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HYSTERESIS IN POTENTIAL OUTPUT AND MONETARY POLICY

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Hysteresis in Potential Output and Monetary Policy*

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Abstract

We show that actively stabilizing economic activity plays a more prominent role in the conduct of monetary policy when potential output is subject to hysteresis. We augment a basic New Keynesian model by hysteresis in potential output and contrast simulation outcomes of this extended model to the standard model. We find that considering hysteresis allows for a more realistic propagation of macroeconomic shocks and persistent movements in output after monetary shocks. Our central policy implication of active output gap stabilization arises from stability analyses and welfare considerations.

Keywords: Monetary Policy, Hysteresis, Potential Output, Output Gap Mismeasurement.

JEL code: E32, E50, E52.

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1 Introduction

Due to the Great Recession the political and academic debate has experienced a revival of the so-called hysteresis phenomenon on a grand scale. In the face of the severe crisis a large number of major economic institutions and think tanks, such as for example the European Commission (2009), Furceri and Mourougane (2009), the IMF (2009) or Pisani-Ferry and van Pottelsberghe (2009), have addressed the negative implications of the economic downturn for the development of potential output. From this debate two important questions arise: Does hysteresis have economic policy implications? If yes, how should economic policy react to hysteresis?

Generally, hysteresis means that pronounced changes in aggregate demand exhibit procyclical, persistent real supply-side effects. While several facets of hysteresis have been documented in the macroeconomic literature since the early 1970s (see section 2 for a short summary of this literature), the question of how to consider such effects in terms of monetary or fiscal policy strategies has hardly been addressed by popular models. Specifically, currently used standard models designed for monetary policy research fade out the stimulus of demand-determined actual output on an economy's potential output. This holds for the logic of the New Keynesian model à la Woodford (2003) or Galí (2008) as well as for rather pragmatic models within the inflation targeting context such as Svensson (2000).[1] However, as Orphanides et al. (2000) argue, ignoring hysteresis effects may involve substantial misjudgement for the conduct of fiscal or monetary policies.

We address this shortcoming by examining the consequences of hysteresis in potential output for monetary policy. To this end, we extend Galí (2008)'s basic New Keynesian model by hysteresis, i.e. by allowing the path of potential output to be influenced by lagged actual output. To work out the relevance of hysteresis for monetary policy, we contrast simulation outcomes of the extended model with Galí (2008)'s benchmark model and with empirical second moments. Moreover, we examine the implications for the conduct of monetary policy with respect to stability and welfare considerations.

We find that the extended model traces more realistic adjustment patterns than the standard New Keynesian model after monetary shocks hit the economy. Specifically, hysteresis helps to reproduce empirically well-documented persistence patterns of output. Furthermore, our model

[1] A notable exception to this is Woodford (2003)'s chapter 5, where he extends the basic model-setup by capital investment and illustrates that productive capacity as well as the equilibrium real rate of interest are affected by monetary policy.

exhibits a number of features that assign a more important role to active output gap stabilization if the economy is subject to hysteresis. First, if the central bank applies a monetary policy rule and the degree of hysteresis is large enough, achieving a unique stable equilibrium requires a reaction to the output gap for certain ranges of the reaction parameter for inflation. Second, if a welfare loss criterion based on the variability of inflation and the output gap is applied, reacting to the output gap in the monetary policy rule yields sizeable welfare loss reductions beyond those that would arise without hysteresis effects. The reason for these results lies in the dynamics of the output gap. If the central bank wants to fight inflation, the procyclical behavior of potential output requires a balancing reaction to the output gap in order to maintain downward pressure on inflation. At the same time, this downward pressure helps to reduce inflation variability.

As a robustness check we also show that, depending on the degree of hysteresis, actively stabilizing the output gap can still be welfare improving even when there is measurement uncertainty with respect to the output gap. Furthermore, if hysteresis is in effect, shifting the focus on output instead of the output gap in the monetary policy rule is not necessarily detrimental to welfare as is the case in the basic New Keynesian model.

Although hysteresis has not played a meaningful role in standard models so far, its relevance for stabilization policy has been addressed by several authors such as Ball (1999), DeGrauwe and Costa Storti (2007), DeLong and Summers (2012), Lavoie (2004) or Solow (2000). More specifically, there are even some approaches that explicitly model such endogenous supply-side adjustment. For example, Mankiw (2001), Fritsche and Gottschalk (2006), and Kapadia (2005) basically share a common reduced form specification for hysteretic adjustment that was originally proposed by Hargreaves Heap (1980). Our model also refers to this specification.

The analysis closest to ours is Kapadia (2005) who also examines the consequences of hysteresis in potential output for monetary policy. However, there are a number of aspects which distinguish our approach from his study. First, Kapadia (2005) analyzes cost-push shocks, while we consider productivity and monetary shocks (which is more common in the business cycle literature and facilitates comparison with the basic New Keynesian model). Second, Kapadia (2005) strongly focuses on different specifications of the Phillips curve. While this is a useful robustness check, it also clouds somewhat the role of hysteresis in the model. Our approach involves the basic New Keynesian Phillips curve but varies the degree of hysteresis to get better insights into the hysteresis mechanism. This is also an important robustness check since little work has been done to quantify the degree of hysteresis so far.[2] Third, while Kapadia (2005) focuses on optimal adjustment paths, we provide a more policy focused view by analyzing stability and welfare issues in the framework of interest rate rules that could potentially be adopted by policy makers. This also involves an analysis of the implications of output gap mismeasurement by the central bank.

The remainder of this paper is organized as follows: Section 2 briefly summarizes the basic mechanisms constituting hysteresis in potential output. Section 3 introduces our model. Section 4 examines the model dynamics after macroeconomic shocks, with a focus on monetary shocks. Section 5 investigates the implications of hysteresis in potential output for monetary policy. Section 6 examines the robustness of our policy implications given output gap uncertainty. Section 7 discusses the plausibility of the magnitude of hysteresis in potential output. Finally, section 8 concludes.

2 Hysteresis in potential output

This section summarizes the most important channels of hysteresis in potential output. Subsection 2.1 addresses the underlying theoretical mechanisms. Subsection 2.2 points at the respective empirical evidence.

2.1 Theoretical mechanisms

From a theoretical perspective pronounced changes in aggregate demand can impact potential output in several ways. As pointed out by Blanchard (2008), DeLong and Summers (2012), Schmid (2010) or Solow (2000), according to the factors within a conventional production function (capital stock, employment and total factor productivity) one may categorize three major channels of hysteretic adjustment: First, varying net capital formation, second, labor market hysteresis, and third, investment-induced efficiency gains. These mechanisms have been extensively documented in the literature and will therefore in the following only be addressed briefly.

First, as a very basic insight from the theory of economic growth, capital investment not only affects aggregate income (multiplier dynamics) but also changes an economy's productive capacity (accelerator mechanism). Hence, for example in case of a severe recession the investment shortfall reduces the future productive potential of an economy.

[2] See section 7 for a discussion of this issue.

Second, as seminally pointed out by Phelps (1972) and, for example also addressed by Ball and Mankiw (2002) or Spahn (2000), labor market hysteresis captures the procyclical impact of recent cyclical unemployment upon the current natural rate of unemployment. Thereby, cyclical changes of the demand for employees lead to the adjustment of the effective labor supply. The most prominent channels behind this phenomenon are insider-outsider mechanisms as highlighted by Blanchard and Summers (1987) and Lindbeck and Snower (1988) and de-qualification (skill loss) as addressed by Hargreaves Heap (1980) and Pissarides (1992).

Third, as pioneered by Arrow (1962), Kaldor (1966), Solow (1960) and Young (1928), changes in aggregate demand may stimulate the growth of labor productivity. Thereby, market expansion during economic upturns pushes the division of labor and stimulates industrial specialization and intersectoral spillovers which raise productivity on a macroeconomic level. Besides this, the application of innovative production techniques and the use of new machinery (vintages approach) - which is itself stimulated by cyclical capacity adjustment - promote learning-by-doing effects and initiate intersectoral knowledge spillovers.

