Limited Information and its Impact on a Policyholder’s Optimal Choice on Deductibles

Jan–Christian Fey*1  Hato Schmeiser1  Florian Schreiber2

1University of St. Gallen, Switzerland
2Lucerne University of Applied Science and Arts, Switzerland

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*Corresponding author: jan-christian.fey@unisg.ch
Research Questions

Policyholders typically face two sources of uncertainty:
- Uncertainty arising from the randomness of future losses
- Limited information about the functional forms of the loss distribution and the utility function

How can we incorporate limited information about the functional forms in traditional models for the optimal deductible choice?

Which information sources can the policyholder leverage to approximate the functional forms?

Given limited information about the functional forms: Which characteristics should good decision rules for the optimal deductible choice have?
Motivation

Wealth of policyholder

\[
Y_D(L) = \begin{cases} 
W - R(D) & \text{if } L = 0 \\
W - R(D) - L & \text{if } 0 < L \leq D \\
W - R(D) - D & \text{if } D < L \leq N 
\end{cases}
\]

Optimal deductible choice

\[
D^*_K = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E}[u(Y_D(L))]
\]
Motivation

- Majority of employees enrolled in a health plan of a Fortune 100 firm choose strictly dominated health plans; average amount saved by changing to non-dominated plan: $375 (Bhargava et al., 2017)
- Customers in the homeowners insurance market purchase low deductibles despite costs significantly above the expected value (Sydnor, 2010)
- Although it is financially profitable, only 11 percent of Dutch health insurance customers choose policies with a deductible feature (Van Winssen et al., 2015, 2016)
- ...
### Related Literature

#### Deductible policies
- Optimality of deductible policies: Arrow (1963), Raviv (1979), Gollier (2013)

#### Decision making under limited information
- Decision making under uncertainty: Kofler and Menges (2013)
- Decision rules under uncertainty: Bamberg et al. (2019)
- Heuristics and decision rules: Gigerenzer and Gaissmaier (2011)
Contribution of the Paper

Modeling limited information

- Modeling sources of information in an expected utility framework
- Deriving approximations of the loss distribution
- Deriving approximations of the utility function

Deriving suitable heuristics for choosing an optimal deductible

- Defining desirable characteristics for decision rules
- Analyzing the performance of different decision rules under limited information
Model

- Loss $L$ with $\mu = \mathbb{E}[L]$, $\sigma^2 = \text{Var}[L]$ and cumulative distribution function

\[
F(x) = \mathbb{P}(L \leq x) = \begin{cases} 
0 & \text{if } x < 0 \\
p & \text{if } x = 0 \\
p + \int_0^x f(z) \, dz & \text{if } 0 < x < N \\
1 & \text{if } x \geq N 
\end{cases}
\]

- Premium $R(D) = (1 + \lambda) C(D)$ with $C(D) = \int_{D}^{N} (z - D) f(z) \, dz$

- Utility function $u$ which is everywhere twice differentiable with $u' > 0$ and $u'' \leq 0$

- Optimal Deductible: $D_K^* = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E}[u(Y_D(L))]$
Cost function $c : \{D_1, \ldots, D_K, N\} \rightarrow \mathbb{R}$ for which $c(D)$ is defined as the value $c \in \mathbb{R}$ which solves

$$\mathbb{E}[u(Y(D^*_K) - c)] = \mathbb{E}[u(Y(D))]$$

Expected costs for a decision rule $\hat{\theta}_{m,n}^{(l)}$:

$$\mathbb{E}[c(\hat{\theta}_{m,n}^{(l)})] = \sum_{i=1}^{K} \mathbb{P}(\hat{\theta}_{m,n}^{(l)} = D_i)c(D_i) + \mathbb{P}(\hat{\theta}_{m,n}^{(l)} = N)c(N)$$

Sources of information:

- Loss observations $L_1, \ldots, L_n$ for which the policyholder knows $\mu$ and $\sigma^2$
- Functional values of utility function $u(x_1), \ldots, u(x_m)$
- Deductible policies with deductibles $D_K = (D_1, \ldots, D_K)$ and prices $R_K = (R(D_1), \ldots, R(D_K))$ for which the policyholder knows $\lambda$
Approximating the Loss Distribution

