Risk aggregation in non-life insurance
: Standard models vs. internal models

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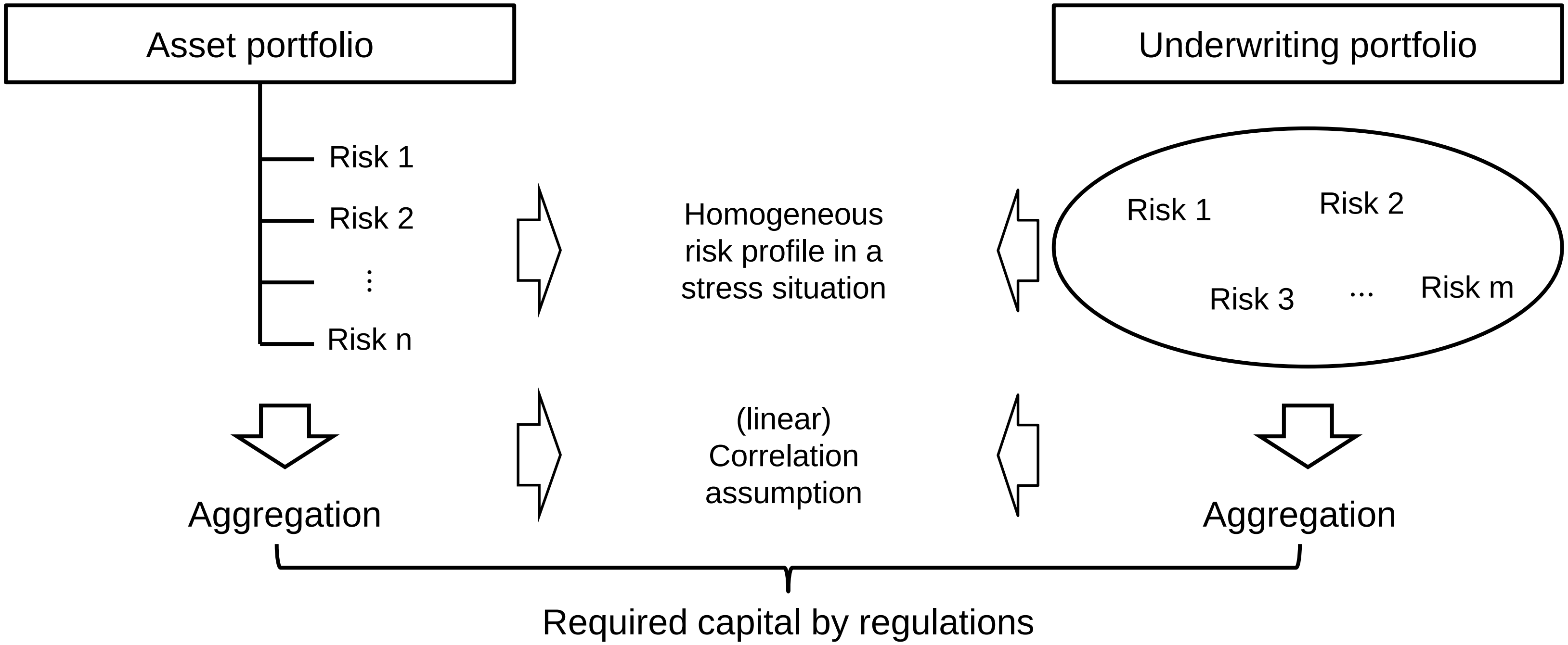
Motivation of the study

