Essays on Premium Payment Options, Liquidity Risk and Behavioural Insurance

DISSERTATION

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Hsiaoyin Chang

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Summary

This doctoral thesis is a collection of four essays covering two main topics. The first two essays relate to the lapse risk, which is caused by the embedded premium payment options in participating life contracts and is especially important during the current low-interest environment. The remaining two essays focus on "on-demand" insurance, an innovative insurtech product. Using experimental data, the essays investigate how reduced-duration insurance changes customers' risk attitudes.

Participating life contracts, which are popular throughout Europe, typically include a cliquet-style option (i.e., a guaranteed yearly interest rate) together with a profit-sharing scheme. These contracts are generally legally required to offer certain premium payment options. This feature predetermines the surrender amount and the adjusted benefit. Hence, premium payment options may come into money and can be of great value if policyholders use them strategically. In the current low-interest environment, this potential risk should be addressed as policyholders exercise their premium payment options with increasing option value once the interest rate rebounds. The first essay examines three different options – paid-up options, resumption options, and surrender options – as well as the combinations among them. Using an extended least square Monte Carlo (LSMC) method to approximate an optimal exercise strategy, these premium payment options increase as interest volatility grows while this increase in value mainly comes from the surrender options. The second essay then combines the liquidity risk and the lapse risk. As these two risks are positively correlated, the lapse risk can be underestimated if it is estimated without considering liquidity risk.

As technology advances, on-demand insurance products are developed to cover risk in a short-term period. A reduced-period insurance plan grants policyholders the freedom to choose when to be insured in a flexible manner. This insurance product may stimulate a decision maker's (DM's) risk aversion as the product reduces their risk evaluation periods. The third essay constructs an extended cumulative prospect theory model to capture this excess risk aversion with two behavior biases: myopic loss aversion and probability miscalculation. The experiment result shows that these biases stimulated by short-term insurance are only obvious when the risk is salient. Otherwise, DMs can make risk-seeking decisions as the evaluation periods are combined. In the fourth essay, the same experiment data set is used to test five hypotheses in order to analyze if numerical literate participants demonstrate fewer decision biases. This essay further examines if more time spent making decisions improves the decision quality.

Zusammenfassung

Die vorliegende Doktorarbeit ist eine Sammlung von vier Aufsätzen zu zwei Hauptthemen. Die ersten beiden Aufsätze beziehen sich auf das Ausfallrisiko eines Lebensversicherers, das durch die Prämienzahlungsoptionen mit Überschussbeteiligung verursacht wird und im aktuellen Niedrigzinsumfeld von besonderer Bedeutung ist. Die verbleibenden zwei Aufsätze konzentrieren sich auf eine On-Demand-Versicherung – ein innovatives Insurtech-Produkt. Anhand experimenteller Daten untersuchen die beiden Beiträge wie das Risikoverhalten von Versicherungskunden verändert werden kann, indem ihnen eine alternative Versicherung mit reduzierter Laufzeit angeboten wird.

Lebensversicherungen mit Überschussbeteiligung, die in ganz Europa beliebt sind, um- ¨ fassen typischerweise eine Option im Cliquet-Stil; dabei wir eine garantierte jährliche Mindestverzinsung für die Versicherungsnehmer durch eine Gewinnbeteiligung an die Eigenkapitalgeber finanziert. Diese Verträge sind im Allgemeinen gesetzlich verpflichtet, bestimmte Prämienzahlungsoptionen anzubieten. Diese Optionen legen den Rückkaufswert und die angepasste Leistung fest. Daher können "Im Geld" – Prämienzahlungsoptionen von grossem Wert sein, wenn die Versicherungsnehmer sie strategisch einsetzen. Im gegenwärtigen Niedrigzinsumfeld sollte dieses potenzielle Risiko in den Fokus rücken, da Versicherungsnehmer dazu neigen ihre Prämienzahlungsoptionen auszuüben, sobald die Zinsen steigen. Der Aufsatz untersucht drei verschiedene Optionen – Prämienzahlungs -einstellung, Prämienwiederaufnahme, und Vertragsstorno – sowie die Kombinationen zwischen diesen. Mit der erweiterten LSMC-Methode (Least Square Monte Carlo) zur Annäherung an eine optimale Ausübungsstrategie zeigen wir auf, dass der Wert dieser Prämienzahlungsoptionen und insbesondere das Rückkaufsrecht mit zunehmender Zins -volatilität steigt. Der zweite Beitrag kombiniert dann das Liquiditäts- mit dem Ausfallrisiko. Da diese beiden Risiken positiv korreliert sind, kann das kombinierte Risiko unterschätzt werden, wenn jedes Risiko separat betrachtet wird.

Mit fortschreitender Technologie werden On-Demand-Versicherungsprodukte entwickelt, um Risiken kurzfristig abzudecken. Ein befristeter Versicherungsplan gibt den Versicherungs - nehmern die Freiheit flexibel zu entscheiden, wann sie versichert sein möchten. Ein solches Versicherungsprodukt kann die Risikoaversion eines Entscheidungsträgers (DM) stimulieren, da das Produkt Risikobewertungszeiträume verkürzt. Der dritte Aufsatz konstruiert ein erweitertes Cumulative Prospect Theory Modell um diese ausgeprägte Risikoaversion mit zwei Verhaltensverzerrungen zu erfassen: Kurzsichtige Verlustaversion und Wahrscheinlichkeitsfehlkalkulation. Das experimentelle Versuchsergebnis zeigt, dass diese durch kurzfristige Versicherungen stimulierten Effekte nur dann wesentlich sind, wenn das Risiko besonders ausgeprägt ist.

Andernfalls können DMs risikofreudige Entscheidungen treffen, da die DM die Bewertungszeiträume kombinieren und diese so länger als die Versicherungsvertragszeitraum des traditionellen Produkts werden. Im vierten Beitrag werden mit demselben Experimentdatensatz fünf Hypothesen getestet um zu analysieren, ob Personen mit guten quantitativen Kenntnissen weniger Entscheidungsverzerrungen aufweisen. Weiter wird geprüft, ob mehr Zeit für Entscheidungen die Entscheidungsqualität verbessert.

Essay I

Should I stay or should I go? Multiple premium-payment option valuation for participating life insurance contracts

Hsiaoyin Chang, Hato Schmeiser

Participating life insurance contracts with investment guarantees typically possess various embedded options. In this paper, we focus on common options with early exercise features, i.e., paid-up options, resumption options, surrender options and the combinations among them. The valuation of these options depends strongly on the exercise strategy applied by the policyholder. We extend the Least Square Monte Carlo (LSMC) method to approximate an optimal exercise strategy and to determine multiple optimal exercise points while incorporating two stochastic sources (asset and interest rate risk). In addition, these findings are compared to results based on two other exercise assumptions. In contrast to earlier studies on this topic, we show in our model setup that premium-payment options can pose a severe risk for life insurers if not priced properly and hence are of great value for policyholders if used strategically. Moreover, we demonstrate that introducing fees for exercising premium-payment options only slightly reduces the option price necessary to finance adequate risk management measures.

Keywords: Participating life insurance contracts · Embedded option pricing · Stochastic interest rate · Risk-neutral valuation · Least Square Monte Carlo

1.1 Introduction

Participating life insurance contracts with investment guarantees are typically offered with several embedded options. This paper focuses on premium-payment options with early exercise features found in essentially all life insurance contracts with investment guarantees. These options are provided in various forms: A paid-up option allows policyholders to stop premium payments while the main contract continues with adjusted benefits. A resumption option permits policyholders to resume payments after the paid-up option has been exercised (benefits again will be adjusted accordingly). With a surrender option, policyholders can terminate their contracts and receive a surrender amount. With a combined paid-up and surrender option, policyholders may surrender their policy with or without previously exercising the paid-up option. The combination of all three of these options allows policyholders to exercise paid-up, resumption, and surrender options during the contract period.

Popular all over Europe, participating life contracts typically include a cliquet-style option, i.e., a guaranteed yearly interest rate together with a profit sharing scheme. With this feature, the surrender amount and the adjusted benefit are predetermined. These predetermined values may exceed actual market values. Hence, the premium-payment options may come into money and can be of great value if policyholders use them strategically. In the current low-interest rate environment, insurance companies are particularly struggling with high long-term interest guarantees, which they previously provided to their policyholders. The situation for the insurers can be even more problematic, as policyholders tend to exercise their surrender or paid-up options once the interest rate rebounds (cf. Feodoria and Förstemann (2015)). Specifically, the exercise behaviour in respect to premium-payment options and, hence, these option values, depend on future interest rate developments. If the options are not priced adequately and hence no proper risk management has taken place, insurance companies may encounter severe difficulties (cf. the cases of Equitable Life in 2000 or the Hartford in 2009). Current solvency regulation schemes, such as Solvency II in the EU, require insurers to consider lapse risk and offer proper risk management and equity capital for options provided to their customers. Proper models for the premium-payment option valuation and the related risk assessment are thus essential for life insurance companies and should be conducted with care.

Besides the insurers' perspective, the aim of this paper is to provide a general framework to policyholders in order to give them the opportunity to use the contracts' embedded options in an optimal way. For instance, the proposed model could be installed on-line and simply used by inserting individual contract data (e.g. premiums, time to maturity, level of interest rate guarantee, and exercising fees).[1] As we show in this paper, the present value of premium payment options in participating life insurance contracts with cliquet-style investment guarantees can easily take up to 5% of the present value of all policyholders' premium payments made in the treaty. If insurance companies base their pricing on the option pricing theory and policyholders do not exercise their premium payment options in an optimal manner, a severe wealth transfer takes part to the disadvantages of the insured. This aspect becomes of large general interest taking into account the premium volumes of this kind of products: According to European Insurance and Occupational Pensions Authority (EIOPA), around 68% of the gross premiums written in 2018 (814.72 billion Euro) within the states of the European Union are invested in products of the type focused in this paper.[2] Hence, even a small percentage of the premium volume caused by a suboptimal exercise strategy sums up to a large amount of money per year lost for old-age provision. The empirical research on policyholders' option exercising behaviours strongly supports the assumption that premium payment options are often used in a suboptimal manner.[3]

To the best of our knowledge, this paper is the first to provide a valuation of multiple premium-payment options with two stochastic risk sources (assets and interest rates). To solve the underlying optimal stopping problems, we develop an extended version of the Least Square Monte Carlo (LSMC) approach. In contrast to an earlier paper on this topic (cf. Schmeiser and Wagner (2011)) with deterministic term structure, our paper shows that the price for multiple premium-payment options is substantial and can cause a severe risk for life insurance companies. However, additional options possess rather little values, regardless of the underlying market risk: That is, a triple combined paid-up, resumption, and surrender option has almost the same value as a double combined paid-up and surrender option, while this double option values only little more than a single surrender option.[4] This surrender option value enlarges strongly with increasing interest rate risk under the optimal-exercise assumption as firstly concluded in Zaglauer and Bauer (2008). We further demonstrate that introducing fees for exercising premium-payment options – as it is commonly done in insurance practice – only slightly reduces the option price necessary to finance adequate risk management measures. This finding stands in contrast to Schmeiser and Wagner (2011) for a deterministic term structure. In the setting of Schmeiser and Wagner (2011), even low fees can eliminate the risk for the insurer and hence no additional payments from the policyholders' side are necessary for the embedded premium-

[1] Fron another perspective, if many policyholders use their option optimally, it could lead to a massive simultaneous lapse behavior, causing instability to the life insurance sector.

[2] Source from https://eiopa.europa.eu – Insurance Statistics.

[3] For an overview cf. Eling and Kochanski (2013).

[4] One can argue that focusing on the pricing of surrender option in this form of participating life insurance contracts is in general sufficient and has been provided in the literature already. However, we think it is an important finding, showing that other premium payment options typically cannot be exercised optimally in the realistic case of multiple premium payment options and hence possess — in contrast to an isolated valuation — no substantial value.

payment options. Hence, in a realistic setting with stochastic interest rates, a state dependent fee structure as proposed by MacKay et al. (2017) does not eliminate the positive value of premium payment options.

It can be argued that, in real life, policyholders cannot easily apply an optimal stopping strategy because of severe market frictions (such as taxation or informational barriers). In such a case, we show that these options value little and can be even negative if policyholders exercise options without certain optimal strategies. However, a systematic change in the market efficiency and policyholders' behaviour may take place in the future. Hence, insurers should consider an optimal behaviour from the policyholders' perspective for risk management purposes.

The remainder of this paper is set out as follows: Section 2 discusses the relevant literature. A general contract framework is introduced in Section 3. Section 4 puts forth different exercise assumptions in respect to the evaluation of premium-payment options. Section 5 provides numerical results. Finally, Section 6 concludes the paper with the central economic implications of our findings.

1.2 Related Literature

Most of the literature dealing with premium-payment options in participating life insurance contracts with investment guarantees centres on a single surrender option with the assumption that one single premium payment is made up front. Life insurance policies without surrender options are classed as European options, while the insurance policies with surrender options embedded are classed as Bermudan or American options (cf. Grosen and Jørgensen (1997)). Surrender options can therefore be valued as the difference between the Bermudan/American option and the European option. To value this option, Bacinello (2005) apply the Recursive Binomial-Tree approach discussed in Cox et al. (1979). First suggested by Longstaff and Schwartz (2001), the Least Square Monte Carlo method (LSMC) is another approach with which to value American options. Bacinello (2008a) and Bacinello et al. (2009) have applied this method for the life insurance case. Andreatta and Corradin (2003) compare the Recursive Binomial-Tree approach with the LSMC method and conclude that these two approaches are similarly accurate, while the LSMC can be better applied for a high-dimensional derivative valuation. Bauer et al. (2010) build a general model and compare these numerical valuation approaches. Again, the LSMC was found to be superior because of its efficiency.

Paid-up and resumption options do not exist under the single premium payment assumption. However, life insurance contracts, typically include multiple premium payments during the contract period with multi-period dynamic features. Modelling a multiple premium payment contract, Kling et al. (2006), Gatzert and Schmeiser (2008), and Schmeiser and Wagner (2011) value both single and combined premium-payment options. With geometric Brownian motion for assets and a deterministic interest rate, these studies base their first step on a fair pricing concept: the net present value (NPV) for insurance contracts without premium-payment options must be zero. In the second step, option values are computed as the NPV of the contracts with premium-payment options. In particular, if the options are exercised at their maximum level, the provider may face severe risk (cf. Gatzert and Schmeiser (2008)). However, as Kling et al. (2006), Gatzert and Schmeiser (2008), and Reuß et al. (2016) point out, this strategy is not feasible from the policyholders' viewpoint because a perfect forecast would be necessary. Nevertheless, an assessment on this basis can be interpreted as an upper bound of the option value. Schmeiser and Wagner (2011) value premium-payment options with an optimal stopping strategy first proposed by Andersen (1999). It is showed that the value of premium-payment options is fairly small. The existing literature related to combined premium-payment options assumes a deterministic interest rate. However, this assumption of a constant interest rate is not realistic if the contract's duration is not short. Life insurance contracts have typically long maturity and the empirical findings confirm the influence of interest rate on policyholders' option exercising behaviours (cf. Russell et al. (2013) and Kuo et al. (2003) for the US market, Kiesenbauer (2012)for the German market and Kim (2005) for the Korean market).

Comparing LSMC and the optimal stopping strategy, LSMC is slightly biased downwards (cf. Douady (2002)). However, the optimal stopping strategy is only feasible when one risk source is considered. Moreover, the LSMC method is fairly efficient, solving the technical challenges which Schmeiser and Wagner (2011) encounter. Thus, the combinations of three or even more options can be handled with this method. Except for the surrender option, which most of the literature focuses on, the payoffs of paid-up and resumption options involve a series of uncertain future cash flows. These payoff schemes and multiple exercise points for combined options make the valuation challenging. Hence, multi-factor models with a closed-form solution, such as those offered by Peterson et al. (2003) cannot be applied in general. The original LSMC method aims to find one optimal exercising point for the option triggering a known cash flow as the case for surrender options in insurance life contracts. In order to value other types of premium-payment options, we develop an adjusted LSMC method, with which policyholders make exercise decisions based on two or more conditional expected values. This adjusted LSMC method allows even more stochastic features (e.g., mortality) and produces multiple optimal exercise decision points.[5]

1.3 The Model Framework

Basic life insurance endowment contracts with a cliquet-style option include two standard features: a guaranteed yearly interest rate (g) and a surplus participation with participation rate (α) (cf. Grosen and Jørgensen (2000)). This basic contract is then extended with different premium-payment options (cf. Schmeiser and Wagner (2011) and Gatzert and Schmeiser (2008)). These premium-payment options are assumed to be exercised only at the end of each year, given that the main basic contracts are still in force (i.e., at the end of each contract year, policyholders are alive and the relevant options can still be exercised). Once the option is exercised, the benefit will be adjusted accordingly. We assume that the insurer faces no default risk and hence legitimate payments to the policyholders can always be achieved.[6]

1.3.1 Basic Contract

A basic life insurance endowment contract lasts T years. Let ${}_tp_x$ be the probability that a policyholder aged x years survives the next t years (with $t = 0...T$ and ${}_0p_x = 1$), while $q_x = 1 - {}_1p_x$ represents the death probability over the next year. Following general actuarial practice, we assume that mortality risk is uncorrelated to financial risk sources and hence is fully diversifiable (cf. Biffis et al. (2010)).

Annual premium payments, B_t for $t = 0...T-1$, are paid by a policyholder at the beginning of year $t+1$ if the policyholder is alive then. Without premium-payment options, premium payments are constant in time, i.e., $B_t = B^*$. The present value (PV) of premium payments can be written as $B^* \sum_{t=0}^{T-1} {}_tp_x (1+r^*)^{-t}$, where r^* is the technical discount rate. Annual premiums are accumulated in the policy account, A_t at the end of year t for the benefit distribution. The investment return of this accumulated account value includes the guaranteed interest rate g and the surplus participation.

5 In our case, the surrender value under the adjusted LSMC method is slightly better than the value under the original LSMC method (cf. appendices of this paper). Whether this adjusted LSMC method provides a better result and improves the downward bias in general may be a subject for future research.

6 For the case of the valuation of the surrender option only, Cheng and Li (2018) takes the insurer's default risk into account. In addition, the authors include solvency rules: Whenever a certain solvency level is hit, the insurer is not allowed to sign new contracts but a run-off has to take place. Both effects — default risk and particular run-off rules — influence the policyholders' cash-flows and hence the surrender option's value.

If the policyholder dies during year t, a death benefit γ_t is payable at the end of year t for $t = 1...T$. Without premium-payment options exercised, the death benefits are constant, i.e., $\gamma_t = \gamma^*$. If the policyholder survives the whole contract period, the survival benefit is paid. This survival benefit is the policy account A_T, guaranteed with the minimum amount γ^*. Hence, the PV of the benefit payment can be written as $\gamma^* \cdot \sum_{t=0}^{T-1} {}_tp_x q_{x+t}(1+r^*)^{-(t+1)} + A_T {}_Tp_x(1+r^*)^{-T}$.

According to the actuarial equivalence principle, the PV of the premium payments and that of the death and survival benefits should be identical. As the benefit is guaranteed, the interest guaranteed rate g is used as the technical discount rate r^* and A_T equals γ^* (guaranteed). γ^*, the benefit guaranteed, can then be derived from a fixed premium payment amount B^*. The relationship between B^* and γ^* is shown via the following equation:

$$B^* \sum_{t=0}^{T-1} {}_tp_x(1+g)^{-t} = \gamma^* \Big(\sum_{t=0}^{T-1} {}_tp_x q_{x+t}(1+g)^{-(t+1)} + {}_Tp_x(1+g)^{-T} \Big). \tag{1.3.1.1}$$

Hence, γ^* is given by:

$$\gamma^* = \frac{B^* \sum_{t=0}^{T-1} {}_tp_x(1+g)^{-t}}{\sum_{t=0}^{T-1} {}_tp_x q_{x+t}(1+g)^{-(t+1)} + {}_Tp_x(1+g)^{-T}}. \tag{1.3.1.2}$$

The guaranteed rate is considered as the lower bound of the contract's interest rate. With a participating scheme, policyholders receive a surplus return whenever the insurer's asset return exceeds the guaranteed rate. This surplus is retained in the policy account.[7]

An annual premium payment can be separated into two parts: B_t^R and B_t^A. B_t^R as $q_{x+t}\max(\gamma_{t+1} - A_t, 0)$, the annual term life premium, is used to pay the expected difference between the death benefits and the policy account accumulated by the end of year t. The remainder, B_t^A, serves as the savings premium:

$$\begin{aligned} B_t &= B_t^R + B_t^A, \\ B_t^A &= B_t - B_t^R = B_t - q_{x+t}\max(\gamma_{t+1} - A_t, 0). \end{aligned} \tag{1.3.1.3}$$

At the beginning of year $t+1$ for $t = 0...T-1$, the accumulated policy account contains two

[7] Generally, policyholders can choose to 1) receive the surplus as cash each year, 2) purchase extra insurance and increase the death benefit amount, or 3) keep the surplus in the policy account and earn the return guaranteed with g. In such a case, the survival benefit is increased. In what follows, we assume that policyholders keep their surplus in the policy account.

parts: the accumulated amount at the end of the previous year, A_t, and the annual savings premium, B_t^A, on the condition that the policyholder is alive at the end of year t. This policy account earns an annual return at the guaranteed interest rate or surplus participation – whichever is greater. The annual participation rate α is a fraction of the annual insurer's investment return in year $t+1$, given by $S_{t+1}/S_t - 1$, where S_t stands for the value of the insurers' asset at the end of year t with $S_t > 0$. The development of the policy account over time can be formally written as:

$$A_{t+1} = (A_t + {}_t p_x B_t^A) \cdot \Big(\max\big(g, \alpha(S_{t+1}/S_t - 1)\big) + 1 \Big) \tag{1.3.1.4}$$

with $A_0 = 0$.

The policy account is subject to investment risk, including two risk sources: the interest rate risk and the asset risk. We assume that the interest rate $\hat{r}_t$ is approximated by r_t, which evolves according to the one-factor Vasicek model (cf. Vasicek (1977)):

$$dr_t = \kappa(\theta - r_t)dt + \sigma^I dZ_t^{\mathbb{P}}. \tag{1.3.1.5}$$

Here, $Z_t^{\mathbb{P}}$ is a Wiener process on a probability space $(\Omega, \phi, \mathbb{P})$. To capture the interest rate risk, σ^I determines how much randomness of $Z^{\mathbb{P}}$ is acquired in the model. κ and θ are positive constants representing the speed of reversion and the long-term mean, respectively. A constant market price of risk, λ, is introduced to transfer the model from an empirical probability space into the risk-neutral probability space. In the case of risk averse market participants, we have $\lambda < 0$. Under the risk-neutral measure, $\mathbb{Q}$, the interest spot rate process given in Equation (1.3.1.5) changes to:

$$dr_t = \kappa(\theta - \frac{\sigma^I \lambda}{\kappa} - r_t)dt + \sigma^I dZ_t^{\mathbb{Q}}, \tag{1.3.1.6}$$

where $Z_t^{\mathbb{Q}}$ denotes the Wiener process under the risk-neutral measure, $\mathbb{Q}$. The solution of the Vasicek model for one period return ($\Delta t = 1$) is approximated by:

$$\hat{r}_t = e^{(-\kappa)} \cdot \hat{r}_{t-1} + (\theta - \frac{\sigma^I \lambda}{\kappa})(1 - e^{-\kappa}) + \frac{\sigma^I}{\sqrt{2\kappa}} \sqrt{1 - e^{-2\kappa}} \cdot (Z_t^{\mathbb{Q}} - Z_{t-1}^{\mathbb{Q}}) \simeq r_t. \tag{1.3.1.7}$$

For the asset risk, we assume that the policy's asset follows a geometric Brownian motion:

$$S_t = S_0 e^{(\mu - \sigma^{s2}/2)dt + \sigma^s dW_t^{\mathbb{P}}},$$

where $W_t^{\mathbb{P}}$ is a Wiener process, and the constants, μ and σ^s are the percentage drift and the percentage volatility. Under the risk-neutral measure $\mathbb{Q}$, combined with the stochastic interest rate derived from Equation (1.3.1.7), the deterministic drift changes into the stochastic spot rate.

S_t, the insurer's asset value at the end of year t is approximated by:

$$S_t \simeq \hat{S}_t = \exp\left(\hat{r}_t - \sigma^{s2}/2 + \sigma^s\left(\rho\cdot(Z_t^{\mathbb{Q}} - Z_{t-1}^{\mathbb{Q}}) + \sqrt{1-\rho^2}\cdot(W_t^{\mathbb{Q}} - W_{t-1}^{\mathbb{Q}})\right)\right)\cdot\hat{S}_{t-1}. \quad (1.3.1.8)$$

In this context, $W_t^{\mathbb{Q}}$ represents another Wiener process under the risk-neutral measure $\mathbb{Q}$. σ^s captures the investment risk related to both the asset risk and the interest risk. ρ indicates the correlation coefficient between these two risk sources.

The net present value (NPV) of the basic insurance contract is the PV difference between two cash flows: the benefit paid to the policyholder and the premiums paid by the policyholder to the insurer. The basic contract's NPV, denoted by $\vartheta^{\emptyset}$ can be formalised as:

$$\vartheta^{\emptyset} = E^{\mathbb{Q}}\Big[\gamma^* \sum_{t=0}^{T-1} {}_t p_x q_{x+t} \prod_{i=1}^{t+1}(1+\hat{r}_i)^{-1} + {}_T p_x A_T \prod_{i=1}^{T}(1+\hat{r}_i)^{-1} - B^* - B^* \sum_{t=1}^{T-1} {}_t p_x \prod_{i=1}^{t}(1+\hat{r}_i)^{-1}\Big]. \quad (1.3.1.9)$$

We call a contract fair whenever its NPV is zero (i.e., $\vartheta^{\emptyset} = 0$). For different parameters, we derive their respective participation rate, α (with $0 \leq \alpha \leq 1$), which leads to a fair condition. The following table summarizes the notations for the basic contracts.

g	guaranteed interest rate
$\hat{r}_t$	stochastic annual spot rate for year t ($r_t \simeq \hat{r}_t$)
α	participation rate ($0 \leq \alpha \leq 1$)
B_t B_t^R B_t^A B^*	annual premium payment, paid at the beginning of year $t+1$ ($B_t = B_t^A + B_t^R$) annual term life premium annual savings premium constant annual premium payment (for a contract without premium-payment options)
γ_t γ^*	death benefit paid at the end of year t constant death benefit (for a contract without premium-payment options)
A_t	policy account value at the end of year t
$\vartheta^{\emptyset}$	NPV of a basic contract without any premium-payment options

Table 1.1: Summary of notation in basic contract and the premium-payment options

1.3.2 Modelling Premium-Payment Options

With the investment guaranteed rate and the surplus participation included in the contract, premium-payment options have certain values given two stochastic sources (asset and interest risks) in the policy. This option value can be positive or negative depending on the market condition at the exercise points.[8]

The present value of premium-payment options is computed as the NPV of the contract with the premium-payment options. Since the NPV of the basic contract (without any premium-payment options) is zero, the premium-payment option's value is regarded as the additional value at the beginning of the contract ($t = 0$).

The premium-payment options considered in this paper are:

[8] If only rational decision makers are assumed in a complete and frictionless capital market, option values are non-negative as option holders will not exercise an option that is out of the money.

1. ϑ_{τ}^{P}, PV of a single paid-up option with the paid-up option exercised at the end of year τ, where $0 < \tau \leq T$: When a paid-up option is exercised, policyholders stop premium payments while the contract continues with adjusted benefits γ_{τ}^{P}.

2. $\vartheta_{\tau,\nu}^{PR}$, PV of a combined paid-up and resumption option with the paid-up option exercised at the end of year τ and the resumption option at the end of year ν, where $0 < \tau < \nu \leq T$, or $\tau = \nu = T$: Compared to the single paid-up option, the extra resumption option allows policyholders to resume the premium payments after exercising the paid-up option. Again, the benefit will then be adjusted to $\gamma_{\tau,\nu}^{R}$.

3. ϑ_{ξ}^{S}, PV of a single surrender option with the surrender option exercised at the end of year ξ, where $0 < \xi \leq T$: This surrender option allows policyholders to terminate the policy and receive a surrender amount.

4. $\vartheta_{\tau,\xi}^{PS}$: PV of a combined paid-up and surrender option with the paid-up option exercised at the end of year τ and the surrender option at the end of year ξ, where either $0 < \xi \leq \tau = T$ or $0 < \tau < \xi \leq T$. With this combined option, policyholders can first exercise the paid-up option and then terminate the contract with the surrender option.

5. $\vartheta_{\tau,\nu,\xi}^{PRS}$: PV of a combined paid-up, resumption, and surrender option with the paid-up option exercised at the end of year τ, the resumption option at the end of year ν, and the surrender option at the end of year ξ, where τ, ν, and ξ fall in one of the three conditions:

1) $0 < \tau < \nu < \xi \leq T$;

2) $0 < \tau < \xi \leq T$ and $\nu = T$;

3) $0 < \xi < T$ and $\tau = \nu = T$.

All of these options permit policyholders to change their premium payments in different ways.

The NPV of the contract with or without premium-payment options can be generalised as:

$$\vartheta = E^{\mathbb{Q}}\Big[\sum_{t=0}^{\xi-1} \gamma_{t+1\,t}p_x q_{x+t} \prod_{i=1}^{t+1}(1+\hat{r}_i)^{-1} + {}_{\xi}p_x \hat{A}_{\xi} \prod_{i=1}^{\xi}(1+\hat{r}_i)^{-1} - B_0 - \sum_{t=1}^{\xi-1} B_{t\,t}p_x \prod_{i=1}^{t}(1+\hat{r}_i)^{-1}\Big]. \tag{1.3.2.1}$$

Thereby, $0 < \tau \leq T$, $0 < \nu \leq T$, and $0 < \xi \leq T$ denote the exercise points for a paid-up option, a resumption option and a surrender option respectively. If none of these options are exercised, $\tau = \nu = \xi = T$. A fee, denoted by Fee, with $0 \leq Fee < 1$, is charged as a percentage of the policy account when any of the options is exercised to cover, e.g., the administration costs of the insurer. With Fee considered, the policy account value becomes $\hat{A}_t$. $\hat{A}_t = A_t$, for $t < \min\{\nu, \tau, \xi\}$ and $\hat{A}_{t+1} = \{\hat{A}_t + {}_tp_x B_t^A(\max(g, \alpha(S_{t+1}/S_t - 1) + 1)$ for $t > \min\{\nu, \tau, \xi\}$ and $t \notin \{\nu, \tau, \xi\}$, as no fee is applied when none of the premium-payment options is exercised. For $t \in \{\nu, \tau, \xi\}$ and $t < T$, $\hat{A}_t = (\hat{A}_{t-1} + {}_{t-1}p_x B_{t-1}^A) \cdot \Big(\max\big(g, \alpha(S_t/S_{t-1} - 1)\big) + 1\Big) \cdot (1 - Fee)$. Beside exercise fees, we do not take other frictions into account that may influence the policyholder's exercise behavior. For instance, in insurance practice, different tax treatments when using premium payment options can have an impact on the exercise strategy.

For a basic contract without any premium-payment options, $\vartheta^{\emptyset}$, $\xi = T$, $B_t = B^*$, and $\gamma_t = \gamma^*$. Equation (1.3.1.9) is, therefore, a special case of Equation (1.3.2.1).

Single Paid-up Option:

ϑ_τ^P denotes the PV of a single paid-up option exercised at the end of year τ with $0 < \tau \leq T$.

With Equation (1.3.2.1), $\xi = T$, as no surrender option is offered. If $\tau = T$, this contract matures without exercising the paid-up option. Thus, $\vartheta_T^P = \vartheta^{\emptyset} = 0$.

Otherwise, if $\tau < T$, $B_t = B^*$, $\gamma_t = \gamma^*$, when $t < \tau$. When $t \geq \tau$, $B_t = 0$ and $\gamma_t = \gamma_\tau^P$, with:

$$\gamma_\tau^P = \frac{\hat{A}_\tau}{\sum_{t=\tau}^{T-1} {}_{t-\tau}p_{x+\tau} q_{x+t} (1+g)^{-(t+1-\tau)} + {}_{T-\tau}\, p_{x+\tau}(1+g)^{-(T-\tau)}}, \tag{1.3.2.2}$$

Combined Paid-up and Resumption Option:

$\vartheta^{PR}_{\tau,\nu}$ signifies the PV of a combined option with the paid-up option exercised at the end of year τ and the resumption option at the end of year ν, where $0 < \tau < \nu \leq T$ or $\tau = \nu = T$. In the latter case, no options are exercised and $\vartheta^{PR}_{T,T} = \vartheta^{\emptyset} = 0$.

Without a surrender option offered, $\xi = T$. If $\nu = T$, this contract matures without exercising the resumption option and $\vartheta^{PR}_{\tau,T} = \vartheta^{P}_{\tau}$.

If $\nu < T$, when $t < \tau$, $B_t = B^*$ and $\gamma_t = \gamma^*$. For $\tau \leq t < \nu$, $B_t = 0$ and $\gamma_t = \gamma^P_\tau$. For $t \geq \nu$, $B_t = B^*$ and $\gamma_t = \gamma^R_{\tau,\nu}$ with:

$$\gamma^R_{\tau,\nu} = \frac{\hat{A}_\nu + B^* \sum_\nu^{T-1} {}_{t-\nu}p_{x+\nu}(1+g)^{-(t-\nu)}}{\sum_{t=\nu}^{T-1} {}_{t-\nu}p_{x+\nu}q_{x+t}(1+g)^{-(t+1-\nu)} + {}_{T-\nu}\, p_{x+\nu}(1+g)^{-(T-\nu)}}.$$

Single Surrender Option:

ϑ^S_ξ denotes the PV of a single surrender option exercised at the end of year ξ with $0 < \xi \leq T$. Without paid-up or resumption options, $B_t = B^*$ and $\gamma_t = \gamma^*$. If $\xi = T$, this contract matures without exercising the option, and hence, $\vartheta^S_T = \vartheta^{\emptyset} = 0$.

Combined Paid-up and Surrender Option:

$\vartheta^{PS}_{\tau,\xi}$ signifies the PV of a combined option with the paid-up option exercised at the end of year τ and the surrender option at the end of year ξ, where $0 < \tau < \xi \leq T$ or $0 < \xi \leq \tau = T$. If $\tau = \xi = T$, no options are exercised and $\vartheta^{PS}_{T,T} = \vartheta^{\emptyset} = 0$. If $\xi = T$, this contract matures without exercising the surrender option. In such a case, $\vartheta^{PS}_{\tau,T} = \vartheta^P_\tau$. If $\tau = T$, this contract matures without exercising the paid-up option and $\vartheta^{PS}_{T,\xi} = \vartheta^S_\xi$.

