative multiple of $\lambda$.

A more intensive labor market program has always a positive direct impact on the probability to find a suitable occupation for its participants. They are thus more likely to gain the consumption utility differential between the two employment states, which is approximated by the income difference $w - t - b > 0$. However, in spite of this positive direct effect, very high marginal program costs $G_\delta$ that lead to $\frac{dW}{d\delta} < 0$ will prevent the implementation of an effective job search assistance program, given that social welfare is concave in $\delta$. It is then optimal to set both program size and intensity to zero.

In contrast, if program costs are rather small initially, the optimal program intensity is determined by the condition

$$\phi s_P m_0 \theta^{1-\alpha} (w - t - b) = \left( e w + \Gamma e \frac{(1 - \alpha) w \omega}{\alpha (y - w)} \right) \frac{\lambda}{\Psi}.$$  \hspace{1cm} (28)

As both the left-hand side and the term in brackets are positive, optimality requires that the net fiscal effect $\lambda$ is also positive. The marginal gains of a more intensive program, consisting of the direct increase in employment prospects and, consequently, expected
utility of participants, are then opposed by the marginal costs in the form of an increase in the labor income tax. This not only reduces the disposable income of taxpayers, but also distorts employment and, if $\Gamma > 0$, labor market tightness.

Comparing the effects of program size and intensity on social welfare in (26) and (27) makes clear that both characteristics affect social welfare through the same equilibrium channels. Thus, in the event that both instruments optimally take interior values, they must jointly satisfy the simple condition

$$\frac{EU_P - EU_N}{\mu_P - \phi_s P m_0 \theta^{1-a}(w - t - b)} = \frac{\xi}{\lambda}.$$ 

The left-hand side shows the ratio of the direct marginal effects of an increase in program size and in program intensity, while the right-hand side shows the ratio of the corresponding direct marginal effects on the tax rate. Thus, if the gain in expected utility of program participants due to an intensification of the program is higher than the gain in expected utility for the marginal participant if the program is expanded, it is also optimal to accept a greater rise in the required tax on labor income in the case of program intensification.

It is also obvious from condition (28) that optimal program intensity depends on the generosity of the unemployment insurance system. As individuals are risk averse and the unemployed have no other source of income than insurance benefits, it is optimal to pay out a positive benefit $b$ in this model.⁴ In the interaction of insurance benefits and optimal program intensity, there are mainly two effects going on. On the one hand, a more generous unemployment insurance system directly reduces the difference in consumption utility between employment and unemployment. Consequently, jobseekers curb their search effort, which reduces the marginal increase in the probability of finding employment that occurs with a rise in program intensity. Both effects imply a reduction in the marginal benefit of a program intensification.

On the other hand, also the participation tax rises with the unemployment compensation. As a higher program intensity then directly raises the number of participants who end up employed instead of out of work, the corresponding fiscal savings turn out to be higher. This effect reduces the marginal cost of a more intensive job search assistance program. By differentiating condition (28) in Appendix A5, we show that if the Hosios

⁴Inserting (12)-(16) into (25) shows that optimal unemployment insurance benefits are defined by

$$(1 - e) \rho_X = et^* \left( \frac{\sigma \rho_X}{1 - \tau^*} + \psi \left( 1 + \frac{\kappa}{\Psi} \right) \right) + \Gamma e (1 - a) \omega \left( 1 + \frac{\kappa}{\Psi} \right).$$ 

The gains from insurance on the left-hand side are opposed by the efficiency costs on the right, which include the distortionary effects of unemployment insurance on job search efforts and employment, and potentially on labor market tightness. In general, neither no nor full insurance are optimal in the model.
condition is fulfilled, i.e. \( \gamma = \alpha \) and therefore \( \Gamma = 0 \), the reduction in marginal benefits dominates the reduction in marginal costs. The optimal program intensity is thus lower the more generous the unemployment insurance system.