Motivation



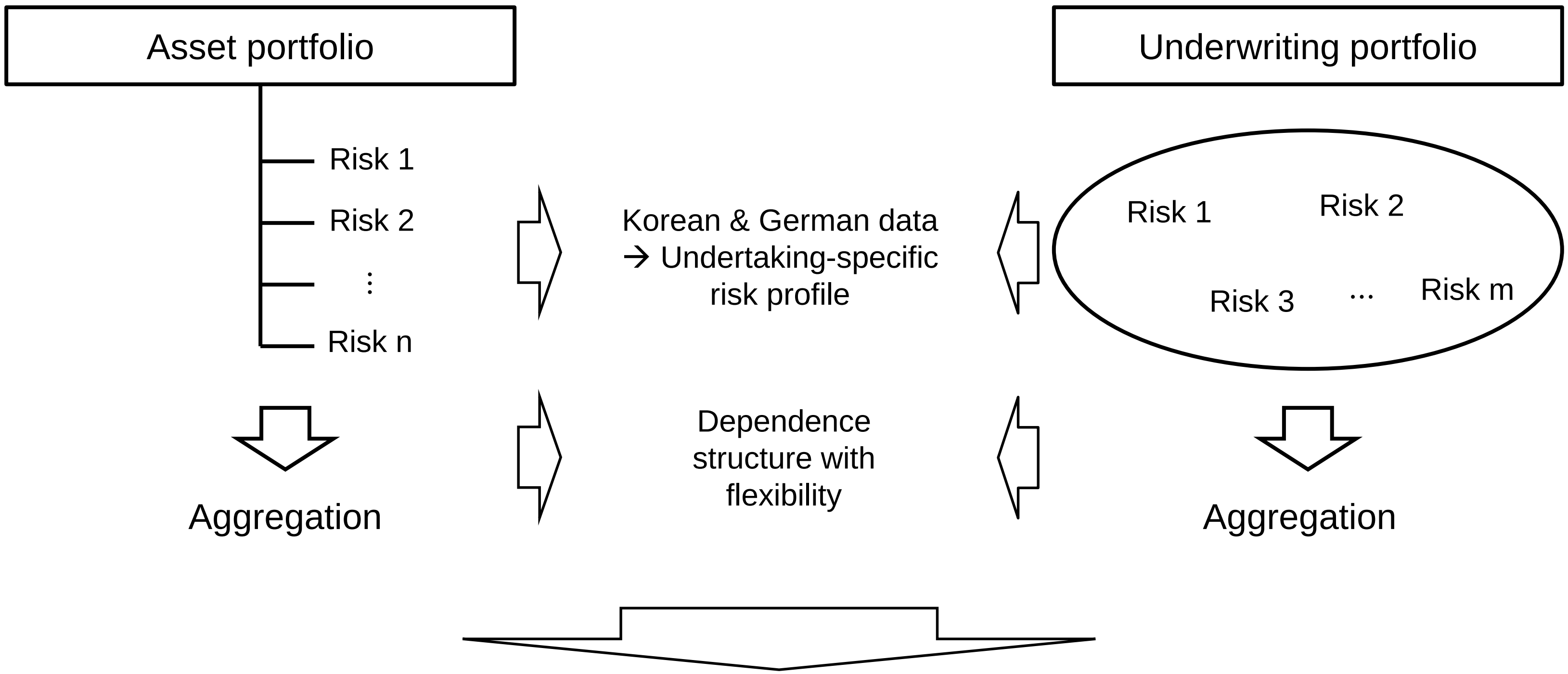
- Thus, there is a need to

  (1) construct an undertaking-specific model (internal model) with an efficient, comprehensive modeling

  (2) investigate how the impact of considering undertaking-specific risk profile and dependence structures of risk factors is on the risk measurement.