2.2 Empirical evidence

On the empirical side, many studies such as Bank of England (2012), Cerra and Saxena (2008), DeLong and Summers (2012), European Commission (2009), Furceri and Mourougane (2009) or Pisani-Ferry and van Pottelsberghe (2009) have provided evidence for hysteresis in potential output in the context of the Great Recession. Most of these analyses go beyond the recent experience and thus also cover earlier time periods. Thereby, it has become common sense that hysteresis, although difficult to quantify, is not only in place in times of severe economic downturns. Rather there has also been evidence for positive hysteresis which is in line with the underlying theoretical considerations. Within this literature the adjustment of potential output to cyclical fluctuations is normally explained by the above mentioned hysteresis channels.

Focussing on the empirical relevance of these specific mechanisms, since the late 1970s there has been a variety of studies exploring the scope of labor market hysteresis and the procyclical character of productivity growth. For example, Blanchard and Summers (1988) as well as Layard and Bean (1989) state empirical evidence for de-qualification and decreasing re-employment options of long-term unemployed persons. Hargreaves Heap (1980) and McGregor (1978) observe a positive relationship between average unemployment duration and the level of unemployment.

Daly et al. (2011) find that during the Great Recession the natural rate of unemployment has risen substantially. Regarding investment-induced efficiency gains, among others Léon-Ledesma and Thirlwall (2002) and Cornwall and Cornwall (2002) provide broad cross-country evidence of positive effects of aggregate demand and particularly capital investment on labor productivity.

3 Model

Our modelling framework to address the implications of hysteresis in potential output for monetary policy is the basic New Keynesian set-up. It consists of a dynamic IS-equation, a forward-looking Phillips curve and a central bank reaction function. For a detailed derivation of the basic model, we refer to chapter 3 in Galí (2008).

The dynamic IS-equation of the model reads

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho), \tag{1}$$

where y_t is (log) output, i_t is the central bank's (nominal) interest rate, π_t is the inflation rate in period t, and ρ is the discount rate. This IS-equation is a log-linearized version of the household's Euler equation combined with the market clearing condition that consumption equals output.

Nominal supply-side dynamics are captured by a forward-looking Phillips curve given by

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^*), \tag{2}$$

where y_t^* is (log) potential output in period t. Its derivation involves aggregating the log-linearized optimal price-setting rules of firms facing a constant probability of resetting prices, in a neighborhood of the zero inflation steady state. In the context of this model, potential output is the equilibrium level of output if prices were completely flexible.

The central bank employs the following interest rate rule:

$$i_t = \rho + \gamma\pi_t + \psi(y_t - y_t^*) + \nu_t, \tag{3}$$

where ν_t is an exogenous component of monetary policy following the AR(1)-process $\nu_t = \rho_\nu \nu_{t-1} + u_t^\nu$. u_t^ν is an error term with mean zero and variance σ_ν. γ and ψ are parameters determining the strength of the central bank's reaction to inflation and the output gap.

In the basic New Keynesian model, potential output evolves according to $y_t^* = a_t$ (we have neglected the constant), where a_t is (log) productivity. Assuming an AR(1)-process for productivity yields

$$y_t^* = \rho_a y_{t-1}^* + u_t^a, \tag{4}$$

where u_t^a is a productivity shock with mean zero and variance σ_a. We deviate from this specification to allow for hysteresis effects. Specifically, we model the process of potential output as

$$y_t^* = (1-\eta)\rho_a y_{t-1}^* + \eta y_{t-1} + u_t^a, \tag{5}$$

where η is the degree of hysteresis. Potential output is thus not only a function of productivity and past potential output but also a function of past actual output. This formulation of hysteresis is meant to capture the various channels described in section 2 on an aggregate level, as similarly applied by Kapadia (2005) and Fritsche and Gottschalk (2006).[3] The higher (lower) η, the more (less) potential output is affected by lagged actual output. Note that the basic New Keynesian model is nested in our specification for $\eta = 0$.

4 Hysteresis and monetary policy shocks

Since we ultimately aim at analyzing monetary policy issues, we need an indication of whether our model is reliable. Therefore, we compare model generated second moments to empirical second moments and show that our model can improve on some dimensions compared to the basic New Keynesian model. The two variables we can evaluate are inflation and output. Unfortunately, such a comparison is not possible for potential output. In the model, the variance of potential output is driven by innovations in productivity, while in the data potential output is some kind of smoothed trend of actual output.[4]

Nevertheless, we can still infer from the characteristics of inflation and output if our specification for potential output adds realism to the model. As we will show, hysteresis helps reconcile model outcomes with well-established stylized facts. In particular, Christiano et al. (2005) docu-

[3] The functional form dates back to Hargreaves Heap (1980).

[4] This also holds if potential output is calculated using the production function approach.

ment a persistent response of output to monetary shocks which cannot be reproduced by the basic New Keynesian model. Our model with hysteresis is particularly suited for studying the effects of monetary shocks since they have an impact on potential output in our specification. This in turn implies richer dynamics of output and inflation in response to monetary shocks than in the standard New Keynesian model. In contrast, we do not expect substantial deviations regarding productivity shocks because, as in the standard model, these impact potential output directly, outweighing the hysteresis effect on this margin.

The fact that hysteresis in potential output mainly has consequences for monetary and hence demand side shocks does not compromise the relevance of the analysis. There is a large literature documenting the importance of demand shocks in general and monetary shocks in particular for real variables. Romer and Romer (1989, 1994) use a narrative approach to show that monetary shocks account for more than a fifth of the variation in real economic activity in the US. Using a structural VAR, Christiano et al. (2005) estimate that the fraction of the variance in US output due to monetary shocks is between 15% and 27% depending on the horizon after the shock. Using the same methodology, Bouakez et al. (2005) get even higher numbers, especially in the short run. More generally, Romer and Romer (1989, 1994) conclude that demand shocks are the primary source of business cycle fluctuations. This result is supported by Smets and Wouters (2007) who find that demand shocks account for more than 50% of the variation in US GDP in the short run and approximately 40% in the long run.

To simulate second moments, we have to calibrate the model's parameters. Following Galí (2008), we set $\beta = 0.99$, $\rho = -log(\beta)$, $\kappa = 0.13$ and $\sigma = 1$, which implicitly assumes an annual steady state real interest rate of 4%, a log utility function of consumers, a degree of price stickiness of 4 quarters, a Frisch elasticity of labor supply of 1, an elasticity of substitution of 6 and a labor elasticity of output of 1/3. For these parameters, we stick to this calibration throughout the rest of the paper. This facilitates direct comparisons of our model outcomes to the basic New Keynesian model in Galí (2008). Besides this, we have to decide on the values of the policy reaction parameters. Like much of the literature that aims at describing a realistic behavior of the Federal Reserve, we set $\gamma = 1.5$.[5] For the reaction to the output gap, we set $\psi = 0.3$ since it is the smallest value for which stability is ensured for all considered degrees of hysteresis.[6] The degree of hysteresis is varied between $\eta = 0$ - in which case our model coincides with the basic

[5] See Taylor (1993).

[6] The issue of stability is separately analyzed and discussed in subsection 5.1.

New Keynesian model - and $\eta = 0.5$. This parameter range for η will be discussed in section 7. For the specification of the exogenous processes we follow Smets and Wouters (2003) and set $\rho_a = 0.81$, $\sigma_a = 0.5\%$ and $\sigma_\nu = 0.15\%$. The last two values are also found in Lechthaler et al. (2010). Moreover, we use different values for ρ_ν to illustrate the persistence properties of our model in response to monetary shocks.

Table 1 contrasts empirical and model generated business cycle statistics for output y and inflation π. The first data row shows empirical unconditional moments. Data are from the OECD and range from 1955Q1 to 2012Q4. For calculating empirical moments we follow Stock and Watson (1999). Empirical moments for output are obtained using the cyclical component of US real GDP, calculated as percentage deviation from HP-filtered trend. Inflation is calculated as the quarter-on-quarter percentage change in the CPI at annual rates. Empirical moments for inflation are calculated using the cyclical component of inflation obtained by an HP filter.

The other second moments in table 1 are model generated moments for joint, monetary and productivity shocks for different degrees of hysteresis, respectively. Looking at productivity shocks, the only notable difference between the various hysteresis specifications and the basic New Keynesian model ($\eta = 0$) is the somewhat larger propagation of the productivity shock as hysteresis effects increase. This is documented by increasing standard deviations for output as η increases, bringing the model predictions closer to the empirical counterpart on this account. Otherwise, as noticed above, hysteresis does not make much of a difference for productivity shocks.