This result connects well to the results of Coe and Snower (1997) on policy complementarity. They find that active labor market policies are more effective at reducing unemployment if insurance benefits are reduced at the same time. We complement this insight by showing that also from a welfare perspective, it is better to have a highly intensive job search assistance program only if the unemployment insurance system is not too generous.

5 Conclusion

Job search assistance programs aim at improving the job search skills of the unemployed and are generally found to be among the most effective active labor market policies for a broad range of participants. Being also relatively inexpensive compared to other activation measures, in many countries a large share of insured jobseekers are assigned to attend these programs. It follows from this that in addition to the direct implications of programs on their participants, their macroeconomic effects must also be expected to be significant, and it is all the more important to design these programs in a way that is beneficial for social welfare.

This paper thus develops the optimal rules for determining the two most important characteristics of such a program, i.e. its size and intensity. It is found that both characteristics have positive direct effects on their participants (or the marginal participants in the case of program size). They follow from the direct stimulation of attendants’ employment probability, which is generally the focus of the microeconometric evaluation literature. These positive direct effects are traded off against the program’s implications for the labor income tax, which consist of a positive component in the form of direct program costs and a negative component in the form of an enlarged tax base due to the direct employment stimulation of the policy. We find that the net tax effects of both instruments should be positive at the margin. This then provokes also a distortion in employment and might further remove labor market tightness from its efficient level.

The generosity of the unemployment insurance system also importantly influences the optimal design of the job search assistance program. High benefits generally reduce job search efforts, but imply higher fiscal savings when the program manages to improve the employment prospects of its participants. We show that the first effect dominates in the
determination of optimal program intensity: the higher the unemployment compensation, the less intensive should the job search assistance program be.

In addition, we find that the implementation of a job search assistance program can enhance social welfare only if it sufficiently raises the job finding rates of participants and is not too costly already for small numbers of participants. Further, if the general level of labor income taxes is high, for instance due to a large welfare state, a program is also more likely to improve welfare. The fiscal gains from the participation tax paid by the additionally employed are then higher and can compensate for the dilution of search incentives.
Appendix

A1 Proof of $EU_P > EU_N$ for $\delta > 1$

Here, we show that $EU_P > EU_N$ for $\delta > 1$. Using the optimality conditions (5) and (7) for job search efforts, the difference between indirect expected utilities of program participants (4) and of non-participants (6) can be written as (remember that $p_P = \delta s_P m_0 \theta^{1-\alpha}$ and $p_N = s_N m_0 \theta^{1-\alpha}$):

$$EU_P - EU_N = p_P (u(w - t) - u(b)) - \varphi(s_P) - [p_N (u(w - t) - u(b)) - \varphi(s_N)]$$

Further, we know that $s_P > s_N$ for $\delta > 1$. It is therefore sufficient to show that the function $\mu(s) = s\phi'(s) - \varphi(s)$ is monotonically increasing. The derivative of this function is $\mu'(s) = s\phi''(s)$. As we have assumed that the search cost function is strictly convex, $\varphi''(s) > 0$, it follows that $\mu'(s) > 0$.

A2 Market Clearing

Walras’ Law implies that the market for the numeraire good must clear when all budget constraints are fulfilled and the labor market is in equilibrium, $e = qV$. Individuals spend all disposable income on the numeraire good, leading to private consumption $C \equiv e(w - t) + (1 - e)b$. Using (11) to eliminate the tax rate and the free entry condition (8) yields the GDP identity

$$qyV = C + G + Vk.$$

Total production $qyV$ of the numeraire good is thus used for private consumption $C$, public investment $G$ in the labor market program, and for capital input $Vk$ to create vacancies.