Objective and contribution of the study

Objective / Contribution



Comparison study with three regulatory frameworks:
Korean Risk-based Capital (RBC), **Solvency II** and **Swiss Solvency Test (SST)**

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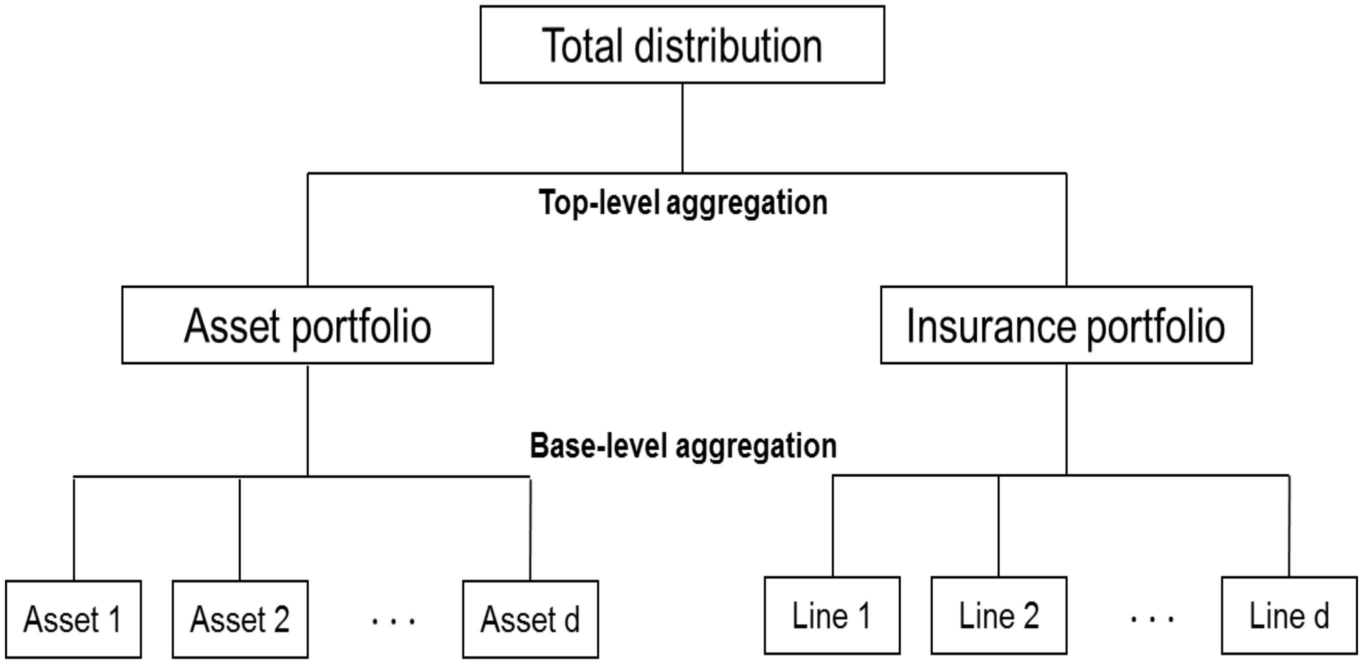
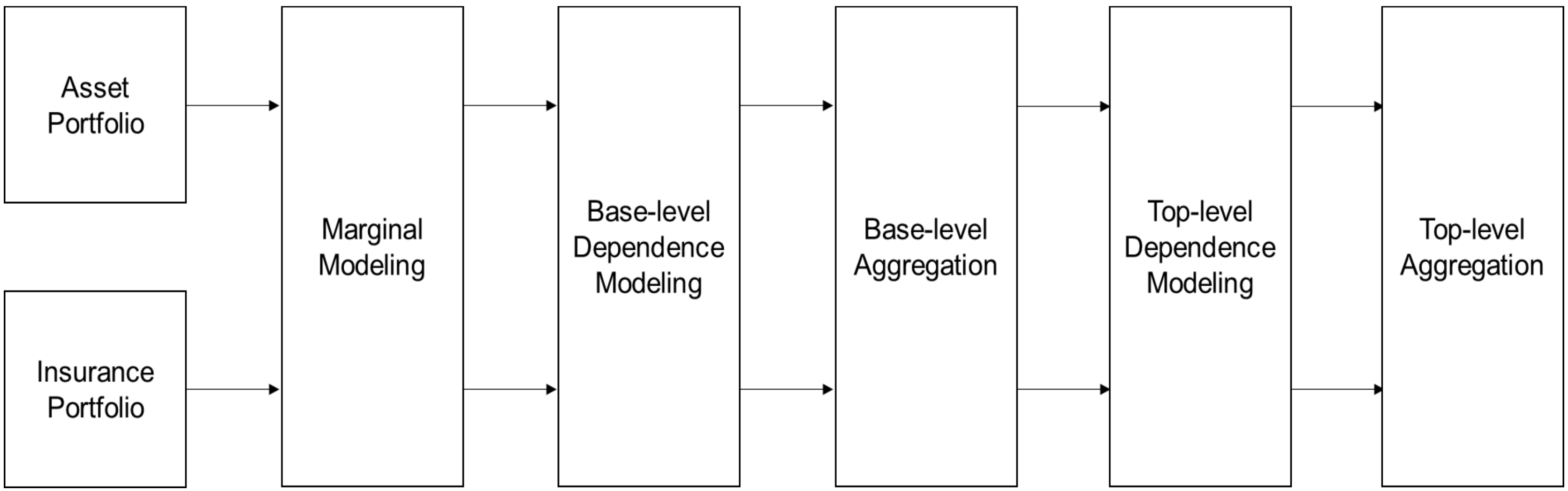
Methodology