Consequently, as the productivity shock is the dominating shock for our calibration[7], the effects of hysteresis for joint shocks seem to be small and only apparent for the reported standard deviations. Note, however, that this is only the case because our relatively small model features only one demand shock (the monetary shock). In medium scale models with several demand shocks, the productivity shock would no longer be as dominant as it is here.

The main effects of hysteresis come to light when looking at monetary shocks. Table 1 shows model moments for an autocorrelated monetary shock with $\rho_\nu = 0.5$. In the literature this is often assumed to incorporate a realistic amount of persistence into the model.[8] As usual, in the standard model the autocorrelation of the monetary shock is passed on to the autocorrelation

7 The productivity shock is approximately 3.5 times stronger than the monetary shock in line with the estimates of Smets and Wouters (2003).

8 See for example Galí (2008).

	Standard dev.		Autocorr.		Corr.
	y	π	y	π	y, π
data	0.0155	0.0215	0.85	0.32	0.27
Joint shock					
$\eta = 0$	0.0106	0.0013	0.89	0.86	-0.88
$\eta = 0.1$	0.0107	0.0013	0.89	0.85	-0.87
$\eta = 0.2$	0.011	0.0013	0.89	0.85	-0.86
$\eta = 0.3$	0.0114	0.0013	0.9	0.85	-0.85
$\eta = 0.4$	0.0121	0.0013	0.91	0.85	-0.84
$\eta = 0.5$	0.0135	0.0014	0.92	0.85	-0.83
Monetary shock					
$\eta = 0$	0.0016	0.0004	0.5	0.5	1
$\eta = 0.1$	0.0021	0.0004	0.64	0.45	0.89
$\eta = 0.2$	0.0027	0.0004	0.76	0.43	0.62
$\eta = 0.3$	0.0037	0.0005	0.84	0.46	0.28
$\eta = 0.4$	0.0052	0.0006	0.89	0.55	-0.07
$\eta = 0.5$	0.0073	0.0007	0.93	0.65	-0.37
Productivity Shock					
$\eta = 0$	0.0104	0.0012	0.9	0.9	-1
$\eta = 0.1$	0.0105	0.0012	0.9	0.9	-1
$\eta = 0.2$	0.0106	0.0012	0.9	0.9	-1
$\eta = 0.3$	0.0108	0.0012	0.91	0.91	-1
$\eta = 0.4$	0.11	0.0012	0.91	0.91	-1
$\eta = 0.5$	0.0114	0.0012	0.91	0.91	-1

Table 1: Business cycle statistics for a persistent monetary policy shock ($\rho_\nu = 0.5$). The first data row shows empirical second moments. Quarterly data from 1955Q1 to 2012Q4 are obtained from the OECD Quarterly National Accounts database and the Main Economic Indicators database. y is the cyclical component of real GDP as percentage deviation from HP-filtered trend. π is the cyclical component of inflation rate obtained by an HP filter. The inflation rate is calculated as quarterly percentage change of consumer price index at an annual rate. The other data rows show model generated second moments for joint, monetary, and productivity shocks, respectively and for different degrees of hysteresis η.

of output and inflation, and the correlation of these two variables is 1. As hysteresis kicks in, the correlation between output and inflation decreases with η and standard deviations rise, which is a desirable feature of the model considering the respective empirical counterparts. More importantly, the persistence of output increases substantially, which is in line with the empirical evidence on monetary shocks. The persistence of inflation first decreases when hysteresis effects become relevant and then increases with the strength of hysteretic adjustment. Note that this feature also brings the model predictions closer to the empirical counterpart for moderate degrees of hysteresis.

To elucidate the implications of hysteresis for the persistence characteristics of the model for a monetary shock, we also consider a one-off (transitory) monetary policy shock. This is illus-

	Standard dev.		Autocorr.		Corr.
	y	π	y	π	y, π
data	0.0155	0.0215	0.85	0.32	0.27
Joint shock					
$\eta = 0$	0.0105	0.0012	0.89	0.89	-0.98
$\eta = 0.1$	0.0106	0.0012	0.89	0.89	-0.98
$\eta = 0.2$	0.0107	0.0012	0.89	0.89	-0.98
$\eta = 0.3$	0.0109	0.0012	0.9	0.89	-0.97
$\eta = 0.4$	0.011	0.0012	0.9	0.9	-0.97
$\eta = 0.5$	0.0116	0.0012	0.91	0.9	-0.97
Monetary shock					
$\eta = 0$	0.001	0.0001	0	0	1
$\eta = 0.1$	0.0011	0.0001	0.13	-0.0489	0.9172
$\eta = 0.2$	0.0013	0.0001	0.29	0	0.68
$\eta = 0.3$	0.0015	0.0002	0.46	0.12	0.35
$\eta = 0.4$	0.0018	0.0002	0.6	0.29	0.185
$\eta = 0.5$	0.0023	0.0002	0.72	0.45	-0.26

Table 2: Business cycle statistics for a one-off (transitory) monetary policy shock ($\rho_\nu = 0$). The explanations for table 1 carry over to this table. Model second moments for the productivity shock are not reported since they are the same as in table 1.

trated in table 2. Without externally induced persistence, the autocorrelation for output and inflation in the standard model is zero. When hysteresis effects are considered, the autocorrelation in output and inflation increases substantially, up to 0.72 and 0.45, respectively, for a high degree of hysteresis. For output, even small to medium degrees of hysteresis bring about notable improvements of the model's internal persistence properties.

The improvements for the propagation of shocks and the internal persistence are well apparent when we look at impulse response functions (IRFs) for monetary policy shocks. Figure 1 shows IRFs for output, inflation, potential output and the policy rate for a transitory monetary policy shock. The different IRFs for each variable refer to different degrees of hysteresis. While there is no output persistence at all for $\eta = 0$, persistence gradually increases for higher degrees of hysteresis. We also see why the autocorrelation coefficient for inflation is negative for a small degree of hysteresis in case of a monetary shock. The reason is that inflation exhibits an "overshooting" behavior. After inflation decreases on impact of the monetary policy shock, the hysteretic adjustment induces a relatively quick decrease of the output gap, attenuating the downward pressure on inflation. For a small value of η, this effect dominates the slow adjustment to equilibrium in the subsequent periods. We also see that potential output is responding quite

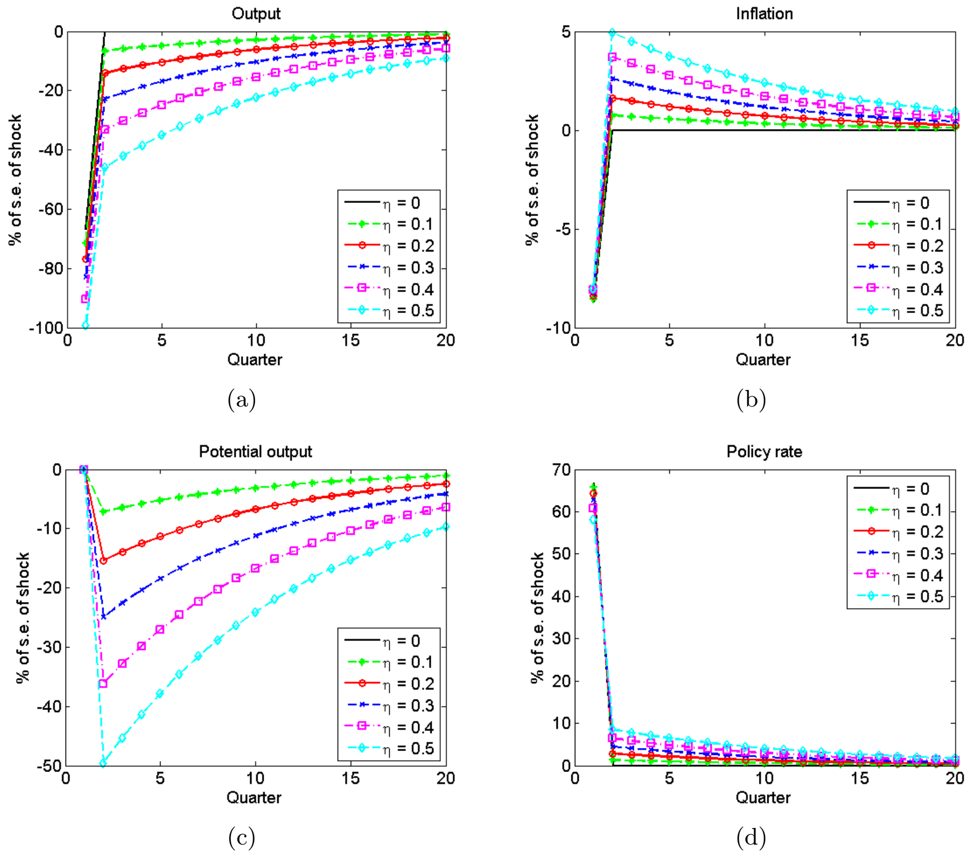


Figure 1: Model generated impulse response functions for output, inflation, potential output and the policy rate in the case of a transitory monetary policy shock and different degrees of hysteresis η. Units are percent of standard error of the underlying shock.