Empirical distribution function

\[ \hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}_{(-\infty,x]}(L_i) \]
Approximating the Loss Distribution

Approximation derived from the available deductible policies

\[ \hat{F}_{K}^{\text{ded}}(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\hat{F}_{K}^{\text{ded}}(0) & \text{if } x = 0 \\
\hat{F}_{K}^{\text{ded}}(0) + \int_{0}^{x} \hat{f}_{K}^{\text{ded}}(y) dy & \text{if } x > 0
\end{cases} \]

where

\[ \hat{f}_{K}^{\text{ded}}(x) = \mathbb{1}_{(\hat{\lambda}_{1}, \hat{\lambda}_{2}]}(x) \hat{z}_{1} + \ldots + \mathbb{1}_{(\hat{\lambda}_{K}, \infty)}(x) \hat{z}_{K} \]
Approximating the Loss Distribution

Figure 1: Illustration of a piecewise constant density function for $K = 3$
Approximating the Loss Distribution

- Weighting of the two approximations $\hat{F}_n$ and $\hat{F}_{K}^{\text{ded}}$ with weight $0 \leq \kappa_{n,K} \leq 1$:

  $$\hat{F}_{n,K}^* = \kappa_{n,K} \hat{F}_{K}^{\text{ded}} + (1 - \kappa_{n,K}) \hat{F}_n$$

- Deriving the weight $\kappa_{n,K}^*$ which minimizes the distance measure

  $$d_{n,K}(\kappa_{n,K}) = \left( \kappa_{n,K} \mu_{K}^{\text{ded}} + (1 - \kappa_{n,K}) \tilde{L} - \mu \right)^2 + \beta \left( \kappa_{n,K} \sigma_{K}^{\text{ded}} + (1 - \kappa_{n,K}) \sigma_n^2 - \sigma^2 \right)^2$$

- Properties of $\kappa_{n,K}^*$:
  - $\lim_{n \to \infty} \kappa_{n,K}^* = 0$ $\mathbb{P}$-almost surely
  - $\lim_{K \to \infty} \kappa_{n,K}^* = 1$ under certain conditions
Approximating the Loss Distribution

(a) $D_K = (500, 1000, \ldots, 2500)$

(b) $D_K = (500, 1000, \ldots, 10000)$

**Figure 2**: Distribution of $\kappa_{n,K}^*$ for different $n$ and $D_K$
Preference Uncertainty

\[ u(x) = W - N \]

Figure 3: Illustration of the functions \( u_m \) and \( \tilde{u}_m \) for \( x_2 = (x_1, x_2) \)
Probability Equivalence (PE) Elicitation Method

Figure 4: Illustration of the hypothetical decision situations faced by the policyholder (Source: Bamberg et al., 2019)

\[ u(x) = (1 - p^*) u(W - N) + p^* u(W) = p^* \]

\[ \begin{align*} 
  := 0 \\
  := 1 
\end{align*} \]
Decision Rules for the Optimal Deductible Choice

Definition

A decision rule $\hat{\theta}_{m,n}^{(l)}$ under the information vector $l = (D_K, R_K)$ is an estimator of the optimal deductible level $D_K^*$. It is called

(i) **consistent** if $\lim_{m,n \to \infty} \hat{\theta}_{m,n}^{(l)} = D_K^*$ $\mathbb{P}$-almost surely.

(ii) **unbiased** if $\lim_{m \to \infty} \mathbb{E}[\hat{\theta}_{m,n}^{(l)}] = D_K^*$ holds for all $n \in \mathbb{N}$.

(iii) **preference-independent** if $\hat{\theta}_{0,n}^{(l)} = \hat{\theta}_{m,n}^{(l)}$ $\mathbb{P}$-almost surely for all $m \in \mathbb{N}$ for a fixed but arbitrary $n \in \mathbb{N}$.
Definition (continued)

If $u'' < 0$ and $D_K^* \neq N$, the decision rule $\hat{\theta}^{(l)}_{m,n}$ is called

(iv) **preference-consistent** if

$$\lim_{n \to \infty} \mathbb{E}[c(\hat{\theta}^{(l)}_{0,n})] - \lim_{m,n \to \infty} \mathbb{E}[c(\hat{\theta}^{(l)}_{m,n})] > 0.$$
Examples of Decision Rules