Overview of the integrated structure



Methodology

Graphical description on the methodology



Methodology

Marginal modeling

- Asset data: ARMA-GARCH(1,1) with normal, student-t, skew-normal and skew-student
- Underwriting data: Risk modeling (distribution fitting) by testing student-t, skew-normal, skew-student, lognormal, gamma, Weibull, inverse Gaussian, Cauchy, Burr, GPD, POT with normal body and POT with lognormal body.

Dependence modeling

- Base-level (High-dimension): Elliptical copulas (Gaussian, Student-t), Simple Archimedean (Gumbel, Clayton), Hierarchical (nested) Archimedean (Gumbel, Clayton), Vine copula (R-Vine), Bernstein (D-Vine) and Independence.
- Top-level (Bivariate): Elliptical copulas (Gaussian, Student-t), Simple Archimedean (Gumbel, Clayton) and Independence.

Methodology

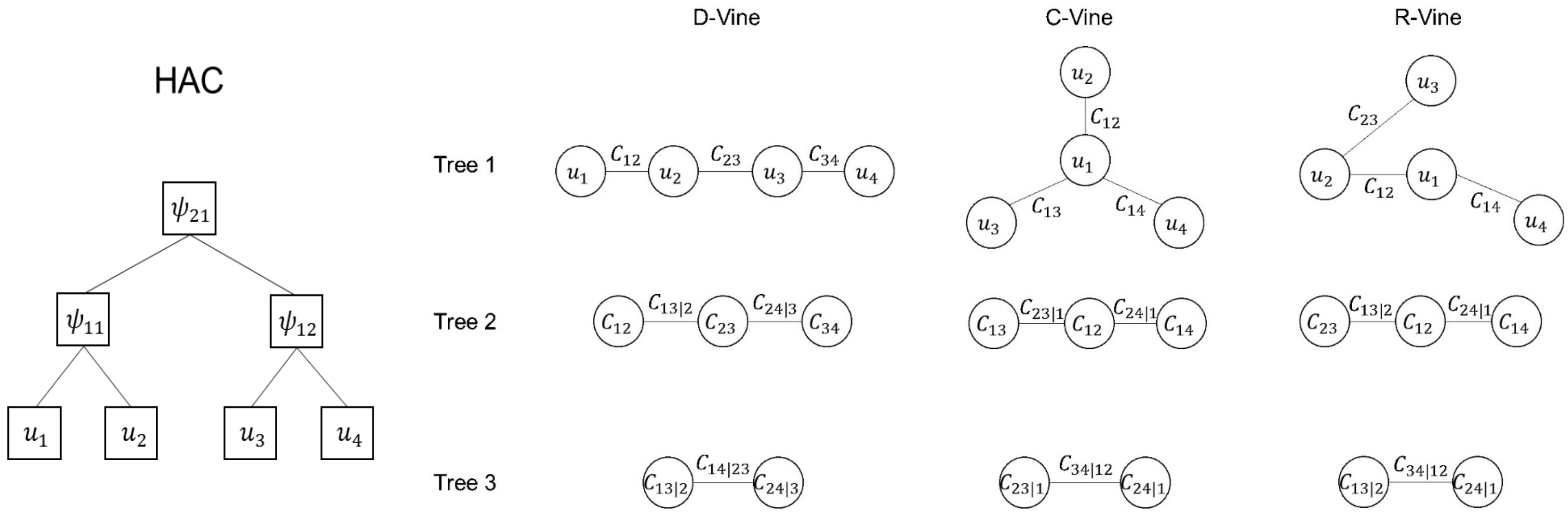
Dependence modeling

- List of dependence models used in the empirical study

Model	# of parameters	Pros	Cons
Gaussian	$\frac{d(d-1)}{2}$	• Easy to use and interpret. • Normally distributed margins.	• No tail dependence. • Limited to symmetric and linear dependency.
Student-t	$\frac{d(d-1)}{2}+1$	• Easy to use and interpret. • Tail dependence.	• Limited to symmetric dependency.
Archimedean	1	• Easy to construct. • A great variety of copula families representing diverse dependence structures exist.	• Difficult interpretation with a single parameter. • Exchangeability.
Hierarchical Archimedean Copulas (HAC)	$d-1$	• Easy to construct by using generating functions of Archimedean copulas. • A more accurate dependence structure by grouping more strongly correlated variables.	• Limited to Archimedean family. • Difficult interpretation. • Complicated form when different functions are used in a structure.
Pair Copula Construction (PCC)	$\frac{d(d-1)}{2}$	• A more accurate high dimensional dependency by pair-wise dependence modeling. • A variety of copulas. • Any type of dependence model (either hierarchical or flexible structure).	• Difficult interpretation. • Distribution function is not explicitly available.
Bernstein copula (D-Vine)	$\frac{d(d-1)}{2}\cdot(m+1)^2$, where *m* is the degree of Bernstein polynomial	• Advantageous when parametric copula functions are misspecified. • Flexible by controlling the polynomial degree.	• Still unknown parameter of optimal polynomial degree *m*. • Not parsimonious (i.e. $(m+1)^2$ parameters for each bivariate case in D-Vine).

Methodology

Dependence modeling (Base-level) – Graphical structures



Methodology

Regulatory frameworks (Korean Risk-Based Capital)

$$Required\ capital = \sqrt{\sum_{i=1} \sum_{j=1} Corr_{i,j} \times Risk_i \times Risk_j},$$

- Factor-based approach
- Risks are estimated by Value-at-Risk 99% with risk coefficients and calibrated parameters for risk thresholds.
- Correlation matrices are given to between risk modules and between risk factors only within the underwriting module (No assumption on the correlation in the market risk module).
- Market risk capital is calculated by the simple summation of individual risk levels.

Methodology

Regulatory frameworks (Solvency II)

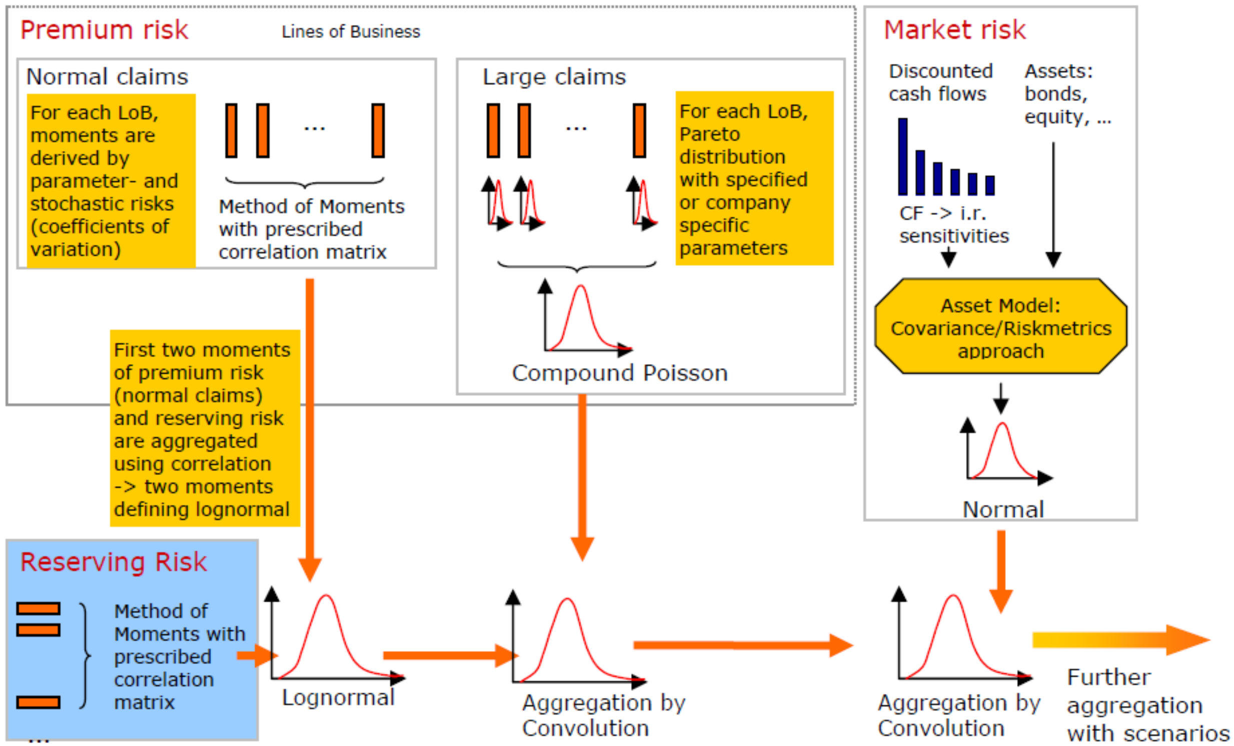
$$Required\ capital = \sqrt{\sum_{i=1}\sum_{j=1} Corr_{i,j} \times SCR_i \times SCR_j}\,,$$