heavily in the hysteresis case, while it is constant after a monetary policy shock in the basic New Keynesian model. Moreover, the IRFs illustrate well the stronger propagation of shocks when hysteresis is in effect which is apparent in the magnitude of the responses.

5 Implications of hysteresis for monetary policy

We study the implications of hysteresis for a monetary authority using an interest rate rule as a policy guideline. That is, the central bank decides on the nominal interest rate according to a reaction function of endogenous variables. The monetary policy decision then boils down to setting the reaction parameters on these endogenous variables.

In this setting, we address two questions: First, how can monetary policy achieve a stable economy when hysteresis effects are in place? In particular, we are looking for constellations of reaction parameters which yield a unique stable equilibrium. Second, for the set of parameter constellation that yields unique stationary equilibria, which policy yields a minimal welfare loss? In addition, we elucidate how policies that yield stable outcomes and minimal welfare losses under hysteresis differ from the respective policies in the baseline model without hysteresis effects.

For the analysis of stability and welfare issues it is convenient to rewrite the model so as to present it in matrix form. As the appendix shows, equations (1), (2), (3) and (5) can be summarized as follows:

$$\begin{pmatrix} \nu_{t+1} \\ y^*_{t+1} \\ E_t\{y_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{pmatrix} = A \begin{pmatrix} \nu_t \\ y^*_t \\ y_t \\ \pi_t \end{pmatrix} + B \begin{pmatrix} u^{\nu}_{t+1} \\ u^{a}_{t+1} \end{pmatrix}, \tag{6}$$

where

$$A = \begin{pmatrix} \rho_\nu & 0 & 0 & 0 \\ 0 & (1-\eta)\rho_a & \eta & 0 \\ \frac{1}{\sigma} & \frac{(-\beta*\psi-\kappa)}{(\beta*\sigma)} & \frac{\beta*(\sigma+\psi)+\kappa)}{\sigma*\beta} & \frac{\beta*\gamma-1}{\sigma*\beta} \\ 0 & \frac{\kappa}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

5.1 Inducing stability in a hysteretic economy

Model (6) has two predetermined variables (y^*_t and ν_t) and two non-predetermined variables (y_t and π_t). Hence, according to Blanchard and Kahn (1980) a stationary unique solution will exist if and only if A has two eigenvalues inside and two eigenvalues outside the unit circle.[9] Since checking this condition analytically is not possible in our model, we apply a numerical procedure to show that the determinacy of the equilibrium depends on the reaction parameters of the central bank given a certain degree of hysteresis η.

Assuming that the central bank can adjust its reaction parameters γ and ψ in 0.1-steps, figure 2 illustrates the determinacy and indeterminacy regions in the (γ, ψ)-space for different degrees of hysteresis, namely $\eta = 0$, $\eta = 0.1$, $\eta = 0.2$, $\eta = 0.3$, $\eta = 0.4$ and $\eta = 0.5$. We look at positive values for γ up to five and for ψ up to two. A wider range would not yield different results. Recall

[9] The required rank conditions are satisfied for all stable parameter constellations, see Blanchard and Kahn (1980).

that for $\eta = 0$, we are back to the basic New Keynesian model, so we can readily compare the hysteresis to the non-hysteresis case.

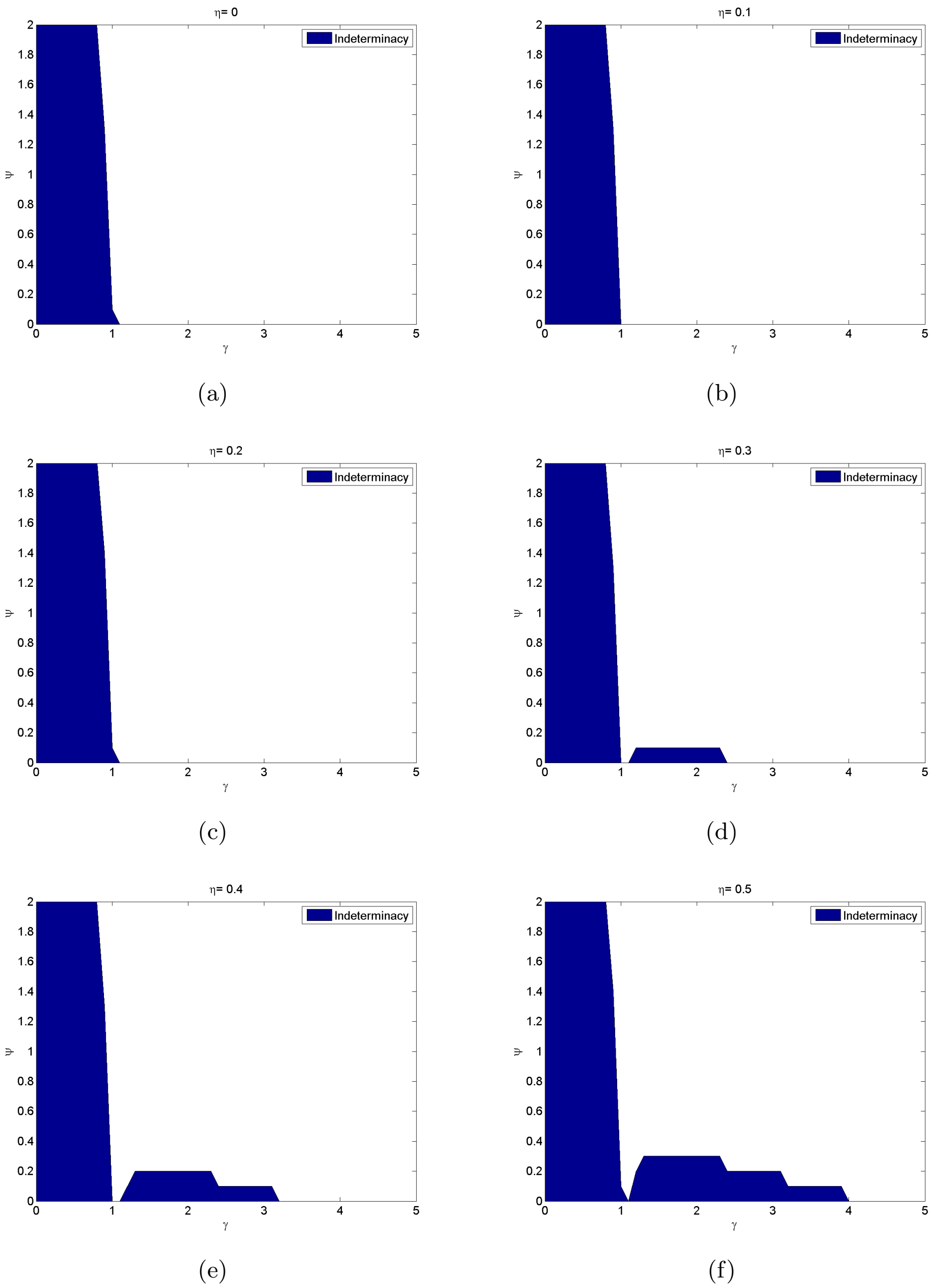


Figure 2: Stability regions for different constellations of central bank reaction parameters to inflation γ and the output gap ψ and for different degrees of hysteresis in potential output η.