If $0 < \tau < \xi < T$, for $t < \tau$, $B_t = B^*$ and $\gamma_t = \gamma^*$. For $\tau \leq t$, $B_t = 0$ and $\gamma_t = \gamma^P_\tau$.

Combined Paid-up, Resumption, and Surrender Option:

$\vartheta^{PRS}_{\tau,\nu,\xi}$ denotes the PV of the combined option including all three options: a paid-up option, a resumption option, and a surrender option exercised at the end of year τ, ν, and ξ.

If $\tau = \xi = \nu = T$, none of the options are exercised and $\vartheta^{PRS}_{T,T,T} = \vartheta^{\emptyset} = 0$.

If $0 < \tau < \nu = \xi = T$, only the paid-up option is exercised at the end of year τ, thus $\vartheta^{PRS}_{\tau,T,T} = \vartheta^{P}_{\tau}$.

If $0 < \tau < \nu < \xi = T$, the surrender option has not been exercised when the contract matures. Thus, $\vartheta^{PRS}_{\tau,\nu,T} = \vartheta^{PR}_{\tau,\nu}$.

If $0 < \xi < \tau = \nu = T$, neither the paid-up option nor the resumption option is exercised. Thus, $\vartheta^{PRS}_{T,T,\xi} = \vartheta^{S}_{\xi}$.

If $0 < \tau < \xi < \nu = T$, the contract matures without the resumption option being exercised. Thus, $\vartheta^{PRS}_{\tau,T,\xi} = \vartheta^{PS}_{\tau,\xi}$.

If $0 < \tau < \nu < \xi < T$, $B_t = B^*$ and $\gamma_t = \gamma^*$ with $t < \tau$. When $\tau \le t < \nu$, $B_t = 0$ and $\gamma_t = \gamma^P_\tau$. When $\nu \le t$, $B_t = B^*$ and $\gamma_t = \gamma^R_{\tau,\nu}$.

Interpretation

The basic contract contains regular premium payments, possesses a net present value of zero, and encloses no premium-payment options. The present values of the different forms of premium-payment options can be interpreted as the fair price the policyholder would need to pay at the beginning of the contract ($t = 0$) in addition to the regular premium payments of the basic contract. For a rational policyholder in a complete and frictionless capital market, this price cannot be negative. In any case, the present value of premium-payment options depends strongly on the policyholder's exercise strategy.

Typically, policyholders provide constant premium payments for participating life insurance contracts with premium-payment options. In the context of our paper, the fair price of premium-payment options can be transferred in a constant add-up on the regular premium payments for the basic contract (i.e., the contract without premium-payment options). Thereby, the term structure, mortality probabilities, and the exercise strategy in respect to the premium-payment options must be taken into account. Another way to obtain constant premium payments is to adjust the participation rate derived in the basic contract in a way that the insurance contract with premium-payment options has a net present value of zero. Again, the adjustment depends very much on the exercise strategy used by the policyholders. However, in order to derive results that are easy to compare for different scenarios and contract lengths, we decided to focus on the present values of premium-payment options only.

1.4 Valuation of Premium-Payment Options

The assumed policyholder's exercise strategy is central for option valuation. We begin by calculating the upper bound of the premium-payment options. This method indicates the options' value range and demonstrates the worst-case scenario from the insurers' viewpoint. However, it is not an accessible value since such a procedure requires information on the future development of the two stochastic sources. Hence, it is not a feasible strategy for policyholders. In the second step, we develop an adjusted LSMC strategy as an approximation of an optimal and feasible exercise approach. With this approach, we assume the policyholder to be rational in the following sense: The policyholder exercises the premium-payment option at its optimal value. It demonstrates the worst-case scenario from the insurers' viewpoint.[9]

1.4.1 Upper Bound of the Option Value (${}^{UP}\overline{\vartheta}$)

Kling et al. (2006), Gatzert and Schmeiser (2008), and Schmeiser and Wagner (2011) calculate an upper bound for premium-payment options and discuss its economic interpretation in detail. Assuming policyholders know future developments, they would exercise the premium-payment option only if it is in the money (ITM if $\vartheta > 0$) and at its maximum value for the whole contract

[9] It is argued that policyholders neither behave rationally nor have access to the hedging tools to manage their interest-rate risk. However, with growing life settlement market, policyholders can sell their contracts to financial institutes, whose surrender behavior is rational and depended on the surrender option value (cf. Braun and Xu (2020)).

period. Using Monte Carlo simulation with $n = 1...N$ paths, we have:

$$^{UP}\overline{\vartheta} = \frac{1}{N}\sum_{n=1}^{N}\left(\max({}^{n}\vartheta_t, 0)\right) \text{ with } t = 1...T-1, \tag{1.4.1.1}$$

where ${}^{n}\vartheta_t$ denotes the different option values if exercised at the end of year t for the n^{th} simulation path. Options are non-negative, as policyholders do not exercise these options if they are out of the money (OTM) for the whole contract period. The upper bound can also be referred to as the PV of the option given perfect information about the future. Although such information clearly does not exist for a human being, the concept still provides useful insight as a reference for the upper bound of the option – or maximum loss from the insurer's viewpoint.

1.4.2 Option Valuation via the Least Square Monte Carlo Strategy ($^{LSMC}\overline{\vartheta}$)

The LSMC method was first presented by Longstaff and Schwartz (2001) to price American options. It has been used to value a single surrender option in life insurance contracts (cf. Andreatta and Corradin (2003), Nordahl (2008), and summarised by Bauer et al. (2010)) with a single premium payment paid at the beginning of the contract.

The LSMC approach aims to find an optimal exercise point t^* with accessible information only. For different points in time t, two values are compared: exercise value and continuation value. Exercise value denotes the value if the option is exercised at t, while continuation value represents the value if the policyholder does not exercise the option and the contract goes forward. Following this strategy, policyholders exercise an option if its exercise value is larger than the continuation value and $t^* = t$.

The original LSMC determines the exercise value as the PV of a defined and deterministic cash flow when the option is exercised. The continuation value is the PV of the future cash flows if the options are not exercised immediately. However, except for the surrender option, exercising premium-payment options does not always cause a defined immediate cash flow. Adjusting the original approach, we define both the exercise value and the continuation value as the PV of future cash flows under the condition that the premium-payment option is exercised or is not exercised. For the special case of surrender options, we compare both the original and the adjusted LSMC in the appendices. Our numerical examples suggest that the optimal option value with the adjusted LSMC strategy is slightly better than that of the original LSMC as the expected future interest development is taken into account. In the following, unless stated otherwise, LSMC refers to the adjusted LSMC strategy.

Future cash flows are stochastic. The original algorithm contains two approximations to estiamte the optimal option value (cf. Clément et al. (2002)). First, the continuation value at the end of year t denoted by ${}_tC(\vartheta)$ is approximated by a combination of finite value functions with accessible relevant information. The second approximation determines the value functions via a least square regression. Two approximations are further added for the exercise value, ϑ_t, i.e., the option value when the option is immediately exercised at the end of year t. All of the values are discounted to the beginning of the contract for the purpose of convenient comparison.

${}_tC(\vartheta)$ is approximated by $f(x_t^1...x_t^J)$, where $x_t^1...x_t^J$ denotes all accessible information at the end of year t. As the option exercise decision is influenced by the financial market condition, we include all relevant variables at the end of year t, such as interest rate, $\hat{r}_t$, the asset value, S_t, and the benefit, γ_t.[10]

The second approximation includes K sets of basis functions, υ^k with $k = 1...K$, to approximate $f(x_t^1...x_t^J)$ with K constant coefficients, $\mathbf{a}_t = (a_t^1...a_t^K)$. Here, υ^k are weighted Laguerre polynomials suggested in Longstaff and Schwartz (2001):

$$ {}_tC(\vartheta) \cong f(x_t^1...x_t^J) \cong \sum_{k=1}^{K} a_t^k \cdot \upsilon^k(x_t^1...x_t^J). \tag{1.4.2.1}$$

The coefficients $\mathbf{a}_t$ are unknown so far. With Monte Carlo simulation paths $n = 1...N$, we estimate $\mathbf{a}_t$ via a least square linear regression. In Longstaff and Schwartz (2001), these estimators are based solely on in-the-money paths to reduce computation effort. However, in our case, all paths should be considered since the option we focus on is not standard (cf. Andreatta and Corradin (2003)). The estimator for $\mathbf{a}_t$ is provided by:

$$\hat{\mathbf{a}}_t = \arg\min_{\mathbf{a}_t} \left\{ \sum_{n=1}^{N} \left[{}_t^nC(\vartheta) - \sum_{k=1}^{K} a_t^k \cdot \upsilon^k({}^nx_t^1...{}^nx_t^J) \right]^2 \right\}. \tag{1.4.2.2}$$

With $\hat{\mathbf{a}}_t = (\hat{a}_t^1...\hat{a}_t^K)$, we can calculate:

$${}_t^n\hat{C}(\vartheta) = \sum_{k=1}^{K} \hat{a}_t{}^k \cdot \upsilon^k({}^nx_t^1...{}^nx_t^J), \tag{1.4.2.3}$$

[10] In our model, both interest and asset risks are simulated based on Wiener processes. With the Markov structure, the future state of the financial market in $t+1$ depends on the its current state in t and the stochastic behaviour of the Wiener process in the time interval $(t, t+1)$.

where ${}^{n}_{t}\hat{C}(\vartheta)$, ${}^{n}_{t}C(\vartheta)$, and ${}^{n}x_t^j$ denote ${}_{t}\hat{C}(\vartheta)$, ${}_{t}C(\vartheta)$, and x_t^j in the n^{th} simulation path.

As explained above, the exercise value is unknown until maturity ($t = T$). Therefore, another two approximations are required to estimate ϑ_t as ${}^{n}\hat{\vartheta}_t$ with accessible information ${}^{n}x_t^1 \ldots {}^{n}x_t^J$ per each simulation n:

$$\begin{aligned}
\vartheta_t &\cong s(x_t^1 \ldots x_t^J) \cong \sum_{k=1}^{K} a_t'^k \cdot \upsilon^k(x_t^1 \ldots x_t^J);\\
\hat{\mathbf{a}} &= \arg\min_{\mathbf{a}'} \left\{ \sum_{n=1}^{N} \left[{}^{n}\vartheta_t - \sum_{k=1}^{K} a_t'^k \cdot \upsilon^k({}^{n}x_t^1 \ldots {}^{n}x_t^J) \right]^2 \right\};\\
{}^{n}\hat{\vartheta}_t &= \sum_{k=1}^{K} \hat{a}_t'^k \cdot \upsilon^k({}^{n}x_t^1 \ldots {}^{n}x_t^J).
\end{aligned} \tag{1.4.2.4}$$

In the following we exam five different cases: ${}^{LSMC}\overline{\vartheta}^{P}$, ${}^{LSMC}\overline{\vartheta}^{S}$, ${}^{LSMC}\overline{\vartheta}^{PR}$, ${}^{LSMC}\overline{\vartheta}^{PR}$ and ${}^{LSMC}\overline{\vartheta}^{PRS}$:

Single Premium-Payment Option Case, ${}^{LSMC}\overline{\vartheta}^{P}$ & ${}^{LSMC}\overline{\vartheta}^{S}$

For single options, namely ϑ_t^P and ϑ_t^S, we aim to find one optimal exercise point, ${}^{n}t^*$, that maximises the option value in each path, n, by using accessible information (${}^{n}x_t^j$). At the end of each year, policyholders decide whether to exercise the option or not. The option should be exercised if the exercise value exceeds the continuation value.

For the backward iteration m_1 with $m_1 = T \ldots 1$, the single-option valuation procedure can be formally described as follows:

Step 1: Initialisation: Start with $m_1 = T$ and set all ${}^{n}t^* = m_1 = T$:
The option value is zero as at $m_1 = T$, the contract matures without exercising the option. The optimal option value is given by ${}^{n}\vartheta_{{}^{n}t^*} = \vartheta_T = 0$.

Step 2: One Year Backward:
One year backward at $m_1 = T - 1$, the continuation value is set zero as ${}^{n}_{T-1}C(\vartheta) = \vartheta_T = 0$. If ${}^{n}\hat{\vartheta}_{T-1}$ is positive and hence exceeds the continuation value, the option should be exercised and the optimal exercise point becomes ${}^{n}t^* = m_1 = T - 1$. Otherwise, the contract continues and ${}^{n}t^*$

remains unchanged. In formal terms, we have ${}^{n}t^{*} = m_1$, if ${}^{n}\hat{\vartheta}_{m_1} > {}^{n}_{m_1}C(\vartheta)$, with ${}^{n}_{m_1}C(\vartheta) = 0$ and ${}^{n}\hat{\vartheta}_{m_1}$ derived from Equation (1.4.2.4). Otherwise, ${}^{n}t^{*}$ remains T.

Step 3: Backward Iteration: for $m_1 = T-2...1$:
(1) ${}^{n}_{m_1}C(\vartheta) = {}^{n}\vartheta_{{}^{n}t^{*}}$

(2) Approximate the continuation value ${}^{n}_{m_1}C(\vartheta)$ and the exercise value ${}^{n}\vartheta_{m_1}$ with Equation (1.4.2.3) and Equation (1.4.2.4).

(3) If ${}^{n}\hat{\vartheta}_{m_1} > {}^{n}_{m_1}\hat{C}(\vartheta)$, ${}^{n}t^{*} = m_1$. Otherwise, ${}^{n}t^{*}$ remains unchanged.

With the algorithm above, the optimal option value equals the average of each path's option value exercised at its respective optimal point, ${}^{n}t^{*}$:

${}^{LSMC}\overline{\vartheta}^{P} = \frac{1}{N}\sum_{n=1}^{N}({}^{n}\vartheta^{P}_{{}^{n}t^{*}})$ for the paid-up option and ${}^{LSMC}\overline{\vartheta}^{S} = \frac{1}{N}\sum_{n=1}^{N}({}^{n}\vartheta^{S}_{{}^{n}t^{*}})$ for the surrender option.

Double Premium-Payment Option Case, ${}^{LSMC}\overline{\vartheta}^{PR}$ & ${}^{LSMC}\overline{\vartheta}^{PS}$

The double premium-payment option includes the combined paid-up and resumption option, $\vartheta^{PR}_{\tau,\nu}$, and the combined paid-up and surrender option $\vartheta^{PS}_{\tau,\xi}$. The main difference between these two options is that the resumption option can only be exercised if the paid-up option has been exercised, i.e., $\tau < \nu$, if $\nu < T$. However, policyholders can exercise the surrender option independently, even if the paid-up option has not yet been exercised.

Combined Paid-up and Resumption Option ${}^{LSMC}\overline{\vartheta}^{PR}$

The double option contains two optimal exercise points. Hence, its value cannot be estimated when deciding whether to exercise the first option, as the second exercise point has not been determined yet. However, for $\vartheta^{PR}_{\tau,\nu}$, while the first paid-up option can be seen as a put option, the resumption option has the call option feature. Therefore, at the end of year t, if it is optimal to exercise the paid-up option, the intrinsic value of the resumption option is 0. Thus, we can first consider the paid-up option only. On the condition that the paid-up option is exercised, we then determine the second optimal exercise point for the resumption option.

First we determine ${}^n\tau^* = {}^nt^*$ with the single paid-up option process described in the previous section. If ${}^n\tau^* = T$, the optimal strategy is not to exercise the paid-up option. In such a case, the resumption option will also expire and ${}^n\nu^* = T$, ${}^n\vartheta^{PR}_{T,T} = 0$. On the condition that ${}^n\tau^* < T$, we find the second optimal exercise point ${}^n\nu^*$ to maximise the second option. The second option value, or the resumption value, is the remaining value derived from ${}^n\vartheta^R_{{}^n\tau^*,\nu} = {}^n\vartheta^{PR}_{{}^n\tau^*,\nu} - {}^n\vartheta^P_{{}^n\tau^*}$. This second optimal exercise point ${}^n\nu^*$ can be computed by considering ${}^n\vartheta^R_{{}^n\tau^*,\nu}$ as a single option with iteration $m_2 = T \ldots {}^n\tau^* + 1$.

The optimal value can then be approximated by ${}^{LSMC}\overline{\vartheta}^{PR} = \frac{1}{N}\sum_{n=1}^{N}({}^n\vartheta^{PR}_{{}^n\tau^*,{}^n\nu^*})$.

Combined Paid-up and Surrender Option ${}^{LSMC}\overline{\vartheta}^{PS}$

For the combination of paid-up and surrender, both options are considered as put options and can be exercised independently. More specifically, the surrender option as the second option can be exercised even if the first paid-up option has not yet been exercised. In formal terms, we have:

$$\vartheta^{PS}_{\tau,\xi} = \vartheta^S_\xi \cdot \mathbb{1}_{\tau=T} + (\vartheta^P_\tau + \vartheta'^S_{\tau,\xi}) \cdot \mathbb{1}_{\tau<T}. \tag{1.4.2.5}$$

Thereby, $\vartheta'^S_{\tau,\xi} = \vartheta^{PS}_{\tau,\xi} - \vartheta^P_\tau$ for $\tau < T$.

At the end of every year t, two put options are compared to determine whether to exercise one of the two options. We begin by finding the optimal exercise point for one of these two options (${}^n\tau^*$ for ${}^n\vartheta^P_{{}^n\tau^*}$ and ${}^n\xi^*$ for ${}^n\vartheta^S_{{}^n\xi^*}$) with the procedure described as follows:

With iteration $m_3 = T \ldots 1$:

Step 1. Initialisation:
For $m_3 = T$ with ${}^n\tau^* = {}^n\xi^* = T$, ${}^n\vartheta^P_{{}^n\tau^*} = {}^n\vartheta^S_{{}^n\xi^*} = 0$.

Step 2. One Year Backward Comparison:

At the end of year $m_3 = T-1$, the option value is ${}^{n}\hat{\vartheta}^{PS}_{{}^{n}\tau^{*},{}^{n}\xi^{*}} = \max\left({}^{n}_{T-1}C(\vartheta), {}^{n}\hat{\vartheta}^{P}_{T-1}, {}^{n}\hat{\vartheta}^{S}_{T-1}\right)$. ${}^{n}\hat{\vartheta}^{P}_{T-1}$ and ${}^{n}\hat{\vartheta}^{S}_{T-1}$ are two estimators of ${}^{n}\vartheta^{P}_{T-1}$ and ${}^{n}\vartheta^{S}_{T-1}$, while the continuation value ${}^{n}_{T-1}C(\vartheta)$ equals zero.

Three scenarios are considered:

(1) If $\max\left({}^{n}_{T-1}C(\vartheta), {}^{n}\hat{\vartheta}^{P}_{T-1}, {}^{n}\hat{\vartheta}^{S}_{T-1}\right) = {}^{n}\hat{\vartheta}^{P}_{T-1}$: The best strategy is to exercise the paid-up option. Hence ${}^{n}\tau^{*} = m_3 = T-1$ and ${}^{n}\xi^{*} = T$.

(2) If $\max\left({}^{n}_{T-1}C(\vartheta), {}^{n}\hat{\vartheta}^{P}_{T-1}, {}^{n}\hat{\vartheta}^{S}_{T-1}\right) = {}^{n}\hat{\vartheta}^{S}_{T-1}$: The best strategy is to exercise the surrender option. Hence, ${}^{n}\tau^{*} = T$ and ${}^{n}\xi^{*} = m_3 = T-1$.

(3) If $\max\left({}^{n}_{T-1}C(\vartheta), {}^{n}\hat{\vartheta}^{P}_{T-1}, {}^{n}\hat{\vartheta}^{S}_{T-1}\right) = {}^{n}_{T-1}C(\vartheta) = 0$: In this case, policyholders should exercise neither of the options, and ${}^{n}\tau^{*} = {}^{n}\xi^{*} = T$.

Step 3. Backward Iteration: For $m_3 = T-2...1$:
Approximate the continuation value ${}^{n}_{m_3}C(\vartheta)$ with ${}^{n}_{m_3}C(\vartheta) = {}^{n}\vartheta_{{}^{n}\tau^{*},{}^{n}\xi^{*}}$ and the exercise value ${}^{n}\vartheta^{P}_{m_3}$, ${}^{n}\vartheta^{S}_{m_3}$ with Equation (1.4.2.3) and Equation (1.4.2.4).

Three scenarios are considered:
(1) ${}^{n}\tau^{*} = m_3$, ${}^{n}\xi^{*} = T$, if $\max\left({}^{n}\hat{\vartheta}^{P}_{m_3}, {}^{n}\hat{\vartheta}^{S}_{m_3}, {}^{n}_{m_3}\hat{C}(\vartheta)\right) = {}^{n}\hat{\vartheta}^{P}_{m_3}$.

(2) ${}^{n}\tau^{*} = T$, ${}^{n}\xi^{*} = m_3$, if $\max\left({}^{n}\hat{\vartheta}^{P}_{m_3}, {}^{n}\hat{\vartheta}^{S}_{m_3}, {}^{n}_{m_3}\hat{C}(\vartheta)\right) = {}^{n}\hat{\vartheta}^{S}_{m_3}$.

(3) ${}^{n}\tau^{*}$, ${}^{n}\xi^{*}$ remain unchanged, if $\max\left({}^{n}\hat{\vartheta}^{P}_{m_3}, {}^{n}\hat{\vartheta}^{S}_{m_3}, {}^{n}_{m_3}\hat{C}(\vartheta)\right) = {}^{n}_{m_3}\hat{C}(\vartheta)$.

After the iteration m_3, we find two optimal exercise points, ${}^{n}\tau^{*}$ and ${}^{n}\xi^{*}$. For ${}^{n}\tau^{*} = T$, the best strategy is either to exercise the surrender option only (${}^{n}\xi^{*} < T$), or not to exercise any of the options (${}^{n}\xi^{*} = T$). For the paths where ${}^{n}\tau^{*} < T$, the optimal strategy is to exercise the paid-up option at the end of year ${}^{n}\tau^{*}$. Given ${}^{n}\tau^{*} < T$, we update the second optimal exercise point, ${}^{n}\xi^{*}$, to maximise the remaining surrender option value, ${}^{n}\vartheta'^{S}_{{}^{n}\tau^{*},\xi} = {}^{n}\vartheta^{PS}_{{}^{n}\tau^{*},\xi} - {}^{n}\vartheta^{P}_{{}^{n}\tau^{*}}$. This second optimal exercise point can be determined as the single option with another iteration procedure $m_4 = T ... {}^n\tau^* + 1$.

This combined option's optimal value is given by ${}^{LSMC}\overline{\vartheta}^{PS} = \frac{1}{N}\sum_{n=1}^{N}({}^n\vartheta^{PS}_{{}^n\tau^*,{}^n\xi^*})$.

Triple Premium-Payment Option Case, ${}^{LSMC}\overline{\vartheta}^{PRS}$

When it comes to extending the algorithm with the combined paid-up, resumption, and surrender option, the decision process becomes complex: The paid-up and surrender options are both put options. Hence, for their combination, which option to exercise depends on whichever value is larger. For the triple option, the surrender option is compared with the combined paid-up and resumption option. The intrinsic value of the resumption option is zero whenever it is optimal to exercise either the surrender or the paid-up option. However, the time values of the resumption option is non-zero if the paid-up option is exercised. Specifically, when policyholders exercise the paid-up option, they can still use the resumption option to readjust the reduced benefit. On the contrary, once the surrender option is exercised, the contract terminates and the time value of all other options become zero. Hence, for $\vartheta^{PRS}_{\tau,\nu,\xi}$, the resumption option's time value attached to the paid-up option should be considered when determining whether to exercise the paid-up option or the surrender option.

Step 1:
With an iteration $m_4 = T ... 1$, we determine whether to exercise the surrender or the paid-up option based on the comparison between ${}^n\hat{\vartheta}^S_{m_4}$ and ${}^n\hat{\vartheta}^{PR}_{m_4,{}^n\nu^*_{m_4}}$, where ${}^n\hat{\vartheta}^{PR}_{m_4,{}^n\nu^*_{m_4}}$ denotes the expected optimal value of the combined paid-up and resumption option with the paid-up option exercised at the end of year m_4. This expected optimal value is computed with another optimal exercise point ${}^n\nu^*_{m_4}$, determined by another LSMC algorithm with $m_5 = T ... m_4 + 1$. The first step determines two optimal exercise points for ${}^n\tau^*$ and ${}^n\xi^*$.

Step 2:
With an iteration $m_6 = T ... {}^n\tau^* + 1$ for the paths that $0 < {}^n\tau^* < T$, we decide either to exercise the resumption option or the surrender option. This decision is based on the two remaining option values:

$${}^n\vartheta'^S_{{}^n\tau^*,m_6} = {}^n\vartheta^{PS}_{{}^n\tau^*,m_6} - {}^n\vartheta^P_{{}^n\tau^*} \text{ and } {}^n\vartheta^R_{{}^n\tau^*,m_6} = {}^n\vartheta^{PR}_{{}^n\tau^*,m_6} - {}^n\vartheta^P_{{}^n\tau^*}$$

Step 3:
On the condition that it is optimal to exercise the resumption option, we run another iteration $m_7 = T ... {}^n\nu^* + 1$ for the paths that $0 < {}^n\tau^* < {}^n\nu^* < T$ to determine whether to exercise the remaining surrender option with its value ${}^n\vartheta''^{S}_{{}^n\tau^*, {}^n\nu^*, m_7} = {}^n\vartheta^{PRS}_{{}^n\tau^*, {}^n\nu^*, m_7} - {}^n\vartheta^{PR}_{{}^n\tau^*, {}^n\nu^*}$.

With all three optimal exercise points determined, the optimal value of this triple option is given by:

$$^{LSMC}\overline{\vartheta}^{PRS} = \frac{1}{N} \sum_{n=1}^{N} \left({}^n\vartheta^{PRS}_{{}^n\tau^*, {}^n\nu^*, {}^n\xi^*} \right).$$

1.5 Numerical Results

This section presents key results of our numerical example for discussion. In particular, we focus on the influence of the interest rate volatility, σ^I, on the value of the various embedded options. For easy comparison purpose, the parameters are chosen based on the parameters given in Schmeiser and Wagner (2011). Due to the current low interest rate environment, we further run another sensitivity test to account for the actual term structure. Unless stated otherwise, the numerical results are gathered using a Monte Carlo simulation with $N = 10^4$ and the LSMC method is employed with $K = 4$.[11]

1.5.1 Basic Contract

We consider a basic contract with the following parameters: A 30-year-old female policyholder enters into a 10-year life insurance contract.[12] The annual premium is 1,200 currency units, with the yearly interest rate guaranteed at 3%. The annual investment return rate combines both the interest and the asset processes laid down in Equation (1.3.1.8). The asset volatility is fixed to $\sigma^s = 0.2$. The correlation between the asset and interest rate risks is $\rho = 0.05$. Under the Vasicek model, to obtain $\hat{r}_t$, we use the parameters $\kappa = 8\%$, $\hat{r}_0 = 4\%$, $\theta = 4\%$, and $\lambda = 0$. Based on these assumptions and using Equation (1.3.1.2), the death benefit is 14,091 currency units. Table 1.2 summarises the initial parameters.

[11] When taking different N and K, our numerical results stabilise as N reaches 10^4 and $K = 4$.

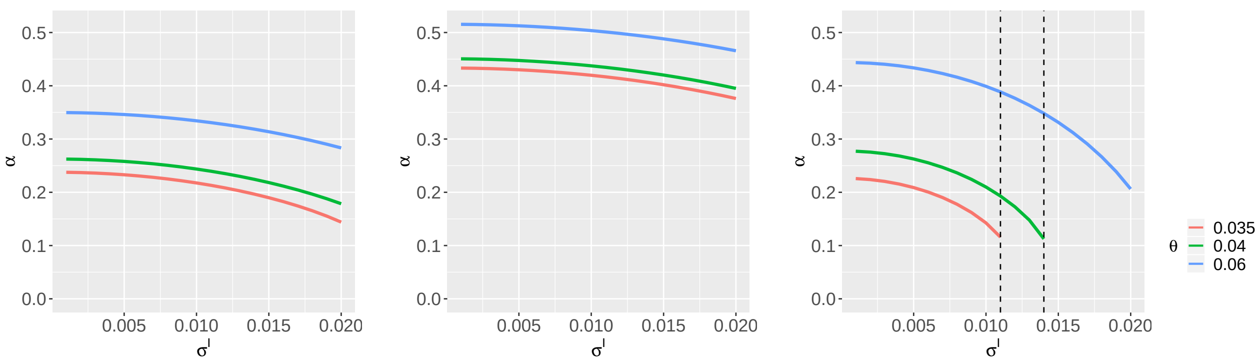
[12] Mortality probabilities are for a 30-year-old US woman in 1994 based on the data from HMD (the Human Mortality Database, 2016).

B^*	constant annual premium payment	1,200
γ^*	constant death benefit	14,091
g	guaranteed interest rate	3%
T	time to maturity	10
x	initial age	30
κ	interest rate reversion speed	8%
σ^s	asset volatility	20%

ρ	correlation coefficient between the asset and the interest rate risks	5%
$\hat{r}_0$	initial interest rate	4%
θ	long-term interest mean	4%
λ	market price of risk	0%

Table 1.2: Parameter table for the base case

Figure (1.1) shows the relationship between the participation rate, α, and interest rate volatility, σ^I, under different conditions. The participation rate, α, is derived such that the basic contract is fair at $t = 0$ (i.e. $\vartheta^{\emptyset} = 0$ in Equation (1.3.1.9)). A clear trend is shown whereby a higher volatility of interest rate results in a lower participation rate. For the same contract with the same interest rate guaranteed, insurers face higher risk and thus must lower the participation rate to ensure a risk-adequate return for the shareholders. In addition, the curves with different θ move in parallel for σ^I from 0.01% to 0.2%. This supports the conclusion drawn in Schmeiser and Wagner (2011), where σ^I is assumed to be 0: Given a fixed guaranteed interest rate, policyholders demand a higher participation rate as the interest rate is higher. As can be seen in Figure (1.1(b)), with $g = 0$, the participation rate is pushed upward in parallel because the value offered by the guaranteed rate decreases. The longer duration emphasizes the σ^I impact on α. If the duration is prolonged to 40 years, Figure (1.1(c)) shows a stronger negative interrelation between α and σ^I. When σ^I is larger than 1.1% for $\theta = 3.5\%$ and 1.4% for $\theta = 4\%$, there exists no α with $0 \leq \alpha \leq 1$ that satisfies the fairness condition.



(a) α and σ^I for different θ with $T = 10$ and $g = 3\%$

(b) α and σ^I for different θ with $T = 10$ and $g = 0\%$

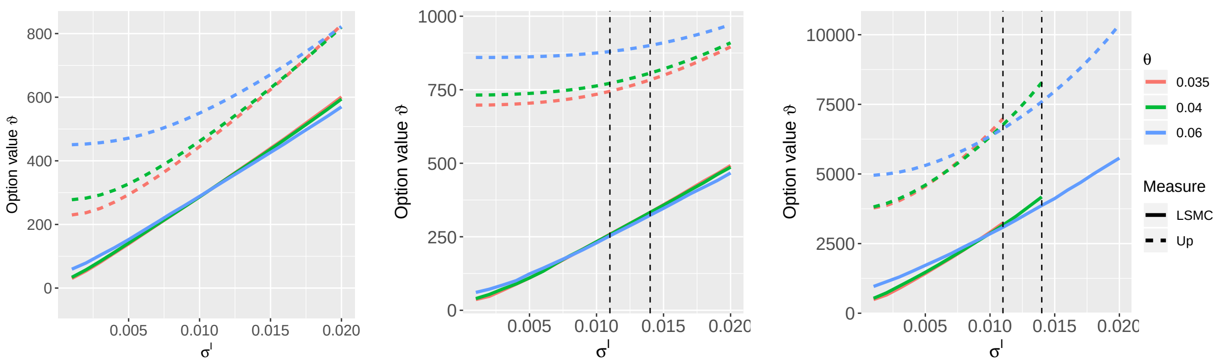
(c) α and σ^I for different θ with $T = 40$ and $g = 3\%$

Figure 1.1: Relationship between participation rate (α) and interest rate volatility (σ^I) for different values of long-term interest mean (θ), time to maturity (T), and the interest guaranteed rate (g)

1.5.2 Premium-Payment Option:

Triple Premium-Payment Option:
Paid-up, Resumption and Surrender Option

In the initial setup, we assume that no fee is applied ($Fee = 0$). Figure (1.2) demonstrates the results of the triple option (${}^{Up}\overline{\vartheta}^{PRS}$, ${}^{LSMC}\overline{\vartheta}^{PRS}$), consisting of paid-up, resumption, and surrender options. This option value is fairly small when σ^I is small, even if policyholders follow the LSMC strategy. However, the option value increases as σ^I enlarges. While σ^I is small, the options' upper-bound value is higher with a higher mean of the interest rate (θ). However, the option value with lower θ grows much faster as σ^I increases. With the guaranteed rate closer to θ, the impact of the interest volatility on the fairness situation becomes much stronger. This influence of θ and σ^I on the option value is captured if policyholders follow the LSMC strategy. Though the option value based on the LSMC strategy is much lower than the upper bound, it shows a similar structure: when σ^I is small, ${}^{LSMC}\overline{\vartheta}^{PRS}$ with $\theta = 6\%$ is slightly higher than the option value with $\theta = 4\%$ or $\theta = 3.5\%$. As σ^I increases, the option value with smaller θ increases faster. However, θ's impact on ${}^{LSMC}\overline{\vartheta}^{PRS}$ is fairly small. As seen in Figure (1.2(b)), the guaranteed value decreases with small g and hence ${}^{Up}\overline{\vartheta}^{PRS}$ increases especially when σ^I is small. This low guaranteed value and thus the high participation rate result in a low contract value variation. Thereby, the optimal exercise strategy becomes less efficient with g much smaller than θ and the difference between ${}^{LSMC}\overline{\vartheta}^{PRS}$ and ${}^{Up}\overline{\vartheta}^{PRS}$ enlarges. Therefore, ${}^{LSMC}\overline{\vartheta}^{PRS}$ with $g = 0\%$ is generally lower than the value if $g = 3\%$. Figure (1.2(c)) shows that with the contract duration prolonged to 40 years, the option value increases substantially and faster with larger σ^I due to an even higher uncertainty (${}^{LSMC}\overline{\vartheta}^{PRS} \simeq 5570$ with $T = 40$ compared to ${}^{LSMC}\overline{\vartheta}^{PRS} \simeq 570$ with $T = 10$ for $\sigma^I = 2\%$, $\theta = 6\%$).