- Factor-based approach
- SCRs are estimated by Value-at-Risk 99.5% with calibrated parameters for risk thresholds.
- Correlation matrices are given to between risk modules and between risk factors within each risk module.
- The correlation matrix for the market risk module is differentiated by the interest rate shock (up and down turns)

Methodology

Regulatory frameworks (Swiss Solvency Test)

- Model-based approach
- Risks are estimated by tail Value-at-Risk (TVaR) 99%



Source: White paper of the Swiss Solvency Test (Swiss Federal Office of Private Insurance, 2004)

Data description

Korean (In-sample estimation): between Jan 2002 and Dec 2016

- Asset: seven benchmark monthly indices
- Underwriting: monthly aggregate claims from five lines of business

Panel A: Asset portfolio

	mean	sd	skewness	kurtosis	Max	median	min	JB-test
KR_stock (MSCI)	0.0081	0.0771	-0.3898	1.1737	0.2341	0.0087	-0.3028	15.81***
KR2Y	-0.0001	0.0118	0.0256	5.3170	0.0610	0.0019	-0.0444	219.55***
KR5Y	0.0001	0.0173	-0.5500	2.1224	0.0569	0.0030	-0.0615	44.87***
KR10Y	-0.0002	0.0252	-0.4863	2.8502	0.1039	0.0038	-0.0835	70.98***
KRcor	-0.0066	0.0528	0.2558	1.1983	0.1970	-0.0098	-0.1474	13.63***
KR3MCD	-0.0065	0.0506	-2.6660	13.0787	0.1130	0.0000	-0.3270	1,535.30***
Wrd_real (MSCI)	0.0071	0.0564	-1.3226	6.7998	0.2050	0.0129	-0.3244	411.43***

Panel B: Insurance portfolio *(billion Korean Won)*

	mean	sd	skewness	kurtosis	Max	median	min
Fire	10.286	7.150	0.957	1.058	35.259	9.796	0.611
Motor	1,197.732	713.675	0.464	-0.437	3,288.883	1,171.932	117.852
Marine	45.707	31.177	0.494	-0.793	123.084	39.943	1.827
Liability	24.113	16.376	0.886	0.399	74.868	21.304	1.548
Accident	82.782	69.597	0.823	-0.460	272.329	58.581	1.578

Data description

German (Out-of-sample estimation and Robustness check): between Jan 1998 and Dec 2006

- Asset: ten benchmark monthly indices
- Underwriting: monthly aggregate claims from six lines of business

Panel A: Asset portfolio

	Mean	sd	skewness	kurtosis	Max	median	min	JB-test
Wrd_stock (MSCI)	0.005	0.041	-0.679	0.673	0.089	0.009	-0.140	11.007***
EMU_stock (MSCI)	0.007	0.054	-0.715	1.136	0.121	0.019	-0.170	16.085***
DE_stock (MSCI)	0.007	0.069	-0.810	2.845	0.213	0.010	-0.279	51.446***
US2Y	-0.001	0.100	0.149	2.337	0.378	0.003	-0.301	27.139***
DE2Y	-0.001	0.069	0.078	-0.319	0.177	-0.002	-0.136	0.436
EMU2Y	0.003	0.009	-0.209	-0.616	0.025	0.004	-0.017	2.275
IBOXX corp	0.003	0.009	-0.173	-0.320	0.024	0.004	-0.022	0.880
EURIBOR3M	-0.000	0.044	-0.452	2.279	0.135	0.001	-0.151	29.245***
Wrd_real (MSCI)	0.009	0.052	0.156	1.860	0.200	0.012	-0.124	17.579***
Euro_real (MSCI)	0.011	0.043	-0.322	0.593	0.136	0.012	-0.102	3.883

Panel B: Insurance portfolio *(€ euro)*

	mean	sd	skewness	kurtosis	Max	median	min
Ind_fire	803,228.2	1,240,581.2	2.521	6.028	6,186,363.1	295,276.2	949.3
HO_fire	2,228,594.6	991,556.9	-0.238	0.042	4,663,559.9	2,251,845.3	25,119.4
Other_fire	1,313,010.5	917,484.8	1.054	0.543	4,146,761.6	984,767.3	3,384.7
HH_storm	44,862.9	74,288.1	4.431	26.823	594,514.1	18,223.3	0.0
HO_storm	1,833,345.2	3,990,165.4	6.633	53.926	37,075,463.2	462,420.4	5,411.4
Water	4,007,250.9	1,445,809.7	-1.097	2.166	7,930,423.2	4,277,080.0	101,092.1

Empirical Results: Dependence modeling

Marginal modeling: Asset portfolio

- **Korean data**

	ARMA-order	AIC for ARMA	Fitted distribution for innovation	AIC for GARCH(1,1)
KR_stock	(0,3)	-416.37	Skew normal	-439.28
KR2Y	(3,3)	-1,119.34	Student-t	-1,171.82
KR5Y	(3,2)	-993.47	Student-t	-1,000.89
KR10Y	(3,1)	-835.83	Skew student	-841.92
KRcor	(0,1)	-552.81	Student-t	-557.40
KR3MCD	(0,3)	-607.65	Student-t	-696.40
Wrd_real	(2,3)	-540.72	Skew normal	-591.04

- **German data**

	ARMA-order	AIC for ARMA	Fitted distribution for innovation	AIC for GARCH(1,1)
Wrd_stock	(0,0)	-379.11	Skew normal	-396.16
EMU_stock	(2,2)	-322.21	Skew student	-351.18
DE_stock	(0,0)	-269.68	Skew student	-282.58
US2Y	(0,0)	-188.98	Student-t	-228.81
DE2Y	(2,1)	-285.28	Normal	-263.37
EMU2Y	(1,1)	-735.19	Skew normal	-706.72
IBOXX corp	(3,0)	-729.12	Normal	-720.41
EURIBOR3M	(3,2)	-385.23	Skew student	-400.88
Wrd_real	(3,1)	-355.91	Skew student	-346.53
Euro_real	(0,1)	-380.42	Skew normal	-367.99