Figure 2(a) depicts the determinacy region for the basic New Keynesian model and represents the well-known Taylor principle: to achieve a unique stable equilibrium, the central bank has to adjust the interest rate overproportionally in response to a change in inflation. This requires

$\gamma > 1$, at least when the central bank is not reacting at all to changes in the output gap. As figure 2(b) and 2(c) show, this principle carries over to economies with mild hysteresis effects.

However, for higher degrees of hysteresis the indeterminacy region expands. In particular, the overproportional change in the interest rate is not a sufficient condition any more. Figures 2(d), 2(e) and 2(f) show that for certain ranges of $\gamma > 1$, a reaction to the output gap is required in order to achieve determinacy. These ranges expand as the degree of hysteresis increases. In addition, the required reaction to output gap variations increases with η. For example, for $\eta = 0.5$ and $\gamma \in [1.2; 2.2]$ the reaction parameter for the output gap, ψ, has to be above 0.3, while for the same range of γ and $\eta = 0.4$, $\psi > 0.2$ suffices for a stable equilibrium.

An additional important observation is that the ranges of the inflation reaction parameter requiring a reaction to the output gap include $\gamma = 1.5$, a value often associated with a good description of the actual behavior of major central banks. In this sense, our model can provide an explanation why it is reasonable for a monetary authority to react to economic activity, which is considered to be common practice among central banks.[10]

The pattern of the instability regions in figures 2(d), 2(e) and 2(f) is very distinctive. The required reaction to the output gap for an increasing reaction to inflation first increases, then is constant and then decreases. To gain intuition, suppose the central bank raises the interest rate in order to bring down inflation. This will cause output and (with an initially constant potential output) the output gap to fall. However, due to hysteresis effects, potential output decreases subsequently. This reduces the originally intended downward pressure on inflation. Hence, the monetary authority needs to react to the output gap variation - which requires an interest rate movement in the opposite direction - in order to cushion the initial interest rate increase and being able to bring inflation to its equilibrium level.

Moreover, the required reaction to the output gap decreases with a larger reaction to inflation because a large γ implies a larger output gap and, consequently, brings down inflation to its equilibrium value quicker. Therefore, the balancing effect of the reaction to the output gap does not have to be that strong for large γ.

[10] See for example Taylor (1999a).

5.2 Hysteresis and welfare implications of monetary policy

Knowing the set of feasible parameter combinations for different degrees of hysteresis, we proceed by analyzing optimal monetary policy when the central bank uses an interest rate rule. Therefore, we require a criterion to assess welfare implications of monetary policy. Much of the literature has adopted a welfare loss criterion based on a second-order approximation of the household's utility function, as in Rotemberg and Woodford (1999). This has the advantage that the welfare criterion is consistent with the specific model at hand. However, the disadvantage is that the policy maker has to know the model in order to employ the "correct" welfare loss function. Paez-Farrell (2012) points out that if this is not the case, using an exogenous quadratic welfare loss function might be less detrimental than using a micro-founded loss function. So far, not much work has been done regarding hysteresis effects in business cycle models, suggesting high model uncertainty. Thus, we use an exogenous quadratic loss function to evaluate welfare consequences of monetary policy.[11] It expresses the welfare loss in terms of a weighted average of the variance of inflation and the variance of the output gap:

$$L = \left[\phi var(y_t - y_t^*) + var(\pi_t)\right]. \tag{7}$$

Here, ϕ is the relative weight of the output gap variance in the welfare loss. Note that this loss function has the same form as the welfare loss function obtained by a second-order approximation of the consumer's utility function in Galí (2008)'s basic New Keynesian model. There, ϕ takes on the value 0.02, and we adopt this calibration for the subsequent simulations.[12]

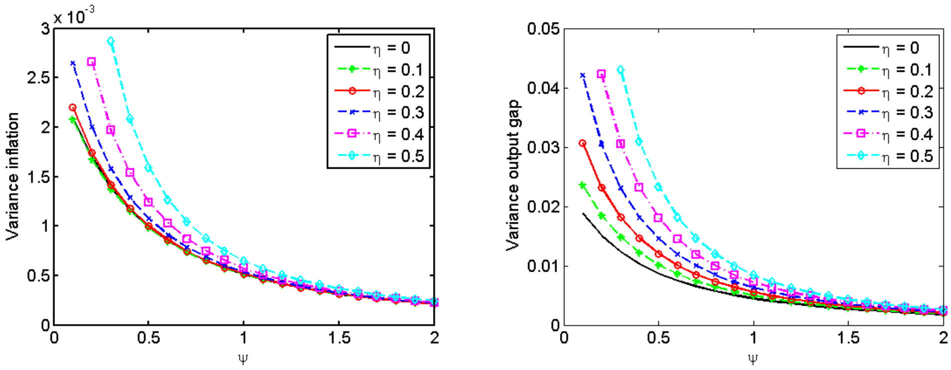
For calculating the welfare loss associated with different policies, we apply the following procedure: For parameter constellations of γ and ψ which yield a determinate system (the ranges for these parameter values correspond to the analysis presented in subsection 5.1), we calculate the welfare loss based on the implied variances of the output gap and inflation according to equation (7). Again, we alter the values for the reaction parameters γ and ψ in 0.1-steps. We then check which reaction parameter constellation yields the minimum welfare loss. In this way, we obtain an optimized monetary policy rule. As before, we consider various degrees of hysteresis

[11] Other papers that use exogenous welfare criteria include Taylor (1979), Orphanides et al. (2000), Angeloni et al. (2003), or Davis and Huang (2011).

[12] The choice of ϕ is not critical for our qualitative results. To illustrate this, we report the variance of both inflation and the output gap separately in the following.

throughout our analysis. For the sake of exposition, we fix γ to 1.5, but the results for different γ's do not change qualitatively.

Figure 3 shows the variances for inflation and the output gap for varying output gap reaction parameters, whereas figure 4 shows the values for the loss function (7) for varying output gap reaction parameters.[13] We can see in figure 3 that the variability of both inflation and the output gap declines as the central bank's reaction to the output gap increases. This translates to the loss function in figure 4.



(a) (b)

Figure 3: Model implied variances for inflation (3(a)) and the output gap (3(b)) for different values of output gap reaction parameter ψ across different degrees of hysteresis η, where the inflation reaction parameter γ is fixed to 1.5.

For all degrees of hysteresis, the minimal welfare loss is attained for a strong reaction to the output gap; in this dimension, the hysteresis ($\eta > 0$) and the non-hysteresis ($\eta = 0$) case do not differ. However, we see that there are substantial costs in not reacting to the output gap when the economy is subject to hysteresis. In particular, in the region of relatively low ψ's, losses are higher for $\eta > 0$ than for $\eta = 0$. Put differently, the marginal gain for reacting to the output gap increases substantially when the economy is subject to hysteresis.

Therefore, output gap stabilization becomes relatively more important when hysteresis effects are in place. In the hysteresis-augmented model, an aggressive reaction to inflation renders potential output to fall considerably in the next period (as opposed to the case without hysteresis that implies no change of potential output). This brings about a relatively quick reduction of the output gap which by itself intensifies the inflationary pressure compared to a situation without

[13] Due to the non-determinacy for small ψ's given comparably high values of η, in figures 4 and 3 the smallest value for ψ is 0.2 and 0.3 for $\eta = 0.4$ and $\eta = 0.5$, respectively.