A3 Derivation of equation (23)

Differentiating equation (6) and applying the optimality condition for job search (7) yields

$$dEU_P = p_P [u_E - u_B] (\hat{p}_P - \hat{s}_P) + p_P w u'_E (\hat{w} - \hat{t}) + (1 - p_P) w u'_B \hat{b}.$$

Using the approximations for $u_B$ and $u'_B$ stated above, dividing by $u'_E$ and substituting $\hat{p}_P = \hat{\delta} + \hat{s}_P + (1 - \alpha)\hat{\theta}$ leads to

$$\frac{dEU_P}{u'_E} = p_P (w - t - b) \left( \hat{\delta} + (1 - \alpha)\hat{\theta} \right) + p_P w (\hat{w} - \hat{t}) + (1 - p_P) w (1 + \rho \chi) \hat{b}$$
Finally, inserting from (16), substituting from the wage bargaining condition (9) and rearranging yields equation (23). Derivation of equation (24) starts out from equations (4) and (5) and then follows exactly the same steps.

A4 Program introduction

This section shows that program introduction is more likely to be welfare improving if the overall level of labor income taxes and thus also the participation tax is high. For simplicity, we assume that $\gamma = \alpha$, implying $\Gamma = 0$. We consider an increase in $t$, assuming that the rise in tax revenues is neither spent on unemployment insurance nor on JSA program (equation (17) therefore does not hold). From (26), we derive

$$d^2W = \frac{dEU_P}{dt} - \frac{dEU_N}{dt} - \frac{u'_E d(ew\xi)}{\Psi dt} - \frac{du'_E ew\xi}{\Psi dt} + \frac{u'_E ew\xi d\Psi}{\Psi^2 dt}. \quad (A.1)$$

From (23) and (24) it follows that $\frac{dEU_P}{dt} - \frac{dEU_N}{dt} = -u'_E(p_P - p_N)$. The third term on the right simplifies to $\frac{d(ew\xi)}{dt} = \frac{d(wt^*)}{dt}(p_P - p_N) - wt^* \frac{dP^P - dP^N}{dt}$, where we have made use of the identity $\hat{S}_P - \hat{S}_N = \frac{p_P - p_N}{e}$. Use $\frac{d(wt^*)}{dt} = 1$ and $\frac{dP^P - dP^N}{dt} = -\frac{\psi}{w}(p_P - p_N)$, which follows from inserting $\hat{P} = \hat{\delta} + \hat{s}_P + (1 - \alpha)\hat{\theta}$ and $\hat{P} = \hat{s}_N + (1 - \alpha)\hat{\theta}$, applying (13), (14), (16) and (12) and using the definition of $\psi$ in (18). We therefore have $\frac{u'_E d(ew\xi)}{\Psi dt} = -u'_E(p_P - p_N)$, where we have used $\Psi = 1 - t^* \psi$. The first three terms on the right of (A.1) thus just cancel.

In the fourth term, $\frac{du'_E}{dt} = -u''_E(1 - \omega)$, which is positive as the utility function $u$ is concave. To derive $\frac{d\Psi}{dt}$, we use $\frac{dt^*}{dt} = \frac{1}{w}(1 - t^* \omega)$ and $\frac{dw}{dt} = \omega$ to get in a first step

$$\frac{d\Psi}{dt} = -\left(\frac{\sigma (1 - \omega)}{(1 - t^*)^2} + (1 + \sigma) \frac{1 - \alpha \omega w \alpha(1 - \omega)}{\alpha(y - w)}\right) 1 - t^* \frac{\omega}{w} - t^* (1 + \sigma) \frac{1 - \alpha \omega^2}{\alpha(y - w)} 1 + \frac{w}{y - w}$$

$$+ \left(\frac{\sigma t^*}{1 - t^*} - t^* (1 + \sigma) \frac{1 - \alpha \omega}{\alpha(y - w)}\right) \frac{dw}{dt}, \quad (A.2)$$

where $\frac{dw}{dt} = -\frac{(1 - \gamma)\rho \chi(1 - \omega)^2}{1 + (1 - \gamma)\rho \chi} \frac{1 - \chi}{w(1 - t^*)}$. Using the approximation $u_B \approx u_E - (w - t - b)u'_E$ in the wage bargaining condition (9) and the assumption $\gamma = \alpha$ lets us summarize the terms in brackets on the second line by $-\frac{t^*}{1 - t^*}$. Thus, the first two terms in (A.2) are negative, while the third is positive. To derive the sign of the overall expression, rearrange (A.2) to summarize all terms that are not multiplied by $\sigma$, while again using the approximation of (9) and the definition of $\omega$ in (12):