Empirical Results: Dependence modeling

Marginal modeling: Underwriting portfolio

- **Korean data**

	Fire	Motor	Marine	Liability	Accident
Skew Normal	6,347.08 (0.447***)	>10,000 (0.999***)	7,294.23 (0.499***)	6,657.01 (0.452***)	9,913.13 (0.565***)
Student-t	6,261.55 (0.420***)	8,138.76 (0.744***)	6,818.89 (0.451***)	6,716.72 (0.743***)	7,098.29 (0.743***)
Skew student-t	6,148.62 (0.080)	7,845.05 (0.113**)	6,684.81 (0.059)	6,452.62 (0.068)	6,924.53 (0.115**)
Lognormal	6,175.85 (0.131***)	7,860.66 (0.115**)	6,722.82 (0.103**)	6,466.80 (0.096*)	6,938.39 (0.084)
Gamma	6,148.70 (0.089)	7,834.37 (0.081)	6,690.58 (0.064)	6,445.75 (0.045)	6,922.64 (0.065)
Weibull	**6,145.10 (0.058)**	**7,826.92 (0.051)**	6,685.07 (0.056)	**6,443.84 (0.034)**	**6,922.35 (0.066)**
Inverse Gaussian	6,193.25 (0.185***)	>10,000 (0.999***)	6,747.59 (0.172***)	6,481.98 (0.148***)	>10,000 (0.999***)
Cauchy	6,251.24 (0.149***)	7,934.52 (0.142***)	6,810.38 (0.166***)	6,548.92 (0.153***)	7,071.04 (0.213***)
Burr	7,057.71 (0.570***)	8,894.04 (0.593***)	7,619.40 (0.566***)	7,395.66 (0.574***)	7,799.34 (0.565***)
GPD	6,153.91 (0.081)	7,838.88 (0.080)	**6,671.22 (0.030)**	6,454.96 (0.066)	6,915.91 (0.092*)
POT 90% (Norm-GPD)	6,360.01 (0.445***)	9,625.44 (0.825***)	6,907.21 (0.438***)	6,666.27 (0.447***)	7,147.97 (0.438***)
POT 90% (Lognorm-GPD)	7,124.12 (0.786***)	8,962.02 (0.804***)	7,687.81 (0.728***)	7,466.74 (0.725***)	7,866.29 (0.735***)

- **German data**

	Ind_fire	HO_fire	Otr_fire	HH_stm	HO_stm	Water
Skew Normal	3,359.63 (0.426***)	3,452.52 (0.549***)	3,369.76 (0.428***)	2,732.08 (0.274***)	3,604.26 (0.426***)	3,572.18 (0.600***)
Student	3,145.12 (0.683***)	3,516.66 (0.637***)	3,390.33 (0.693***)	2,795.55 (0.487***)	3,295.83 (0.655***)	3,635.47 (0.633***)
Skew student	**3,128.23 (0.056)**	3,337.36 (0.309***)	3,246.36 (0.109)	2,514.79 (0.075)	3,292.71 (0.093)	3,457.85 (0.441***)
Lognormal	3,137.60 (0.103)	3,391.84 (0.267***)	3,285.51 (0.148**)	2,529.24 (0.077)	**3,289.55 (0.095)**	3,520.44 (0.377***)
Gamma	3,140.02 (0.133**)	3,337.01 (0.210***)	3,247.31 (0.087)	2,514.24 (0.079)	3,310.62 (0.164***)	3,455.35 (0.345***)
Weibull	3,134.27 (0.100)	3,311.14 (0.141**)	**3,244.18 (0.068)**	2,509.65 (0.073)	3,300.03 (0.149**)	3,409.59 (0.282***)
Inverse Gaussian	3,256.48 (0.411***)	3,462.41 (0.428***)	3,418.60 (0.421***)	2,958.41 (0.776***)	3,337.78 (0.213***)	3,582.77 (0.506***)
Cauchy	3,203.37 (0.220***)	3,321.02 (0.078)	3,289.30 (0.139**)	2,606.33 (0.252***)	3,381.99 (0.264***)	**3,309.34 (0.062)**
Burr	3,499.17 (0.509***)	3,907.38 (0.540***)	3,759.83 (0.540***)	2,813.60 (0.501***)	3,669.61 (0.529***)	4,048.19 (0.536***)
GPD	3,128.36 (0.070)	3,315.54 (0.240***)	3,247.90 (0.114)	**2,509.01 (0.061)**	3,289.94 (0.114)	3,430.96 (0.369***)
Norm-GPD	3,267.40 (0.215***)	**3,296.22 (0.070)**	3,266.07 (0.160***)	2,639.78 (0.201***)	3,451.95 (0.220***)	3,371.74 (0.246***)
LN-GPD	3,513.02 (0.657***)	3,403.94 (0.389***)	3,797.46 (0.741***)	2,552.05 (0.207***)	3,709.15 (0.694***)	4,085.97 (0.742***)

Empirical Results: Dependence modeling

Decision Criteria and Goodness-of-fit (Korean data)