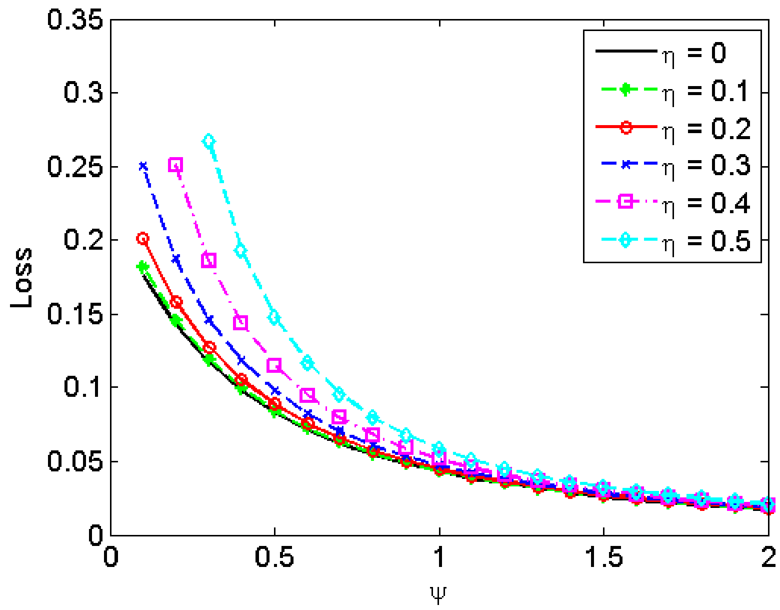


Figure 4: Welfare loss for different values of the output gap reaction parameter ψ across different degrees of hysteresis η, where the inflation reaction parameter γ is fixed to 1.5.

hysteresis. Therefore, a reaction to the output gap - which signals an interest rate change of opposite sign compared to the response to inflation - balances the reaction to inflation in the sense that the output gap is stabilized in order to bring down inflation, hence reducing the variability of both variables.

6 How to deal with output gap uncertainty?

Our analysis so far makes a case for active output gap stabilization rather than only focusing on inflation. However, a popular critique of output gap stabilization is that in reality the output gap is measured with error and thus may not be a suitable variable to base policy decisions upon. Due to incomplete information about the current state of the economy and the unobservability of potential output, the central bank faces uncertainty with regard to output gap dynamics. As output gap data is only available with a considerable time lag, monetary policy has to rely on estimated values. Diverging estimation results provided by different measuring techniques as well as the frequent and considerable data revisions extensively illustrate the disputable reliability of output gap measures. Thus, as for example pointed out by Orphanides (1999) or the European Central Bank (2000), the usefulness of output gap measures for monetary policy might be questionable.

In particular, overestimation of potential output in times of downturns and underestimation during macroeconomic expansion bears the risk of procyclical overreaction regarding interest rates. Therefore, the central bank may have to fear an unanticipated course of productive capacity that ultimately classifies the original interest rate reaction as inadequate. Hence, an interest rate policy that attributes less or no weight to output gaps in interest rate rules is suggested by authors such as Onatski (2000), McCallum (2001), or Willems (2009).

In light of this critique, we analyze whether we can maintain our result that reacting to economic activity is desirable from a welfare point of view when hysteresis effects are in place. We approach this issue in two ways: First, we examine our model's welfare properties when output gap uncertainty is explicitly taken into account in the form of a measurement error and check if output gap stabilization remains a desirable feature of monetary policy.

Second, an influential literature, for example Taylor (1999b), suggests to consider output itself (or deviations of output from steady state) instead of the output gap in monetary policy rules. It is a well-known result that in the absence of real frictions, this specification of monetary policy suggests to refrain completely from reacting to output since it drives up welfare losses.[14] We examine if this is still true when hysteresis effects are in place, as our previous results advocate for a more prominent role of directly stabilizing economic activity.

6.1 Taking uncertainty explicitly into consideration

To evaluate the effective risk of suboptimal policy reactions to output gap mismeasurement in the context of hysteresis, we first analyze the robustness of our model's welfare implications when the output gap is measured with error.[15] As proposed by Orphanides et al. (2000), we capture output gap mismeasurement by an additive observation distortion ξ_t within the central bank's reaction function:

$$i_t = \rho + \gamma\pi_t + \psi[(y_t - y_t^*) + \xi_t] + \nu_t, \tag{8}$$

[14] See for example Galí (2008).

[15] The concept of robustness examines the ability of a central bank's strategy to guarantee desirable results for different macroeconomic specifications. Given the uncertainty regarding the exact state of macroeconomic aggregates robust policy rules are preferable. See, for example, McCallum (1988) and McCallum (1997).

where $\xi_t = \rho_\xi \xi_{t-1} + \epsilon_t^\xi$. ξ_t can be thought of as the process describing the ex-post revisions with regard to the ex-ante estimate of the output gap. ρ_ξ represents the persistence of the observation distortion and ϵ_t^ξ is assumed to be white noise with variance σ_ξ.

Within this framework, we can examine if the advantages of active output gap stabilization that arise if hysteresis is in effect are outweighed by the disadvantages that come along with output gap measurement errors. We employ the following procedure: Based on the estimates of Orphanides et al. (2000), we look at a "best case", a "base case" and a "worst case" with a relatively low ($\rho_\xi = 0.8$), medium ($\rho_\xi = 0.84$) and high ($\rho_\xi = 0.96$) persistence of the measurement error process, respectively. Within each case, we fix the central bank reaction parameter to inflation γ to 1.5[16] and calculate the output gap reaction parameter that yields the lowest welfare loss ψ^* for varying intensities of the measurement error shock σ_ξ. Again, we consider different degrees of hysteresis between $\eta = 0$ and $\eta = 0.5$.[17]

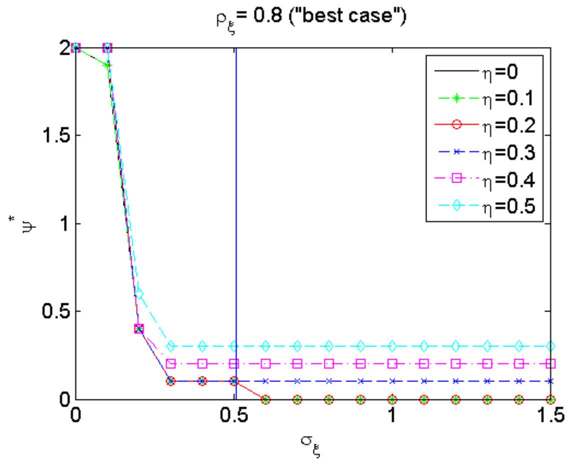
Figure 5 illustrates the results; figures 5(a), 5(b) and 5(c) refer to the best, base and worst case described above. The loss-minimizing reaction to the output gap is shown on the vertical axis, while the intensity of the measurement error is shown on the horizontal axis. Different line styles refer to different degrees of hysteresis.

As expected, as the effect of the measurement error kicks in, the importance of output gap stabilization declines with higher values of σ_ξ. The higher the persistence of the measurement error, the faster the optimal strength of output gap stabilization falls. However, while for the basic New Keynesian model ($\eta = 0$) and mild degrees of hysteresis ($\eta = 0.1$ and $\eta = 0.2$) the optimal reaction to the output gap remains at zero for increasing measurement error shocks, $\eta \geq 0.3$ suggests a positive reaction to the output gap in order to minimize welfare losses. This is true for all persistence patterns of the measurement error. The demanded reaction to the output gap rises with the degree of hysteresis.

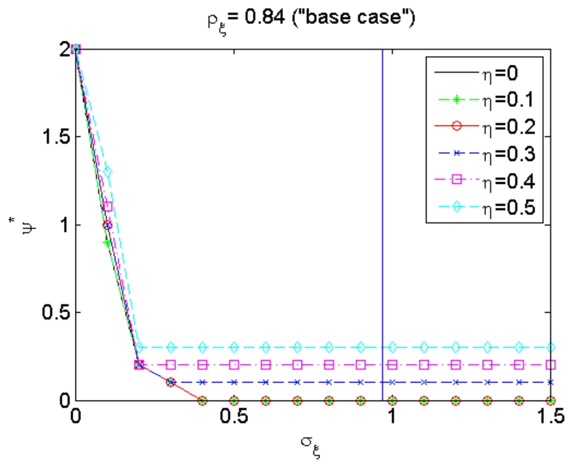
In particular, for the values of σ_ξ estimated by Orphanides et al. (2000) - indicated by the vertical lines in the graphs - a reaction to the output gap can be optimal depending on the persistence of the measurement error and the degree of hysteresis. In the best case, i.e. for a relatively small persistence in the measurement error, even small hysteresis effects suffice to render a reaction to the output gap beneficial. Depending on the strength of hysteresis, the optimal ψ

[16] The results do not change qualitatively for higher values of γ.