Preference-dependent decision rules

\[ \hat{\theta}_{m,n}^{(1)} = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \frac{1}{n} \sum_{i=1}^{n} \tilde{u}_m(Y_D(L_i)) \]

\[ \hat{\theta}_{m,n}^{(2)} = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \int_0^N \tilde{u}_m(Y_D(l)) d\hat{F}_{n,K}(l) \]
## Simulation Study for the US Homeowners Insurance Market

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter for claims frequency</td>
<td>$\lambda_P$</td>
<td>5.64%</td>
</tr>
<tr>
<td>Parameter for claim amount</td>
<td>$\lambda_E$</td>
<td>$\frac{1}{13,814}$</td>
</tr>
<tr>
<td>Loss limit</td>
<td>$N$</td>
<td>$40,000$</td>
</tr>
<tr>
<td>Available deductible levels</td>
<td>$D_1, \ldots, D_K$</td>
<td>$500, 1000, \ldots, 2500$</td>
</tr>
<tr>
<td>Loading factor</td>
<td>$\lambda$</td>
<td>0.05</td>
</tr>
<tr>
<td>Weighting factor for the approximation of the loss distribution</td>
<td>$\beta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Wealth</td>
<td>$W$</td>
<td>$49,806$</td>
</tr>
<tr>
<td>Relative risk aversion parameters</td>
<td>$\eta_1, \ldots, \eta_n$</td>
<td>0.1, 0.5, 1, 2, 3, 5</td>
</tr>
<tr>
<td>Probability weighting factor for gains</td>
<td>$\gamma^+$</td>
<td>0.61</td>
</tr>
<tr>
<td>Probability weighting factor for losses</td>
<td>$\gamma^-$</td>
<td>0.69</td>
</tr>
<tr>
<td>Loss aversion parameter</td>
<td>$\lambda_L$</td>
<td>2.25</td>
</tr>
</tbody>
</table>
## Distribution of Risk Preferences

<table>
<thead>
<tr>
<th>Specification</th>
<th>Absolute risk aversion</th>
<th>Relative risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean estimates from insurance contexts:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sydnor (2006)</td>
<td>$1.9 \cdot 10^{-3}$</td>
<td>54.07</td>
</tr>
<tr>
<td>Cohen and Einav (2007)</td>
<td>$6.7 \cdot 10^{-3}$</td>
<td>97.22</td>
</tr>
<tr>
<td>Handel and Kolstad (2015): Base Model</td>
<td>$1.6 \cdot 10^{-3}$</td>
<td>60.97</td>
</tr>
<tr>
<td>Handel and Kolstad (2015): Full Model</td>
<td>$8.6 \cdot 10^{-5}$</td>
<td>3.28</td>
</tr>
<tr>
<td><strong>Benchmark model of Cohen and Einav (2007):</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25th percentile</td>
<td>$2.3 \cdot 10^{-6}$</td>
<td>0.03</td>
</tr>
<tr>
<td>Median individual</td>
<td>$2.6 \cdot 10^{-5}$</td>
<td>0.37</td>
</tr>
<tr>
<td>75th percentile</td>
<td>$2.9 \cdot 10^{-4}$</td>
<td>4.27</td>
</tr>
<tr>
<td>90th percentile</td>
<td>$2.7 \cdot 10^{-3}$</td>
<td>39.02</td>
</tr>
<tr>
<td>95th percentile</td>
<td>$9.9 \cdot 10^{-3}$</td>
<td>143.27</td>
</tr>
<tr>
<td><strong>Friction adjusted benchmark model of Cohen and Einav (2007):</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25th percentile</td>
<td>$3.0 \cdot 10^{-8}$</td>
<td>0.0011</td>
</tr>
<tr>
<td>Median individual</td>
<td>$3.3 \cdot 10^{-7}$</td>
<td>0.0125</td>
</tr>
<tr>
<td>75th percentile</td>
<td>$3.7 \cdot 10^{-6}$</td>
<td>0.1410</td>
</tr>
<tr>
<td>90th percentile</td>
<td>$3.5 \cdot 10^{-5}$</td>
<td>1.33</td>
</tr>
<tr>
<td>95th percentile</td>
<td>$1.3 \cdot 10^{-4}$</td>
<td>4.95</td>
</tr>
</tbody>
</table>
## Costs of Choosing a Wrong Deductible