- **Base-level**

		Asset Portfolio			Insurance Portfolio		
Family	Copula	Log-lik	AIC	GoF	Log-lik	AIC	GoF
Elliptical	Gaussian	164.04	-286.07	0.018**	619.23	-1,218.45	0.144***
	Student-t	191.07	-338.14	0.012	620.75	-1,219.49	0.114***
Archimedean	Gumbel	0.183	1.634	0.041***	344.48	-686.96	0.810***
	Clayton	3.617	-5.234	0.025*	418.30	-834.59	2.735***
HAC	Gumbel	109.28	-206.56	0.082*	474.65	-941.30	0.039**
	Clayton	99.43	-186.87	0.082*	488.22	-968.44	0.039**
PCC	**R-Vine**	**210.81**	**-377.62**	**161.03**	**660.53**	**-1,303.06**	**75.44**
Bernstein (D-Vine)		207.70	-60.47	-	638.96	-504.67	-

- **Top-level**

		Statistics	P-value
Independence test		0.8586	0.3906
Elliptical	Gaussian copula	0.3860**	0.0400
	Student-t copula	0.3893**	0.0300
Archimedean	Gumbel copula	0.3907**	0.0200
	Clayton copula	0.3952***	0.0000

Empirical Results: Dependence modeling

Decision Criteria and Goodness-of-fit (German data)

- **Base-level**

		Asset Portfolio			Insurance Portfolio		
Family	Copula	Log-lik	AIC	GoF	Log-lik	AIC	GoF
Elliptical	Gaussian	269.35	-448.70	0.033**	175.74	-321.47	0.119***
	Student-t	285.41	-478.82	0.024	175.75	-319.49	0.100***
Archimedean	Gumbel	48.86	-95.72	0.036**	76.87	-151.73	0.230***
	Clayton	68.03	-134.07	0.022*	122.78	-243.56	0.055
HAC	Gumbel	243.39	-468.78	0.175	142.77	-275.54	0.642
	Clayton	208.74	-339.48	0.875	162.36	-314.73	0.759
PCC	**R-Vine**	**311.27**	**-580.53**	**71.36**	**203.00**	**-374.00**	**56.43**
Bernstein (D-Vine)		310.30	-32.34	-	181.88	-73.72	-

- **Top-level**

		Statistics	P-value
Independence test		0.7593	0.4477
Elliptical	Gaussian copula	0.2334	0.1300
	Student-t copula	0.2782	0.1500
Archimedean	Gumbel copula	0.2356*	0.0900
	Clayton copula	0.2714**	0.0400

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Applications

Capital requirement (Korean data)

(in ₩ trillion)			Top-level dependence (Bivariate) Indep	Gauss	t	Gumbel	Clayton
Base-level dependence (High-dimension)	VaR at 99%	R-Vine	**25.33**	25.61	25.62	25.54	25.50
		Gauss	25.19	25.40	25.43	25.37	25.33
		t	25.59	25.82	25.83	25.78	25.72
		HAC-best	24.41	24.73	24.75	24.64	24.61
		HAC-worst	27.03	27.27	27.27	27.22	27.18
		Independence	20.71	20.97	21.01	20.91	20.88
		Bernstein	22.10	22.19	22.14	22.18	22.12
	VaR at 99.5%	R-Vine	**28.87**	29.21	29.33	29.45	29.05
		Gauss	28.62	28.93	29.03	29.07	28.83
		t	29.38	29.67	29.80	29.84	29.58
		HAC-best	27.22	27.38	27.50	27.54	27.32
		HAC-worst	30.91	31.25	31.38	31.46	31.10
		Independence	23.63	23.77	23.85	24.01	23.70
		Bernstein	25.07	25.24	25.29	25.27	25.17
	TVaR at 99%	R-Vine	**29.32**	29.60	29.64	29.72	29.55
		Gauss	28.78	29.06	29.10	29.13	28.98
		t	29.67	29.97	30.00	30.07	29.89
		HAC-best	27.73	27.97	27.97	28.03	27.91
		HAC-worst	31.08	31.37	31.42	31.48	31.30
		Independence	24.07	24.33	24.32	24.39	24.29
		Bernstein	25.24	25.52	25.56	25.53	25.39

	K-RBC (VaR 99%)	SII (VaR 99.5%)	SST (TVaR 99%)
Regulations (in ₩ trillion)	39.07	38.12	35.34

Applications

Capital requirement (German data)

(in € million)			Top-level dependence (Bivariate): Indep	Gauss	t	Gumbel	Clayton
Base-level dependence (High-dimension)	VaR at 99%	R-Vine	**242.72**	259.60	267.97	270.56	267.97
		Gauss	218.98	237.95	247.69	250.03	247.69
		t	224.38	240.81	248.52	250.68	248.52
		HAC-best	152.89	168.96	179.42	182.46	179.42
		HAC-worst	283.33	299.01	306.44	309.16	306.44
		Independence	46.41	69.68	79.05	81.33	67.79
		Bernstein	297.03	301.44	305.89	307.05	299.10
	VaR at 99.5%	R-Vine	**411.32**	415.71	416.93	422.23	413.48
		Gauss	388.33	392.05	392.95	399.76	389.09
		t	401.40	407.06	407.57	414.53	404.65
		HAC-best	307.97	311.76	312.00	315.89	309.58
		HAC-worst	477.37	480.44	484.26	494.37	476.84
		Independence	203.66	207.25	207.70	212.18	205.13
		Bernstein	355.50	363.49	363.11	365.52	363.39
	TVaR at 99%	R-Vine	**417.22**	420.57	423.64	428.45	417.48
		Gauss	397.17	401.04	404.58	409.43	398.66
		t	412.50	416.45	420.26	425.55	413.56
		HAC-best	310.63	314.21	317.43	321.15	311.99
		HAC-worst	486.15	490.43	494.24	500.88	487.66
		Independence	204.38	207.38	210.24	213.99	205.80
		Bernstein	374.92	374.74	375.98	379.79	371.96