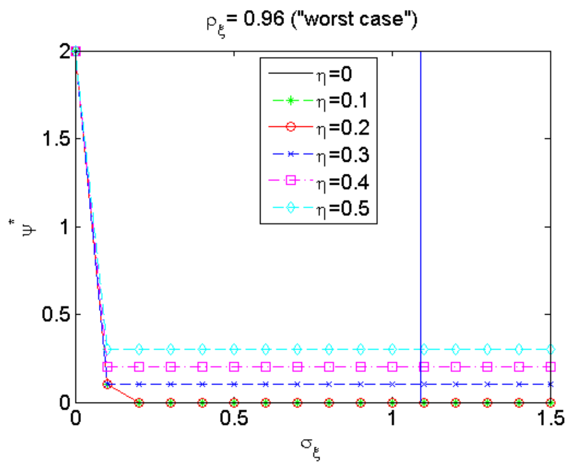
[17] Note that the analysis in Orphanides et al. (2000) and our examination in section 5.2 are based on the same welfare loss function which is illustrated in equation (7).



(a)



(b)



(c)

Figure 5: Optimal output gap reaction parameter ψ^* plotted against intensity of measurement error. Figures 5(b), 5(a), and 5(c) show low, medium and high persistence of measurement error, respectively. Vertical lines indicate estimates for the standard deviations of the measurement error shock by Orphanides et al. (2000). Different line types in each graph refer to different degrees of hysteresis η.

ranges from 0.1 to 0.3. In the base and worst case, medium to strong, but not small hysteresis effects demand for active output gap stabilization. The range of ψ^* is again between 0.1 and 0.3.

Thus, we find that even when the output gap is measured with error, hysteresis effects can imply a beneficial role for active output gap stabilization. It depends on the strength of the hysteresis effect and the size of the measurement error to what extent the central bank should target the output gap.

6.2 Dispensing with the output gap in the monetary policy rule

At least since Taylor (1993)'s influential work, many researchers have studied monetary policy rules in which the policymaker reacts to output (or the deviation from output from its steady state) rather than the output gap. While, in the context of the New Keynesian model, the monetary authority would like to employ the output gap in the reaction function (since it is the variable that influences the inflation process), it is not feasible to do so. The argument is that the output gap is not directly observable and should therefore be replaced by output in the monetary policy rule. As our previous results suggest a beneficial role for the stabilization of the output gap when hysteresis is in effect, the question appears whether we can generalize our results to the active stabilization of economic activity (be it output or the output gap).

This question is particularly interesting because the standard result is that a reaction to output inevitably reduces the economy's welfare performance in the absence of real imperfections.[18] In the following, we examine if this finding can be maintained when hysteresis is considered. The analysis can be viewed as a further robustness check for our model implications.

We proceed similar as in subsection 5.2, except that the monetary policy rule contains output instead of the output gap.[19] Figure 6 shows the variances of the output gap and inflation for this case. Again, since the weight on inflation is high in the welfare loss function, the pattern of inflation variances translates into welfare losses, shown in figure 7.

The unmarked line in figure 7 reproduces the above mentioned result of the basic New Keynesian model with a strong (but slightly diminishing) marginal increase in the welfare loss as the reaction to output increases. When hysteresis is considered (marked lines), the slope of the loss curve for every ψ decreases substantially for small degrees of hysteresis and disappears completely

[18] For a detailed exposition of this result, see Galí (2008), chapter 4.

[19] Note that the stability regions for $\gamma > 1$ are very similar in subsection 5.1. Therefore, we do not discuss them here again.

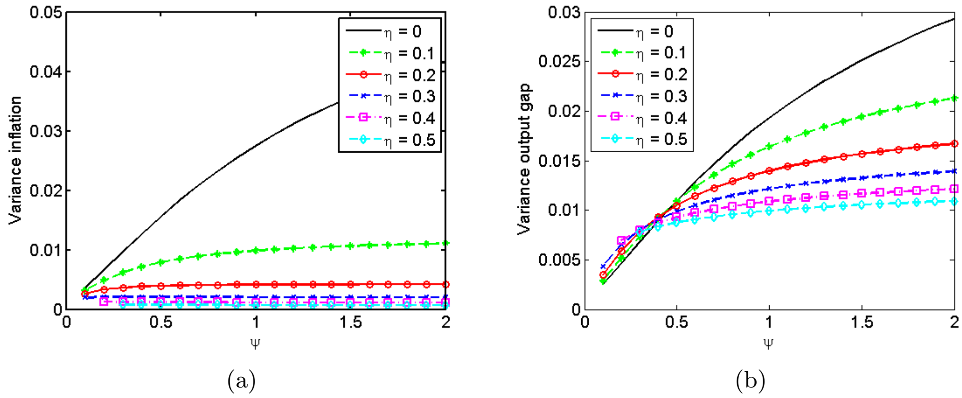


Figure 6: Model implied variances for inflation (6(a)) and the output gap (6(b)) for different values of the output gap reaction parameter ψ across different degrees of hysteresis η, where the inflation reaction parameter γ is fixed to 1.5. The central bank reacts to output instead of the output gap.

for $\eta \geq 0.3$. For $\eta = 0.2$, the increase is very small. That is, a reaction to the output gap only involves small or no welfare losses if hysteresis is in effect.

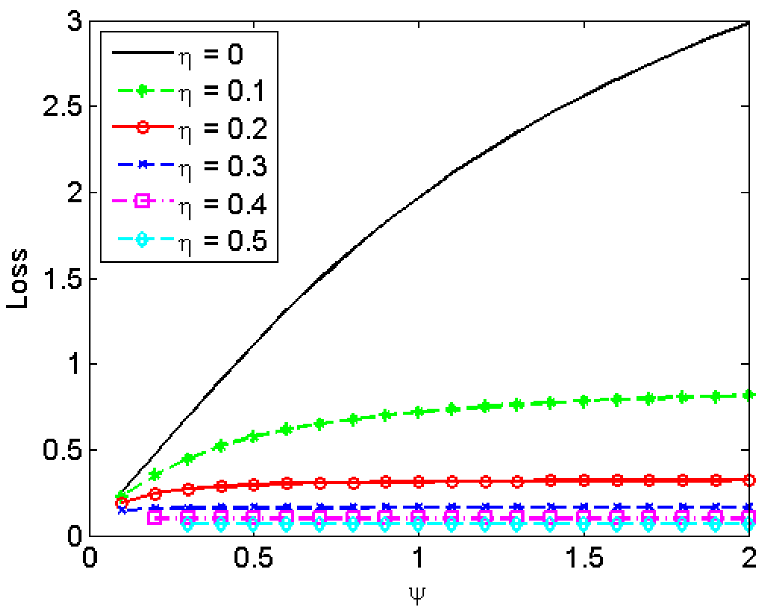


Figure 7: Welfare loss for different values of the output gap reaction parameter ψ across different degrees of hysteresis η, where the inflation reaction parameter γ is fixed to 1.5. The central bank reacts to output instead of the output gap.

Intuitively, when the central bank stabilizes output, the output gap - the variable which determines inflation dynamics - could in principle fluctuate heavily due to movements in potential output. These fluctuations would then be passed on to inflation via the Phillips curve. This is exactly what happens in the standard model. However, in the hysteresis case, potential output depends positively on lagged actual output, as equation (5) illustrates. This is why they cannot drift apart strongly as the central bank stabilizes output. Consequently, for small degrees of hysteresis, the central bank only induces little variation in the output gap and inflation by stabilizing output. For larger degrees of hysteresis ($\eta \geq 0.3$), no additional loss is created by reacting to output.

Hence, while reacting to output does not yield welfare gains as is the case for output gap stabilization, it produces small or no welfare losses if the economy exhibits hysteresis effects.

7 Discussion: what is a plausible degree of hysteresis?

So far, we have considered the degree of hysteresis η to range from 0 to 0.5. This has been done to obtain the best possible insights into the characteristics of our model, i.e. to learn how the dynamics in the economy change when different intensities of hysteresis are in effect. Clearly, the question arises what could actually be a plausible degree of hysteresis. We address this issue from three perspectives: First, we point to empirical evidence for the degree of hysteresis in potential output. Second, we summarize the values of η used in similar models. Third, we deduct plausible parameter values for η from the comparison of our model dynamics with the data.