(a) Option value for $T = 10$ and $g = 3\%$

(b) Option value for $T = 10$ and $g = 0\%$

(c) Option value for $T = 40$ and $g = 3\%$

Figure 1.2: Value of the triple option, ${}^{LSMC}\overline{\vartheta}^{PRS}$ and ${}^{Up}\overline{\vartheta}^{PRS}$ for different values of long-term interest mean (θ), time to maturity (T), and the interest guaranteed rate (g)

To avoid a high premium for policyholders and to cover administrative costs arising on the insurers' side when the premium-payment option is exercised, insurers usually charge a certain fee when the option is used. In policyholders' perspective, Fee can be considered as market friction. Figure (1.3) demonstrates the option value with $Fee \geq 0$. The fee's impact on ${}^{LSMC}\overline{\vartheta}^{PRS}$ enlarges with growing σ^I. When σ^I is small, ${}^{LSMC}\overline{\vartheta}^{PRS}$ has little value even if no fee is charged. When $Fee > 0$, it is less likely to exercise a premium-payment option under the LSMC strategy. However, even with a fee equal to 10% of the policy account value, the premium-payment option value increases to more than 250 (almost a quarter of the annual premium) with $T = 10$ if σ^I exceeds 2%. The triple option is OTM, or zero intrinsic value most of the time. Nevertheless, with large σ^I, certain extreme cases may occur, where policyholders benefit greatly from the embedded premium-payment options.

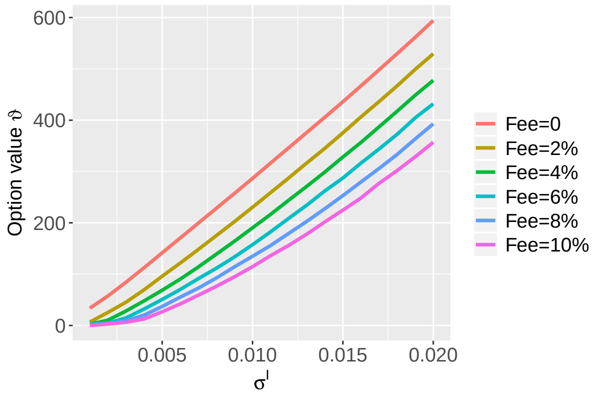
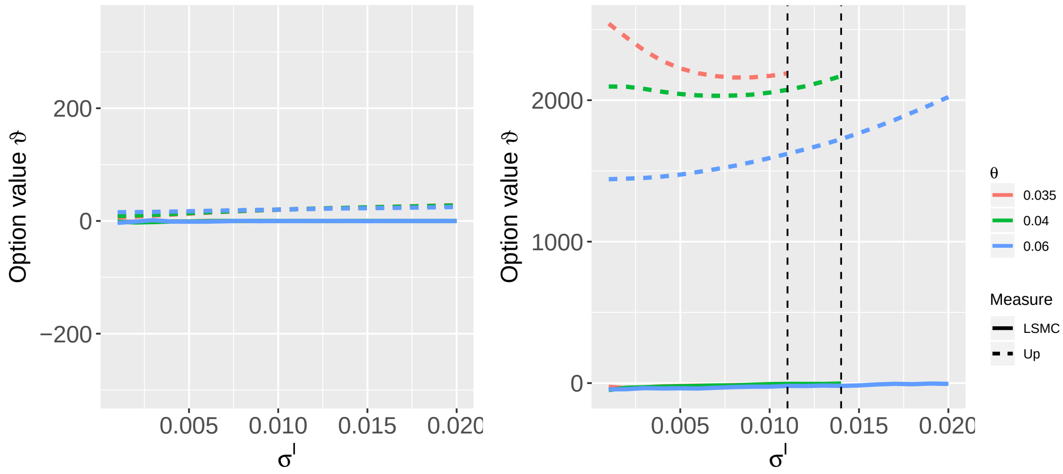


Figure 1.3: Fee structure within ${}^{LSMC}\overline{\vartheta}^{PRS}$ with $\theta = 4\%$, $T = 10$, and $g = 3\%$

Figure (1.4) shows the discrepancy between ${}^{LSMC}\overline{\vartheta}^{PRS}$ and ${}^{LSMC}\overline{\vartheta}^{PS}$, or the value of the extra resumption option compared to the combined paid-up and surrender option. Though the upper bound of this resumption option's value is non-negative and more than 2,000 if the maturity is long ($T = 40$), it is not possible to exercise this option efficiently with a feasible exercise strategy.



(a) $T = 10$ (b) $T = 40$

Figure 1.4: Value of the extra resumption option as the difference between ${}^{LSMC}\overline{\vartheta}^{PRS}$ and ${}^{LSMC}\overline{\vartheta}^{PS}$ for $T = 10$ and $T = 40$

Figure (1.5) presents the triple option value for specific stopping points with $\overline{\vartheta}^{PRS}_{\tau,\nu,\xi} = \frac{1}{N}\Sigma_{n=1}^{N}({}^{n}\vartheta^{PRS}_{\tau,\nu,\xi})$. This value drops even deep OTM as an extra option being exercised ($\nu < T$ and $\xi < T$). Without following any optimal strategy, this triple option is only positive when the paid-up option is exercised during the early years without the surrender option being used. In general, it is not worth exercising two or even all of the three options if no optimal exercise strategy is followed.

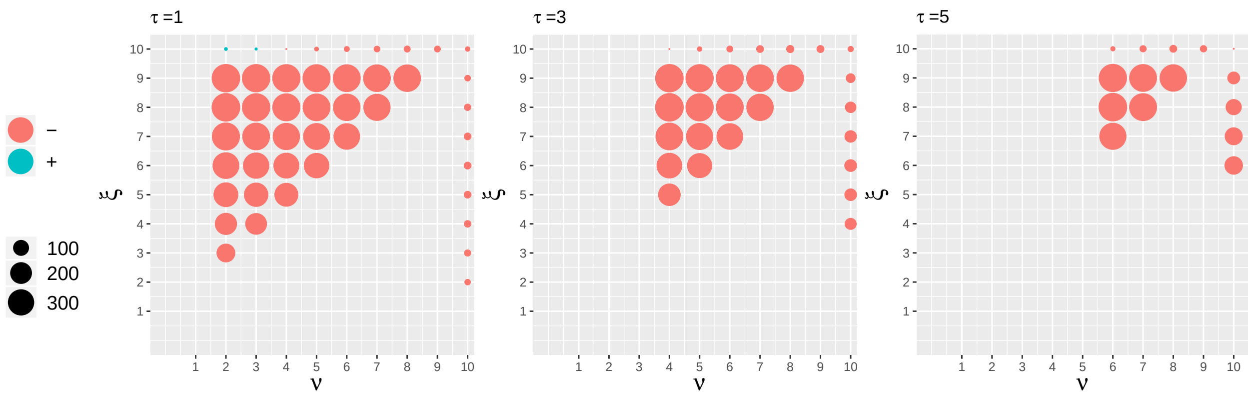
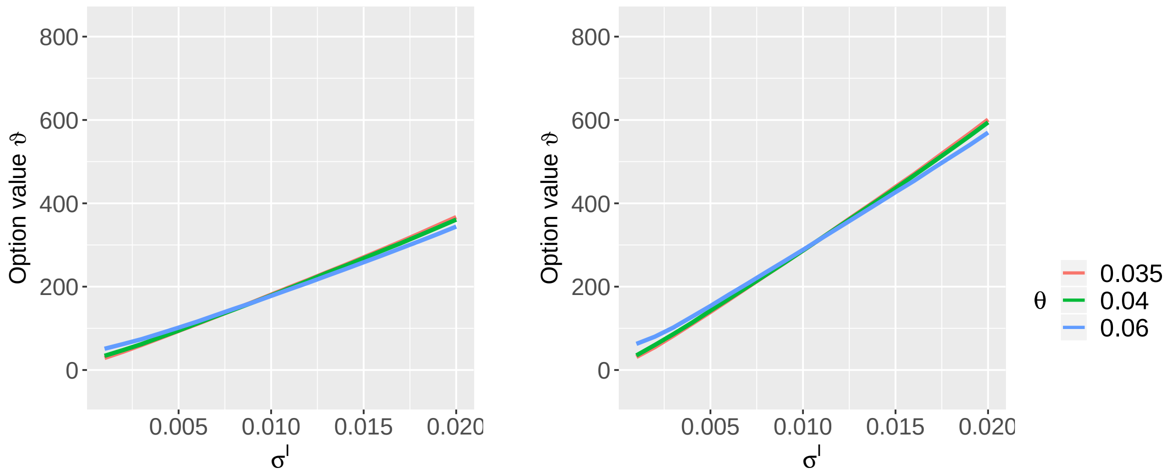


Figure 1.5: The triple option value with respect to different exercise points τ, ν, and ξ with $\sigma^{I} = 2\%$, $\theta = 4\%$, and $T = 10$

Double Premium-Payment Option:
Paid-up and Resumption Option & Paid-up and Surrender Option

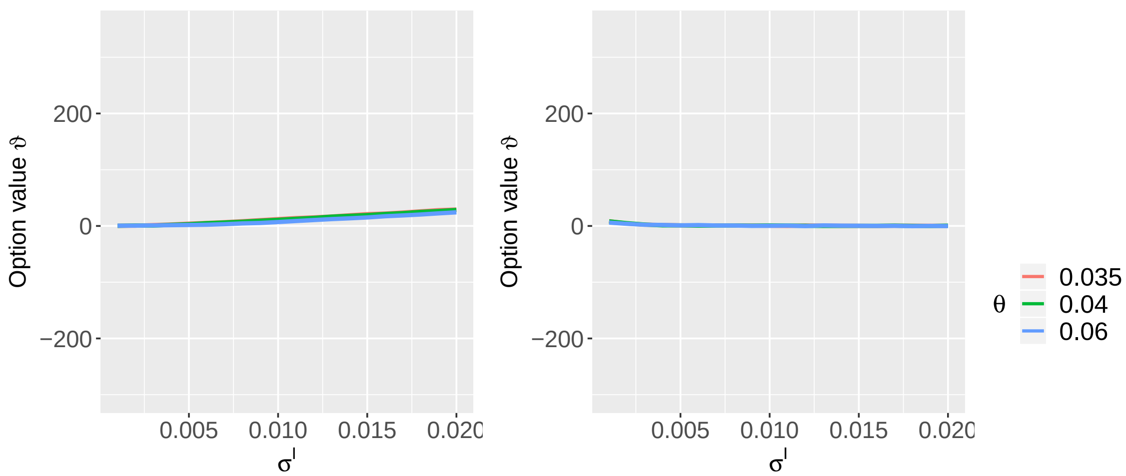
Figure (1.6) demonstrates the value of the double option: the combined paid-up and resumption (${}^{LSMC}\overline{\vartheta}^{PR}$) and the combined paid-up and surrender (${}^{LSMC}\overline{\vartheta}^{PS}$). ${}^{LSMC}\overline{\vartheta}^{PS}$ possesses much higher value compared to ${}^{LSMC}\overline{\vartheta}^{PR}$, especially when σ^I is large. Figure (1.7) compares the double and the single option: ${}^{LSMC}\overline{\vartheta}^{PR}$ with ${}^{LSMC}\overline{\vartheta}^{P}$ and ${}^{LSMC}\overline{\vartheta}^{PS}$ with ${}^{LSMC}\overline{\vartheta}^{S}$. The extra resumption option generated by the difference between ${}^{LSMC}\overline{\vartheta}^{PR}$ and ${}^{LSMC}\overline{\vartheta}^{P}$ is only slightly positive even when σ^I is fairly large. As a call option, the value of the resumption option is small when the paid-up option as a put option is in the money in the previous contract years. The extra paid-up option generated by the difference between ${}^{LSMC}\overline{\vartheta}^{PS}$ and ${}^{LSMC}\overline{\vartheta}^{S}$ is negligible regardless of σ^I. It is thus concluded that the double option value comes dominantly from a single option.



(a) Value of the combined paid-up and resumption option, ${}^{LSMC}\overline{\vartheta}^{PR}$

(b) Value of the combined paid-up and surrender option, ${}^{LSMC}\overline{\vartheta}^{PS}$

Figure 1.6: Value of the double option: ${}^{LSMC}\overline{\vartheta}^{PR}$ and ${}^{LSMC}\overline{\vartheta}^{PS}$ with $T = 10$ while the LSMC strategy is followed

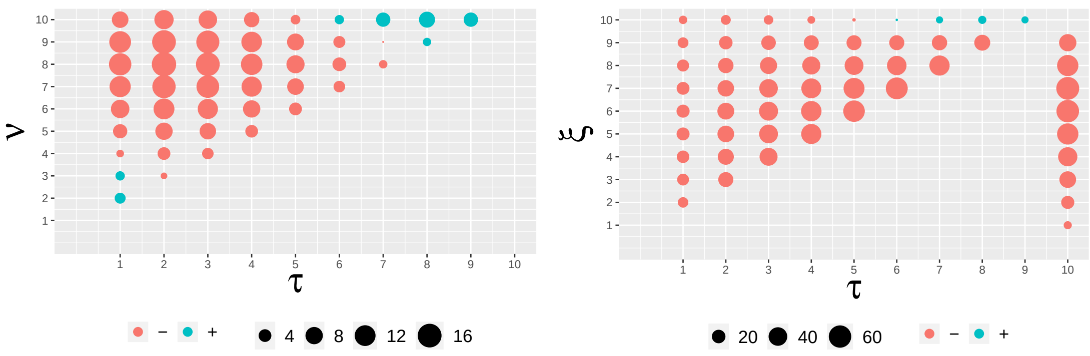


(a) The extra resumption option value

(b) The extra paid-up option value

Figure 1.7: Value of the extra options: the extra resumption option and the extra paid-up option if the LSMC strategy is followed

The double option values with different exercise points, $\overline{\vartheta}^{PR}_{\tau,\nu} = \frac{1}{N}\Sigma^{N}_{n=1}({}^{n}\vartheta^{PR}_{\tau,\nu})$ and $\overline{\vartheta}^{PS}_{\tau,\xi} = \frac{1}{N}\Sigma^{N}_{n=1}({}^{n}\vartheta^{PS}_{\tau,\xi})$ are presented in Figure (1.8). Without following any exercise strategy, both double option values are negative most of the time. In addition, the highest value occurs when only one option is used ($\overline{\vartheta}^{PR}_{\tau,T}$ and $\overline{\vartheta}^{PS}_{\tau,T}$).



(a) Option value, $\overline{\vartheta}^{PR}_{\tau,\nu}$ (b) Option value, $\overline{\vartheta}^{PS}_{\tau,\xi}$

Figure 1.8: The double option value with respect to different exercise points τ, ν, and ξ with $\sigma^{I} = 2\%$, $\theta = 4\%$, and $T = 10$

Single Premium-Payment Option:
Paid-up option & Surrender Option

Figure (1.9) demonstrates the single option value: the paid-up option and the surrender option. These two options possess put option features while the surrender option influences a larger cash flow. Hence, both ${}^{LSMC}\overline{\vartheta}^{S}$ and ${}^{Up}\overline{\vartheta}^{S}$ are much greater than ${}^{LSMC}\overline{\vartheta}^{P}$ and ${}^{Up}\overline{\vartheta}^{P}$, especially when σ^{I} is fairly large. Thus, while the triple option value is dominated by the double option, ${}^{LSMC}\overline{\vartheta}^{PS}$, ${}^{LSMC}\overline{\vartheta}^{PS}$ provides little extra value to the single option ${}^{LSMC}\overline{\vartheta}^{S}$. Specifically, even if policyholders following an optimal exercise strategy, insurers do not need to charge extra when offering additional options.

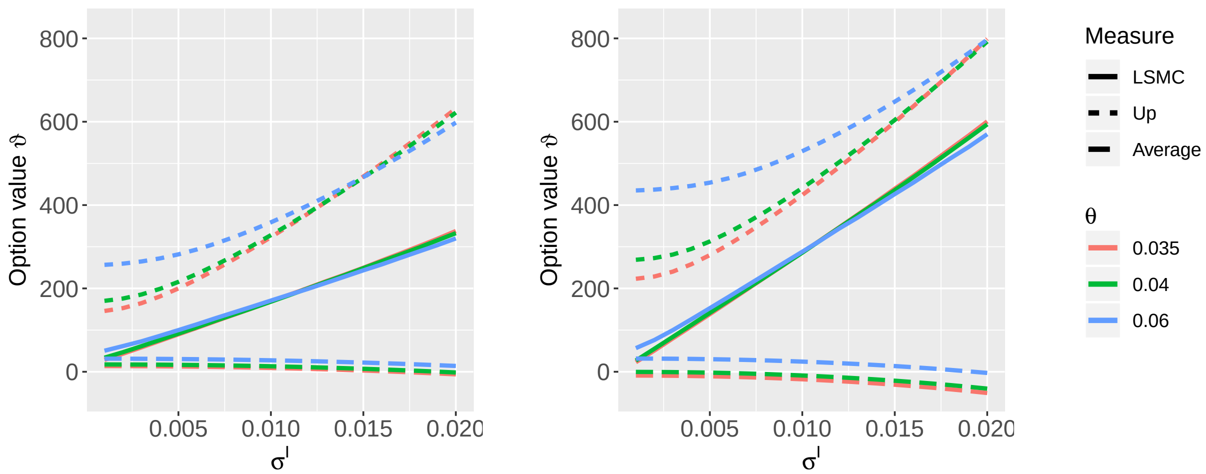
Following an optimal strategy may seem a rather strong assumption, as policyholders are typically subject to several constraints (e.g., liquidity constraint) or other frictions.[13] Additionally, the value of the premium-payment option is not the only concern when policyholders decide whether to exercise their options or not. Instead, this decision can be influenced by not only the external features (such as financial market developments) but also certain personal reasons. We therefore introduce a concept denoted as “Average”. Assuming that the options are equally

[13] For the case of the valuation of the surrender option only, Li and Szimayer (2014) shows that the rationality of the policyholders significantly influences the option value.

likely to be exercised throughout the contract period, the option value is the average of the yearly option valued at t with $0 < t < T$. In formal terms, the option value under this measurement is calculated by:

$$^{Average}\overline{\vartheta} = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{T-1}\left(\sum_{t=1}^{T-1} {}^{n}\vartheta t\right).$$

In Figure (1.9), the option value in "Average" is around zero and decreases mildly as σ^I increases. Exercising the paid-up or surrender option means to give up partial (for the paid-up option) or entire (for the surrender option) value provided by the guaranteed rate (g). Without following a certain exercise strategy, this positive guaranteed value increases as σ^I increases. Hence, $^{Average}\overline{\vartheta}$ decreases with larger σ^I. In addition, the guaranteed value is larger with θ closer to g. Hence, as concluded if the LSMC strategy is followed, on average, lower θ leads to a lower premium-payment option value.



(a) Value of the single paid-up option (b) Value of the single surrender option

Figure 1.9: Value of the single option with $T = 10$

The previous section shows that the triple option has a fairly large value even if a 10% fee is charged when the option is exercised under the LSMC strategy. As the triple option values almost the same as the single surrender option, Figure (1.10) shows the single surrender option value with $Fee \geq 0$ assuming that policyholders exercise their options according to the LSMC strategy ($^{LSMC}\overline{\vartheta}^S$ as a solid line) or equally likely throughout the contract years ($^{Average}\overline{\vartheta}^S$ as a dotted line). $^{Average}\overline{\vartheta}^S$ drops significantly, while the impact of the fee on $^{LSMC}\overline{\vartheta}^S$ is relatively small, especially when σ^I is low. Following the LSMC strategy, it is less likely that the options will be exercised if a fee is charged as shown in Figure (1.11). As it is not always optimal to exercise the option, the option fee charged when exercising the option does not significantly influence the option price in the LSMC setting. Especially when σ^I is large, extreme cases may occur, in which the surrender option goes deep ITM and becomes fairly valuable even with a

10% fee.

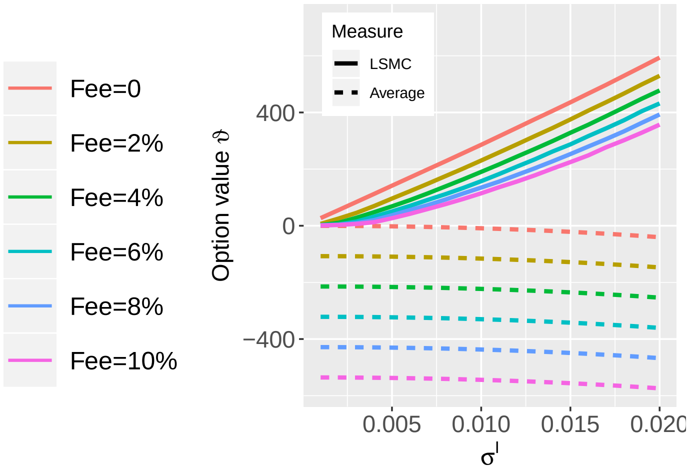


Figure 1.10: ${}^{LSMC}\overline{\vartheta}^{S}$ and ${}^{Average}\overline{\vartheta}^{S}$ with $Fee \geq 0$

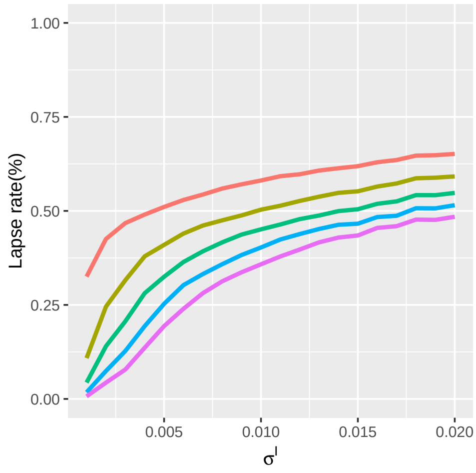


Figure 1.11: Lapse rate of the single surrender option when LSMC is followed with $Fee \geq 0$

1.5.3 Sensitivity Test

Adjusting to the current low-interest rate environment, we reset our parameters with the guaranteed rate $g = 0\%$, the initial interest rate $\hat{r}_0 = 0.5\%$, and the long-term interest rate mean $\theta \in \{0.5\%, 1\%, 1.5\%\}$, as described in Table (1.3).

The results show that as long as there is α with $0 \leq \alpha \leq 1$ such that $\vartheta^{\emptyset} = 0$ (i.e., it is possible to offer a fair contract), the relation between σ^I and the option value, and the relation among different option values remain mostly unchanged.

Figure (1.12) shows the participation rate, α, with various long term interest means. Larger σ^I leads to a lower participation rate, while the participation rate is higher with a larger θ. With a guaranteed rate much closer to θ, α is fairly small compared to the previous example. For $\theta = 0.5\%$, there exists no α, with $0 \leq \alpha \leq 1$ when σ^I reaches 2%. The interest impact on the option values is trivial if LSMC is assumed followed. As concluded in the previous example, even if following an optimal exercise strategy, an extra resumption option and an extra paid-up option contribute little to the triple option. Specifically, the triple option possesses around the same value as the single surrender option. Regarding the single surrender option, ${}^{Average}\overline{\vartheta}^{S}$ is negative especially when σ^I is large and θ is small.

B^*	constant annual premium payment	1200
γ^*	constant death benefit	11944.96
g	guaranteed interest rate	0%
$\hat{r}_0$	initial interest rate	0.5%
θ	long-term interest mean	0.5%/1%/1.5%

Table 1.3: Parameter table for the low-interest rate case

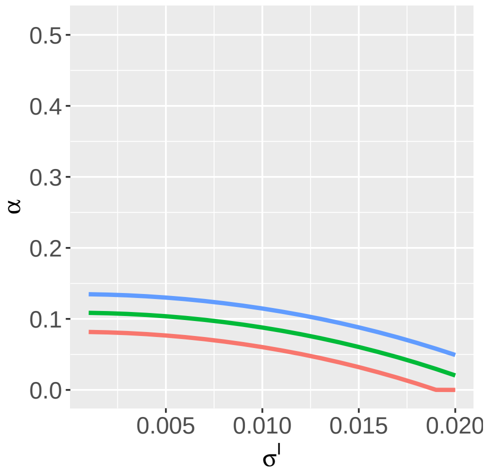


Figure 1.12: α/σ^I combinations for different θ resulting in a fair contract condition

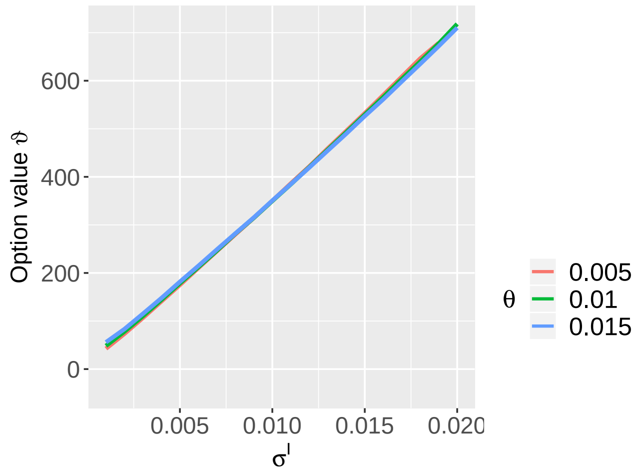


Figure 1.13: Option value, ${}^{LSMC}\overline{\vartheta}^{PRS}$ with different σ^I

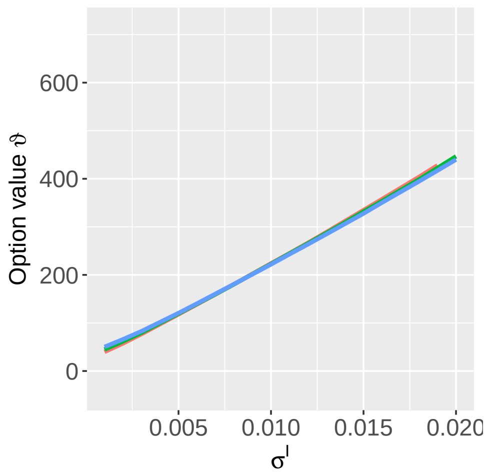


Figure 1.14: Option value, ${}^{LSMC}\overline{\vartheta}^{PR}$ with different σ^I

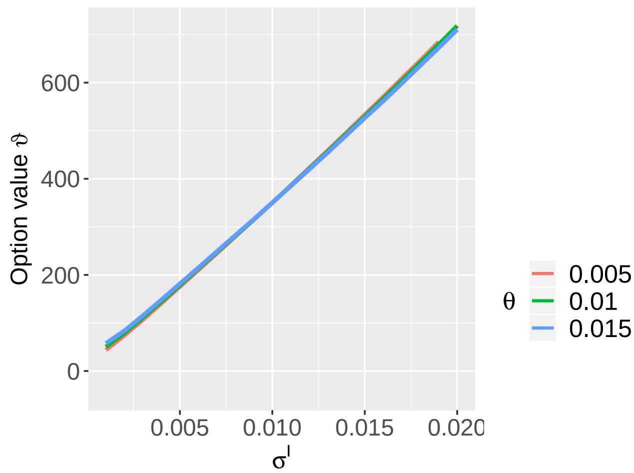


Figure 1.15: Option value, ${}^{LSMC}\overline{\vartheta}^{PS}$ with different σ^I

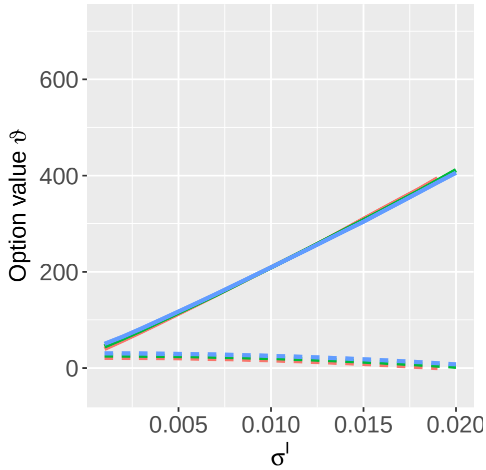


Figure 1.16: Option value, ${}^{LSMC}\overline{\vartheta}^{P}$ and ${}^{Average}\overline{\vartheta}^{P}$ with different σ^{I}

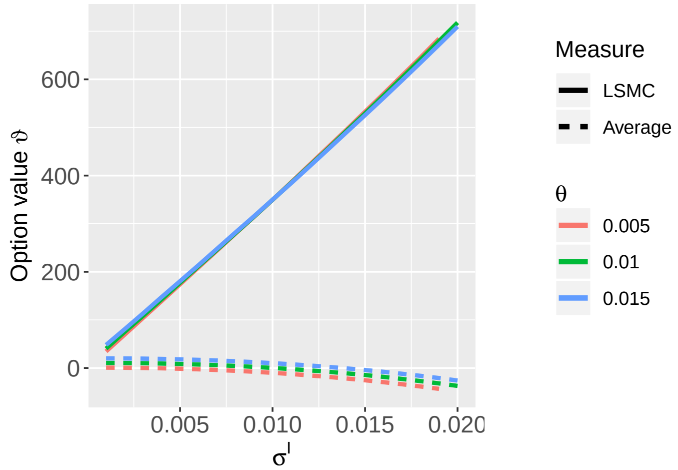


Figure 1.17: Option value, ${}^{LSMC}\overline{\vartheta}^{S}$ and ${}^{Average}\overline{\vartheta}^{S}$ with different σ^{I}

1.6 Economic Interpretation and Outlook

The numerical results show that, if stochastic interest rates are taken into account and the policyholder follows an exercise strategy based on LSMC, the values of premium-payment options can be substantial. For instance, even with a short contract duration $T = 10$, a surrender option values up to two thirds of the annual premium if interest becomes rather volatile. Hence, insurers will need to charge extra for the options provided in order to finance adequate risk management measures. In addition, we demonstrate that the necessity of charging for premium-payment options cannot be eliminated by a fee structure dependent on the account value.

Beside the risk sources introduced in our model, life insurance companies typically face substantial model and parameter risk regarding the valuation of embedded options. Additionally, Swishchuk (2004) suggests that the risk is underestimated whenever the volatility is assumed to be constant in time. Considering these factors, insurers may need to charge even higher premiums than those proposed in our model setup.

If an optimal exercise strategy is followed, the multiple premium-payment option values only little more than a single option. The valuation of a single option, however, strongly depends on the policyholders' exercise behaviour. Real insurance markets may be incomplete and face frictions such as taxation and information asymmetries. These kinds of effects may hinder policyholders' attempts to take full advantage of future market developments -– which are assumed to take place via a LSMC strategy — by using premium-payment options. In addition, policyholders may not act in a fully rational way when it comes to exercising options. To approximate such a behaviour, we assume in the paper that a fixed fraction of policyholders would exercise their single option each year. In such a case, the average option value is around zero or even negative if the interest rate is rather volatile. Hence, if such an exercise behaviour takes place, paid-up and surrender options could be offered free of charge. However, in the future, policyholders could be better advised on how to use embedded premium-payment options or the market efficiency may alter. Hence, if insurance companies base their pricing purely on the empirical exercise behaviour of policyholders (or any other assumption of a suboptimal exercise strategy), they can face a considerable risk of underpricing.

In most cases, insurance companies are not free to choose whether to offer premium-payment options or not. For instance, a participating life insurance contract must have at least a surrender option by law in all insurance markets we know. As pointed out, in order to provide these options, insurers must charge, in addition to the savings premium and the term life premium, a substantial price to finance adequate risk management measures. Given a financial market with alternative products in the field of old-age provision, such as fixed-income mutual funds (cf. e.g., Koijen et al. (2009)), this additional charge may reduce the attractiveness of participating life insurance contracts.

In practice, insurance companies charge policyholders a fee whenever a premium-payment option is exercised. The general idea of such a fee model is that only those policyholders who exercise a premium-payment option would need to pay. This fee structure may discourage policyholders from exercising their options. In such a case, the introduction of a fee model can be beneficial for both the insurers and those policyholders who do not or cannot follow an optimal exercise strategy: On the one hand, insurers face less uncertainty regarding the exercise behaviour. On the other hand, as, for instance, average option values are often negative especially when interest rate volatility is large, policyholders are better off if they exercise their options more seldom. For policyholders using a LSMC strategy, the fee model only slightly reduces the price of premium-payment options. Clearly, under fair pricing conditions, the situation of these policyholders remains unchanged (the NPV of the contract is zero with or without fees) even though fewer exercises take place.

As another approach, insurers may also introduce a lockup period, during which policyholders are not allowed to exercise premium-payment options. With a lockup period, the duration in which the premium-payment options can be exercised is shortened. Hence, the value of these options will decrease substantially. Another way to tackle the issue raised in this paper is to pay back only the market value if a premium-payment option is exercised before maturity. The insurer would then face no risk from premium-payment options and need not charge any additional premium because the option has no value under any exercise assumption. Clearly, the insurer would then be unable to promise policyholders a fixed payback at certain points in time as is the current practice in participating life insurance contracts with cliquet-style investment guarantees.

Beside the insurers' viewpoint, this paper also offers a framework to policyholders to use their contracts' embedded options in a better way. As we show in this paper, the present value of premium payment options in participating life insurance contracts with cliquet-style investment guarantees can often become 5% of the present value of all policyholders' premium payments made in the contract. If insurance companies base their pricing on the option pricing theory while policyholders do not exercise their premium payment options in an optimal way, a severe wealth transfer takes part to the disadvantages of the insured. Even a small percentage of the large premium volume invested in this kind of products sums up to a large amount of money per year, which is lost for the policyholders' old-age provision. Empirical research to policyholders' option exercising behaviour in the life insurance sector strongly supports the thesis, that premium payment options are typically used in a suboptimal manner.[14]

As shown in this paper, premium-payment options in general, and the surrender option in particular, can be of great value if stochastic interest rates are taken into account and an optimal exercise strategy is followed. In particular, with a large volatility of the term structure, extreme cases can happen, where the premium-payment options will go deep ITM. In this case, policyholders, even without explicit knowledge of an optimal exercise strategy, can easily benefit from interest rate fluctuations. For example, current contracts with low interest rate guarantees seem to be very unattractive when the interest rate rebounds. If this happens, policyholders can enjoy a large premium-payment option value simply by surrendering their contracts. From this point of view, introducing a lockup period, particularly in the case of surrender options, or surrender amount based on the market condition is necessary. However, regulatory authorities in many EU countries are currently attempting to set minimum levels for surrender values to thwart such approaches. Given the findings of this paper, such regulatory steps could negatively influence the financial stability of life insurance companies if a large proportion of policyholders surrender their contracts at the same time whenever the interest rate rebounds. Hence, similar to a bank run scenario, a life insurance run scenario could take place.

[14] For an overview cf. Eling and Kochanski (2013)

Appendix: Least Square Monte Carlo Method (LSMC)

Figure A1 compares the LSMC strategy discussed in this paper, termed ”adjusted LSMC”, with the Longstaff and Schwartz (2001) method, termed ”original LSMC”. For our numerical example, we demonstrate that the adjusted LSMC, leading to a slightly higher option value, is more efficient than the original LSMC strategy.