<table>
<thead>
<tr>
<th>η</th>
<th>500</th>
<th>1,000</th>
<th>1,500</th>
<th>2,000</th>
<th>2,500</th>
<th>40,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>14.06</td>
<td>12.88</td>
<td>11.76</td>
<td>10.70</td>
<td>9.71</td>
<td>0*</td>
</tr>
<tr>
<td>0.5</td>
<td>3.27</td>
<td>2.23</td>
<td>1.34</td>
<td>0.60</td>
<td>0*</td>
<td>56.40</td>
</tr>
<tr>
<td>1</td>
<td>1.90</td>
<td>1.04</td>
<td>0.44</td>
<td>0.10</td>
<td>0*</td>
<td>161.73</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.01</td>
<td>0*</td>
<td>0.48</td>
<td>1.42</td>
<td>499.16</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0*</td>
<td>0.60</td>
<td>1.92</td>
<td>3.95</td>
<td>1,177.60</td>
</tr>
<tr>
<td>5</td>
<td>0*</td>
<td>0.63</td>
<td>2.49</td>
<td>5.59</td>
<td>9.92</td>
<td>5,639.40</td>
</tr>
</tbody>
</table>

Note: η = 0 indicates no deductible chosen.

* denotes the value exceeds the deductible amount.
**Preference-Dependent Decision Rules**

<table>
<thead>
<tr>
<th>Costs by degree of risk aversion</th>
<th>Population costs per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta = 0.1$</td>
<td>$\eta = 0.5$</td>
</tr>
<tr>
<td>$n = 5$</td>
<td>2.8</td>
</tr>
<tr>
<td>$n = 25$</td>
<td>5.15</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>6.27</td>
</tr>
</tbody>
</table>

**Expected costs for $\hat{\theta}_{n,1}^{(l)}$:**

<table>
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<th>Expected costs for $\hat{\theta}_{m,n}^{(l)}$:</th>
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<td>$n = 5$</td>
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<td>$n = 25$</td>
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<td>$n = 100$</td>
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**Expected costs for $\hat{\theta}_{m,n}^{(l)}$:**

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<td>$n = 100$</td>
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Preference-Dependent Decision Rules

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</thead>
<tbody>
<tr>
<td>$\eta = 0.1$</td>
<td>$\eta = 0.5$</td>
</tr>
<tr>
<td>$n = 5$</td>
<td>0.72</td>
</tr>
<tr>
<td>$n = 25$</td>
<td>1.28</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>1.72</td>
</tr>
</tbody>
</table>

Expected costs for $\hat{\theta}_n^{(I),2}$:

Expected costs for $\hat{\theta}_m,n^{(I),2}$ for $m = 5$:

| $n = 5$ | 0.72 | 45.2 | 5.1 | 3.54 | 5.53 |
| $n = 25$ | 1.24 | 30.54 | 5.22 | 3.1 | 4.60 |
| $n = 100$ | 1.65 | 24.77 | 5.17 | 3.11 | 4.32 |

Expected costs for $\hat{\theta}_m,n^{(I),2}$ for $m = 20$:

| $n = 5$ | 0.72 | 44.99 | 5.1 | 3.5 | 5.36 |
| $n = 25$ | 1.28 | 28.73 | 5.22 | 3.01 | 4.41 |
| $n = 100$ | 1.7 | 21.72 | 5.17 | 2.94 | 4.06 |
Conclusion

- Including limited information in traditional models for the optimal deductible choice
- Derivation of approximation for loss distribution from different sources of information
- Discussion of approximations for the utility function
- Identification of suitable decision rules for the optimal deductible choice
- Valuable insights for designing decision aids:
  - Today’s decision aids are mostly expected costs minimizer
  - Our analysis suggests that this approach could be improved by accounting for risk preferences and the loss distribution (not only expected costs)
Appendix
Approximating the Loss Distribution