$$\frac{d\Psi}{dt} = -\left(\frac{\sigma (1 - \omega)}{(1 - t^*)^2} + (1 + \sigma) \frac{1 - \alpha \omega w \alpha(1 - \omega)}{\alpha(y - w)}\right) 1 - t^* \frac{\omega}{w} - t^* \sigma \frac{1 - \alpha \omega^2}{\alpha(y - w)} 1 + \frac{w}{y - w}$$

$$- \frac{\omega}{w(1 - t^*)} \left(1 + \frac{(1 - \omega) t^*}{1 - t^*} \left(1 + \rho \chi - \frac{\rho \chi}{1 + \rho \chi} (1 - \chi)(1 - \omega)\right)\right) < 0.$$
Clearly, as \(0 < \omega, \chi < 1\), the third term is also negative and the whole derivative is smaller than zero. Consequently, the sign of \(\frac{d^2W}{d\delta dt}\) is solely determined by the sign of \(\xi\):

\[
\text{sign} \left( \frac{d^2W}{d\delta dt} \right) = -\text{sign}(\xi).
\]

In situations of program introduction when we have a high participation tax, leading to \(\xi < 0\), the effect on welfare is thus more beneficial than when the participation tax is low.

**A5 Dependence of optimal \(\delta\) on \(b\)**

This section shows that optimal program intensity \(\delta\) decreases in \(b\) when \(\gamma = \alpha\), implying \(\Gamma = 0\). Note that in the optimality condition (27), we used the approximation \(u_E - u_B \approx u'_E(w - t - b)\). Resubstituting leads to the condition

\[
\Omega(\delta, b) = \phi spm_0 \theta^{1-\alpha}(u(w - t) - u(b)) - u'_E \frac{ew\lambda}{\Psi} = 0. \tag{A.3}
\]

The optimal \(\delta\) implied by (A.3) locally depends on \(b\) in the following way: \(\frac{d\delta}{db} = -\frac{d\Omega}{db} \frac{1}{d\delta/\frac{d\Omega}{db}}\). \(\delta\), being chosen to maximize social welfare \(W\) implies that \(d\Omega/db < 0\), thus the sign of \(d\delta/db\) only depends on

\[
\frac{d\Omega}{db} = \frac{d}{db} \left( \phi spm_0 \theta^{1-\alpha}(u(w - t) - u(b)) \right) - \frac{du'_E}{db} \frac{ew\lambda}{\Psi} - \frac{u'_E}{db} \frac{d(ew\lambda)}{db} + u'_E \frac{ew\lambda}{\Psi^2} \frac{d\Psi}{db}. \tag{A.4}
\]

In deriving the first term on the right in (A.4), use equations (14), (16), (12), \(\frac{dt}{db} = \frac{\kappa}{\Psi}\) and the approximation of \(u'_B \approx (1 + \rho\chi)u'_E\) to get

\[
\frac{d}{db} \phi spm_0 \theta^{1-\alpha}(u(w - t) - u(b)) = -u'_E \frac{\sigma + 1}{w(1 - t^*)} \left( 1 + \frac{\kappa}{\Psi} + \rho\chi \right) \frac{ew\lambda}{\Psi}, \tag{A.5}
\]

where we have substituted \(\frac{ew\lambda}{\Psi}\) from (A.3) and again used \(u_E - u_B \approx u'_E(w - t - b)\). Further, note that \(\frac{du'_E}{db} = -\frac{u'_E \rho \chi}{w(1 - t^*)} (\omega - (1 - \omega) \frac{\Psi}{\Psi})\), where we have used \(\rho \equiv -cu''(c)/u'(c)\). The third term on the right yields \(\frac{d(ew\lambda)}{db} = -\frac{(\sigma + 1)w_s^*}{S}\), as \(\frac{d}{db} \left( \frac{w}{\Psi} \right) = 0\). By applying (A.3) and \(u_E - u_B \approx u'_E(w - t - b)\), this term can be written as \(\frac{d(ew\lambda)}{db} = -\frac{\sigma + 1}{w(1 - t^*)} \frac{ew\lambda}{\Psi}\). Summarizing the first three terms on the right in (A.4) yields