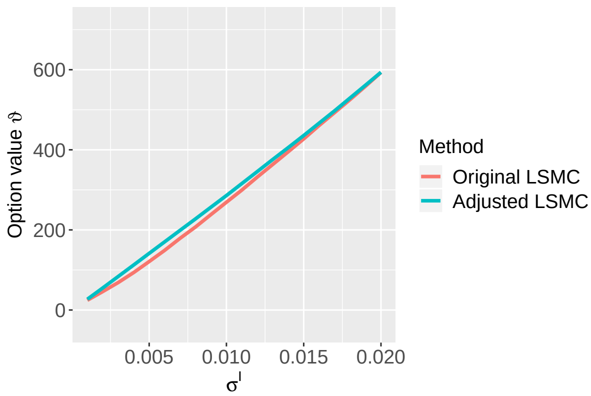


Figure A1: Comparison of surrender option value using different LSMC methods

Essay II

Life Insurance Surrender and Liquidity Risks

Hsiaoyin Chang, Hato Schmeiser

Surrender options in endowment life insurance contracts can result in a surrender risk for the insurer. This risk is closely related to investment and liquidity risks. Consequently, the surrender risk is underestimated if it is assessed without consideration of all major risk sources. Using different risk measures, this paper shows that the surrender risk increases if the liquidity constraint is considered. Additionally, in an extreme event, mass surrender and a liquidity crisis can trigger each other. Therefore, the surrender risk could grow even faster. In this case, the solvency capital calculated within the Solvency II regime may not be enough to provide the intended protection. Furthermore, if policyholders surrender their contracts rationally, insurers face an even higher default threat.

Keywords: Endowment life insurance · Risk management · Liquidity risk · surrender risk

2.1 Introduction

Liquidity in the financial market fluctuates. Bookstaber (2000) studies liquidity cycles and concludes that the market dislocation of a liquidity-based crisis is difficult to model and thus unpredictable. The liquidity crisis in the financial market is interrelated to the personal liquidity strain. More specifically, the insurance run may lead to a liquidity crisis in the financial market, while a financial downturn causes a mass surrender. Due to this interdependency, the combination of the surrender and liquidity should be considered together to avoid an underestimation of the true surrender risk for life insurance companies. Consequently, this paper investigates the surrender risk influenced by the liquidity constraint, which fluctuates along with the financial market.

Liquidity is the capability to meet an immediate cash demand with an ongoing cash inflow or the prompt sale of an asset (see Claire et al. (2000)), while liquidity risk involves the uncertainty of this capability. An entity faces a liquidity problem if one of the following two conditions occur. Firstly, the entity does not have sufficient liquid assets to meet its immediate obligations. Secondly, to meet its immediate obligations, the entity is forced to sell its asset at a discounted price. Consequently, its asset value is reduced to less than its liability value. In such a case, the entity is forced to declare bankruptcy as the equity value becomes negative.

Liquidity risk is important for financial institutions for two primary reasons:

1. Financial institutions are highly interconnected; hence, liquidity problems can easily spill over. For the banking sector, Greenwood et al. (2015) models spillover effects, showing that a “fire sale” of assets in one bank negatively influences another bank’s balance sheet. In the insurance sectors, Ellul et al. (2011) and Becker and Ivashina (2015) record the price discount of downgraded bonds due to fire sales, especially when the insurance industry is distressed.

2. Financial institutions generally hold a substantial amount of liabilities. Once financial institutions face liquidity problems, their clients lose confidence in them, which worsens the situation (the so-called bank-run problem). Compared to the banking industry, an “insurance run” problem is often argued to be minor. On the one hand, no ”insurance run” risk exists in the non-life sector, since payments are only made to policyholders in the event of claims. On the other hand, the liabilities undertaken by life insurers are usually long-term.

However, surrender options embedded in most life insurance contracts create cashflow uncertainty and may shorten the contract duration (see Paulson et al. (2012)). Back in 2000, Claire et al. (2000) addressed liquidity issues in response to the National Association of Insurance Commissioners (NAIC), following the concern of a downgraded put option on Guaranteed Investment Contracts (GIC) and the unexpected default of the General American Insurance Company in 1999. According to Claire et al. (2000), a well-managed company faces little liquidity risk when dealing with a large, but expected, amount of cash outflow (e.g., well estimated claim payment at a specific point in time). The embedded surrender option, triggering cash outflow uncertainty, imposes a great liquidity risk to the life insurer. This liquidity issue, caused by surrender options and augmented by the contagion effects among policyholders, may lead to systemic risk. Thus, it has attracted attention from regulatory authorities (see European Systemic Risk Board (2015), p.16; International Monetary Fund (2016), p.91). Supervisors in some countries (e.g., France, Japan) are even granted powers to suspend surrender payments (see Haefeli and Ruprecht (2012)).

Participating life insurance contracts are generally required by law to offer surrender options. The surrender payout amounts are fixed at the beginning of the contracts and may be higher or lower than the contract's market value at the time of the surrender. This disparity between the amount fixed in the insurance contract and the market value, combined with the uncertainty of when the policyholders surrender their contracts, creates a surrender (lapse) risk to the insurer.[15]

In the Solvency II framework, the lapse risk is the most important part of the underwriting risk for life insurance companies. After the market risk, the underwriting risk is the second most serious risk source (see EIOPA (2011)). In addition, the market risk, which includes the liquidity risk, is closely related to the surrender risk.

[15] Slight differences between the two terms, lapse and surrender, are defined in Kuo et al. (2003) and Gatzert et al. (2009). The former indicates the early termination of insurance contracts, usually without a payment to the policyholder. The latter refers to the surrender value paid to the policyholders upon termination before the end of maturity. According to Kuo et al. (2003) and Eling and Kiesenbauer (2014), lapse risk generally relates to the uncertainty caused by both surrender and lapse.

Country	Traditional Contract(*) (million Euro)	Unit-Linked Contract(**) (million Euro)	Traditional Contract Portion
U.K.	80,299.11	137,230.96	36.91%
France	111,611.97	42,056.59	72.63%
Italy	75,320.43	30,711.85	71.04%
Germany	73,423.51	16,836.11	81.35%
Ireland	6,045.92	31,326.96	16.18%
Spain	22,545.94	5,928.48	79.18%

The aggregated gross written premiums (excluding reinsurance business) are shown in million Euro; *Traditional contracts are life products, excluding health and index-linked & unit-linked; ** Unit-Linked contracts are index-linked & unit linked products (seeEIOPA Insurance Statistics (2019))

Table 2.1: Top six life insurance markets in Europe in terms of the gross written premium

Facing the current low-interest challenge, insurers attempt to adjust their products towards those with lower equity capital requirements (e.g., unit-linked products). However, due to the competitive insurance market and policyholder needs and expectations, traditional insurance products still play a major role in several European markets (see Dany (2018)). Table (1) shows that the traditional life products dominate the major life insurance markets in continental Europe. On the one hand, if the interest rate continues sinking, insurers face difficulty via rising liabilities caused by embedded investment guarantees. On the other hand, once the interest rate rebounds, policyholders may decide to (or be advised to) surrender for better investment opportunities. Therefore, insurers are threatened by both the guaranteed interest rates and the surrender options. When the interest rate remains low, the overweighting of high-risk and illiquid bonds can be encouraged ("reach-for-yield"). As such, insurers become increasingly vulnerable to market turbulence (see Ellul et al. (2018)). Thereby, the liquidity risk occurs and exacerbates once the surrender rate spikes alongside the interest rate (see Paulson et al. (2012)).

The surrender option value increases with the market interest rate. Accordingly, the so-called interest rate hypothesis assumes that the surrender rate rises whenever the market interest rate increases. This hypothesis is confirmed by Kuo et al. (2003), Russell et al. (2013), Kiesenbauer (2012), and Kubitza et al. (2020). In addition, Barsotti et al. (2016) modelled copycat behavior and confirmed the correlation and contagion effects among policyholders. This effect results in a mass surrender in which many policyholders surrender their policies after learning that a large number of policyholders have already exercised their surrender options. More specifically, the first group surrenders for a certain reason (e.g., personal financial difficulties, rising interest rates), whereas the second group follows the first group.

Biagini et al. (2020) built a model based on extreme value theory and calibrated an extreme lapse scenario using data from the US and German insurance market. In addition to the modeling of a mass lapse scenario, the effects of such an event on the financial stability of insurance companies and the adequacy of the current Solvency II regulatory requirements are analyzed. The mass surrender scenario, as modelled by the author, gives rise to an "insurance run". In contrast to our paper, which examines the influence of the policyholder's exercise behavior on the value of the surrender option and the insurer's liquidity risk caused by a mass surrender, Biagini et al. (2020) do not analyze these two central risk sources for an insurer in the context of the termination of a life insurance contract.

In Förstemann (2018), an insurance run occurs when the interest rate rises to a critical level, at which the policyholders are prone to exercise their options. However, in contrast to the model setup in our paper, such an exercise behavior is not based on an optimal exercise strategy. This entails that the true risk may be underestimated even if the triggered liquidity risk is neglected. Kubitza et al. (2020) built a granular empirically calibrated theoretical model of a life insurer with different cohorts of policyholders to examine the insurance liquidity exposure due to policyholders' surrender behavior. The aim of the paper of Kubitza et al. (2020) is not to analyze the impact of surrender with respect to the financial stability of the insurer but, rather, to model empirically the connection between interest rate developments, triggered surrenders by policyholders and the impact of mass surrender on the capital market.

The surrender risk is influenced by the interest rate development and the liquidity constraint. This constraint fluctuates with the conditions of the capital market. In this paper, we contribute to the existing body of knowledge by documenting that the surrender risk can be largely underestimated if the potential liquidity risk is neglected. The paper starts with the modeling of a life insurance endowment contract with an embedded cliquet-style investment guarantee. The setup captures two risk sources: asset risk and interest rate risk. The valuation of the contract is provided via risk-neutral valuation assuming fair conditions (i.e., the present value of the claim benefits to the beneficiary equals the present value of the premium payments). We take the policyholder's surrender option into account and analyze how the insurer's financial stability is influenced by the combined risk of early surrender and a liquidity constraint. There is a large body of literature on the valuation of the surrender options in participating life insurance contracts in case of one risk source (asset risk) and a deterministic term structure (for a literature overview see Schmeiser and Wagner (2011)). In this paper, we use the approach first introduced by Chang and Schmeiser (2020), in which the authors develop an extended version of the Least Squares Monte Carlo (LSMC) approach to price surrender options taking stochastic interest rates into account (hence, a second risk source is considered). We extend the line of reason of Chang and Schmeiser (2020) by analyzing how the insurer's financial stability is influenced by the combined risk of early surrender and a liquidity constraint. The valuation of the insurer's investment portfolio with the liquidity constraint is based on the Marginal Supply Demand Curve (MSDC) concept first presented by Acerbi and Scandolo (2008). In this context, we analyze different exercise strategies with respect to the surrender option. Using different risk measures, this paper shows that the surrender risk increases if the liquidity constraint is considered. If policyholders surrender their contracts rationally by solving an optimal stopping problem, insurers face a substantial bankruptcy threat. For this case, the solvency capital calculated within the Solvency II regime may not be sufficient to provide the intended protection.

The rest of this paper is structured as follows. A life insurance contract with a surrender option is constructed and the modelled surrender rate is described in the second section. The third section discusses the liquidity constraints and how the portfolio value evolves when liquidity is considered. The risk measurement is defined in the next section to quantify the surrender risk, with or without consideration of the liquidity constraint. The fifth section presents and analyzes the numerical results and the final section concludes this paper.

2.2 Participating Life Insurance Contracts

2.2.1 General Contract Modeling and Fair Pricing

We consider a life insurance endowment contract as discussed, for example, in Schmeiser and Wagner (2011) and Chang and Schmeiser (2020). This contract features two standard options: a cliquet-style investment guarantee and a surplus participation. In addition, the policyholder has the right to surrender the contract. The contract duration T in years can be separated in single years using a time index t. In the beginning of year t, the policyholder pays an annual premium B_t under the condition that, at the end of year $t˘1$, the policy is still in force (i.e., the policyholder is alive and the surrender option has not been exercised). As is typically the case, the annual premium B ($\equiv B_t$) is constant in time.

With x we denote the age of the policyholders. ${}_tp_x$ (with ${}_0p_x = 1$) stands for the probability that the x-aged policyholder survives the next t years. $q_x = 1 -_1 p_x$ refers to the probability that the focal policyholder dies within the coming year. We assume that mortality risk is independent from asset and interest rate risk. Following actuarial practice, we assume that mortality risk is of pure unsystematic nature; that is, mortality risk can be (approximately) eliminated by writing a large number of insurance treaties.

The policyholder/beneficiary of the contract is entitled to either death or survival payments. The death benefit γ is paid to the beneficiary at the end of year t if the policyholder dies in year t. In our setting, the death benefit is constant in time (hence $\gamma \equiv \gamma_t$). If the policyholder survives till T, the survival benefit will be paid out by the insurer. Survival benefits are guaranteed with a minimum payment given by the death benefit γ plus a surplus participation.

In general, policyholders can choose to 1) receive the surplus as cash each year, 2) purchase extra insurance coverage and increase the death benefit amount, or 3) keep the surplus in the policy account and earn a stochastic return with a minimum level g, which is guaranteed by the insurer. In the third case, the survival benefit is strictly increasing in time if the investment guarantee rate g is positive. In what follows, we assume that policyholders keep their surplus in the policy account.[16]

The death benefit γ is calculated according to the actuarial equivalence principle (see (Linnemann, 2003)). Thereby, the expected premiums paid to the insurer (left-hand side of Equation (2.2.1.1)) must equal the expected payments to the beneficiary (right-hand side of Equation (2.2.1.1)). Using the yearly time-discrete guaranteed interest rate g provided in the contract[17] to discount future payoffs, the following relation can be formulated:

$$B\Sigma_{t=0}^{T-1}\, {}_t p_x (1+g)^{-t} = \gamma\Big(\sum_{t=0}^{T-1} {}_t p_x q_{x+t} (1+g)^{-(t+1)} + {}_T p_x (1+g)^{-T}\Big). \qquad (2.2.1.1)$$

The two terms in the brackets on the right-hand side of Equation (2.2.1.1) describe the case

[16] For the contract setup used in our paper, see Schmeiser and Wagner (2011) and the primary sources given in this contribution.

[17] The guaranteed rate is considered as the lower bound of the contract's interest rate. With a participating scheme, policyholders receive a surplus return whenever the insurer's asset return exceeds the guaranteed rate. This surplus is retained in the policy account (see Bacinello (2003a), Bacinello (2003b), Gatzert and Schmeiser (2008), and Schmeiser and Wagner (2011)).

of death within the maturity of the contract and the case of survival paid at year T. Based on Equation (2.2.1.1), the constant death benefit and the minimum guaranteed survival benefit γ can be derived as follows:

$$\gamma = \frac{B\sum_{t=0}^{T-1} {}_tp_x(1+g)^{-t}}{(\sum_{t=0}^{T-1} {}_tp_x q_{x+t}(1+g)^{-(t+1)} + {}_Tp_x(1+g)^{-T})}. \tag{2.2.1.2}$$

If the policyholder survives the duration of the contract T, the accumulated policy's account A_T is paid out. A_T consists of minimum payments based on the yearly interest rate guarantee g and the surplus participation. To calculate A_T, we separate the premium payment B into two parts: B_t^R and B_t^A. B_t^R is the premium for the embedded term life contract calculated as the death probability (q_{t+x-1}) times the difference between the death benefits γ and the accumulated account A_{t-1}. The remaining premium part B_t^A denotes the saving premium at year t and is given by:

$$B_t^A = B - B_t^R = B - q_{x+t-1}\max(\gamma - A_{t-1}, 0). \tag{2.2.1.3}$$

B_t^A is part of the accumulated account at the beginning of year t. At the beginning of year t, the sum of A_{t-1} and ${}_{t-1}p_xB_t^A$ evolves with a return rate containing both the guaranteed rate g and the surplus with the participation rate α via the following equation:

$$A_t = (A_{t-1} + {}_{t-1}p_xB_t^A)\cdot(\max(g, \alpha\cdot r_t)+1). \tag{2.2.1.4}$$

Thereby, $A_0 = 0$, and r_t denotes the insurer's stochastic investment portfolio return in year t.

Assuming that no short-selling is possible, this portfolio contains a π ($0 \leqslant \pi \leqslant 1$) portion of risky assets with a return rate of r_t^A and a $1-\pi$ portion of a government bond with a return rate of r_t^f. While r_t^f is only subject to the spot interest rate risk, r_t^A is related to the investment risk, including both the spot rate risk and the asset risk. As such, r_t can be written as:

$$r_t = (1-\pi)r_t^f + (\pi)r_t^A. \tag{2.2.1.5}$$

The spot rate risk element is assumed to evolve according to the one-factor Vasicek Model (see

Vasicek (1977)):[18]

$$(dr_t^f)^{\mathbb{P}} = \kappa(\theta - r_t)dt + \sigma_I dZ^{\mathbb{P}},$$

where $Z^{\mathbb{P}}$ is a Wiener process on a probability space $(\Omega, \phi, \mathbb{P})$. σ_I determines how much randomness of $Z^{\mathbb{P}}$ is acquired, while κ and θ are two positive constants representing the speed of the reversion and the long-term mean, respectively. For the risk-neutral measure $\mathbb{Q}$, a constant market price of risk λ is introduced. The interest spot rate under the risk-neutral measure $\mathbb{Q}$ is changed to:

$$(dr_t^f)^{\mathbb{Q}} = \kappa(\theta - \frac{\sigma_I \lambda}{\kappa} r_t)dt + \sigma_I dZ^{\mathbb{Q}},$$

where $Z^{\mathbb{Q}}$ denotes a Wiener process under the risk-neutral measure $\mathbb{Q}$.

$\hat{r}_t^f$, the one-period spot rate under the Vasicek model for the neutral measure $\mathbb{Q}$ and the real-world measure $\mathbb{P}$, can be approximated as follows:

$$\begin{aligned} (r_t^f)^{\mathbb{P}} &\cong (\hat{r}_t^f)^{\mathbb{P}} = \hat{r}_0 \cdot e^{(-\kappa\Delta t)} + \theta(1 - e^{-\kappa\Delta t}) + \frac{\sigma_I}{\sqrt{2\kappa}}\sqrt{1 - e^{-2\kappa\Delta t}} Z_t^{\mathbb{P}}, \\ (r_t^f)^{\mathbb{Q}} &\cong (\hat{r}_t^f)^{\mathbb{Q}} = \hat{r}_0 \cdot e^{(-\kappa\Delta t)} + (\theta - \frac{\sigma_I \lambda}{\kappa})(1 - e^{-\kappa\Delta t}) + \frac{\sigma_I}{\sqrt{2\kappa}}\sqrt{1 - e^{-2\kappa\Delta t}} Z_t^{\mathbb{Q}}, \end{aligned} \tag{2.2.1.6}$$

where Δ_t stands for the time span (one single year) in focus.

The investment risk denotes the risk the insurer faces when investing in the capital market. This risk source includes the spot rate risk and the asset risk. We assume that the asset risk follows a geometric Brownian motion with a deterministic asset drift μ and volatility σ_A. For the real-world measure $\mathbb{P}$, $\hat{r}_t^A$ can be formally described as:

$$(\hat{r}_t^A)^{\mathbb{P}} = \mu - \sigma_A/2 + \sigma_A(\rho(Z_t^{\mathbb{P}} - Z_{t-1}^{\mathbb{P}}) + \sqrt{1-\rho^2}(W_t^{\mathbb{P}} - W_{t-1}^{\mathbb{P}})), \tag{2.2.1.7}$$

where ρ indicates the correlation coefficient between the spot rate and the asset risks, with W denoting a Wiener process.

Under the risk-neutral measure $\mathbb{Q}$, the deterministic drift for the asset risk becomes the stochas-

[18] The Vasicek model allows for a negative interest rate, which has currently occurred in many countries.

tic interest rate $\hat{r}^f$ derived in Equation (2.2.1.6):

$$(\hat{r}_t^A)^{\mathbb{Q}} = \hat{r}_t^f - \sigma_A/2 + \sigma_A(\rho(Z_t^{\mathbb{Q}} - Z_{t-1}^{\mathbb{Q}}) + \sqrt{1-\rho^2}(W_t^{\mathbb{Q}} - W_{t-1}^{\mathbb{Q}})). \tag{2.2.1.8}$$

δ, the contract's net present value (NPV) from the policyholders' viewpoints under the risk-neutral $\mathbb{Q}$ measure, is defined as the present value (PV) difference between two cashflows: the benefit payments paid by the insurer to the insured and the premium payments paid by the insured to the insurer. δ of a basic contract is:[19]

$$\delta = E^{\mathbb{Q}}(\gamma \sum_{t=0}^{T-1} {}_t p_x q_{x+t} \prod_{i=1}^{t+1}(1+\hat{r}_i^f)^{-1} + A_T \prod_{i=1}^{T}(1+\hat{r}_i^f)^{-1} - B - B \sum_{t=1}^{T-1} {}_t p_x \prod_{i=1}^{t}(1+\hat{r}_i^f)^{-1}). \tag{2.2.1.9}$$

With the predetermined parameters, we can derive the participation rate α ($0 \leq \alpha \leq 1$) such that the basic contract is "fair" for policyholders (i.e., $\delta = 0$).

In the real-world measure, the insurer's liability at the end of year t (L_t) is determined by the expected future cashflow discounted at the spot rate at year t. With the investment and interest development conditioned at year t, the expected future cashflow includes the expected death benefit payout ($\tilde{\gamma}_t$), the expected survival benefit payout ($\tilde{A}_t$) and the expected premium income ($\tilde{B}_t$) of the in-force policy contracts at year t (of the policyholders who are alive and have not surrendered their contracts at year t).

$$\begin{aligned}
L_t &= \tilde{\gamma}_t + \tilde{A}_t - \tilde{B}_t, \\
\tilde{\gamma}_t &= E^{\mathbb{P}}(\sum_{i=t}^{T-1} (\gamma_i p_x q_{x+i} \prod_{j=t+1}^{i+1} ((r_j^f))^{\mathbb{P}} + 1)^{-1})), \\
\tilde{A}_t &= E^{\mathbb{P}}(A'_T \prod_{i=t+1}^{T} (1 + r_i^f)^{-1}), \\
&\quad \text{with } A'_{i+1} = A'_i + {}_i p_x (B - q_{x+i} \max(\gamma - A'_i, 0)) \cdot (1+g) \text{ and } A'_t = A_t, \\
\tilde{B}_t &= \begin{cases} B_t p_x, & t = T-1, \\ E^{\mathbb{P}}(B({}_t p_x + \sum_{i=t+1}^{i=T-1} {}_i p_x \prod_{j=i}^{i} ((\hat{r}_j^f)^{\mathbb{P}} + 1)^{-1})), & t < T-1. \end{cases}
\end{aligned} \tag{2.2.1.10}$$

[19] Note that we assume that the contract can only be started if the policyholder is alive and the first premium is paid at year $t = 0$.

With the surrender option, at the end of each year, the policyholders can choose whether or not to surrender their contracts. ρ_t denotes the surrender rate in year t, with $\rho_t > 0$. Thereby, the insurer's liability at year t becomes $L_t \cdot \prod_{i=1}^{i=t}(1-\rho_i)$ after the surrender payment is made.

We assume in the paper that there is no default risk for the policyholder. All justified claims to the beneficiaries can be provided by the insurer. Hence, the valuation of the life insurance contract, including embedded options laid down in this chapter, is done without taken the insurer's default risk into account. In real markets, life insurance companies enjoy limited liability. In chapter 4, different amounts of an insurer's equity capital are analyzed in the context of surrender and liquidity risk. Also, the connection to the solvency capital requirements under Solvency II (based on the insolvency probability) are investigated. In the insolvency case where the insurer cannot fulfill justified claims, we assume that all payments to beneficiaries can be provided through insurance guaranty funds, which we find in many countries in the life insurance industry. Insurance guaranty funds are typically ex-ante financed by all life insurance companies in a particular country directly and hence by the policyholder indirectly.

2.2.2 Modelled Surrender Rate with LSMC Method

Insurers encounter a potential cash outflow due to both the death benefits and the surrender payment. In contrast to the mortality risk, which is unsystematic and can be properly estimated, the surrender risk is difficult to predict. It is subject to various systematic and unsystematic factors (seeKuo et al. (2003), Russell et al. (2013), and Kiesenbauer (2012)). Thereby, the surrender rate ρ is related to the individuals' financial status and the financial market conditions.

The emergency fund hypothesis assumes that individuals leave their contract to acquire the surrender amount when a personal emergency (e.g., unemployment) occurs. Figure (2.1) shows the surrender rates in the German market from 2005 to 2014. This rate spikes in 2008, during the financial crisis. Based on the rational behavior assumption, the interest hypothesis suggests that surrender options are more often exercised when interest rates increase, as the options become more valuable. On the contrary, the surrender rates dropped when the interest rate was cut as the case after year 2009 demonstrated in Figure (2.1).

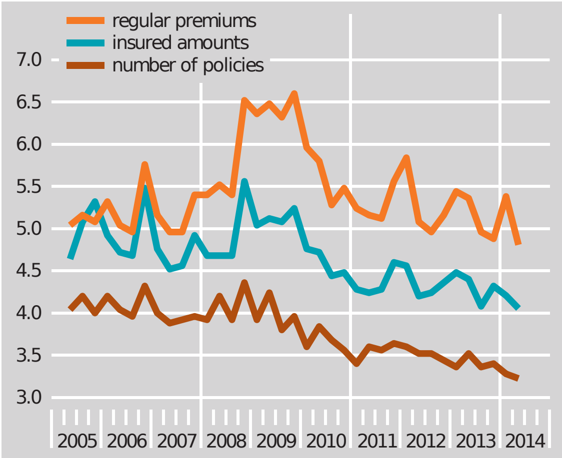


Figure 2.1: Surrender rate (%) in the German life insurance market from year 2005 to 2014 (Sources: BaFin (2014))

To capture the interest rate impact, we begin with the rational exercise assumption[20] and apply the Least Squares Monte Carlo strategy (LSMC). The LSMC was first introduced in Longstaff and Schwartz (2001) for American option pricing. It has been applied in surrender option valuations in life insurance contracts (see e.g., Andreatta and Corradin (2003), Bacinello (2008b), Chang and Schmeiser (2020)). This strategy aims to find an optimal exercise point, where the expected option value is at its maximum.

With a surrender option exercised at year t ($t < T$), the policyholder terminates her contract and receives a surrender amount at the end of year t. Assuming that no surrender fee is applied, the surrender amount equals the policy's accumulated account at year t (A_t). ϑ_t, the PV of the surrender option exercised at year t can be determined from the policyholder's viewpoint as the NPV difference at year $t = 0$ of a contract with and without the surrender option.[21] It is also equal to the NPV of the contract terminated at year t. In formal term, we have:

[20] Assuming that there are no liquidity constraints, rational policyholders will exercise their surrender option at its optimal value (if the option is in the money throughout the contract period).

[21] A contract without the surrender option is the basic contract δ ($\delta = 0$) defined in Equation (2.2.1.9).

$$\vartheta_t = E^{\mathbb{Q}}[\gamma \sum_{i=0}^{t-1} {}_i p_x q_{x+i} \prod_{j=1}^{i+1} (1+\hat{r}_j^f)^{-1} + A_t \prod_{j=1}^{t} (1+\hat{r}_j^f)^{-1} - B - B \sum_{i=1}^{t-1} {}_i p_x \prod_{j=1}^{i} (1+\hat{r}_j^f)^{-1}] - \delta$$

$$= E^{\mathbb{Q}}[-\gamma \sum_{i=t}^{T-1} {}_i p_x q_{x+i} \prod_{j=1}^{i+1} (1+\hat{r}_j^f)^{-1} + A_t \prod_{j=1}^{t} (1+\hat{r}_j^f)^{-1}$$

$$- A_T \prod_{j=1}^{T} (1+\hat{r}_j^f)^{-1} + B \sum_{i=t}^{T-1} {}_i p_x \prod_{j=1}^{j} (1+r_j^f)^{-1}]. \tag{2.2.2.1}$$

By the end of every contract year t, ϑ_t and the continuation value ${}_tC(\vartheta)$ are compared. The continuation value ${}_tC(\vartheta)$ is defined as the present value of the surrender option at year t if the option is not exercised at year t but at the optimal stopping point t^* ($T \geq t^* > t$). Based on the adjusted LSMC described by Chang and Schmeiser (2020), ${}_tC(\vartheta)$ and ϑ_t are estimated via ${}_t\hat{C}(\vartheta)$ and $\hat{\vartheta}_t$ at the end of each year t. These estimated values are based on J relevant information pieces: x_t^j with $j = 1,...,J$, accessible at year t. In our model, x_t^j, with $j = 1,2$ includes the discount rate given by the spot rate $\hat{r}_t^f$ and the surrender amount at year t, A_t. With K coefficients c^k, c'^k and the basis functions, v^k, v'^k (with $k = 1,...,K$), ${}_tC(\vartheta)$ and ϑ_t are approximated by the following equations:

$${}_tC(\vartheta) \cong \sum_{k=0}^{k=K} c_t^k v^k(x_t^1,...,x_t^J),$$

$$\vartheta_t \cong \sum_{k=0}^{k=K} c'^k_t v'^k(x_t^1,...,x_t^J).$$

To derive ${}_t\hat{C}(\vartheta)$ and $\hat{\vartheta}_t$, we estimate $\mathbf{c_t} = (c_t^1,...,c_t^K)$ and $\mathbf{c}'_t = (c'^1_t,...,c'^K_t)$ with $\hat{\mathbf{c}}_t = (\hat{c}_t^1,...,\hat{c}_t^K)$ and $\hat{\mathbf{c}}'_t = (\hat{c}'^1_t,...,\hat{c}'^K_t)$:

$$\hat{\mathbf{c}}_t = \arg\min_{\mathbf{c}_t}\{\sum_{n=1}^{N} [{}_t^nC(\vartheta) - \sum_{k=0}^{K} c_t^k v^k({}^n x_t^1,...,{}^n x_t^j)]^2\},$$

$$\hat{\mathbf{c}}'_t = \arg\min_{\mathbf{c}'_t}\{\sum_{n=1}^{N} [\vartheta_t - \sum_{k=0}^{K} c'^k_t v'^k({}^n x_t^1,...,{}^n x_t^j)]^2\},$$

where n denotes the n^{th} simulation path with $n = 1,...,N$.

Thereby,

$$ {}_t\hat{C}(\vartheta) = \sum_{k=0}^{k=K} \hat{c}_t^k v^k(x_t^1, ..., x_t^J), $$

$$ \hat{\vartheta}_t = \sum_{k=0}^{k=K} \hat{c}'^k_t v'^k(x_t^1, ..., x_t^J). $$

Rational policyholders surrender their contracts if $\hat{\vartheta}_t$ is larger than ${}_t\hat{C}(\vartheta)$. ${}_t\hat{C}(\vartheta)$, depending on the optimal stopping point t^*, is derived backwards given in the following steps:

1. Set all optimal exercise points at year T, (i.e., ${}^n t^* = T$); thereby, the options expire with a value of zero and ${}^n\vartheta_{{}^n t^*} = 0$.

2. One year backwards at year $T-1$, we have the continuation value, ${}^n_{T-1}C(\vartheta) = {}^n\vartheta_{{}^n t^*} = 0$. The exercise value ${}^n\hat{\vartheta}_{T-1}$ can be estimated with the previous formulas. The option should be exercised if ${}^n\hat{\vartheta}_{T-1} > {}^n_{T-1}C(\vartheta)$ and ${}^n t^* = T-1$. Otherwise, ${}^n t^*$ remains unchanged.

3. Backwards iteration for $t = T-2, ..., 1$, with estimated ${}^n\hat{\vartheta}_t$ and ${}^n_t\hat{C}(\vartheta)$: the option should be exercised if ${}^n\hat{\vartheta}_t > {}^n_t\hat{C}(\vartheta)$ and ${}^n t^* = t$. Otherwise, ${}^n t^*$ remains unchanged.[22]

As both ${}^n\hat{\vartheta}_t$ and ${}^n_t\hat{C}(\vartheta)$ are estimated by x_t^j, the optimal exercise strategy as described incorporates the accessible information at year t with respect to the capital market development.

Typically, an insurer issues life contracts with different guaranteed rates and participation rates for various policyholders across generations. Additionally, not all policyholders act rationally when it comes to exercising financial options. Hence, it is unlikely that all the policyholders exercise their contracts at one single optimal point. However, the surrender option is more likely to be exercised as $\hat{\vartheta} - \hat{C}(\vartheta)$ increases. We use the probit model $\Phi(Y_t)$ to capture the market influence on the surrender activity with $Y_t = (X_t - \bar{X}_t)/\sigma_{X_t}$. $X_t = \hat{\vartheta}_t - {}_t\hat{C}(\vartheta)$ and $\bar{X}_t$, σ_{X_t} are the average and the standard deviation of X_t, respectively.

[22] More details are given in Chang and Schmeiser (2020).

Thereby, the modelled surrender rate ρ_t^M, $0 \leq \rho_t^M \leq 1$, is computed as:

$$\rho_t^M = \phi \cdot \Phi(Y_t), \tag{2.2.2.2}$$

where ϕ controls the median of the surrender rate distribution.

2.3 Liquidity Risk in Insurers' Asset

In the classical asset-pricing framework, two assets with identical expected cashflows are traded at the same price; otherwise, an arbitrage opportunity arises. However, Hibbert et al. (2009) reviews the theoretical and empirical literature and confirms the existence of a liquidity risk premium.

An asset's liquidity constraint relates to the asset's market characteristics (e.g., tightness, depth, and resilience). The classic asset pricing model suggests that the capital market is presumably frictionless and competitive.[23] Relaxing both assumptions, Acerbi and Scandolo (2008) values assets with their respective Marginal Supply Demand Curve (MSDC) and an investment portfolio by solving an optimization problem with a certain cash constraint. Tian et al. (2013) applies the MSDC using empirical data and proposes an exponential function for the MSDC approximations.

The MSDC is a downward curve, combining the traditional asset pricing model with the trading volume impact. This curve demonstrates the realizable price discounted by two sources: exogenous transaction costs and endogenous search frictions. The former includes brokerage fees and other processing costs. For the latter, endogenous search frictions, also known as market impact, occur when investors have difficulties finding counterparties, and thus, are forced to make price concessions (see Hibbert et al. (2009)).

The search friction depends on the trading asset size and the market intensity. The realizable value is close to the fair market price when the trading size is negligible compared with the entire market size. In contrast, investors face difficulties in liquidating their assets when the

[23] In a frictionless market, no transaction cost exists. In a competitive market, buying or selling any amount of a security can be conducted without any restrictions and without influencing its market price (see e.g., Jarrow and Protter (2005))

market has no capacity to absorb the relatively large trading size. This situation results in a fire sale, where investors are forced to accept a realizable discounted price that is far below the fair market price.

For the valuation of the insurer's investment portfolio with liquidity constraint, we use the concept first presented in Acerbi and Scandolo (2008) and also used in Tian et al. (2013). Thereby, an investment portfolio is assumed to consist of two types of assets: 1) buy and hold (BAH) and 2) mark to market (MTM). While BAH is priced with the fair value using, e.g, risk-neutral valuation, the MTM's value is the realizable value lying on its respective MSDC.