	K-RBC (VaR 99%)	SII (VaR 99.5%)	SST (TVaR 99%)
Regulations (in € million)	613.63	600.35	561.67

Conclusion

Findings and Implications

- R-Vine copula model with flexible choice on dependence functions is superior to any other high-dimensional models for both Korean and German datasets.
- 35% overestimation on average among three regulatory frameworks takes place.
- In the size of the overestimation, 18 percentage points out of 35 result from the correlation assumption and 17 points from the calibrated risk parameters.
- Specifically, Korean RBC generates the highest gap between the estimated model and the standard model among three regulations, which we conclude might come from no diversification in the market risk module and less categorization in the underwriting module.
- The size of the overestimation can be larger for small- and mid-sized company.

Limitations and Future research

- Life underwriting module
- Other risk modules in SII, e.g., operational risk

Thank you for your attention!

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Applications (Back-up)

Capital requirement: asset allocation

Panel A: Asset allocations

		Allocation 1	Allocation 2
Equity	KR_stock	10%	30%
Fixed income	KR2Y	20%	10%
	KR5Y	20%	10%
	KR10Y	20%	10%
	KRcor	15%	10%
Money Market	KR3MCD	0%	0%
Real estate	Wrd_real	15%	30%

Panel B: Application to the economic capital (in ₩ trillion)

Base-level dependence		Top-level dependence: Indep	Gauss	t	Gumbel	Clayton	K-RBC
Allocation 1 (VaR 99%) • μ=4.09% • σ=25.06% • RP=0.46%	PCC	**26.42**	26.69	26.88	26.74	26.46	36.82
	Gauss	26.45	26.71	26.85	26.73	26.50	
	t	26.66	26.95	27.12	26.95	26.70	
	HAC-best	25.34	25.61	25.81	25.71	25.39	
	HAC-worst	27.90	28.16	28.31	28.16	27.93	
	Independence	21.91	22.15	22.36	22.24	21.95	
	Bernstein	22.45	22.60	22.59	22.48	22.46	
		Indep	Gauss	t	Gumbel	Clayton	K-RBC
Allocation 2 (VaR 99%) • μ=5.04% • σ=28.02% • RP=0.60%	PCC	**27.53**	27.74	27.75	27.74	27.60	42.98
	Gauss	27.32	27.47	27.51	27.53	27.39	
	t	27.61	27.80	27.82	27.85	27.70	
	HAC-best	26.22	26.45	26.48	26.45	26.28	
	HAC-worst	28.61	28.78	28.80	28.83	28.68	
	Independence	23.31	23.54	23.50	23.48	23.36	
	Bernstein	26.23	26.23	26.25	26.25	26.24	

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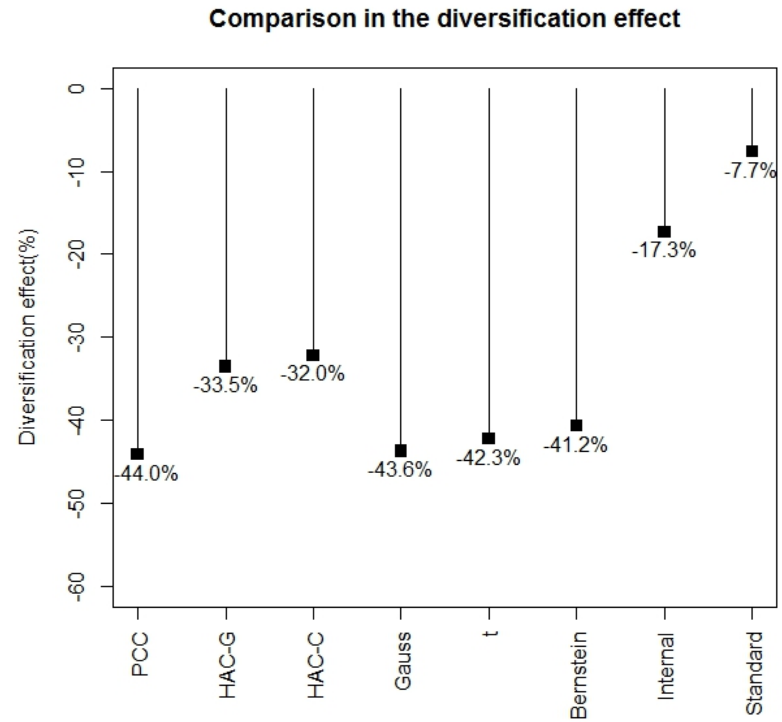
Applications: Diversification effect

Diversification Effect (Korean data) for the asset portfolio

$$div = \frac{SCR\left(\sum_{i=1}^{d} X_i\right) - \sum_{i=1}^{d} SCR_i}{\sum_{i=1}^{d} SCR_i},$$

where $SCR\left(\sum_{i=1}^{d} X_i\right)$ is the solvency capital requirement of a portfolio with d-dimensional risks and $\sum_{i=1}^{d} SCR_i$ is the sum of individual SCRs for the portfolio.



- "Internal" indicates the diversification benefit from the internal model in Gatzert and Martin (2012) and "Standard" from the standard model in the same paper.