\[
\frac{d}{db} \phi spm_0 \theta^{1-\alpha}(u(w - t) - u(b)) - \frac{du'_E}{db} \frac{ew\lambda}{\Psi} - \frac{u'_E}{db} \frac{d(ew\lambda)}{db} = -\frac{u'_E \rho \chi}{w(1 - t^*)} \frac{ew\lambda}{\Psi} \left( \sigma + 1 \right) \left( 1 + \frac{\sigma t^*}{\Psi(1 - t^*)} \right) - \omega + (1 - \omega) \frac{\kappa}{\Psi} < 0.
\]

As \(\lambda > 0\) at the optimal level of \(\delta\), the whole expression is negative. Finally, to determine the sign of \(\frac{dW}{db}\) in (A.4), use \(\frac{d\psi}{db} = \frac{1}{w}(1 - t^* \omega) \left( 1 + \frac{\Psi}{\Psi} \right)\), \(\frac{d\omega}{db} = \frac{(1 - \gamma) \rho (1 - \omega)}{1 + (1 - \gamma) \rho \chi} \frac{d\chi}{db}\) and \(\frac{d\chi}{db} = \frac{d\delta}{db}\).
\[-\chi \frac{1-\omega(1-\chi)}{w(1-t^*)} (1 + \frac{\chi^2}{w(1-t^*)} \frac{\kappa}{\Psi}) \] to get
\[
\frac{d\Psi}{db} = - \left[ \left( \frac{\sigma(1-\omega)}{w(1-t^*)^2} + \frac{\sigma + 1}{1-t^*} \right) \frac{1-t^* \omega}{w} + \frac{\sigma + 1}{w(1-t^*)^2} \frac{\omega^2}{1 - \omega} \right] \left( 1 + \frac{w}{y - w} \right) \left( 1 + \frac{\kappa}{\Psi} \right)
\]
\[\quad - \frac{t^*}{w(1-t^*)^2} \frac{1-\gamma}{1 + (1-\gamma)\rho^2\chi^2} \left[ \frac{\kappa}{\Psi} - (1 - (1-\chi)\omega) \left( 1 + \frac{\kappa}{\Psi} \right) \right],
\]
where we have also used the wage bargaining condition (9) together with the approximation \( u_E - u_B \approx u'_E(w - t - b) \) and \( \gamma = \alpha \). The expression in the first row is negative, while the sign of the expression in the second line is ambiguous. By summarizing all terms that are not multiplied by \( \sigma \) and applying the definition of \( \omega \) from (12), the derivative can be rearranged to
\[
\frac{d\Psi}{db} = - \left[ \left( \frac{\sigma(1-\omega)}{w(1-t^*)^2} + \frac{\sigma}{1-t^*} \right) \frac{1-t^* \omega}{w} + \frac{\sigma t^* \omega^2}{w(1-t^*)} \left( 1 + \frac{w}{y - w} \right) \right] \left( 1 + \frac{\kappa}{\Psi} \right)
\]
\[\quad - \frac{t^*}{w(1-t^*)^2} \frac{1-\gamma}{1 + (1-\gamma)\rho^2\chi^2} \frac{\kappa}{\Psi} \left[ \frac{\kappa}{\Psi} - (1 - (1-\chi)\omega) \left( 1 + \frac{\kappa}{\Psi} \right) \right] < 0.
\]
Thus, also the last expression in (A.4) is negative and we have \( d\Omega/db < 0 \). This implies
\[
\frac{d\delta}{db} = - \frac{d\Omega/db}{d\Omega/d\delta} < 0,
\]
i.e. optimal program intensity decreases with the level of unemployment insurance benefits.
Bibliography