The MSDC function, hereinafter denoted by $m(x)$ is a decreasing function of trading quantities x with $\mathbb{R}_* \longrightarrow \mathbb{R}$, which satisfies two conditions:

1. $m(x)$ is non-increasing, (i.e., $m(x_1) \geq m(x_2)$ if $x_1 < x_2$).

2. $m(x)$ is càdlàg, left-continuous with right limits for $x > 0$.[24]

$v(q_i)^{MTM}$, the value of the MTM asset i with q_i units can be expressed as:

$$v(q_i)^{MTM} \triangleq \int_0^{q_i} m_i(x)dx.$$

$v(q_i)^{BAH}$, the value of the BAH asset i, is its quantity q_i times its fair price $m_i(0)$ (i.e., trading size has no influence on the unit price):

$$v(q_i)^{BAH} \triangleq q_i \cdot m_i(0).$$

The relationship between two asset types can be formulated as:

$$v(q_i)^{MTM} \leqslant v(q_i)^{BAH} = m_i(0) \cdot q_i. \tag{2.3.0.1}$$

For cash and other cash-equivalent liquid assets, $i = 0$ and q_0 units of cash values exactly q_0 ($m_0(x) = m_0(0) = 1$). In formal terms, we receive:

[24] In our model, only the long position is considered.

$$m_0(0) = 1 \text{ and } v(q_0)^{MTM} = v(q_0)^{BAH} = q_0. \tag{2.3.0.2}$$

Each investment asset i possesses its own MSDC. Following Tian et al. (2013), MSDC is assumed to be an exponential function: $m_i(x) = m_i^+ e^{-k_i\sqrt{x}}$. Hereby, m_i^+ denotes the traditional asset price of asset i (without the liquidity constraint) and $m_i(0) = m_i^+$. A liquidity index, $k_i \geq 0$, represents a liquidity level. Influenced by several factors (e.g., market intensity), this index k_i captures the value discount due to the trading size. Hence, the liquidity risk depends not only on its liquidity index k_i, but also on the traded quantity x.

Figure (2.2) illustrates the impact of the liquidity index k_i on an asset's realizable value $v(q_i)$. With a unit price of $m_i^+ = 1$, $v(q_i)^{BAH}$ lies on the diagonal line, where $k_i = 0$, as no liquidity constraint is considered and $m_i(x) = m_i^+$. The distances between this diagonal line and the other curves reflect the price discounts due to the liquidity constraints. These distances grow as q_i increases and rise even faster with a larger k_i.

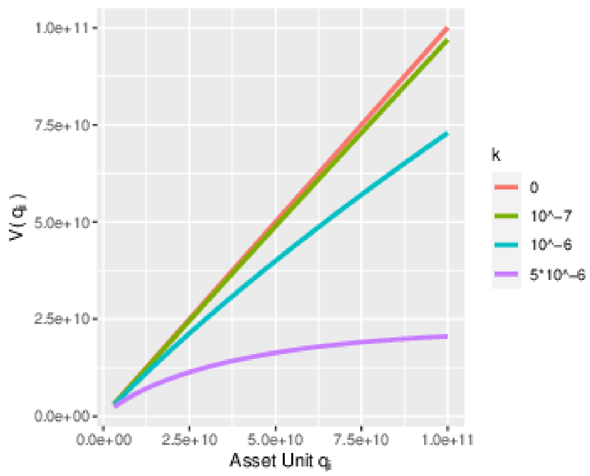


Figure 2.2: Liquidity impact with asset unit price $m_i^+ = 1$: with $m_i(x) = m_i^+ e^{-k_i\sqrt{x}}$, the distances between this diagonal line and the other curves reflect the price discounts due to the liquidity constraints. They grow as q_i enlarges and rise even faster with larger k_i.

A portfolio contains $I + 1$ asset classes with an asset size of q_i $(i = 0, ..., I)$. Vector $q = (q_0, ..., q_I)$, $q \in \mathscr{P}$, represents this portfolio. $\mathscr{P}$, $\mathscr{P} \in \mathbb{R}^{I+1}$ denotes the portfolio space. Every asset class i possesses its own MSDC curve, m_i. $V(q)$ defines the value of the portfolio bounded by the maximum value $V^{\max}(q)$ and the minimum value $V^{\min}(q)$. For the maximum value, all assets in this portfolio are classified as BAH, valued as $v(q_i)^{BAH}$. The minimum value occurs when there is a tremendous cash demand; at this point in time, all assets should be immediately liquidated. Hence, all assets are classified as MTM, with a discount value $v(q_i)^{MTM}$.

For the maximum value of an investment portfolio, we have:

$$V^{\max}(q) = \sum_{i=0}^{I} v_i(q_i)^{BAH} = \sum_{i=0}^{I} m_i(0) \cdot q_i. \tag{2.3.0.3}$$

For the minimum value of an investment portfolio, we receive:

$$V^{\min}(q) = \sum_{i=0}^{I} v_i(q_i)^{MTM} = \sum_{i=0}^{I} \int_0^{q_i} m_i(x)dx. \tag{2.3.0.4}$$

The portfolio value $V(q)$ boundaries are given by:

$$V^{\max}(q) \geq V(q) \geq V^{\min}(q). \tag{2.3.0.5}$$

Our portfolio optimization is done under a cash-position constraint. Such a constraint can be a requirement by the insurer's risk management department (in order to allow unexpected payouts) or by the regulatory authority. In our context, the holding of a cash-position can be important in order to fulfill the unexpected surrender payments.

Subject to a cash-position constraint a, $\Gamma(a)$, a closed and convex subset of $\mathscr{P}$ represents a risk management strategy to satisfy this constraint. The formal definition can be written as:

$$\Gamma(a) := \{q \in \mathscr{P} | q_0 \geqslant a \geqslant 0\}.$$

$V^{\Gamma(a)}$ defines the maximum value attainable in this constraint. If $\Gamma(a) = \emptyset$, $V^{\Gamma(a)}$ is defined as $-\infty$. In such a case, it is not possible to meet the cash constraint within $\mathscr{P}$. Ignoring the case of $V^{\Gamma(a)} = -\infty$, $V^{\Gamma(a)}$ can be determined by vector $s = (s_0, ..., s_I)$, where $s \in \mathscr{P}$ is the solution of an optimization problem. In economic term, s determines the proportion of each asset class so that the portfolio value is maximized while the cash constraint is satisfied. With s, $V^{\Gamma(a)} = V^{\max}(q-s) + V^{\min}(s)$. This optimization problem solution, s, is unique and given as follows:[25]

[25] A detailed proof can be found in Acerbi and Scandolo (2008).

$$s_i = \begin{cases} m_i^{-1}(\frac{m_i(0)}{1+\Lambda}) & q_0 < a \\ 0 & q_0 > a, \end{cases} \tag{2.3.0.6}$$

where m_i^{-1} denotes the inverse of the MSDC function m_i. The Lagrange multiplier Λ can be seen as the marginal cost of liquidation per trading currency. This optimization solution is derived as:

$$s_i = (\frac{\log(1+\Lambda)}{k_i})^2,$$

where $\Lambda = e^x - 1$, $(1 - a/\sum_{i=1}^{I}(2m_i^+/k_i^2))e^x - x - 1 = 0$ for $x > 0$.

Thereby, $V^{\Gamma(a)}$ is shown as:

$$V^{\Gamma(a)} = V^{\max}(q-s) + V^{\min}(s) = \sum_{i=0}^{I} m_i(0) \cdot (q_i - s_i) + \sum_{i=1}^{I} \frac{2m_i^+}{k_i^2}(1 - (1 + (k_i\sqrt{s_i})(e^{-k_i\sqrt{s_i}})). \tag{2.3.0.7}$$

To begin with, the insurer is assumed not to hold any initial capital[26] while giving out no dividends during the contract period. The insurer invests all of the premium minus the death benefit payout. This investment portfolio contains 2 asset categories: $1-\pi$ portion into the government-bond category with q_1 units and π into risky assets with q_2 units.[27] At $T = 0$, the unit prices for both government bonds and risky assets are 1. Hence, we have:

$$m_1^0 = m_2^0 = 1,\ q_1^0 = (1-\pi) \cdot B, \text{ and } q_2^0 = (\pi) \cdot B$$

The unit prices develop with $\hat{r}_t^f$ and $\hat{r}_t^A$: $m_1^t = m_1^{t-1} \cdot (1 + \hat{r}_t^f)$ and $m_2^t = m_2^{t-1} \cdot (1 + \hat{r}_t^A)$.

[26] This paper focuses on the relative change in the default/insolvency probabilities if liquidity is considered. Therefore, an initial capital value can be considered irrelevant in this relative measure.

[27] This simplified investment strategy is implemented so that the contract's NPV for the insurers and the policyholders is zero with a predetermined α. In reality, an investment strategy would include duration matching and other hedging positions. The management of the interest rate risk is, however, not the main focus of this paper.

At the end of each year, a surrender payment of $a_t = \rho_{tt} p_x \cdot A_t$ is required. If no liquidity is taken into account, all assets can be traded at their fair value:

$$
\begin{aligned}
Asset_t^{W/O} &= \sum_{i=1}^{2} q_i^t m_i^t(0), \\
\text{where, } q_1^t &= q_1^{t-1} +_{t-1} p_x(1-\pi)(B - q_{x+t}\cdot\gamma - a_t)/m_1^t(0) \\
q_2^t &= q_2^{t-1} +_{t-1} p_x\pi(B - q_{x+t}\cdot\gamma - a_t)/m_2^t(0)
\end{aligned}
\tag{2.3.0.8}
$$

Otherwise, if the liquidity is considered, the insurer's asset is valued as:

$$
\begin{aligned}
Asset_t^{W} &= V^{\Gamma(a_t)} = \sum_{i=1}^{2} m_i(0)\cdot(q_i^t - s_i^t) + \sum_{i=1}^{2} \frac{2m_i^+}{k_i^2}(1-(1+(k_i\sqrt{s_i^t})(e^{-k_i\sqrt{s_i^t}})) \\
\text{with } q_1^t &= q_1^{t-1} - s_1^{t-1} +_{t-1} p_x(1-\pi)(B - q_{x+t}\cdot\gamma)/m_1^t(0) \\
q_2^t &= q_2^{t-1} - s_2^{t-1} +_{t-1} p_x(\pi)(B - q_{x+t}\cdot\gamma)/m_2^t(0))
\end{aligned}
\tag{2.3.0.9}
$$

with s^t denoting the optimization solution as in Equation (2.3.0.6) when facing a surrender payment obligation as cash constraint a_t.[28]

2.4 Risk Measurement for Surrender and Liquidity Risk

Liquidity risk can be considered from two perspectives. First, a sudden large surrender payout amount could drive insurers into immediate default, even though they are still solvent based on the balance sheet level. To be more specific, insurers are forced to declare bankruptcy before the contract matures because the surrender payout cannot be delivered. Second, a liquidity crisis could reduce an insurers' asset value dramatically if a fire sale is necessary. An insurer has a solvency problem once its assets become lower than its liabilities.

[28] As the mortality risk is assumed to be fully diversified, the death and survival benefits are based on expected cash outflows only. These cash flows can be replicated by continuous trading. Given continuous trading, a large-volume trade can be divided into infinitely small transactions, each having a negligible impact on its market price (seeCetin et al. (2004).) As a consequence, in our model setting, liquidity risk only plays a role in the context of surrender risk and not in the case of pay-outs for death or survival.

From the first perspective, the surrender risk considering the liquidity constraint can be measured by the probability of the insurer's immediate default. This is caused by the cashflow mismatch. From the second perspective, to stay solvent on the balance sheet level (i.e., the market value of the assets exceeds the present value of the liabilities), the solvency capital is required to cover the surrender risk. Under the Solvency II regime, the solvency capital requirement (SCR) is determined based on the 99.5% confidence level over one year. When the surrender risk is correlated with the liquidity risk, SCR should be increased accordingly, otherwise the 99.5% protection cannot be achieved.

2.4.1 Immediate Default Probability ($P(\beta)$) and the Maximum Surrender Rate ($max(\rho)$)

An insurer faces an immediate default threat at t when there are not enough assets to cover the surrender payment (i.e., $a_t = \rho_{t\,t}p_x \cdot A_t$). If the liquidity constraint on the asset side is not considered, all assets can be traded immediately at their fair value (i.e., $Asset_t^{W/O}$). Thereby, the insurer goes default when $Asset_t^{W/O} < a_t$. With N simulation paths, the immediate default probability is calculated as:

$$P^{W/O}(\beta) = \sum_{n=1}^{N} (^n\beta)/N, \text{ where}^n\beta = \max(\mathbb{I}_{Asset_t^{W/O} < a_t}), \text{ for } t = 1, ..., T. \tag{2.4.1.1}$$

where $^n\beta = 1$ signifies that the default event has occurred ($\mathbb{I}_{Asset_t^{W/O} < a_t} = 1$) at some point during the contract duration for simulation path n.

In Equation (2.4.1.1), $\max(\rho)^{W/O}$ defines the annual maximum surrender rate an insurer can afford without facing a default threat. In formal terms, we have:

$$\max(\rho)^{W/O} = \arg\max_{\rho}\{P^{W/O}(\beta) = 0\}.$$

Thereby, it is assumed that no policyholder has surrendered her contract in the previous years. However, mortality rates are taken into account at this point. More precisely, the portfolio size has decreased because some policyholders had died, but not because of surrender.

Considering the liquidity constraint, to meet the cash outflow once a portion of policyholders exercise their surrender options, some parts of the insurer's assets need to be realized at a discounted price. If the surrender payment cannot be met even if all assets are liquidated, $\Gamma(a_t) = \emptyset$ and the insurers go default. In more formal terms, we have:

$$P^W(\beta) = \sum_{n=1}^{N} ({}^n\beta)/N, \text{ where}{}^n\beta = \max(\mathbb{I}_{\Gamma(a_t)=\emptyset}) \tag{2.4.1.2}$$

The annual maximum surrender rate an insurer can afford without facing a default threat when considering liquidity constraint becomes:

$$\max(\rho)^W = \arg\max_{\rho}\{P^W(\beta) = 0\}. \tag{2.4.1.3}$$

2.4.2 Conditional Insolvency Probability $PC(IS)$ and Extra SCR $_{Extra}SCR$

The insurer becomes insolvent if the liabilities exceed the assets, while solvency capital serves as an extra buffer. Solvency capital charged for the surrender risk under the Solvency II regime is based on the scenarios (see EIOPA (2014)). The standard formula calculates this capital as the adverse change in the insurer's liability due to the unexpected surrender behavior in the extreme scenario (99.5% worst scenario). SCR^{Lp} can be defined as the solvency capital for the isolated lapse risk as in the formula below:

$$SCR^{Lp} = \max((\rho^{ext} - \rho^{nor}) \cdot L, 0).$$

Thereby, L denotes the present value of the liability, as shown in Equation (2.2.1.10). ρ^{nor} and ρ^{ext} signify the surrender rate in the normal period of time and in the extreme scenario (which takes place with 0.5% probability). Thereby, SCR^{Lp} aims to provide sufficient protection to cover the surrender risk with a 99.5% survival rate. Solvency II considers three extreme scenarios: the increase in the surrender rate, the mass surrender event, and the decrease in the surrender rate. SCR^{Lp} is calculated as the highest amount among these scenarios. In our model, the risk enlarges as the surrender rate increases. Thereby, we only consider the first and second extreme scenarios (seeEIOPA (2014)):

I. The increase in the surrender scenario: a 50% increase with respect to the assumed surrender rate ($\rho^{ext} = 1.5\rho^{nor}$);

II. The mass surrender scenario: a 30% surrender rate is suggested for the retail business

$(\rho^{ext} = 30\%)$.[29]

According to the standard formula, the surrender risk influences the liabilities only. However, in reality, surrender risk can heavily influence both the insurer's liabilities and assets. For instance, the value of the assets can be heavily reduced under a severe liquidity constraint. Considering the surrender risk under the liquidity constraint, during the extreme scenario, $V^{L(\rho^{ext}A)} - \rho^{ext} \cdot L + SCR^{Lp} < 0$ indicates a situation in which SCR^{Lp} is not sufficient to cover the extreme surrender case. $PC(IS)^{Lp}$ stands for the conditional probability — i.e., under the condition that the extreme case occurs — that $V^{L(\rho^{ext}A)} - \rho^{ext} \cdot L + SCR^{Lp} < 0$.

To improve the insurer's safety level against the surrender risk while considering the liquidity constraint, extra risk capital is needed to avoid the insolvency once the extreme case occurs. Extra SCR — ${}_{Extra}SCR^{Lp}$ – is defined as the value at risk (VaR) based on the conditional probability $PC(IS)^{Lp}$ for a certain confidence level. The additional solvency capital ${}_{Extra}SCR^{Lp}$ comprises the surrender risk with liquidity constraint considered under the condition that the extreme case occurs.

A liquidity crisis in the financial market may be interrelated with a personal liquidity constraint, which has an impact on the policyholders' surrender behavior. This complex dependency between the liquidity risk and the surrender risk can increase the insolvency threat. To be more specific, when the surrender rate increases dramatically, a liquidity crisis is likely to arise (with the liquidity index k_i in Equation (2.2) spiking up). Thereby, $PC\,(IS)^{Lp+Lq} > PC\,(IS)^{Lp}$, because for $PC\,(IS)^{Lp+Lq}$, it is taken into account that the mass surrender and the liquidity crisis trigger each other in the extreme scenario. Undertaking the combined risk of surrender and liquidity, ${}_{Extra}SCR^{Lp+Lq}$ is necessary to ensure the confidence level of $PC\,(IS)^{Lp+Lq}$ during the extreme scenario.

[29] Biagini et al. (2020) applied their model to German insurance data. Their result concludes that the 99.5% quantile of the lapse rate lies between 20% and 25%, while the mass lapse rate suggested in Solvency II can be overly conservative.

2.5 Numerical Results

2.5.1 Parameter Choices

In this section, we consider a life insurance endowment contract with an embedded clinquet-style investment guarantee. We use the following parameters: The annual premium B equals 12,000 currency units and the duration is $T = 20$. If not stated otherwise, the parameters for Equations (1.3.1.7), (2.2.1.7), and (2.2.1.8) and the guaranteed interest rate g are used from Braun et al. (2015). With Equation (2.2.1.2), the death benefit yields 270,525 currency units. The insurance company sells this life contract to 10^6 policyholders with the same mortality probabilities.[30]

In Equation (2.3.0.7), the liquidity index k_i, are positive for $i = 1,2$, as both the government bonds and the risky assets possess liquidity constraints. k_1 is set to $1.5 \cdot 10^{-8}$, for which a bond with a value of 1.2 billion (annual premium income for the insurer) is liquidated at a 10 basis points (BPS) price discount.[31] We set $k_2 = 7.42 \cdot 10^{-8}$, as estimated in Tian et al. (2013). Thereby, the risky asset value of the annual premium is discounted by 54 BPS if immediate liquidation is required. During the liquidity crisis, the liquidity dries up in the market with $k_1' = 5 \cdot 10^{-8}$ and $k_2' = 7.42 \cdot 10^{-7}$. Accordingly, government bonds worth the annual premium are liquidated at a discount of 33 BPS. The risky asset with the same value is sold at a substantial discount of 525 BPS. The results are generated by the Monte Carlo simulations with 10^5 iterations for all simulations. Table (2.2) lists the relevant parameters.

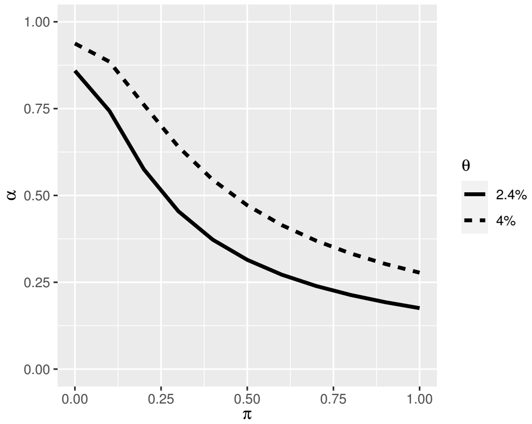
[30] The mortality probabilities are for a 30-year-old US woman based on the data from the HMD (University of California Berkeley (USA), and Max Planck Institute for Demographic Research (Germany)).

[31] This discount is close to the recent empirical studies in Ellul et al. (2011) and Newman and Rierson (2011). The discount has been applied in Feldhütter (2012), Förstemann (2018), and Kubitza et al. (2020). However, the discount estimated in Ellul et al. (2011) and Newman and Rierson (2011) is triggered by the bond market itself. In our model, the fire sale is caused by individual insurers due to their policyholder surrender behaviors. Therefore, we acknowledge that these discount parameters could be overestimated in our model.

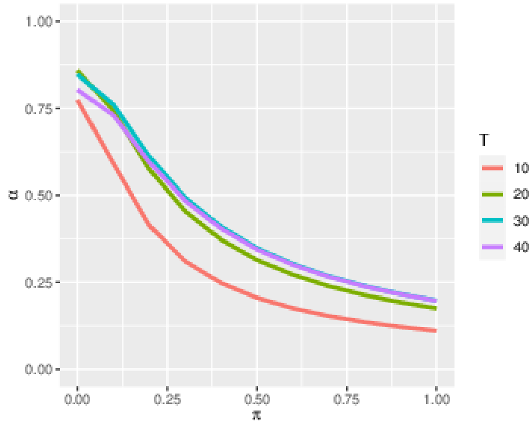
variables and the definitions		value
B	annual premium per contract (in currency units)	$12,000$
γ	death benefits (in currency units)	$270,525$
g	guaranteed interest rate	1.25%
x	initial age (in years)	30
T	time to maturity (in years)	20
σ_I	interest rate volatility	0.6%
κ	interest rate reversion speed	8%
$\hat{r}_0$	initial interest rate	0.5%
θ	long-term interest rate mean	2.4%
λ	market price of risk	-0.18
σ_A	asset volatility	19.1%
μ	deterministic drift for the investment return under the empirical measure $\mathbb{P}$	8.2%
k_i	the liquidity index in normal times: $k_1 = 1.5 \cdot 10^{-8}$ and $k_2 = 7.42 \cdot 10^{-8}$	
k_i'	the liquidity index in a liquidity crisis: $k_1' = 5 \cdot 10^{-8}$ and $k_2' = 7.42 \cdot 10^{-7}$	

Table 2.2: Parameters used for model

Figure (2.3) demonstrates the participation rate α, under the fairness condition ($\delta = 0$ in Equation (2.2.1.9)) for different π (asset allocation) with a different long-term interest mean θ and contract year T. For the same guaranteed rate, policyholders demand a higher α for a higher interest rate. When investing more in risky assets, insurers bear a higher investment risk, and thus, offer a lower participation rate to reach a risk-adequate return for the shareholders. The future benefits that the policyholders are entitled to, as well as the shareholders' return become more volatile as the contract duration extends. Figure (2.3(b)) shows that the contracts with $T \geq 20$ are offered with higher participation rate compared to the contracts with $T = 10$. However, when $T \geq 20$, it is not necessary that the contract with a longer duration should provide with a higher participation rate.



(a) For different long-term interest mean (θ) with $T = 20$



(b) For different contract year (T) with $\theta = 2.4\%$

Figure 2.3: Relationship between the risky asset portion (π) and the participation rate (α)

With this basic contract, the following section examines the surrender risk and how the risk profile changes when the liquidity constraint is considered and when this considered constraint deteriorates. The first case assumes annual constant surrender rates, which are independent from the capital market conditions. In the second case, we apply the modelled surrender rate presented in the Section 2.2. Further on, we examine how the risk profile changes if the interest rate develops unexpectedly and the life contract becomes unfair.

2.5.2 Results under the Constant Surrender Rate Assumption

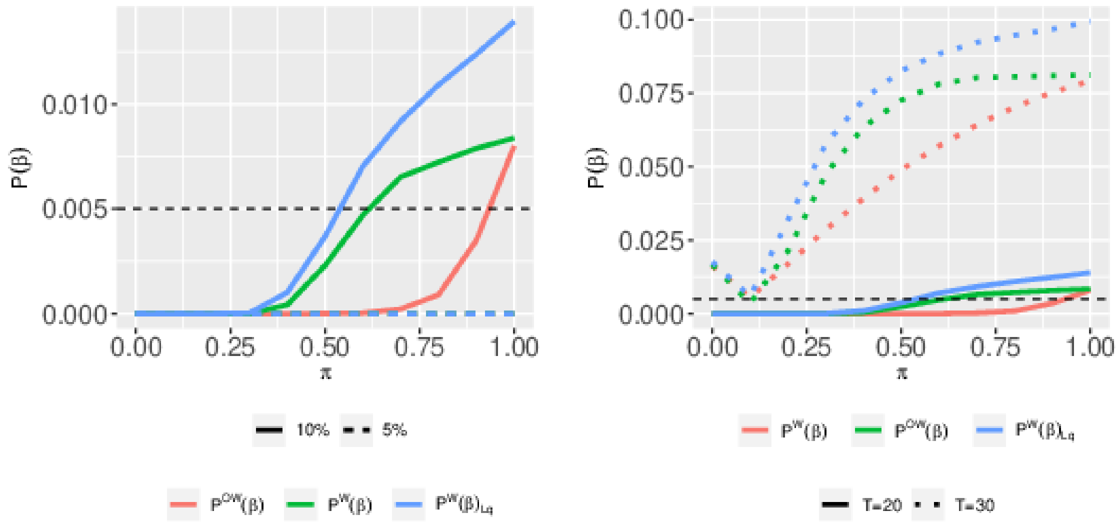
Tables (2.3) shows the maximum surrender rate $\max(\rho)$ that the insurer can survive without going default immediately. Investment returns are more stable with small π, where the insurer invests most of their premium in government bonds, whose returns follow the Vasicek model. Thus, the insurer can survive a large portion of policyholders surrendering their contracts. Generally, the insurer can cope with an annual surrender rate of 40% even if only risky investments are done ($\pi = 100\%$).[32] The maximum surrender rate $\max(\rho)$ stays relatively the same even with an increasing duration of the insurance contract. Additionally, a fire sale exerts a minor price discount given the liquidity index k_i (see Equation (2.2)). Therefore, the difference between cases with (W) and without (W/O) liquidity consideration is negligible.

[32] With $\pi = 100\%$, we focus here for illustrative purpose on the extreme strategy, where the insurer invests in risky assets only. For various reasons - in particular because of regulatory requirements - this is not an asset allocation which we find in insurance practice.

	$T = 20$			$T = 30$		
π	W/O	W	W/O-W	W/O	W	W/O-W
0.00	0.95	0.95	0.00	0.95	0.95	0.00
0.10	0.95	0.95	0.00	0.95	0.95	0.00
0.20	0.90	0.85	0.05	0.90	0.90	0.00
0.30	0.80	0.80	0.00	0.85	0.80	0.05
0.40	0.75	0.75	0.00	0.75	0.75	0.00
0.50	0.70	0.70	0.00	0.70	0.70	0.00
0.60	0.65	0.65	0.00	0.65	0.65	0.00
0.70	0.60	0.60	0.00	0.60	0.60	0.00
0.80	0.55	0.55	0.00	0.55	0.55	0.00
0.90	0.50	0.50	0.00	0.50	0.50	0.00
1.00	0.45	0.40	0.05	0.45	0.45	0.00

Table 2.3: Maximum surrender rate $\max(\rho)$ per π without going default ($P(\beta) = 0$) with different T. (W/O and W for $\max(\rho^{W/O})$ and $\max(\rho^{W})$, maximum surrender rate without and with liquidity consideration; W/O-W: difference as $\max(\rho^{W/O}) - \max(\rho^{W})$)

The insurer faces no severe immediate liquidity threat even if the surrender rate is high in one single year, assuming no surrender took place until the end of the previous year. However, Figure (2.4(a)) illustrates that the immediate default probability is positive if, during each year, 10% of the policyholders steadily surrender their contracts. With a liquidity constraint, the insurer tends to sell the high-liquid assets to provide the surrender payment. The portfolio would then be dominated by the risky asset, instead of the original investment strategy based on the fair contract set-up at year $T = 0$. Therefore, without rebalancing the asset allocation consistently, with the premium partially invested in the government bonds and partially in the risky asset ($0 < \pi < 1$), the immediate default probability with the liquidity constraint ($P^{W}(\beta)$) is substantially higher than the probability without the constraint ($P^{W/O}(\beta)$). This probability enlarges during a liquidity crisis, as shown with $P^{W}(\beta)_{Lq}$. Even without the asset reallocation concern, for $\pi =$ 100%, the default probability $P^{W}(\beta)_{Lq}$ reaches more than 1%, while the probabilities $P^{W/O}(\beta)$ and $P^{W}(\beta)$ are less than 1% (with a constant surrender rate of 10%). Additionally, in Figure (2.4(b)), long-term contracts ($T = 30$) result in an even higher default probability. Without diversification, investing 100% in government bonds may cause higher liquidity risk, while the default probability reaches 10% if 100% of the premiums are invested in risky assets.



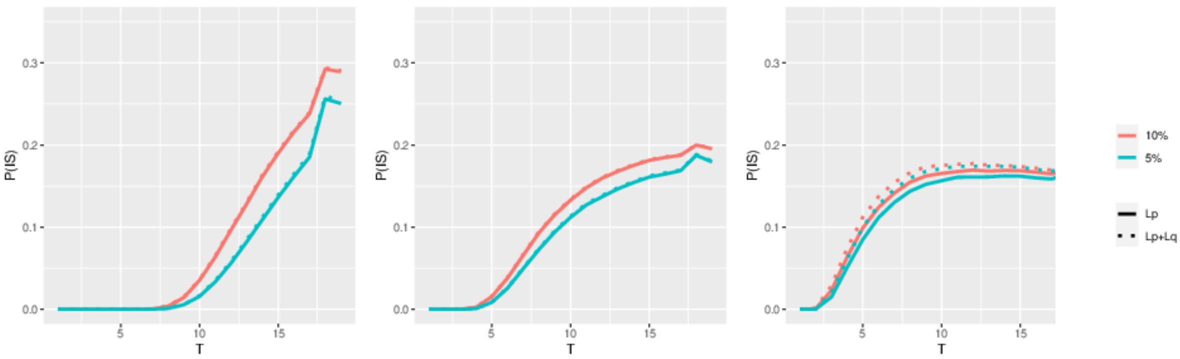
(a) For T=20 with constant surrender rate: 5% and 10%

(b) For constant surrender rate 10% with different T

Figure 2.4: Immediate default probability: $P^{W/O}(\beta)$: no liquidity constraint is considered; $P^{W}(\beta)$: liquidity constraint in normal time is considered; $P^{W}(\beta)_{Lq}$: liquidity constraint during the liquidity crisis is considered; The horizontal dotted line specifies 0.5% of the default probability under the Solvency II regime.

SCR^{Lp} is required in Solvency II to allow for a 99.5% safety level over one year to cover an extensive loss whenever the surrender rate spikes (extreme scenario). As the loss during the extreme scenario becomes a conditional distribution, this SCR^{Lp} cannot always provide the required protection. Figure (2.5) presents $PC(IS)^{Lp}$ and $PC(IS)^{Lp+Lq}$, the conditional insolvency probabilities that SCR^{Lp} or SCR^{Lp+Lq} is not large enough to cover the loss during an extreme event. These probabilities increase as the contract proceeds and the policy account accumulates, except during the last year ($T-1$), where the surrender amount is close to the survival benefit payout after one year when the contract matures.

The insolvency probabilities are higher for a smaller π, where the investment portfolio mainly consists of government bonds and generates lower investment returns. Additionally, if the surrender rate is generally low, the insurers possess a larger sized portfolio with a greater buffer. Therefore, they face a smaller insolvency threat when a mass surrender occurs. As $PC(IS)^{Lp}$ and $PC(IS)^{Lp+Lq}$ almost overlap with one another, the worsening liquidity constraint, due to the liquidity crisis, has little impact on the insolvency probability.

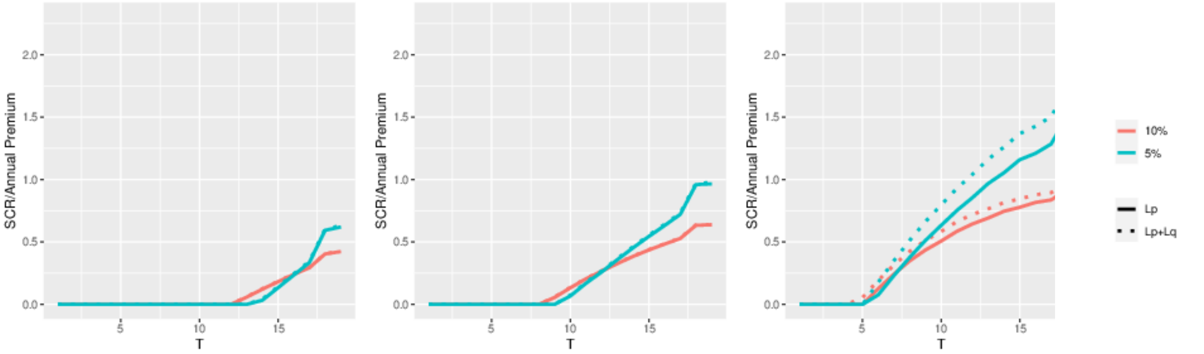


(a) $\pi = 10\%$ (b) $\pi = 50\%$ (c) $\pi = 100\%$

Figure 2.5: Conditional insolvency probability $PC(IS)$: the probability that SCR^{Lp} is not large enough to prevent insolvency under the extreme scenario of mass surrender (Lp) and of the combination of mass surrender and liquidity crisis (Lp+Lq); constant surrender rates of 5% and 10% are assumed

Figure (2.6) shows the extra SCR needed for the insurer to stay solvent with a 90% confidence level conditioned that the mass surrender occurs as discussed in Section 4.2. In contrast to the results in Figure (2.5), an extra SCR is higher for a low surrender rate assumption ρ^{nor} and a high π. Hence, if the surrender rate is assumed low and most of the premium income is invested in the risky asset, the portfolio generates, on average, a high but volatile investment result. Therefore, a larger extra SCR is necessary to provide the corresponding predetermined protection.

Considering the interdependency between the surrender spike and the liquidity crisis in the extreme event, ${}_{Extra}SCR^{Lp+Lq}$ could be even higher. However, this only occurs if π is large (c.f., Figure (2.6(c))). For $\pi = 10\%$ and $\pi = 50\%$, straight lines for ${}_{Extra}SCR^{Lp}$ and dotted lines for ${}_{Extra}SCR^{Lp+Lq}$ overlap with each other. Thereby, the extra risk caused by a risky investment strategy is relatively small, even during a liquidity crisis.