First, empirically, the degree of hysteretic adjustment in potential output is difficult to quantify. This is for several reasons: (a) Economic up- and downturns may themselves be triggered by long-lasting changes in the economy. Hence, it is hard to identify supply-side developments that are due to hysteresis and not partly driven by technological impulses or exogenous shifts in labor force participation. (b) Time series' information of potential output is itself simply some kind of filtered, cyclically changing production pattern. Thus, the impact of changes in the output gap upon potential output cannot be measured in a straight forward way as potential output data is itself highly shaped by output. (c) In the course of economic downturns, it is hard to abstract from the stabilizing impact of policy responses upon the pure hysteresis mechanism, i.e. the adjustment of future potential output to actual output. Thereby, the degree of hysteretic adjustment is likely to be moderated by mitigating demand-side policies. One way to approach the magnitude of hysteresis in potential output despite these troubles has recently been suggested by DeLong and Summers (2012). Taking a production function perspective, these authors approximate hysteresis in potential output by the procyclical adjustment of the capital stock and the labor supply. Their study covers US data for the adjustment of the capital stock from 1967 to 2012 as well as labor market dynamics for France, Germany, Italy and the United Kingdom since 1970 and for the US since 1990. The authors provide evidence that a one percent output shortfall may induce a reduction of potential output by up to 0.3 percent or even more.

Besides this, there is evidence from studies that focus on the adjustment of the natural unemployment rate to changes in lagged cyclical unemployment. For example, Logeay and Tober (2006) report labor market hysteresis for the euro area from 1973-2002, suggesting a value of $\eta = 0.26$. Moreover, Jäger and Parkinson (1994) measure a value of $0.18 \leq \eta \leq 0.22$ for the United Kingdom and for West Germany from 1961-1991.

Second, studies that use a similar specification for hysteresis as our model are Fritsche and Gottschalk (2006), Mankiw (2001) and Kapadia (2005). The former two studies work with $\eta = 0.1$, the latter applies a value of $\eta = 0.25$.

Third, from the comparison of our model second moments to the empirical second moments for inflation and output (see tables 1 and 2 in section 4) we learn that values of $0.2 \leq \eta \leq 0.3$ seem plausible. Considering the correlation between output and inflation for monetary policy shocks, the degree of hysteresis matching the data best is $\eta = 0.3$. Although the propagation of shocks is still too weak to match the empirical moments, increasing values of η lead to a somewhat better approximation of the empirical standard deviations. This holds for a persistent as well as for a transitory monetary policy shock. Regarding the autocorrelation of output (inflation), the model matches the data fairly well for $\eta = 0.3$ ($\eta = 0.2$) in case of a persistent and for $\eta = 0.5$ ($\eta = 0.4$) in case of a transitory monetary policy shock. However, while it seems that for some statistics a high degree of hysteresis seems to be favorable, there are several indications that $\eta > 0.3$ is not plausible. For example, the correlation between inflation and output for a monetary policy shock becomes negative for high values of hysteresis, which contradicts common empirical and theoretical considerations. Furthermore, the autocorrelations of inflation and output become too high compared to the data.

Summarizing these different viewpoints, a value of η around 0.25 seems to be a reasonable assumption for the degree of hysteresis in potential output. The fact that we also consider lower and higher degrees of hysteresis can be understood as a robustness check in the light of uncertainty about the true value for η. Against the background of our analysis, a magnitude of $\eta > 0.2$ indicates that hysteretic adjustment of potential output indeed exhibits substantial implications for the conduct of monetary policy.

8 Conclusion

Due to the recent economic downturn of the Great Recession the topic of hysteresis has re-entered the economic agenda. However, most standard models designed for monetary policy research do not consider hysteresis effects and are of little help for the assessment of policy strategies when potential output is subject to hysteresis.

Our paper addresses this shortcoming by examining the consequences of hysteresis in potential output for monetary policy within the standard New Keynesian framework. We model hysteresis by allowing the path of potential output to be influenced by lagged actual output. To work out the relevance of hysteresis for monetary policy, we contrast simulation outcomes of our model - which nests the standard New Keynesian model - with empirical second moments for output and inflation. Furthermore, we examine the implications of hysteresis for the conduct of monetary policy with respect to stability and welfare considerations.

We find that hysteresis helps to enhance the model's performance: the propagation of macroeconomic shocks to inflation and output improves and the adjustment of output after monetary shocks is more persistent. These observations are in accordance with empirical evidence. Moreover, our model exhibits a number of features that assign a more important role to the stabilization of economic activity by the central bank if the economy is subject to hysteresis. First, for a sufficiently high degree of hysteresis and certain, empirically plausible ranges for the inflation parameter in the interest rate rule of the policy maker, a reaction to the output gap is required to obtain a unique stable equilibrium. Second, the marginal reduction of welfare losses by reacting to the output gap is particularly high when the economy is subject to hysteresis. Robustness checks show that actively stabilizing the output gap can reduce welfare losses even when the output gap is measured with error. Furthermore, reacting to output instead of the output gap does not necessarily increase welfare losses, as is inevitably the case in the standard New Keynesian model.

We consider our analysis as a first step towards a better understanding of the consequences of hysteresis for monetary policy. Our findings point out that hysteresis in potential output bears important implications for the conduct of monetary policy and that ignoring hysteresis effects may be costly. Thus, more research is required to enhance the reliability of policy recommendations. Continuing work in this field may comprise the consideration of hysteretic adjustment in larger models. This would facilitate the analysis of adjustment dynamics and policy implications in a more realistic macroeconomic setting. Besides this, as also pointed out by DeLong and Summers (2012), further empirical evidence for the quantification of the degree of hysteresis in potential output is another important step to learn more about the hysteresis mechanism and its implications for economic policy. Thereby, as mentioned by O'Shaughnessy (2011), the potential asymmetry of hysteretic adjustment with respect to the direction of shock impulses might be an important issue. Further research could thus differentiate between expansionary and contractionary demand shocks with regard to the magnitude and the timing of the hysteretic adjustment.

Appendix

Plugging (3) into (1) and rearranging yields

$$E_t\{y_{t+1}\} = \frac{\sigma+\psi}{\sigma} y_t - \frac{1}{\sigma} E_t\{\pi_{t+1}\} + \frac{\gamma}{\sigma}\pi_t - \frac{\psi}{\sigma} y_t^* + \frac{1}{\sigma}\nu_t. \tag{A.1}$$

Rearranging (2) gives

$$E_t\{\pi_{t+1}\} = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}(y_t - y_t^*). \tag{A.2}$$

Plugging this in (A.1) and collecting terms gives

$$E_t\{y_{t+1}\} = \frac{\beta(\sigma+\psi)+\kappa}{\sigma\beta} y_t + \frac{\beta\gamma-1}{\sigma\beta}\pi_t - \frac{\beta\psi+\kappa}{\sigma\beta} y_t^* + \frac{1}{\sigma}\nu_t. \tag{A.3}$$

Additionally, we can iterate (5) one period forward to obtain

$$y_{t+1}^* = (1-\eta)\rho_a y_t^* + \eta y_t + u_{t+1}^a. \tag{A.4}$$

We assume an autoregressive process for the exogenous monetary policy component according to

$$\nu_{t+1} = \rho_\nu \nu_t + u_{t+1}^\nu. \tag{A.5}$$

We can now summarize equations (A.2), (A.3), (A.4), and (A.5) compactly by the following matrix representation:

$$\begin{pmatrix} \nu_{t+1} \\ y_{t+1}^* \\ E_t\{y_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{pmatrix} = A \begin{pmatrix} \nu_t \\ y_t^* \\ y_t \\ \pi_t \end{pmatrix} + B \begin{pmatrix} u_{t+1}^\nu \\ u_{t+1}^a \end{pmatrix}, \tag{A.6}$$

where

$$A = \begin{pmatrix} \rho_\nu & 0 & 0 & 0 \\ 0 & (1-\eta)\rho_a & \eta & 0 \\ \frac{1}{\sigma} & \frac{(-\beta*\psi-\kappa)}{(\beta*\sigma)} & \frac{\beta*(\sigma+\psi)+\kappa)}{\sigma*\beta} & \frac{\beta*\gamma-1}{\sigma*\beta} \\ 0 & \frac{\kappa}{\beta} & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

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