- **Step 1:** Solve the system of $K + 1$ linear equations given by

$$
\tilde{f}_K^{\text{ded}}(x) = \mathbb{1}_{(D_1, D_2]}(x) z_{K,1} + \ldots + \mathbb{1}_{(D_K, N]}(x) z_{K,K}
$$

$$
\tilde{F}_K^{\text{ded}}(0) = \tilde{p}_K = 1 - \int_0^N \tilde{f}_K^{\text{ded}}(z)dz
$$

$$
= 1 - \sum_{i=1}^{K-1} z_{K,i} (D_{i+1} - D_i) - z_{K,K} (N - D_K)
$$

$$
C(D_k) = \sum_{i=k}^{K-1} z_{K,i} \int_{D_i}^{D_{i+1}} (y - D_k) dy + z_{K,K} \int_{D_K}^N (y - D_k) dy
$$

- **Step 2:** Transform $\tilde{F}_K^{\text{ded}}$ to $\hat{F}_K^{\text{ded}}$ with the help of an algorithm
Approximating the Loss Distribution

Figure 5: Example for $\hat{F}^{\text{ded}}_K$ and $\tilde{F}^{\text{ded}}_K$
Optimal Survey Strategy

- Set of all possible utility functions is described by the probability space \((\mathcal{U}, \mathcal{F}, \mathbb{P}_U)\)
- Determining the set of survey questions \(\mathbf{x}^{**} = (x_1^{**}, \ldots, x_m^{**}) \in \mathcal{X}^m\) with the help of the following procedure:

\[
\tilde{x}_1^{**} = \arg \min_{x \in \mathcal{X}} \sum_{v \in \mathcal{U}} \mathbb{P}_U(v) \int_{W-N}^W (v(z) - \tilde{v}_1(z))^2 dz
\]

\[
\tilde{x}_m^{**} = \arg \min_{x \in \mathcal{X}} \sum_{v \in \mathcal{U}} \mathbb{P}_U(\tilde{u}_{m-1}(v)) \int_{W-N}^W (v(z) - \tilde{v}_m^{(h)}(s((\tilde{x}_1^{**}, \ldots, \tilde{x}_{m-1}^{**}, x), z))^2 dz
\]

\[
\mathbf{x}^{**} = (x_1^{**}, \ldots, x_m^{**}) = s((\tilde{x}_1^{**}, \ldots, \tilde{x}_m^{**}))
\]
Bio in the Elicitation Method

- In theory, widely used elicitation methods like the PE method and the certainty equivalence (CE) method should lead to the same utility function.

- Experimental evidence for difference in the estimated utility functions (cf., e.g., Bleichrodt et al., 2001).

- Assume that the relationship between biased utility function $u^{(b)}$ and $u$ is given by

$$u(x) = s(u^{(b)}(x)) \iff u^{(b)}(x) = s^{-1}(u(x))$$

- Deductible choice: $D^{(b)}_K = \arg \max_{D \in \{D_1, \ldots, D_K, N\}} \mathbb{E} [u^{(b)}(Y_D(L))]$

- Impact on optimal deductible choice:
  - If $s' > 0$, $s'' < 0$, $D^*_K \leq D^{(b)}_K$.
  - If $s' > 0$, $s'' > 0$, $D^*_K \geq D^{(b)}_K$. 

Example for a Correction Function $s$

- Decision maker exhibits preferences which are a combination of prospect theory and expected utility theory.
- Utilities for actions $a_1$ and $a_2$ are given by $u(x)$ and
  \[
  u(x) + w^+(p)(u(W) - u(x)) - \lambda L w^-(1 - p)(u(x) - u(W - N))
  \]
- Bleichrodt et al. (2001) derive the following correction function:
  \[
  s(y) = \frac{w^+(y)}{w^+(y) + \lambda L w^-(1 - y)}
  \]
Example for a Correction Function $s$

Figure 6: Impact of the correction function $s$ of Bleichrodt et al. (2001) on a CRRA utility function with relative risk aversion $\eta$

Limited Information and its Impact on a Policyholder’s Optimal Choice on Deductibles
### Biased Preference-Dependent Decision Rules