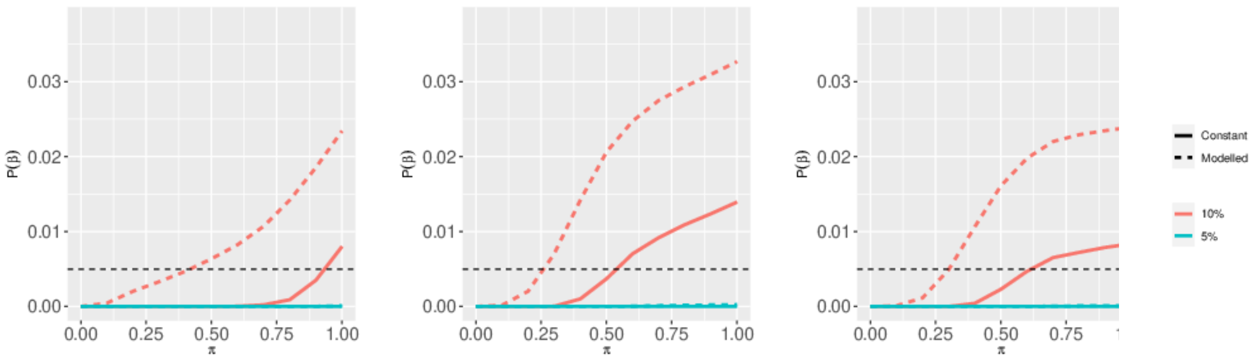


(a) $\pi = 10\%$ (b) $\pi = 50\%$ (c) $\pi = 100\%$

Figure 2.6: Extra SCR to stay solvent for a confidence level of 90% under the condition of an extreme scenario: Lp stands for ${}_{Extra}SCR^{Lp}$ for the extreme scenario of mass surrender and Lp+Lq for ${}_{Extra}SCR^{Lp+Lq}$ when the liquidity crisis is interrelated with the mass surrender in the extreme scenario.

2.5.3 Results under the Modelled Surrender Rate Assumption

Section 5.2 assumes constant surrender rates, regardless of the financial market status. However, as discussed, the surrender rate is typically influenced by market conditions. This section assumes that the surrender rate is determined by the surrender value, which is subject to financial market conditions. Thereby, the surrender rate rises/declines if its surrender value increases/decreases. As modelled by Equation (2.2.2.2), ϕ is determined such that the median of the surrender distribution is equal to 5% and 10%. Figure (2.7) compares the modelled surrender rate to the constant surrender rate. For a median surrender rate of 10%, the insurer faces a much higher default threat if the surrender behavior is influenced by the surrender option value as modelled in Section 2.1.



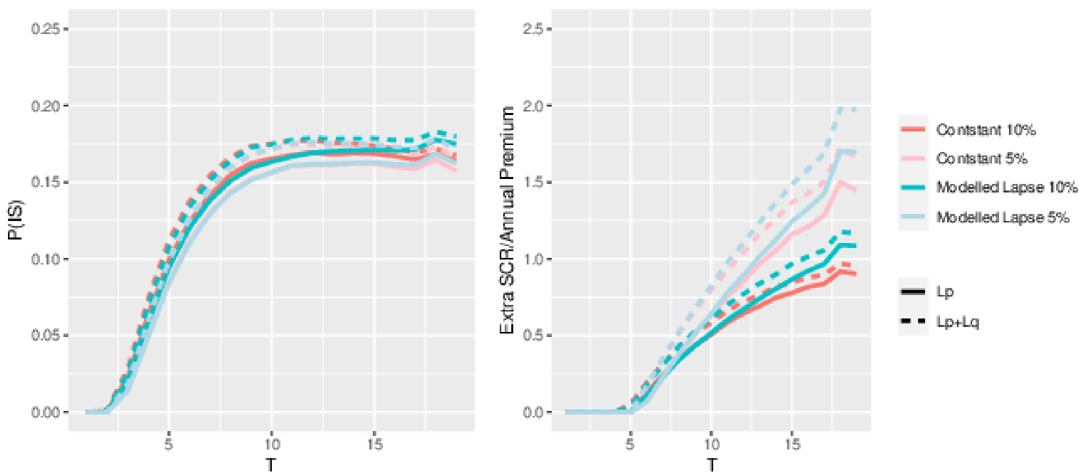
(a) $P^{W/O}(\beta)$ no liquidity constraint is considered

(b) $P^{W}(\beta)$ liquidity constraint in normal time is considered

(c) $P^{W}(\beta)_{Lq}$ liquidity constraint during the liquidity crisis is considered

Figure 2.7: Immediate default probability $P(\beta)$ with constant surrender rates and a modelled surrender rate with a median 5% and 10%, respectively; the horizontal dotted line specifies 0.5% of the default probability under the Solvency II regime.

Figure (2.8) illustrates that $PC(IS)^{Lp}$ and $PC(IS)^{Lp+Lq}$ with constant and modelled rates are very close to one another for the extreme strategy of $\pi = 100\%$. However, ${}_{Extra}SCR^{Lp}$ and ${}_{Extra}SCR^{Lp+Lq}$ increase if the modelled surrender rates are assumed. More precisely, if the policyholders follow an optimal strategy to exercise their surrender options, the tail risk becomes more pronounced. Therefore, a larger amount of additional solvency capital is required to stay solvent for a confidence level of 90% once an extreme case occurs.



(a) $PC(IS)$ for $\pi = 100\%$

(b) Extra SCR for $\pi = 100\%$

Figure 2.8: Conditional insolvency probability PC(IS) and extra SCR for a confidence level of 90% under the extreme scenario of mass surrender (Lp) and of the combination of mass surrender and liquidity crisis (Lp+Lq)

2.5.4 Results given Unfair Insurance Contracts

This section examines the situation when the interest rate changes in the long run. There exists interest rate stochastic development in the short term. In the long term, the mean θ is even harder to estimate (θ is subject to various macroeconomic uncertainties). Once the interest rate developed unexpectedly in the long run, life contracts are no longer fair for the predetermined participation rate α.

Two cases are considered in what follows:

1. Decrease: $\theta = 4\%$ is expected and the participation rate α is calculated accordingly, so that with $\theta = 4\%$, $\delta = 0$ (Equation (2.2.1.9)). However, when the contract starts, the interest rate dncreases to $\theta = 2.4\%$.

2. Increase: $\theta = 2.4\%$ is expected and the participation rate α is calculated accordingly. However, when the contract starts, the interest rate increases to $\theta = 4\%$.

Figure (2.9) shows that in the Decrease Case, the contract's NPV from the insurer's perspective plunges. This case represents the current life insurer's situation. As the interest rate falls to nearly zero, this NPV of the existing life insurance contract drops substantially. On the contrary, the contract's NPV increases in the Increase Case. This interest rate risk (the change in the contract's NPV due to the unexpected interest rate development) enlarges as the contract duration extends.

Figure (2.10) shows that in the Decrease Case, the insurer meets a higher immediate default threat, as the insurer has difficulties in fulfilling the guarantee obligation. However, in the Increase Case, the increase in NPV does not significantly reduce the immediate default probability.

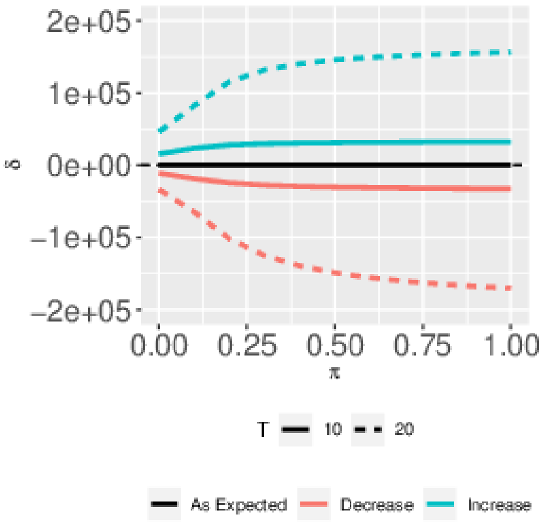


Figure 2.9: The contract's NPV, $-\delta$, from the insurer's perspective (see Equation (2.2.1.9)).

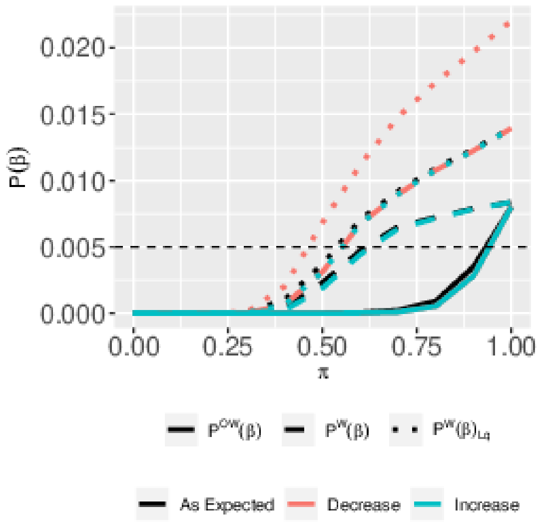
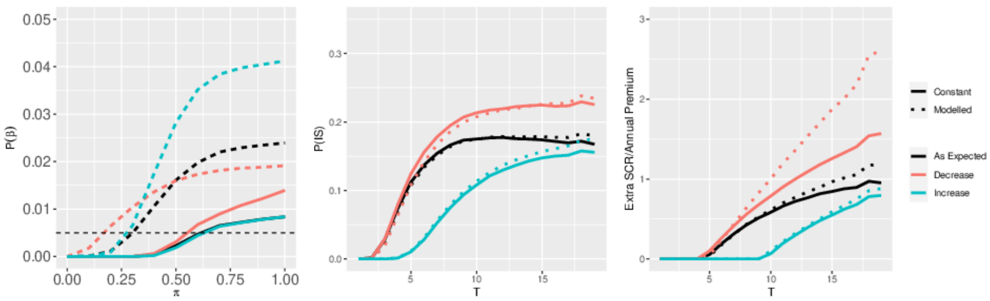


Figure 2.10: Immediate default probability $P(\beta)$ for fair and unfair contracts with constant surrender rate 10%: The horizontal dotted line specifies 0.5% of the default probability under the Solvency II regime.

To model the surrender rate, the probit model $\Phi(Y_t)$ described in Equation (2.2.2.2) is used to reflect the market influence on the surrender activity. $Y_t = (X_t - \bar{X}_t)/\sigma_{X_t}$, where X_t represents the difference between the exercise value ($\hat{\vartheta}$) and the continuation value ($\hat{C}(\vartheta)$) of the option within the fair contracts. $\bar{X}_t$ and σ_{X_t} are the average and the standard deviation of X_t. ϕ is determined so that the median of $\phi \cdot \Phi(Y_t)$, the surrender rate distribution within fair contracts, equals 5% or 10%, respectively. As the contract becomes unfair, $\rho_t^M = \phi \cdot \Phi(Y_t')$, where $Y_t' = (X_t' - \bar{X}_t)/\sigma_{X_t}$ and $X_t' = \hat{\vartheta} - \hat{C}(\vartheta)$ within the unfair contracts.

As the interest increases unexpectedly, the contract value from the policyholder's perspective declines and the surrender option value rises. Thereby, the surrender rate in the Increase Case is generally higher than if the contract is offered fairly. In the Decrease Case, rational policyholders take advantage of the unfair contracts and are less likely to surrender their contracts. This modelled surrender rate incorporates the interest hypothesis, which explains the decrease in the surrender rate after 2009 in the German insurance market, as shown in Figure (2.1).

In Figure (2.11(a)), for the extreme strategy of $\pi = 100\%$, in the Increase Case, the default probability grows, since a higher surrender rate is expected, given that the surrender value increases. Figure (2.11(b)) and (2.11(c)) present $PC(IS)$ and extra SCR for the three cases with constant surrender rate and modelled surrender rate (10%). Generally, $PC(IS)$ and extra SCR are the highest in the Decrease Case. The interest influence for the surrender behavior has a rather small impact on $PC(IS)$. Thereby, $PC(IS)$ for the modelled surrender rate is close to that of the constant surrender rate. However, extra SCR grows significantly in the Decrease Case if the interest influence on the surrender behavior is taken into account. It suggests that in the Decrease Case, policyholders are expected to surrender their unfair contracts less frequently. However, once an extreme event takes place, where a mass surrender and liquidity crisis simultaneously occur, almost 3 times of the annual premium is required as extra SCR. On the contrary, in the Increase Case, extra SCR for the modelled and constant surrender rate almost overlap.



(a) $P^W(\beta)_{Lq}$ (b) $PC(IS)$ for $\pi = 100\%$ (c) ${}_{Extra}SCR^{Lp+Lq}$ for $\pi = 100\%$

Figure 2.11: Immediate default probability $P(\beta)$, Conditional insolvency probability $PC(IS)$, and ${}_{Extra}SCR^{Lp+Lq}$ under the extreme scenario of the combination of mass surrender and liquidity crisis for a confidence level of 90% for the fair and unfair contracts with the constant and modelled surrender rate

2.6 Conclusions

Insurers are forced to realize their assets at a discounted price when an unexpected cash outflow occurs due to a liquidity constraint on the asset side. This paper shows that a life insurer's surrender risk can be underestimated if this liquidity constraint is not considered. Moreover, this constraint varies according to the capital market conditions. Hence, even if the insurer stays solvent, based on accounting standards, a severe immediate default threat remains unconsidered whenever the surrender and liquidity risks are evaluated individually.

This threat increases as the interest rate volatility increases if policyholders strategically exercise their surrender option by means of an optimal stopping strategy. On the one hand, the increase in interest elevates the surrender option value and the policyholders tend to use these options. Therefore, the insurer is more likely to face insolvency. A string liquidity issue can even arise if the asset allocation is not appropriately adjusted. On the other hand, whenever the interest rate drops unexpectedly, the insurer faces a higher danger if the surrender rate remains unchanged.

When the interest rate is raised, Förstemann (2018) and Kubitza et al. (2020) examine the liquidity risk insurers are exposed to due to the surrender option, once the interest rate increases or is expected to rebound. According to our investigation and the results in Kubitza et al. (2020), the liquidity risk is immaterial as long as the liquidity remains deterministic. However, we further demonstrate that during the liquidity crisis, insurers can become vulnerable to a mass surrender event, while a mass surrender and liquidity crisis is interrelated, especially in the extreme scenario.

In the unexpected low interest-rate environment, as the surrender option value diminishes, rational policyholders will maintain their contracts. Accordingly, the surrender rate is expected to decrease. However, even in this case, this paper shows that the default/insolvency threat is only slightly reduced.

Under the Solvency II regime, the SCR against the surrender risk is determined by the change in the liability once an extreme scenario occurs. This paper shows that, in the extreme scenario, the calculated SCR may not be enough, as the actual risk is greatly enlarged by the investment and liquidity risk, affecting both sides of the insurer's balance sheet. Therefore, an additional SCR is required to provide the intended safety level prescribed by Solvency II. This extra protection is especially crucial if the interest rate is very low, as is the current case.

Insurers usually have a greater tolerance toward liquidity risk. Thus, they enjoy the liquidity premium. The liquidity risk, though decisive, has been assumed to be a minor issue in the insurance sector in the past, and hence, is not considered when calculating either the Solvency Capital Requirement (SCR) for Solvency II or the Risk Bearing Capital (RBC) for the Swiss Solvency Test (SST). Instead, these two regimes require insurers to perform a qualitative evaluation of their liquidity risk and their risk management procedure. However, an insurance run may occur when the liquidity problem is triggered by, or leads to, a mass surrender. As the policyholder's behavior and the liquidity conditions are correlated, when calculating SCR, the interrelation between the surrender rate and the liquidity crisis should be considered. Otherwise, the risk results from the interdependence of these two extreme cases, which is underestimated and may reduce the life insurer's financial stability.

Essay III

Risk Attitude towards On-Demand Insurance: An Experiment Study

Hsiaoyin Chang, Hato Schmeiser

As technology advances, on-demand insurance products are developed to cover risk in a short-term period. A reduced-period insurance contract grants policyholders the freedom to choose, in a very flexible manner, when to be insured. Such contracts change the way the relevant risk is perceived. We run an experiment and show that individuals can become extremely risk-averse when short-term insurance is offered. Segregated salient risk is overestimated, and the evaluation period reduced by the short-term insurance leads to myopic loss aversion.

Keywords: On-demand insurance · Insurance demand anomaly · Behavioral economics · Cumulative prospect theory

3.1 Introduction

Insurance is based on the assumption of risk aversion: because people are risk-averse, they are willing to pay a risk premium to transfer potential losses. This risk aversion is typically modelled by the concavity of an underlying utility function. However, the commonly used expected utility theory faces criticism, since voluminous literature records various decision anomalies. For instance, Hershey and Schoemaker (1980) conducted a lab experiment in the loss domain, and Baker et al. (1988) did so in both the profit and loss domains. Both studies provide results that are incompatible with the expected utility theory. For the insurance market, Kunreuther and Pauly (2006) discuss anomalies from the demand and supply sides. Some of these anomalies – especially on the insurance demand side – can be explained by people's misperception of risks. Tversky and Kahneman (1974) describe this risk misperception in terms of four biases: representativeness, availability, adjustment and anchoring. Additionally, myopic loss aversion, as discussed in Thaler (1999), relates to loss aversion and mental accounting. Availability and representativeness explain one of the demand anomalies presented in Kunreuther and Pauly (2006): the popularity of overcharged insurance against "named events", such as cancer risks. Rabin and Thaler (2001) assert that individuals are particularly loss-averse when facing small risks. However, several expensive policies, each covering a small risk, are only attractive if they are presented individually. Otherwise, those covered risks are regarded as one combined risk and individuals become less loss-averse.

Hence, how risk is presented determines whether the risk is considered separately. When risk is divided in a temporal sequence, each risk is assessed within a reduced evaluation period. In contrast, with an expanded evaluation period, the risk is aggregated with a delayed outcome. Experiment studies show that people become more risk-tolerant with delayed results. For example, Noussair and Wu (2006) and Abdellaoui et al. (2011) suggest that risk tolerance increases with delayed lotteries. Additionally, even if the outcome is resolved in the longer term, the concerned risk can still be segregated if its resolution is presented frequently, reducing the evaluation period. Instead of hyperbolic discounting, Read (2001) shows via three experiments strong evidence of sub-additive discounting: the discounting over a delay is greater when the delay is separated into smaller intervals than when its left undivided. Studies in behavioral finance show that decision makers (DMs) tend to make conservative investment choices if the investment performance is assessed more frequently (cf. Gneezy and Potters (1997), Thaler et al. (1997), Gneezy et al. (2003), and Bellemare et al. (2005)). In financial market, an evaluation period can be controlled by the asset's maturity. Specifically, assets with short maturities are shown to possess higher risk premiums compared to assets with long maturities (cf. Van Binsbergen et al. (2012), Andries et al. (2014), and Eisenbach and Schmalz (2016)).

For the insurance sector, Epper and Fehr-Duda (2018) explain the underinsurance puzzle with a rank-dependent probability-weighting function. Low popularity in life insurance is explained by the underweighted probability due to its long evaluation period.

Risk attitude is influenced by the evaluation period, which depends on the timing of the uncertain outcome and the frequency of uncertain resolution. With an insurance contract offered against a certain risk event, this contract period defines the event period, which influences the evaluation period. Currently, a traditional insurance policy protects policyholders from certain risk in a given time period, usually with a minimum of one year. As technology advances, on-demand insurance provides an innovative solution, covering risk for a limited time. For example, Slice offers daily policies for home insurance. Cuvva provides short-term auto insurance covering hourly risk. Trov allows clients to choose when to insure their electronic gadgets, such as mobiles and photography equipment. When facing this on-demand insurance, the DM is expected to become risk averse as the contract reduces the evaluation period. However, a counterexample in Kunreuther and Michel-Kerjan (2015) and Botzen et al. (2013) shows that the insurance demand in the hurricane insurance market can be boosted if multi-year insurance instead of annual insurance is issued. Namely, long-term insurance can increase risk aversion, as the focus risk is aggregated and becomes salient while the evaluation period remains the same. Facing several insurance contracts with various periods on the market, each contract period only partially influences the evaluation period. Risk attitudes and the insurance purchase decision are dependent on how DMs restructure the focus risk in their own self-defined evaluation period during the decision-making process.

To show how the DM edits the risk when facing one risk with multiple periods, we extend the Cumulative Prospect Theory (CPT) model with two additional factors – the myopic loss aversion and the loss probability miscalculation. Compared to the general expected utility function, the CPT model in Tversky and Kahneman (1992) is more suitable for the myopic loss aversion topic. In this model, with a reference point, the DM is expected to behave differently in the gain and the loss domains. Additionally, increasing overestimation of risk due to a reduced evaluation period can be captured from the probability weighting function.

We further conducted a laboratory experiment to quantify these two additional factors (biases), where a Bayesian model is implemented using Markov Chain Monte Carlo algorithms (MCMC). In the experiment, when facing one risk, the participants make insurance purchase decisions in which the insurance products are provided with various durations. Therefore, we investigate whether the participants' own evaluation periods can be influenced merely by the insurance product design.

Contrary to expectations, we show that myopic loss aversion may not always be triggered by a shortened insurance period. If the covered risk is salient, the evaluation period plays a larger role when determining the insurance period. In this case, the reduced insurance period increases the insurance demand due to myopic loss aversion. However, non-salient risk in the reduced insurance period tends to be aggregated. The evaluation period is thereby longer than the insurance period. In this case, when facing non-salient risk, demand for insurance decreases as the insurance period is reduced.

The rest of the paper is structured as follows. The next section introduces our extended Cumulative Prospective Theory Model. The experiment design is described in the third section. The fourth section demonstrates the model results. Finally, the fifth section discusses other bias on-demand insurance products may stimulate and concludes the paper.

3.2 Extended Cumulative Prospect Theory Model

3.2.1 Cumulative Prospect Theory (CPT)

CPT in Tversky and Kahneman (1992) suggests that the risk attitude of the DM differs according to his or her reference point. This risk attitude is controlled by a probability distorted weighting function (w) and a value function (v). The distorted weighting function, as in rank dependent utility theory (Quiggin (1982)), is an inverse-sigmoid curve with which the DM tends to overweight/underweight the probability when the probability is small/large. The value function is strictly increasing with the first derivative being positive. Nevertheless, with the second derivative being negative when the value is above the reference point and positive when below the point, the DM is risk-averse in the gain domain and risk-seeking in the loss domain. Extra loss aversion, captured by v, is steeper in the loss domain. These two functions, v and w, lead to a fourfold pattern of risk attitude: in the gain domain, DM is risk-averse towards outcomes with high probability but risk-seeking with low probability. In contrast, risk-seeking behavior is expected with a high probability of loss and risk-averse behavior is expected with a low probability of loss.

In CPT, the distorted probability weighting function, w, is formed as

$$w(p) = p^{\gamma}/(p^{\gamma} + (1-p)^{\gamma})^{(1/\gamma)}, 0 \leq p \leq 1. \quad (3.2.1.1)$$

Thereby, γ controls the weighting function shape while $w(1) = 1$, $w(0) = 0$. The red curve in Figure (3.1(a)) demonstrates the inverse-sigmoid curve. This curve overweights low probabilities and underweights high probabilities.

The value function v in CPT is suggested as

$$v(x) = \begin{cases} x^{\alpha}, & x > 0, \\ 0, & x = 0, \\ \lambda^{CPT}(-x)^{\alpha}, & x < 0, \end{cases} \quad (3.2.1.2)$$

where α determines the sensitivity of variation and λ^{CPT} (> 1) represents the loss aversion degree. At the reference point, $x = 0$ and $v(0) = 0$. Figure (3.1(b)) shows that this value curve is concave when $x > 0$ and convex and steep when $x < 0$. Decisions on insurance consumption, unlike investment or gambling including both gain and loss domains, are a trade-off between a certain small loss (premium payment) and a rare but severe one (loss event). As both prospects are in the loss domain, the loss aversion degree λ^{CPT} does not play a role in this decision framework. Additionally, with decreasing sensitivity of the increasing loss amount, the demand for insurance (i.e., risk aversion) is mainly caused by the overweighted loss probability.

An uncertain prospect is denoted with a series of pairs (x_k, A_k), $p_k = \mathbb{P}[A_k]$, and $n \leq k \leq 0$. Thereby,$x_l < x_k$ if $l < k$ and $x_0 = 0$. The value of the prospect is calculated as:

$$V = \sum_{i=n}^{0} \pi_i v(x_i),$$

with $\pi_n = w(p_n)$ and $\pi_k = w(p_n + ... + p_k) - w(p_n + ... + p_{k-1})$ for $n < k \leq 0$.

A DM faces two possible outcomes: A loss $L < 0$ with probability p_L, $0 \leq p_L \leq 1$, and no loss with probability $1 - p_L$.

The expected utility without insurance, $V^{w/o}$, is defined as:

$$V^{w/o} = (1 - w(p_L))v(0) + w(p_L)v(L) = w(p_L)v(L). \qquad (3.2.1.3)$$

A full insurance contract is priced with a proportional loading c. Thereby, this insurance premium equals $(1+c)Lp_L$. If insurance is purchased, only one outcome (i.e. loss of premium) is possible and the expected utility with insurance, V^w, can be written as:

$$V^w = w(1)v((1+c)Lp_L) = v((1+c)Lp_L). \qquad (3.2.1.4)$$

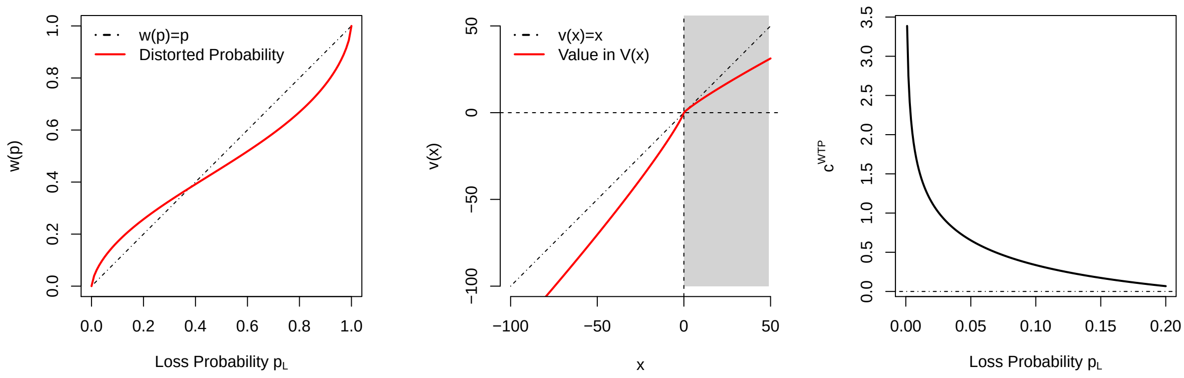
The DM obtains insurance only if

$$V^w = v((1+c)Lp_L) \geq w(p_L)v(L) = V^{w/o}. \qquad (3.2.1.5)$$

Thereby, the maximum willingness to pay (WTP) for the loading, c^{WTP}, can be derived as:

$$c^{WTP} = w(p_L)^{(1/\alpha)}/p_L - 1. \qquad (3.2.1.6)$$

c^{WTP} is controlled by α and γ, while λ^{CPT} has no influence. A negative interrelation between c^{WTP} and p_L, as shown in Figure (3.1(c)), is caused by a higher degree of probability overweight in $w(p_L)$ as p becomes small.



(a) The distorted probability weighting function $w(p)$

(b) The value function $v(x)$

(c) c^{WTP} for different p_L

Figure 3.1: The distorted probability weighting function, the value function, and the maximum willingness to pay for the insurance loading according to CPT ($\alpha = 0.88$, $\gamma = 0.69$, $\lambda = 2.25$ from Tversky and Kahneman (1992))

On the one hand, for an insurance contract with a reduced period (such as on-demand insurance), the insurance contract period can be shorter than the risk evaluation period, in which the DM wishes to be protected. On the other hand, the evaluation period can be adjusted to the insurance contract's duration. In the following, we extend the CPT model to value the prospect with on-demand insurance.

3.2.2 Myopic Loss Aversion with Narrow Framing

In Samuelson (1963), the author asked his colleague to take a bet offering a 50% chance to gain USD 200 and 50% to lose USD 100. Despite the positive expected value, his colleague turned down the offer. The colleague would, however, take this bet if the game were repeated 100 times. This classic myopic example is named and discussed in Thaler (1999) and other articles as myopic loss aversion. Myopic loss aversion combines narrow bracketing and loss aversion: people are more loss averse when the risk is presented separately or narrowly bracketed (cf. Thaler (1999)). On the contrary, risk tolerance increases as multiple risk being accumulated or broadly bracketed.

Introduced in Read et al. (1999), choice bracket designates the grouping set of individual choices. The DM broadly brackets the risks when the individual risks are aggregated and considered together. In contrast, one risk is narrowly bracketed when it is perceived in isolation. Temporal bracketing applies as risk is bracketed along the time horizon. To aggregate the risk, multiple short-term events can be combined into a long-term event, whose result is evaluated in a long-term future. In contrast, a long-term event can also be segregated with several events, each with a short evaluation period. Therefore, the evaluation period determines how the risk is bracketed, while several factors control the length of the evaluation period. In the financial market, risk can be defined by the financial product, and thus the evaluation period is aligned with the product duration. Van Binsbergen et al. (2012) and Andries et al. (2014) exhibit a decreasing term structure of the risk premium due to horizon-dependent risk aversion. In insurance, Epper and Fehr-Duda (2018) explain the underinsurance in life insurance products due to their long-term scheme. Additionally, demand for flight insurance is stimulated as this insurance reduces the evaluation period risk only to the fleeting flight (cf. Eisenbach and Schmalz (2016) and Epper and Fehr-Duda (2018)).

With on-demand insurance, at each point in time, the DM makes an insurance purchase decision against the risk in a short-term period. Therefore, the focus risk can be narrowly bracketed by the on-demand insurance. For example, when the DM decides whether to purchase daily insurance today, tomorrow's risk becomes irrelevant. With this narrowly-bracketed risk, c^{WTP} increases substantially as the loss probability is divided and thus reduced (cf. the negative interrelation between p_L and c^{WTP} in Figure (3.1(c))).

A traditional insurance contract period is valid for time unit 1 (considered as an objective risk period), covering a potential loss L with a loss probability p_L in this time unit. Assuming the time until a loss event occurs (T) is an exponential distribution with parameter, Λ ($\Lambda > 0$), p_L can be defined by:

$$P(T \leq 1) = 1 - \exp(-\Lambda \cdot 1) = p_L.$$

For a different contract period, t, the loss probability during t can be derived as

$$p_L^t = P(T \leq t) = 1 - \exp(-\Lambda \cdot t).$$

The DM frames the risk narrowly if the evaluation period is adjusted by the insurance contract duration. Namely, the subjective evaluation period is determined by the contract duration t with

$t < 1$. The prospects with or without insurance against this narrowly bracketed risk, ${}^{N}V^{w}$ and ${}^{N}V^{o/w}$ become:

$$
\begin{aligned}
{}^{N}V^{w} &= v(C), \\
{}^{N}V^{w/o} &= w(p_L^t)v(L),
\end{aligned}
\tag{3.2.2.1}
$$

where $C = (1+c)L \cdot p_L^t$ denotes the insurance premium with loading. c^{WTP} is derived if ${}^{N}V^{w} = {}^{N}V^{w/o}$. In Figure (3.2), if annual risk is narrowly bracketed according to the insurance duration, c^{WTP} increases substantially as the period is shortened to one month ($t = 1/12$) or even to one day ($t = 1/365$). To be more specific, the DM becomes more risk averse if the evaluation period, aligned with the insurance period, is shortened.

The subjective evaluation period can be influenced by factors other than insurance contracts. For instance, we tend to buy whole-life insurance as the risk in concern is evaluated within a life-long period. However, the period of mobile phone or laptop risk is much shorter as they are replaced frequently. Additionally, different insurance products with distinct contract durations control the subjective evaluation period jointly. For example, with both traditional insurance ($t = 1$) and on-demand insurance ($t < 1$) on the market, on-demand insurance segregates the risk while traditional insurance re-aggregates these segregated risks.

When the DM broadly brackets the risk with the evaluation period 1, while making a decision regarding on-demand insurance with contract duration t ($t < 1$), risk not covered by insurance is considered. The prospects with and without this on-demand insurance change into:

$$
\begin{aligned}
{}^{B}V^{w} &= (1 - w(p_L^{1-t}))v(C) + w(p_L^{1-t})v(L+C), \\
{}^{B}V^{w/o} &= w(p_L)v(L).
\end{aligned}
\tag{3.2.2.2}
$$

p_L^{1-t} denotes the probability that a loss occurs outside on-demand insurance contract coverage. Figure (3.2) shows that the DM becomes less risk-averse, or even risk-seeking ($c^{WTP} < 0$), when broadly bracketing monthly/daily risks into annual risk.

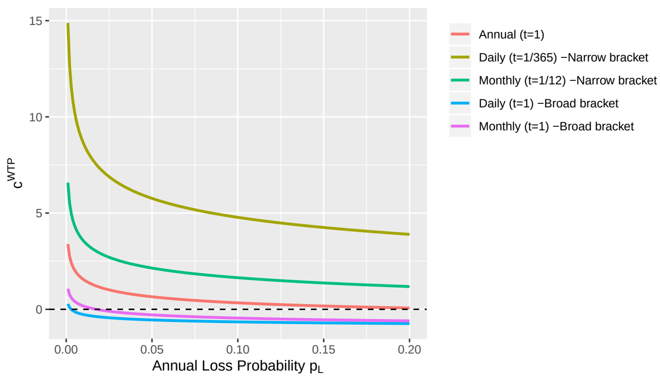


Figure 3.2: c^{WTP} for broadly/narrowly bracketed risk ($\alpha = 0.88$ and $\gamma = 0.69$)

3.2.3 Loss Probability Miscalculation

The parameter Λ in the exponential distribution presumes that loss occurs randomly at a fixed rate regardless of the underlying situation. However, in practice, the loss probability fluctuates. For example, the DM may be exposed to a greater hazard of breaking a leg at a ski resort for one week than at home for the same week. Therefore, the demand for insurance in a risky period (e.g., the week at the ski resort) is expected to be higher than in a less-risky period (e.g., the week at home). c^{WTP} is thus influenced by the DM's perception of the risk in the focus period. Despite the probability distortion effect captured by w in the CPT model, the loss probability can be miscalculated due to the loss probability miscalculation.

The loss probability in a risk period, 1, is $p_L = 1 - \exp(-\Lambda)$. This risk period is separated into two periods with distinct loss probabilities: t^R $(0 < t^R \leq 1)$ and $1 - t^R$. The loss occurrence rate, Λ^R in t^R, is n times the rate in $1 - t^R$. Thereby, $p_L^{t^R}$, the loss probability during t^R, can be shown as:

$$p_L^{t^R} = 1 - \exp(-\Lambda^R) = 1 - \exp(-\Lambda \cdot R),$$

with $R = \dfrac{nt^R}{nt^R + 1 - t^R}$ dependent on n, the relative risk degree, and t^R, the relative length of the risk period. The calculation of $p_L^{t^R}$ dependent on R can be counter-intuitive as a result of the following biases:

1. Representativeness bias:

As discussed in Tversky and Kahneman (1974), representativeness bias includes the insensitivity of sample size. Subject to the base rate fallacy in Kahneman and Tversky (1973), people tend to ignore the difference in sample sizes when calculating posterior probability (i.e., the conditional probability assigned within a certain state). Accordingly, customers estimate Λ^R while neglecting the power of the time length difference (the influence of t^R). For example, each year, the DM is exposed to a higher hazard outside the ski resort, as they spend only a few days per year in the resort. As the difference in sample sizes is neglected, the loss probability in the risky period is overestimated.

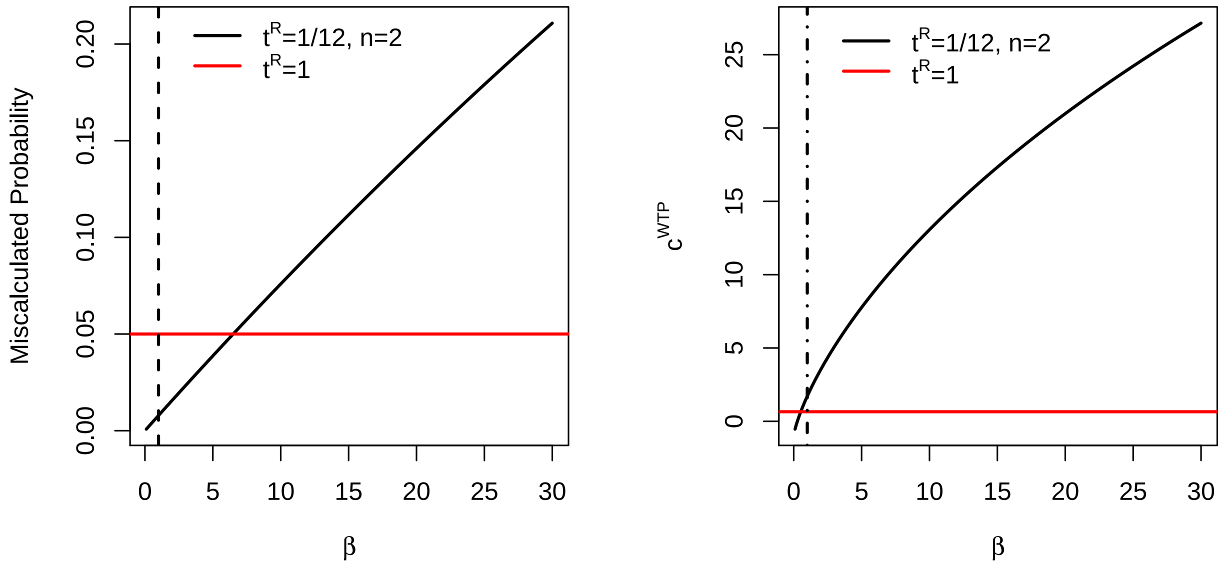
2. Availability:

As another bias mentioned in Tversky and Kahneman (1974), availability bias indicates that the ease of recalling a certain event leads to overestimation of the event's likelihood. Salience is one of the factors influencing this ease. For instance, with several skiing accidents being reported, the DM tends to overestimate the hazard. Kunreuther and Pauly (2006) recorded the anomaly that more insurance is purchased after a disaster, as the disaster risk becomes more salient. Schwarcz (2010) also uses availability and salience to explain the overweighting of low-probability risk such as terrorism.

These two behavior biases lead to a miscalculation of $p_L^{t^R}$, caused by the difference between R and the miscalculated R^*. β (> 0) represents the degree of this miscalculation with $\beta = R^*/R$. The miscalculated probability $p_L^{*t^R}$ is hence formulated as:

$$p_L^{*t^R} = 1 - \exp(-\Lambda \cdot R \cdot \beta). \tag{3.2.3.1}$$

As suggested in Barberis (2013), people overestimate a hazard's low likelihood (captured by β as subjective probability), and overweight this overestimated likelihood (captured by γ in CPT model as probability distortion). Figure (3.3) demonstrates the impact of this bias. For an objective risk period equal to one year, the case of $t^R = 1/12$, $n = 2$ signifies the risk viewed within one month, where the loss occurrence rate is double compared to the rest of the year. A higher β leads to a higher $p_L^{*t^R}$ and thus results in an even higher value for c^{WTP}.



(a) Biased probability for $p_L = 0.05$

(b) c^{WTP} with different β ($p_L = 0.05$, $\alpha = 0.88$ and $\gamma = 0.69$)

Figure 3.3: Loss probability miscalculation

3.3 Experiment and Model Approach

3.3.1 Experimental Setup

To identify the bias parameters, β and m, we ran an experiment in the University of St. Gallen with 176 students. The experiment lasted around 45 minutes, consisting of two sessions and 16 insurance scenarios (decisions) together with a questionnaire recording the participants' basic information. The first session was conducted through an online survey, and then the participants played a computer-based game in the second session. The rewards of the experiment depended mainly on the results of the game in the second session.

Session I

Three scenarios were included in Session I, wherein the participants were asked to specify their WTP for a full coverage mobile phone insurance within a certain period. A hypothetical mobile phone costs CHF 1,000 with a probability of total loss in one year of 5%. After one year, this phone can be sold for CHF 1,000. In contrast with the scenarios in Session II, the decisions for these three scenarios did not influence the participants' final rewards. Thereby, the decision results from the two sessions can be compared to understand the incentive impact on the decision bias.

Scenario A: WTP for a one-year insurance coverage.

Scenario M: WTP for an insurance contract valid for one single month in which the loss probability is double the probability for each of the other months.

Scenario Mp: WTP for an insurance contract valid for one single month in which the loss probability is 0.79%.[33]

The risks in Scenarios M and Mp are the same, while in Scenario Mp, the probability is specified. Here, we describe these scenarios within the model presented in the previous section: As the hypothetical phone can be sold without discount in one year, the objective risk period is assumed to be one year with $p_L(A) = p_L(M) = p_L(Mp) = 5\%$. Thereby, $p_L^{t^R}(A) = p_L(A) = 5\%$ in Scenario A and $p_L^{t^R}(M) = p_L^{t^R}(Mp) = 0.79\%$ in Scenario M and Mp. If DM narrowly frames the scenarios, the evaluation period is determined by the contract period. Thereby, WTP is calculated from Equation (3.2.2.1). Otherwise, the prospects are modelled with Equation (3.2.2.2), where $p_L^{1-t^R}(A) = 0\%$ and $p_L^{1-t^R}(M) = p_L^{1-t^R}(Mp) = 4.25\%$ in Scenario M and Mp.

By the end of this session, the participants were told that they would receive CHF 32 for their effort in completing the survey.[34] The purpose of this statement was to reinforce the participants' perception that they earned their rewards.

Session II

In Session II, the participants made a series of decisions in a bridge-crossing game, the results of which determined their final rewards:

[33] For all the scenarios, we elicit WTP through a series of Yes/No choices, as choice answers generate less noise (cf. Hey et al. (2009)). To eliminate anchoring bias among the participants, the starting price for Scenario A was CHF 55, or WTP equals 1.1 for all participants, while for Scenarios M and Mp, the starting price was randomly assigned for each participant. More details regarding these questions can be found in the Appendix.

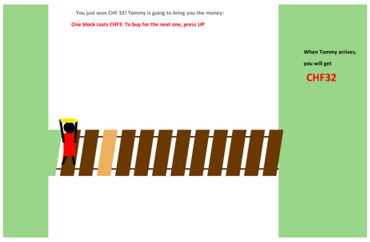
[34] Local students earned CHF 25 per hour on average for participating in a study in the lab at the University of St.Gallen.

In the game, a character called Tommy “delivers" the participants’ rewards (CHF 32) after successfully crossing a bridge consisting of 12 blocks representing the months of the year. The probability that the bridge breaks down is 5%. If the bridge breaks, Tommy falls and no rewards are earned.[35]

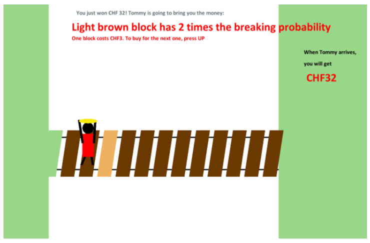
The participants are separated into three groups randomly: Group I contained 93 students, while Groups II and III serve as control groups and are participated by 50 and 33 students respectively. All three groups made their first decision on whole bridge insurance (Scenario W). A whole bridge insurance was offered for CHF 22; hence, participants could choose to receive CHF 10 without taking any risk. Additionally, Groups I and II offered block insurance with a cost of CHF 3 for each bridge block if the participants chose to take a risk and reject the whole bridge insurance offer. For Group I, the third block was described as a risky block (Scenario BR_3), where the probability of breaking was double the probability for any of the the other individual blocks. All the remaining blocks (Scenario B_i, $i = 1...12$, $i \neq 3$) possessed the same probability of breaking. For Group II, all blocks (Scenario B_i, $i = 1...12$) were equally likely to break.

Figure (3.4) shows the screenshots of Group I’s computer-based game. At each block, the participants made a decision of whether to insure Tommy for the next block. In Figure (3.4(c)), when the participants made the decision for the risky block, a warning was shown on the screen to intensify the risk salience.

(a) A whole bridge insurance, (Scenario W)



(b) Block insurance for less-risky blocks



(c) Block insurance for the risky block (Scenario BR_3)

Figure 3.4: The screenshot of the bridge-crossing game in Session II for Group I

As mentioned, a participant received his/her reward once Tommy crossed the whole bridge successfully. On the bridge, the participants compared two prospects, getting insured while giving up CHF 3 or taking the risk to gain the full reward (CHF 32 minus the premium spent on the previous blocks). To describe these scenarios within the extended CPT model, for the participants in Group I, all less-risky blocks (B_i, $i = 1...12$, $i \neq 3$) possess identical risk with

35 The participants were warned before the experiment that it would be possible to receive no rewards (i.e., CHF 0).

$p_L^{t^R}(B_i) = 0.39\%$, and for the third block, $p_L^{t^R}(BR_3) = 0.79\%$. The evaluation period equals one single block if the participants narrowly bracket the risk. If the participants broadly bracket the risk, this period equals the remaining blocks yet to cross. As the participants go further on the bridge, the loss probability decreases with fewer remaining blocks. Hence, $p_L(s)$ and $p_L^{1-t^R}(s)$ per each scenario (block) in Group I can be calculated as:

For Scenario $B_i\,(i = 1,2)$:

$$p_L(B_i) = 1 - \exp(-\Lambda \cdot (14 - i)/13)$$
$$p_L^{1-t^R}(B_i) = 1 - \exp(-\Lambda \cdot (13 - i)/13).$$

For Scenario BR_3:

$$p_L(BR_3) = 1 - \exp(-\Lambda \cdot 11/13) = 4.25\% \tag{3.3.1.1}$$
$$p_L^{1-t^R}(BR_3) = 1 - \exp(-\Lambda \cdot 10/13) = 3.48\%.$$

For Scenario $B_i\,(i = 4...12)$:

$$p_L(i) = 1 - \exp(-\Lambda \cdot (13 - i)/13)$$
$$p_L^{1-t^R}(i) = 1 - \exp(-\Lambda \cdot (12 - i)/13),$$

where $\Lambda = -\ln(0.95)$.

For Group II, all blocks have the same breaking probability. With $i = 1...12$, $p_L^{t^R}(B_i) = 0.43\%$, while $p_L(B_i)$ and $p_L^{1-t^R}(B_i)$ are shown as:

$$\begin{aligned} p_L(B_i) &= 1 - \exp(-\Lambda \cdot (13 - i)/13), \\ p_L^{1-t^R}(B_i) &= 1 - \exp(-\Lambda \cdot (12 - i)/12). \end{aligned} \tag{3.3.1.2}$$

Table (3.1) and (3.2) summarize all the scenarios in Session I and II.

s: Scenario	Probability Specified	Loss Probability		
A: Annual insurance	Yes	$p_L(A) = 5\%$	$p_L^{t^R}(A) = 5\%$	$p_L^{1-t^R}(A) = 0\%$
M: Monthly insurance	No	$p_L(M) = 5\%$	$p_L^{t^R}(M) = 0.79\%$	$p_L^{1-t^R}(M) = 4.25\%$
Mp: Monthly insurance	Yes	$p_L(Mp) = 5\%$	$p_L^{t^R}(Mp) = 0.79\%$	$p_L^{1-t^R}(Mp) = 4.25\%$

Table 3.1: Summary of experiment scenarios: Session I (Hypothetical loss amount: CHF 1000)

Group	s: Scenario	Probability Specified	Loss Probability	Premium
I, II, III	W: Whole bridge insurance	Yes	$p_L(W) = p_L^{t^R}(W) = 5\%$ $p_L^{1-t^R}(W) = 0\%$	CHF 22
I	B_i: $i = 1...12$, $i \neq 3$ ith block insurance Less-risky block	No	$p_L^{t^R}(B_i) = 0.39\%$ $p_L(B_i)$, $p_L^{1-t^R}(B_i)$ are computed with Equation (3.3.1.1)	CHF 3
I	BR_3: 3rd block insurance Risky block	No	$p_L(BR_3) = 4.25\%$ $p_L^{t^R}(BR_3) = 0.79\%$ $p_L^{1-t^R}(BR_3) = 3.48\%$	CHF 3
II	$B_i : i = 1...12$ the ith block insurance Less-risky block	No	$p_L^{t^R}(B_i) = 0.43\%$ $p_L(B_i)$, $p_L^{1-t^R}(B_i)$ are computed with Equation (3.3.1.2)	CHF 3

Table 3.2: Summary of experiment scenarios: Session II (Monetary loss amount: CHF 32 minus premium paid for the insurance of the previous blocks)

3.3.2 Baysian Model Approach

The Bayesian model approach is employed to apply the collected data to the extended CPT model described. Our focus bias, (i.e., myopic loss aversion and loss probability miscalculation), can then be explored with the bias parameters β and m. The Bayesian approach allows us to combine our prior assumptions with the observations. More specifically, the model output is a posterior distribution, integrating prior distributions with collected data through a likelihood function. The hierarchy structure within this model improves the estimated parameters' stability, especially when the data are limited (cf., e.g., Scheibehenne and Pachur (2015)). According to Nilsson et al. (2011), CPT parameters extracted from the Bayesian model are more stable with lower standard deviations than those from the MLE method. Furthermore, the posterior distribution, as a combination of joint probabilities of each parameter, grants us the flexibility to analyse the parameters separately.

Considering the probabilistic nature of human choice behavior, an error terms ϕ, suggested in Rieskamp (2008) is added to the model. With the Luce choice rule (cf. Luce et al. (1963)), the probability of purchasing insurance with the loading c instead of bearing the risk by themselves becomes:

$$P(c^{WTP}) = \frac{1}{1+\exp^{-\phi(m \cdot {}^{B}V + (1-m\cdot)^{N}V)}},$$

where m denotes the way the risk is bracketed. If $m = 1$, the participants broadly bracket the risk, and the subjective evaluation period equals the objective risk period. The difference in the prospects with or without the insurance is computed as ${}^{B}V = {}^{B}V^{w} - {}^{B}V^{w/o}$ using Equation (3.2.2.2). $m = 0$ if the evaluation period is determined by the insurance period. More specifically, the participants take into account the risk covered by the insurance only. ${}^{N}V = {}^{N}V^{w} - {}^{N}V^{w/o}$ is computed with Equation (3.2.2.1), where p_L^t is “miscalculated" as $p_L^{*t^R}$ derived in Equation (3.2.3.1) using the miscalculation factor β.

We assume all the participants possess identical parameter. For Scenario A, Mp and W, the probability is specified without miscalculation bias. Thus, β_A is fixed at 1 for these scenarios. For other scenarios, where the loss probability is not specified, we estimate β_M for Scenario M, β_R for the risky block (Scenario BR_3 in Group I), and β_N for all other less-risky blocks. As $\beta_S > 0$, β_S, $S \in \{M,R,N\}$ follows a Log-normal distribution with $\mu_\beta = \sigma_\beta$, generated by a uniform distribution ranging between 0 and 5.[36] This distribution possesses the mode of $\beta_S = 1$ with the prior assumption that no miscalculation bias exists. The myopic parameter $m_S \in \{0,1\}$ follows a Bernoulli distribution with $p = 0.5$. m_A, m_M, m_{Mp} m_W capture the myopic impact for Scenarios A, M, and Mp in Session I, and Scenario W in Session II. The bridge is separated into various parts: m_I for B_1, B_2 and B_4 in Group I, and for the first four blocks in Group II, m_{II} for the fifth to eighth blocks, and m_{III} for the ninth to eleventh blocks in both Group I and II. The results in Scenario BR_3 and B_{12} are distinct from the rest of the blocks. Therefore, m_R is assigned for Scenario BR_3 in Group I and m_{B12} for Scenario B_{12} in both groups. Table (3.3) summarizes the bias parameters, β_S and m_S.[37]

[36] $\sigma_\beta > 0$ and based on experience, the maximum of β can be set as 150 to approximate $\exp(5)$.

[37] Two additional models are constructed as a sensitivity test in the Appendix.

Bias Parameters		Prior Distribution
β_S	Miscalculation degree:	$\beta_s > 0$
	No bias exists if $\beta_S = 1$	β_S, with $S \in \{A, M, R, N\}$
	β_A for Scenario A, Mp, W	$\beta_A = 1$
	β_M for Scenario M	β_M β_R, $\beta_N \sim LogN(\mu_\beta, \sigma_\beta)$
	β_R for Scenario BR_3	$\sigma_\beta = \mu_\beta$ and $\mu_\beta \sim \mathscr{U}(0,5)$
	β_N for Scenario B_i $i = 1, ..., 12$	
m_S	Myopic factor:	$m_S \in \{0, 1\}$
	Narrowly bracket if $m_S = 0$	m_S with $S \in \{A, M, Mp, W, B_I, B_{II}, B_{III}, BR_3, B_{12}\}$
	Broadly bracket if $m_S = 1$	$m_S \sim Bern(p)$, with $p = 0.5$
	m_A for Scenario A	
	m_M for Scenario M	
	m_{Mp} for Scenario Mp	
	m_W for Scenario W	
	m_I for Scenario B_1, B_2, B_3, B_4	
	m_{II} for Scenario B_5, B_6, B_7, B_8	
	m_{III} for Scenario B_9, B_{10}, B_{11}	
	m_R for Scenario BR_3 in Group I	
	m_{B12} for Scenario B_{12}	

Table 3.3: Summary of bias parameter

3.4 Numerical Results

This section is organized as follows. The first subsection presents the descriptive statistics of the experiment results. In the second subsection, a Bayesian model is applied to estimate the extended CPT model parameters. Two extra models are run in the Appendix as sensitivity tests.

3.4.1 Descriptive Statistics

Experimental results from Session I are summarized in Figure (3.5). The participants are more likely to pay a high premium per risk unit (defined as insurance premium divided by the expected loss) in Scenario M. For the same premium, fewer participants in Scenario Mp than in Scenario M take monthly insurance, where the loss probability is specified. This indicates that the excessive insurance demand in Scenario M is caused by the miscalculation of the loss probability. Additionally, with the loss probability given in both Scenarios A and Mp, more participants purchase insurance for the same premium per risk unit in Scenario Mp due to the myopic loss aversion.

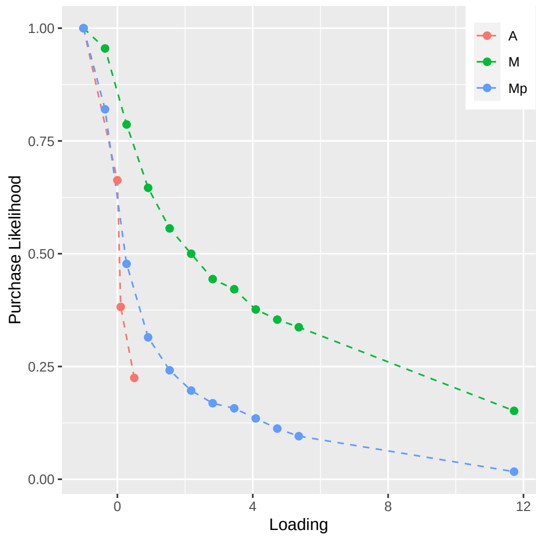
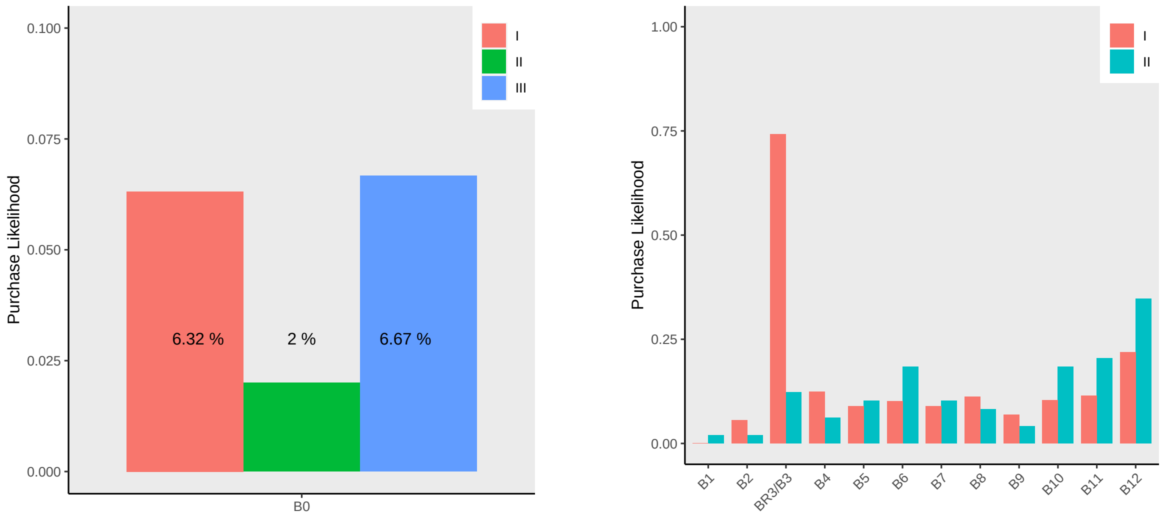


Figure 3.5: The proportion of the participants willing to buy insurance with corresponding premiums per risk unit in Session I, Scenario A, M, and Mp

In Session II, 6.32% of the participants in Group I, 2.00% in Group II and 6.67% in Group III purchase the whole-bridge insurance in Scenario W (cf. Figure (3.6(a))). Thereby, they prefer receiving CHF 10 with 100% to CHF 32 with 95% (and CHF 0 with 5%). For this whole-bridge decision, the Chi-square test does not show a strong difference among these groups (the p-value equals 49%). The block insurance results are presented in Figure (3.6(b)) and summarized in Table (**??**). Few participants (none in Group I and only one in Group II) purchase block insurance for the first block, immediately after rejecting the whole bridge insurance offer, even though the whole bridge insurance is much cheaper in terms of premium per risk unit. Around 75% of the participants purchase block insurance for the risky block in Group I (Scenario BR_3), while in Group II, only around 15% of the participants take insurance for the third block. Hence, 17% of the participants in Group I purchase no insurance and earn CHF 32, which is significantly lower than the proportion of 44% in Group II. For the rest of each block, the participants, especially in Group II, tend to buy insurance as Tommy approaches to the end of the bridge (22% of the participants in Group I and 35% in Group II purchase insurance for the last block). The difference in the average number of block insurance purchased per participant in Group I and II is not significant. Therefore, the final rewards received by these two groups are not significantly different. However, the participants in Group III earn significantly higher rewards when block insurance is not possible to obtain. Namely, additional short-term insurance products offered to the participants worsen the participants final pay-off.



(a) The proportion of whole bridge insurance (Scenario W) for Group I, II, III

(b) The proportion of block insurance decisions for Group I and II

Figure 3.6: Insurance decisions in Session II

Scenario	Group I[1] (%)		Group II[1] (%)		Difference (%)	Significance[2]	
B_1	0.00	(0)	2.04	(0.04)	-2.04	0.36	
B_2	5.62	(0.05)	2.04	(0.04)	3.58	0.42	
BR_3/B_3	**74.16**	**(0.09)**	**12.24**	**(0.09)**	**61.91**	**0.00**	**
B_4	12.36	(0.07)	6.12	(0.07)	6.24	0.38	
B_5	8.99	(0.06)	10.20	(0.09)	-1.22	1.00	
B_6	10.11	(0.06)	18.37	(0.11)	-8.25	0.19	
B_7	8.99	(0.06)	10.20	(0.09)	-1.22	1.00	
B_8	11.24	(0.07)	8.16	(0.08)	3.07	0.77	
B_9	6.90	(0.05)	4.08	(0.06)	2.81	0.71	
B_{10}	10.34	(0.07)	18.37	(0.11)	-8.02	0.20	
B_{11}	11.49	(0.07)	20.41	(0.12)	-8.91	0.21	
B_{12}	21.84	(0.09)	34.69	(0.14)	-12.85	0.11	
	Group I		Group II		Difference	Significance[2]	
None[3]	**17.89**	**(0.08)**	**44.00**	**(0.14)**	**-26.11**	**0.00**	*
Insurance[4]	1.72	(3.57)	1.44	(3.77)	0.28	0.40	

Note: [1]*Proportion (standard deviation (sd) in parentheses) of the participants in Group I and II purchasing insurance for the specific Scenario;* [2]*P-value for Fisher's exact test for the null hypothesis that Group I and Group II are equally likely to purchase insurance (* $*\, p < 0.05$*;* $**\, p < 0.01$*);* [3]*Proportion (sd in parentheses) of the participants in Group I and II purchasing no insurance for the whole bridge (receiving CHF 32);* [4]*Average number of insurance blocks (sd in parentheses) purchased per participants in Group I and II*

Table 3.4: Summary of insurance decision in Session II and the final rewards

Final rewards (CHF)			Paired T-test					
			Group II			Group III		
			Mean.Diff	t	P-Value	Mean.Diff	t	P-Value
Group I	24.82	(7.48)	-1.19	-0.87	0.39	**-5.71****	**-4.48**	**0.00**
Group II	26.02	(8.13)				**-4.51****	**-2.94**	**0.00**
Group III	30.53	(5.58)						

Note: Mean of the final rewards (sd in parentheses) per group; Paired T-test is applied to test the null hypothesis that the difference between two groups is zero ($*p < 0.05$; $**p < 0.01$)

Table 3.5: Summary of final rewards in Session II

3.4.2 Extended CPT Model Results

With the focus on the bias parameters, the CPT parameters α and γ are fixed regardless of the scenarios. Following Nilsson et al. (2011), weak assumptions are applied for α and γ, each following a uniform distribution ranging between 0 and 1. Φ ($= \phi\lambda^{CPT}$) represents the loss aversion times the error term. Φ_I is assigned for Scenarios A, M, and Mp in Session I, where the risks are only hypothetical. Scenario W share common traits with both Session I and block insurance in Session II: As the block insurance in Scenario B_i and BR_3, the decision made in Scenario W influences the participants' final rewards. However, Scenario W is a separate decision, independent from the previous decisions as those in Session I. Φ_W is assigned for Scenario W. Regarding the block insurance, the participants tend to make consistent decisions for the first block (Scenario B_1) as it is determined almost simultaneously with Scenario W. To be more specific, if the participants refuse to buy the whole-bridge insurance and thus enter the bridge game, they are unlikely to purchase the relatively expensive insurance immediately at the first block. Thereby, Φ_{B_1} is set to 20 (a large number). For other block insurance in Session II, Φ_{II} is assigned to Scenarios $B_2,...,B_{12}$ and BR_3. Φ_I and Φ_{II} (both > 0) follow a log normal distribution controlled by μ_Φ and σ_Φ, estimated with two uniform distributions with the range of (-4.6,3.2) and (0,1.13) respectively.[38] As Φ_W is related to both Φ_I and Φ_{II}, $\Phi_W = \exp(\mu_\Phi)$. Table (3.6) summarizes the CPT parameters and the error terms.

[38] Based on the CPT experience in Nilsson et al. (2011), 1.13 represents a reasonable upper bound for σ_Φ, and plausible value for ϕ lies in an interval between 0.1 to 5 or $\exp(-2.3)$ and $\exp(1.6)$. We further double the range to consider the impact of λ^{CPT} on Φ.

CPT Parameters and Error Term		Prior Distribution
α	Sensitivity of value variation:	$0 < \alpha \leq 1$
	Risk neutral if $\alpha = 1$	$\alpha \sim \mathscr{U}(0,1)$
γ	Shape of probability weighting function:	$0 < \gamma \leq 1$
	No probability is distorted if $\gamma = 1$	$\gamma \sim \mathscr{U}(0,1)$
Φ	Error term with $\Phi = \phi \cdot \lambda^{CPT}$:	$\Phi > 0$, Φ_S with $S \in \{B_1, I, II, W\}$
	Φ_{B_1} for Scenario B_1	$\Phi_{B_1} = 20$
	Φ_I for Scenario A,M Mp	$\Phi_I, \Phi_{II} \sim LogN(\mu_\Phi, \sigma_\Phi)$
	Φ_{II} for Scenario $B_2, ..., B_{12}, BR_3$	$\Phi_W = \exp(\mu_\Phi)$
	Φ_W for Scenario W	$\mu_\Phi \sim (-4.6, 3.2)$, $\sigma_\Phi \sim (0, 1.13)$

Table 3.6: Summary of CPT parameters

The Bayesian model is implemented by Markov Chain Monte Carlo algorithms (MCMC) with a total of 500,000 MCMC samples, generated from three chains after a burn-in of 1000 samples (cf. Denwood (2016) and Kruschke (2014)). Table (3.7) summarizes the derived distributions of the CPT parameters and the error term, including modes as the point estimates, coefficients of variation (CV), and effective sample sizes (ESS).[39] With Φ_{II} larger than Φ_I, the participants make choices more consistently when monetary incentives are involved.

Figure (3.7) compares our point estimates, α and γ (denoted as Mode) with those derived from Tversky and Kahneman (1992) ($\alpha = 0.88$ and $\gamma = 0.69$ denoted as TK in Figure (3.7)). The participants in our experiment are less risk averse, leading to a lower WTP for insurance coverage. One of the reasons for this may be that very risk-averse participants who purchase the whole-bridge insurance do not make decisions on block insurance along the bridge. Additionally, the separated decision-making process in Tversky and Kahneman (1992) could lead to a higher degree of risk aversion. As myopic bias is not addressed in their experiment setting, each prospect is narrowly bracketed, elevating the risk aversion degree. Figure (3.7) shows that c^{WTP} of annual insurance computed with TK parameters is close to c^{WTP} of monthly insurance

[39] Marginal distributions among parameters are shown in Appendix.

computed with mode parameters.

Parameters		mode	CV	ESS
α	$\hat{\alpha}$	0.61	0.02	22868.00
γ	$\hat{\gamma}$	0.52	0.03	24312.20
Φ_S	$\hat{\Phi}_I$	0.38	0.06	26934.70
	$\hat{\Phi}_W$	0.49	0.12	120352.10
	$\hat{\Phi}_{B_1}$	fixed at 20		
	$\hat{\Phi}_{II}$	1.24	0.05	102527.50

Table 3.7: CPT Parameter results: mode as point estimators, CV for coefficients of variation, and ESS as effective sample sizes

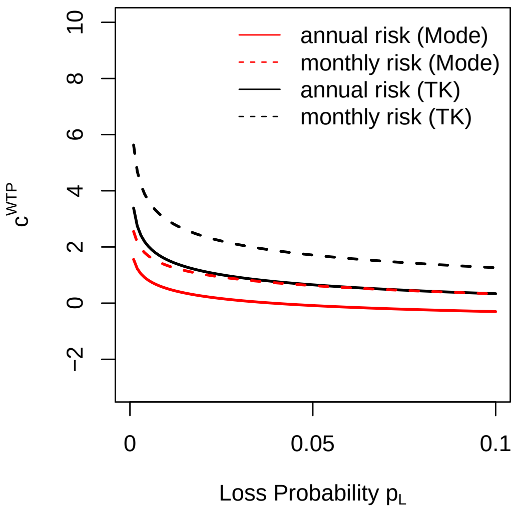


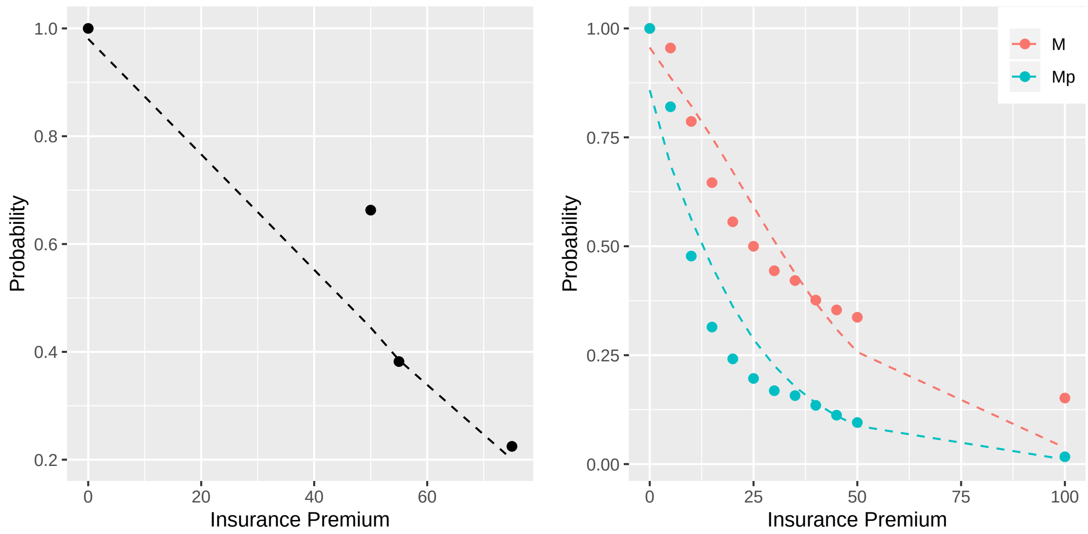
Figure 3.7: Comparison of c^{WTP} of annual insurance and monthly insurance with different probabilities and different CPT parameters (TK: $\alpha = 0.88$, $\gamma = 0.69$) and our model (Mode: $\hat{\alpha} = 0.61$, $\hat{\gamma} = 0.52$)

Table (3.8) lists the distribution summary of the bias parameters in Session I and Figure (3.8) compares the observations and the model output.[40] The mean of m_A equals 0.5 as ^{B}V and ^{N}V are the same in Scenario A, where the insurance period equals the objective risk period of one year. All the three chains generate $m_M = m_{Mp} = 0$ with zero variance. That suggests that the participants are unlikely to broadly bracket the risk when facing a monthly insurance choice. In Session M, the participants calculated the loss probability by themselves. With β_M greater than 1, this loss probability is overestimated, leading to a higher c^{WTP}. The results confirm the existence of both myopic loss aversion and probability miscalculation.