<table>
<thead>
<tr>
<th></th>
<th>Costs by degree of risk aversion</th>
<th>Population costs per capita</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \eta = 0.1 )</td>
<td>( \eta = 0.5 )</td>
<td>( \eta = 5 )</td>
</tr>
<tr>
<td>Expected costs for ( \hat{\theta}^{(I),b,1}_n )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>3.11</td>
<td>44.6</td>
<td>4,387.5</td>
</tr>
<tr>
<td>( n = 25 )</td>
<td>8.61</td>
<td>23.83</td>
<td>2,075.3</td>
</tr>
<tr>
<td>( n = 100 )</td>
<td>12.59</td>
<td>6.55</td>
<td>282.63</td>
</tr>
<tr>
<td>Expected costs for ( \hat{\theta}^{(I),b,1}_{m,n} ) for ( m = 5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>2.8</td>
<td>45.82</td>
<td>4,517.2</td>
</tr>
<tr>
<td>( n = 25 )</td>
<td>5.96</td>
<td>30.68</td>
<td>3,221.5</td>
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<tr>
<td>( n = 100 )</td>
<td>9.46</td>
<td>14.8</td>
<td>1,339.4</td>
</tr>
<tr>
<td>Expected costs for ( \hat{\theta}^{(I),b,1}_{m,n} ) for ( m = 20 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>2.95</td>
<td>45.82</td>
<td>4,517.2</td>
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<tr>
<td>( n = 25 )</td>
<td>7.78</td>
<td>28.21</td>
<td>2,861.1</td>
</tr>
<tr>
<td>( n = 100 )</td>
<td>11.62</td>
<td>11.29</td>
<td>967.7</td>
</tr>
</tbody>
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<tr>
<td>n = 5</td>
<td>11.35</td>
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<tr>
<td>n = 25</td>
<td>12.58</td>
</tr>
<tr>
<td>n = 100</td>
<td>13.21</td>
</tr>
</tbody>
</table>

Expected costs for $\hat{\theta}_{n}^{(l),b,2}$:

- n = 5  
  - $\eta = 0.1$: 11.35  
  - $\eta = 0.5$: 1.1  
  - $\eta = 5$: 0.57  
  - $P_{U}$: 8.89  
  - $P_{CE}^{U}$: 5.42
- n = 25  
  - $\eta = 0.1$: 12.58  
  - $\eta = 0.5$: 2.05  
  - $\eta = 5$: 0.35  
  - $P_{U}$: 9.91  
  - $P_{CE}^{U}$: 5.98
- n = 100  
  - $\eta = 0.1$: 13.21  
  - $\eta = 0.5$: 2.54  
  - $\eta = 5$: 0.42  
  - $P_{U}$: 10.45  
  - $P_{CE}^{U}$: 6.32

Expected costs for $\hat{\theta}_{m,n}^{(l),b,2}$ for $m = 5$:

- n = 5  
  - $\eta = 0.1$: 10.09  
  - $\eta = 0.5$: 0.37  
  - $\eta = 5$: 5.1  
  - $P_{U}$: 8.17  
  - $P_{CE}^{U}$: 6.13
- n = 25  
  - $\eta = 0.1$: 10.1  
  - $\eta = 0.5$: 0.59  
  - $\eta = 5$: 5.21  
  - $P_{U}$: 8.2  
  - $P_{CE}^{U}$: 6.18
- n = 100  
  - $\eta = 0.1$: 10.3  
  - $\eta = 0.5$: 0.7  
  - $\eta = 5$: 5.15  
  - $P_{U}$: 8.36  
  - $P_{CE}^{U}$: 6.27

Expected costs for $\hat{\theta}_{m,n}^{(l),b,2}$ for $m = 20$:

- n = 5  
  - $\eta = 0.1$: 11.34  
  - $\eta = 0.5$: 0.84  
  - $\eta = 5$: 5.1  
  - $P_{U}$: 9.17  
  - $P_{CE}^{U}$: 6.73
- n = 25  
  - $\eta = 0.1$: 12.35  
  - $\eta = 0.5$: 0.79  
  - $\eta = 5$: 5.21  
  - $P_{U}$: 9.94  
  - $P_{CE}^{U}$: 7.2
- n = 100  
  - $\eta = 0.1$: 12.74  
  - $\eta = 0.5$: 0.83  
  - $\eta = 5$: 5.15  
  - $P_{U}$: 10.25  
  - $P_{CE}^{U}$: 7.36