[40] All Markov Chain Monte Carlo algorithms (MCMC) representations of the parameter posterior distributions have Effective Sample Sizes (ESS) larger than 10,000. In Kruschke (2014), $ESS > 10,000$ is recommended to have stable estimates for the 95% highest density interval.

Scenario	Derived c^{WTP}	m_s	mean	CV	95% HDI	β_S	mode	CV	95% HDI
A	-0.09	$\hat{m}_A$	0.50	1.00	(0, 1)	β_A	fixed at 1		
Mp	0.63	$\hat{m}_{Mp}$	0.00	NA	(0, 0)				
M	1.35	$\hat{m}_M$	0.00	NA	(0, 0)	$\hat{\beta}_M$	3.50	0.07	(3.07, 4.00)

Table 3.8: Bias parameter results in Session I: mean as point estimators, CV for coefficients of variation, and HDI as highest density interval



(a) Scenario A (b) Scenario M and Mp

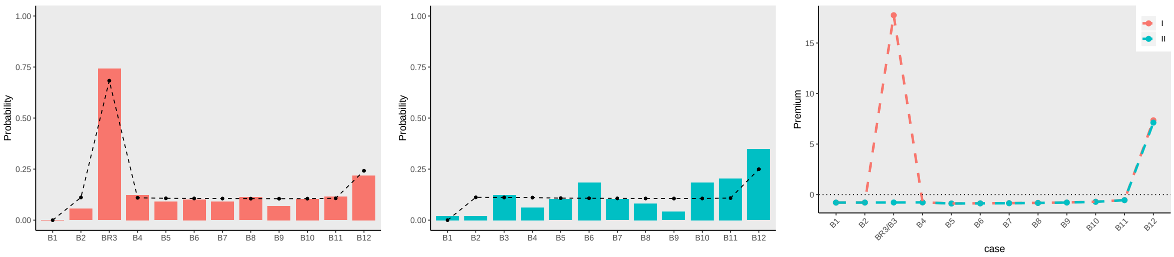
Figure 3.8: Comparison between model output and the observation in Session I

Table (3.9) summarizes the bias parameter distributions and exhibits the corresponding c^{WTP} derived with the point estimates of the parameters in Session II. As in Scenario A, in Scenario W, the insurance period equals the objective risk period (i.e., the whole bridge length). Thus, $\hat{m}_W = 0.5$ as ${}^{B}V = {}^{N}V$. For the risky block, Scenario BR_3, $\hat{m}_R = 0$ with zero variance, confirming the participants' tendency to bracket the salient risk narrowly. Additionally, the 95% highest density interval (HDI) of β_R falls between 32.28 and 93.53, which is significantly larger than 1 and the 95% HDI of β_M and of β_N. Therefore, the influence of the myopic loss aversion and of the probability miscalculation increases the likelihood of insurance purchase for the risky block. For other less-risky blocks, if they narrowly bracket the risk and focus on the insured risk only, they suffer from the miscalculation bias with $\hat{\beta}_N = 7.89$. However, only the risk of the last block (Scenario B_{12}) is overestimated with $\hat{m}_{B12} = 0$. The decision making on Scenario B_{12} is subject to the myopic loss aversion, as even the objective period is the lowest compared to the rest of the scenarios in Session II. Additionally, with $m_{12} = 0.02$, the participants estimate the loss probability and are exposed to the loss probability miscalculation. If the risk is not salient while the insurance period is much shorter than the objective risk period, it can be broadly bracketed. Thereby, at B_{12}, the participants are more likely to purchase insurance with higher derived c^{WTP} even though the expected loss is the same for all B_i, $i = 1,..,12$. Based on the derived c^{WTP}, except for scenarios B_{12} and BR_3, where risk-averse decisions are expected (with $c^{WTP} > 0$), people behave as risk-seeking agents with $c^{WTP} < 0$ (marked red in Table (3.9)).

Scenario	Derived c^{WTP}		m_s	mean	CV	95% HDI	β_S	mode	CV	95% HDI
	Group I	Group II								
W	-0.09		$\hat{m}_W$	0.50	1.00	(0, 1)	β_A	fixed at 1		
BR_3	17.75	NA	$\hat{m}_R$	0.00	NA	(0, 0)	$\hat{\beta}_R$	53.78	0.27	(32.28, 93.53)
B_1	-0.79	-0.79	$\hat{m}_I$	0.98	0.14	(1, 1)	$\hat{\beta}_N$	7.89	0.44	(1.97, 16.82)
B_2	-0.78	-0.78								
B_3	NA	-0.78								
B_4	-0.76	-0.77								
B_5	-0.87	-0.88	$\hat{m}_{II}$	0.99	0.07	(1, 1)				
B_6	-0.86	-0.87								
B_7	-0.84	-0.85								
B_8	-0.81	-0.82								
B_9	-0.77	-0.78	$\hat{m}_{III}$	0.99	0.07	(1, 1)				
B_{10}	-0.70	-0.71								
B_{11}	-0.54	-0.55								
B_{12}	7.35	7.12	$\hat{m}_{B12}$	0.02	7.77	(0, 0)				

Table 3.9: Bias parameter results in Session II ($c^{WTP} < 0$, negative premium per risk unit is marked in red)

Figure (3.9(a)) and Figure (3.9(b)) compare the observation and the model output for Groups I and II, while Figure (3.9(c)) presents c^{WTP} derived with the point estimates. The participants are willing to pay almost 18 times the expected value (CHF 4.60) in Scenario BR_3 and 7.35 times (CHF 1.80) in Scenario B_{12} due to both the myopic loss aversion and the probability miscalculation. For the rest of the less-risky blocks, the participants broadly bracket the risk, reducing the demand for the insurance. In such a case, even a fair-priced insurance contract will be rejected with negative derived values for c^{WTP}.



(a) Comparison between model output and the observation in Group I (b) Comparison between model output and the observation in Group II (c) Derived c^{WTP} in Group I and II

Figure 3.9: Comparison between model output and the observation and the derived c^{WTP} in Session II

This experiment shows that people may be subject to both myopic loss aversion and probability miscalculation. Myopic loss aversion occurs when the evaluation period is shortened. This evaluation period is normally determined by the objective risk period. Therefore, even though the breaking probabilities are the same for Scenario $B_1 ... B_{12}$, the participants are more likely to purchase insurance at the later stage where the objective risk period is the shortest. Additionally, the shrinkage of the insurance period can also reduce the evaluation period if the covered risk is salient, as in Scenario M Mp and BR_3. Even with loss probability information given in both Scenarios A and Mp, c^{WTP} is higher for the monthly insurance than for the annual insurance. In Scenarios Mp and BR_3, where the risk is salient, this myopic loss aversion, together with loss probability overestimation, pushes c^{WTP} upwards. In contrast, if the objective risk period is longer than the insurance period while the risk that the insurance covers is not salient, people take into account the risk not covered by the insurance, reducing their willingness to pay for the short-term insurance. With negative c^{WTP} in Scenario $B_1 ... B_{11}$, the experiment participants behave as risk seeking agents.

This experiment framework controls the objective risk period for the participants. In Session I, the hypothetical mobile phone can be sold at full price after 1 year. In Session II, the participants will be paid once Tommy reaches the other side of the bridge. In reality, however, the risk period is not objective but is determined rather arbitrary. For example, since we change our phones or other electronic gadgets frequently, the risk periods regarding these items can be fixed and decrease as time passes. As a result, the insurance contracts or warranties protecting such gadgets within a short risk period can be sold with a large loading. However, generally, most risks possess long periods, up to the maximum of lifelong. In the experiment conducted in Thaler et al. (1997), the participants are asked to make an investment decision, with which the investment would be binding for 400 periods. Thaler et al. (1997) concludes that the participants are found to either have an infinite horizon or else be radically myopic, since an infinite horizon is not attainable. As it is not possible to broadly bracket a lifelong risk period, a subjective evaluation can be easily influenced. This article shows that, with a salient risk, short-term insurance can reduce the DM's evaluation period. However, with a non-salient risk, the evaluation period remains, and the DM becomes risk-seeking as the evaluation period becomes longer than the insurance period. Therefore, for a catastrophe with an extremely small loss probability, there is low willingness to pay for insurance coverage even if the insurance is subsidized and offered with negative loading. With a long-term evaluation period, policyholders consider the long-term risk, which is not fully covered by the annual insurance. Moreover, the fluctuations of this annual insurance premium make the uncovered risk more prominent. Therefore, according to Kunreuther and Michel-Kerjan (2015) and Botzen et al. (2013), for low-probability risk, the insurance products gain popularity when they are offered for a longer duration. Additionally, the occurrence of certain catastrophe can make the risk salient and boost the insurance demand in the next few terms. As in Scenarios M, and B_3, due to the joint influence of myopic loss aversion and probability miscalculation, risk aversion arises once the focus risk becomes salient. This joint influence also explains the popularity of “named-event" insurance: flight crash tragedies posted on insurance booths at airports make the risk salient, nudging the passengers to focus on this risk for the next single journey instead of the whole-life hazard.

3.5 Further Discussion and Conclusion

Short-term insurance, such as on-demand insurance, may separate risk into small pieces by reducing the evaluation risk period. This paper shows that people become more risk averse when the divided covered risk is salient. This change in the risk attitudes is triggered by the myopic loss aversion as well as the loss probability miscalculation. Due to the myopic loss aversion, if a short-term insurance contract reduces people's evaluation period, they tend to pay more for the transfer of the focused risk. A relatively high risk is even overestimated due to the loss probability miscalculation.

Traditionally, insurers have designed “named-event” insurance contracts sold at an elevated premium. The acceptance of such a contract depends on how salient the risk of this event is to the customer. With on-demand insurance, individuals can freely choose the risk period to focus on according to their specific experience, memory or even superstitious belief (e.g., unlucky Friday the 13th). In this shortened evaluation period, in which the loss probability is overestimated, decision makers typically become very risk-averse with increasing WTP for the corresponding insurance. However, when facing non-salient risk, people can even be risk-seeking as they broadly bracket the risk (i.e., risk not covered by insurance is considered). Other biases may be found in the on-demand insurance market. Consumption categories in budgeting discussed in Thaler (1999) can make on-demand insurance even more appealing, when people engage in private bookkeeping, small and routine expenses are generally ignored. In reality, marketing activities exploit this bias and often frame an annual fee as "pennies-a-day" or "coverage for only pennies a day..." (cf. Johnson et al. (1993)). Despite the high-risk unit rates, the absolute payout amount for the risk per day or even per hour is tiny and can be categorized as negligible "petty cash" (cf. Thaler (1999)). Additionally, Kunreuther and Pauly (2006) indicate that individuals focus more on the coverage-premium ratio while neglecting the loss-probability factor. With a much lower loss probability, on-demand policies can be offered with an attractive coverage-premium ratio. Finally, as on-demand insurance is easy to acquire online or through mobile-phone applications, customers are prone to demonstrate the behavior biases mentioned above by using heuristics or "system 1"[41] to make decisions.

Nonetheless, on-demand insurance does not promise a certain success for the provider. The following factors can hinder the insurer's "money-pump" opportunities: First, the payment decoupling in Thaler (1999) suggests that one integrated loss is usually better tolerated than several small expenses. This explains the popularity of a "flat rate" in the telecommunication industry. Though more expensive, the flat rate makes the payment less salient. Therefore, if the on-demand insurance premium is not small enough to be ignored, customers suffer repeatedly when making premium payments. In this case, traditional insurance may be preferred. Secondly, though the premium amount of on-demand insurance is small, the loss probability is also divided and thus becomes insignificant. The threshold model suggests that individuals treat small probabilities lower than a certain threshold as equal to zero (cf. Kunreuther and Pauly (2006)). Namely, individuals may not consider getting insured at all if the loss probability is negligible. Therefore, in our experiment, the participants behave as risk-seeking agents for the less-risky blocks. Thirdly, adverse selection, moral hazard or even fraud in on-demand insurance could be much more serious than in traditional insurance. Underwriting each risk and detecting frauds while maintaining its convenient trait thus becomes a crucial point. Therefore, so far, only small and rather simple insurance contracts are offered on the market. Currently, only few on-demand insurers exist, providing relatively unique products. As monopolies in

[41] Kahneman and Frederick (2002) describes two modes of thoughts: "System 1" is instinctive and fast but subject to cognitive biases, while "System 2" is slower, but more deliberative and logical.

each segmented market, these “insurtech" companies can charge high risk unit premiums. As more insurers join, this market will become more competitive and the premium may then decrease substantially.

This paper focuses on the change in the insurance demand as the insurance contract term is reduced. With a reduced-period insurance contract and the freedom to choose when to be insured, the underlying risk is presented and thus is typically differently spotted by the policyholders. Other behavioral biases can be triggered by this reconstructed risk. The influence of other biases on the demand for insurance and the interaction among them are subjects for further research.

A Session I, Scenarios A, M, and Mp

Scenarios A, M and Mp in Session I are as follows:

“You are going to spend some years in Wonderland with CHF 4000. You need a mobile phone there and you buy it for CHF 1000. You know you can sell it for CHF 1000 in one year. A full-coverage insurance contract protects you if you lose the phone. That is, if you lose your phone, the insurance company would cover your loss and pay you CHF 1000 back for you to acquire a new phone. Otherwise, without the insurance contract, you will have to pay CHF 1000 for a new phone yourself if you lose yours.

Scenario A:

“In Wonderland, the average probability that you lose your phone in one year is 5%. Are you willing to pay CHF X for the full-coverage insurance valid for one year for the first year?"

(To extract the maximum willingness to pay, X first equals 55. If the participant answers yes, X increases to 75 and then 100. If the participant rejects the offer, X drops to 50 and then 45.)

Scenario M:

“After one year, your phone is now not insured. You sell the old phone and buy a new one for CHF 1000. You know you can sell this new phone in one year for CHF 1000.

Every year in Wonderland, you spend 1 month at the seacoast. In that month, the chance that you will lose your phone is double the chance in other single months. The average probability that you lose your phone in 1 year in Wonderland is 5%.

Next week you are going to the seacoast for one month. Are you willing to pay CHF X for the full-coverage insurance valid for one single month? (For this monthly insurance, you do not pay and are not covered for other 11 months.)"

(To extract the maximum willingness to pay, X ranges from 5 to 100 in random order. Additionally, once the participant agrees/does not agree to pay X for the insurance, it is assumed that they agree/reject the offer with premium lower/higher than X.)

Scenario Mp:

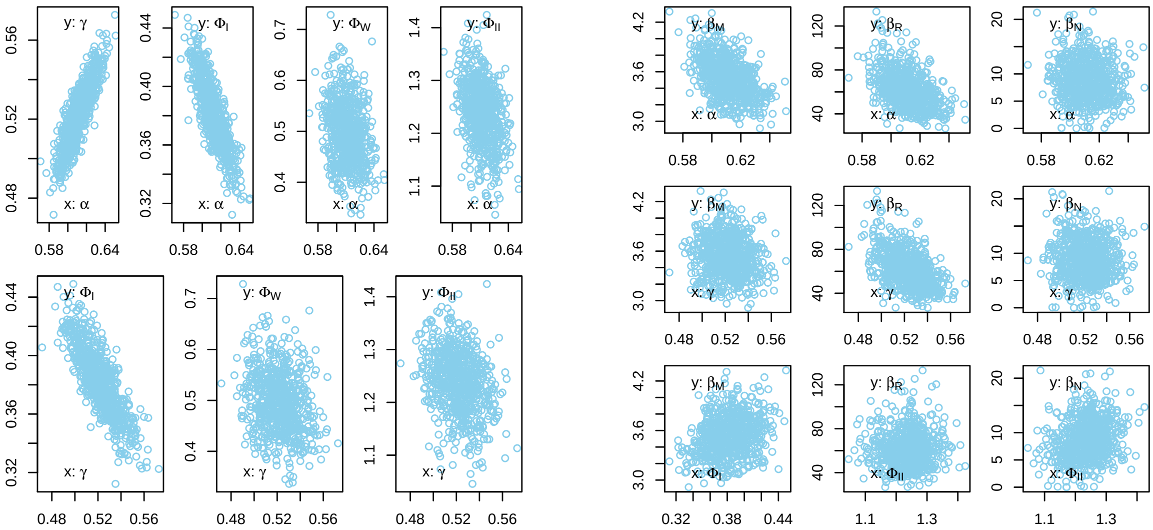
"In the third year, you sell the old phone and buy a new one for CHF 1000. You know you can sell this new phone in one year for CHF 1000.

Every year, you spend 1 month at the seacoast. In that month, the chance that you lose will your phone is 0.78%.

Next week you are going to the seacoast for one month. Are you willing to pay CHF X for the full-coverage insurance valid for one single month? (For this monthly insurance, you do not pay and are not covered for other 11 months.)"
(X varies as in Scenario M.)

B Marginal Correlations among Parameters in the Bayesian Models

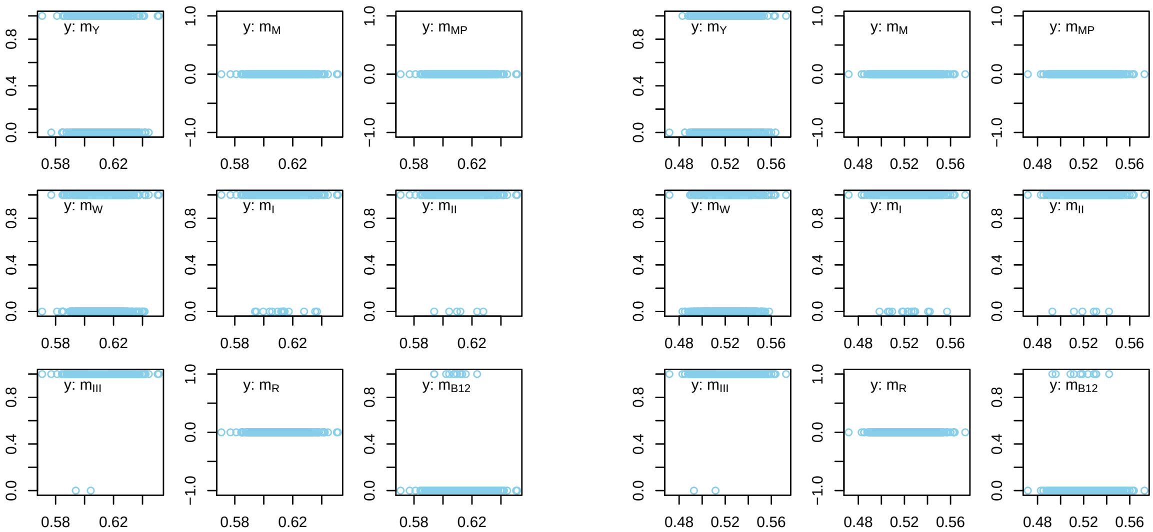
Generally, it takes at least 25 questions/choices with a wide range of risk probabilities and value amounts to estimate the CPT parameters (considering the loss domain only, Tversky and Kahneman (1992) includes 28 prospects and Rieskamp (2008) includes 60). Our parameters are estimated with few probability points. These limited points lead to a strong correlation among α, γ, and Φ as shown in Figure (B.1(a)). However, from Figure (B.1(b)), these CPT parameters and the error terms are less correlated with β_S. Moreover, no strong correlation is shown among m_S and the other parameters (cf. Figure (B.2) and Figure (B.3)).



(a) Among CPT (α, γ) and error parameters (Φ_S)

(b) among β_S and CPT/error parameters

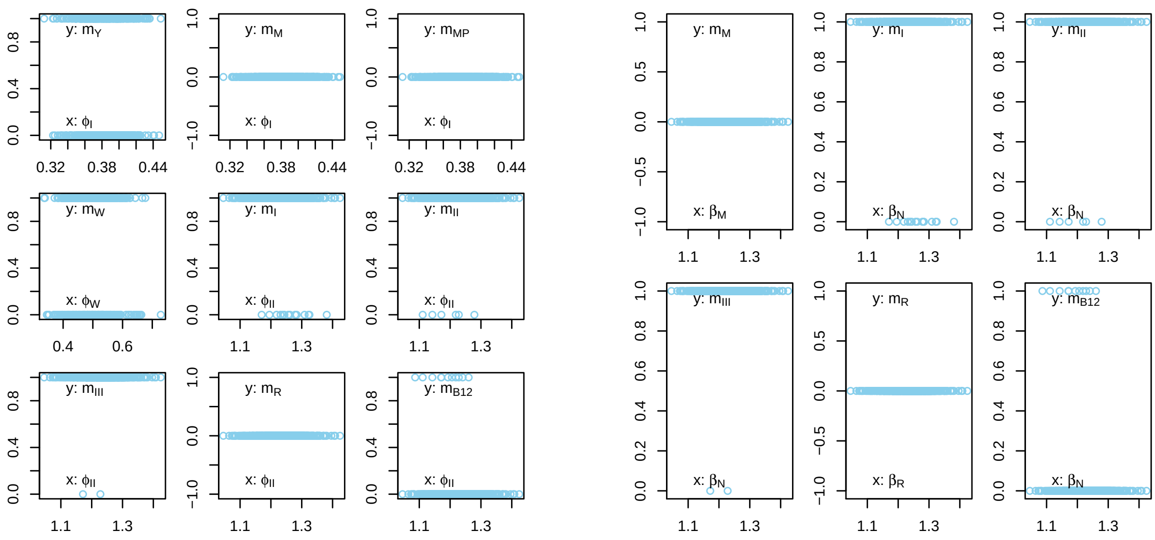
Figure B.1: Marginal distributions among CPT and error parameters and among β_S and CPT/error parameters



(a) Among m_S and α

(b) Among m_S and γ

Figure B.2: Marginal distributions among m_S and α and among m_S and γ



(a) Among m_S and Φ_S (b) Among m_S and β_S

Figure B.3: Marginal distributions among m_S and Φ_S and among m_S and β_S

Figure (B.4) presents the marginal distributions among the parameters in the “Fixed γ" model. With γ fixed at 0.69, no strong correlation is shown between Φ_s and α. Except for β_M in Scenario M, there exists no strong correlation among β_S and other CPT/error parameters.

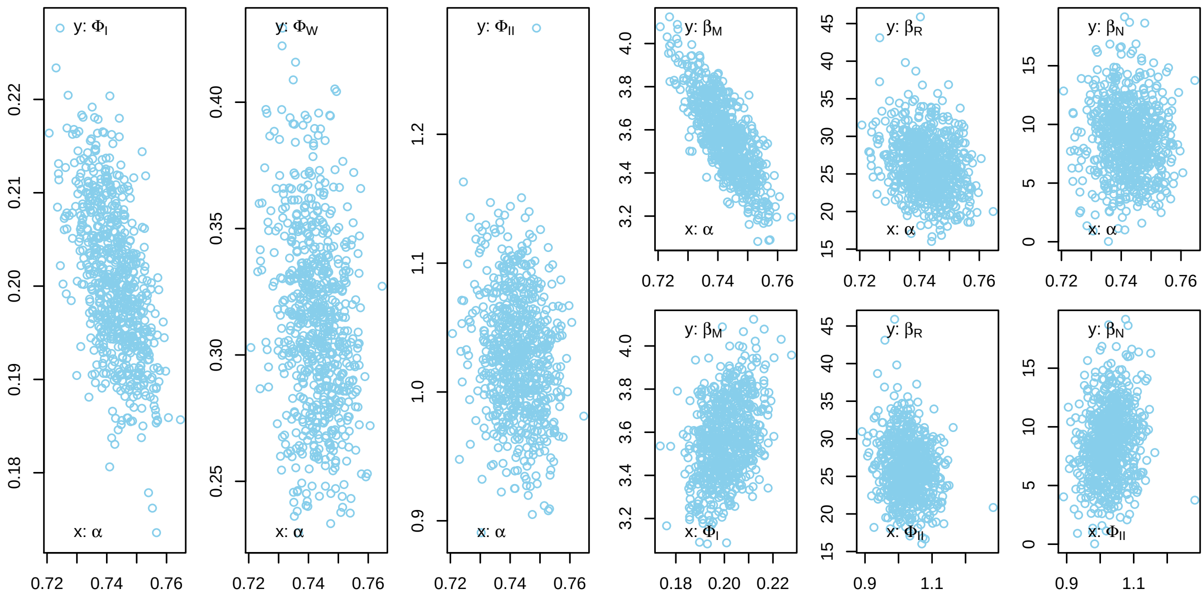


Figure B.4: Marginal distributions for “Fixed γ"

C Sensitivity Test

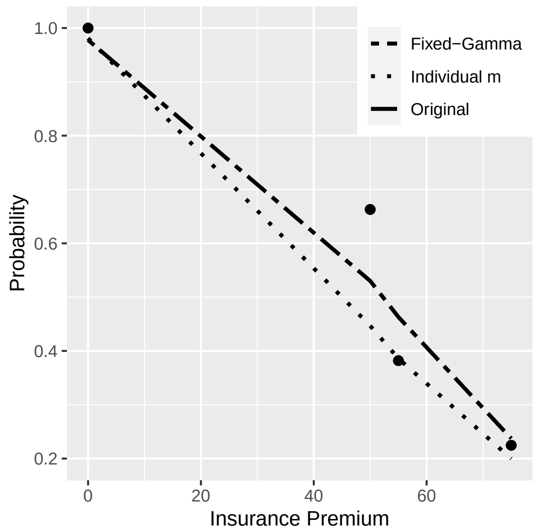
Two additional models are constructed as a sensitivity test. To address the potential problem of strong correlation among the CPT parameters in the original model (as shown in the previous section), we fix γ to 0.69 (as estimated in Tversky and Kahneman (1992)), and refer to this case as “Fixed γ" in what follows.

For the second model – indicated as “Individual m_S" – parameters m_S are assigned differently: except for B_1 and B_2,[42] m_S is estimated per each scenario. In formal terms, we have m_S with $S \in \{A, M, Mp, B_I, BR_3, B_3, ..., B_{12}\}$. Table (C.1) summarizes the two models and the original model. Figure (C.5) compares the three models with the empirical observations. The three models generate similar results except for Scenario B_{12}, where “Individual m_S" fails to depict an increasing demand in B_{12} compared to B_i, $i < 12$.

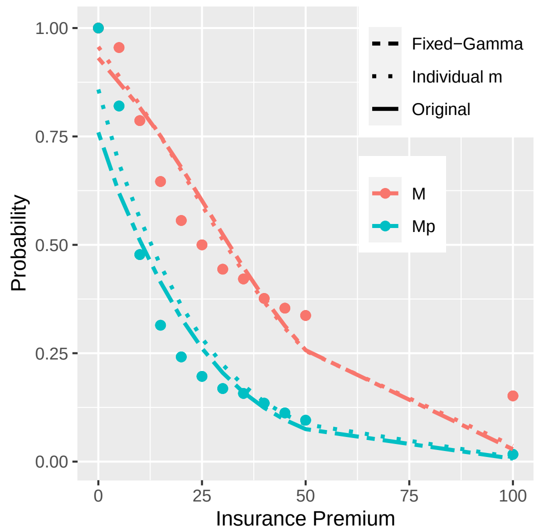
<table>
<tr><td>Original Model</td><td>Fixed γ</td><td>Individual m</td></tr>
<tr><td>$\gamma \sim \mathscr{U}(0,1)$</td><td>$\gamma = 0.69$</td><td>$\gamma \sim \mathscr{U}(0,1)$</td></tr>
<tr><td colspan="3">m_A for Scenario A
m_M for Scenario M
m_{Mp} for Scenario Mp
m_W for Scenario W</td></tr>
<tr><td colspan="2">m_I for Scenario B_1, B_2, B_3, B_4
m_{II} for Scenario B_5, B_6, B_7, B_8
m_{III} for Scenario B_9, B_{10}, B_{11}</td><td>m_I for Scenario B_1, B_2
m_{B_i} for Scenario m_{B_i}
$i = 3...11$</td></tr>
<tr><td colspan="3">m_R for Scenario BR_3
$m_{B_{12}}$ for Scenario</td></tr>
</table>

Table C.1: Summary of the Original Model and the two additional models for the sensitivity test (the assumption difference is emphasized in red)

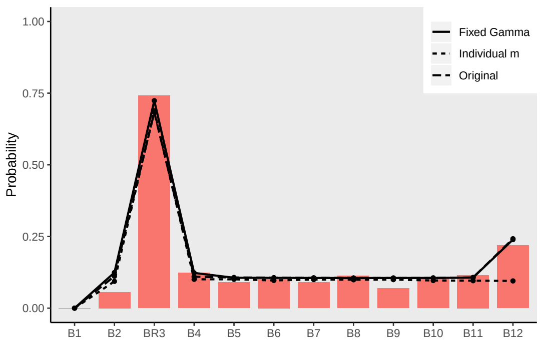
[42] As Φ_{B_1} is controlled exclusively for Scenario B_1, m_I is estimated for scenario B_1 and B_2.



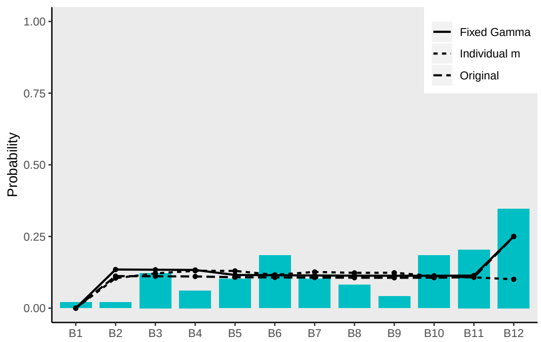
(a) Scenario A in Session I



(b) Scenario M and Mp in Session I



(c) Scenario B_i and BR_3 in Session II Group I



(d) Scenario B_i in Session II, Group II

Figure C.5: Comparison between different model outputs and the observation

Table (C.2) presents the output of these two models.[43] With γ fixed at 0.69, the small loss probabilities less overweighted. In this model, the estimated α increases to 0.74, suggesting that the participants are more conservative, or more vulnerable to the loss event. However, the "Fixed γ" model generates a lower Φ_S as the participants act less consistently to this model. The main findings for the bias parameters remain: Salient risk tends to be bracketed narrowly with m_M, m_{Mp} and m_{B_3} equal to zero with zero variance. β_M, β_R, and β_N are larger than 1, showing that the loss probability is overestimated if the risk is narrowly bracketed. In "Individual m", the estimated α, γ, and Φ_S are close to the output from the original model, indicating that the original model is robust.

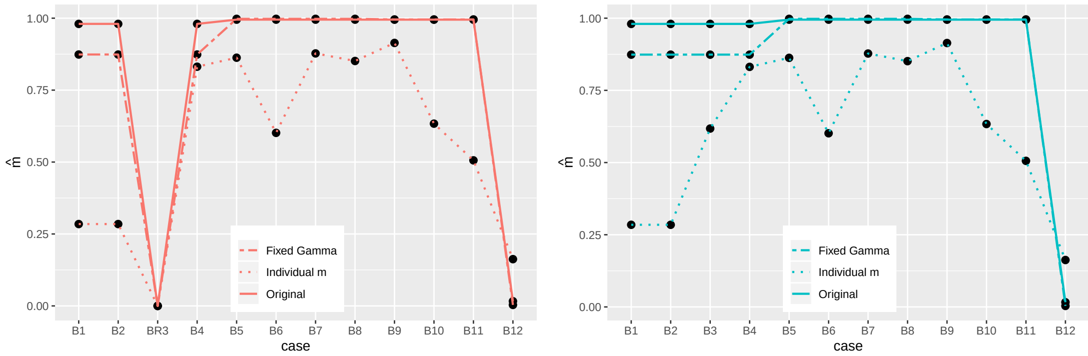
[43] The values for c^{WTP} for each models are presented in the next section.

Parameter in Scenario S		Original Model	Fixed γ	m
α (mode)		0.61	0.74	0.61
γ (mode)		0.52	fixed at 0.88	0.52
Φ_S (mode)	Φ_A for Scenarios A, M, and Mp	0.38	0.20	0.38
	Φ_W for Scenario W	0.49	0.31	0.48
	Φ_{B_1} for Scenario B_1	Φ_{B_1} fixed at 20		
	Φ_{II} for Scenario B_2,...,B_{12}	1.24	1.03	1.26
β_S (mode)	β_A for Scenarios A, Mp, and W	β_A fixed at 1		
	β_M for Scenarios M	2.92	3.54	3.40
	β_R for Scenarios B_3	53.78	25.51	44.56
	β_N for Scenarios B_i, $i \neq 3$	7.89	8.66	0.06
m_S (Mean)	m_A for Scenario A	0.50	0.50	0.50
	m_M for Scenario M	0.00	0.00	0.00
	m_{Mp} for Scenario Mp	0.00	0.00	0.00
	m_W for Scenario W	0.50	0.50	0.50
	m_S for Scenario BR_3	0.00	0.00	0.00
	m_S for Scenario B_1 and B_2	$\hat{m}_I = 0.98$	$\hat{m}_I = 0.98$	0.28
	m_S for Scenario B_3			0.62
	m_S for Scenario B_4			0.83
	m_S for Scenario B_5	$\hat{m}_{II} = 0.99$	$\hat{m}_{II} = 0.99$	0.86
	m_S for Scenario B_6			0.60
	m_S for Scenario B_7			0.88
	m_S for Scenario B_8			0.85
	m_S for Scenario B_9	$\hat{m}_{III} = 0.99$	$\hat{m}_{III} = 0.99$	0.91
	m_S for Scenario B_{10}			0.63
	m_S for Scenario B_{11}			0.50
	$m_{B_{12}}$ for Scenario B_{12}	0.02	0.02	0.16

Table C.2: The parameters' point estimates in the three models

Figure (C.6) shows for all models, that the participants tend to narrowly bracket the prospect

($\hat{m} = 0$) as Tommy (the comic figure in our game) approaches the end of the bridge for all three models – except for Scenario BR_3 in Figure (C.6(a)). The "Individual m" model indicates that the participants do not necessarily overestimate the loss probability during the less-risky period when the risk is narrowly bracketed. However, the model fails to depict the growing tendency of insurance purchase in the last block (Scenario B_{12}).



(a) Group I

(b) Group II

Figure C.6: $\hat{m}_S$ for Scenario $B_1 ... B_{12}$, BR_3 in Session II: For all three models, the participants tend to narrowly bracket the prospect ($\hat{m} = 0$) as the risk is salient (Scenario B_3) and as Tommy (the comic figure in our game) approaches the end of the bridge.

D Derived c^{WTP} for Three Bayesian Models

Table (D.3) compares the derived c^{WTP} for the three models per each scenario.

Scenario	Y	M	Mp	W
Original	-0.09	2.92	0.63	-0.09
Fixed γ	0.04	3.01	0.34	0.04
Individual m	-0.08	2.91	0.62	-0.08

		B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}
Original	I	-0.79	-0.78	17.75	-0.76	-0.88	-0.86	-0.84	-0.82	-0.78	-0.70	-0.54	7.35
	II	-0.79	-0.78	-0.78	-0.77	-0.88	-0.87	-0.85	-0.82	-0.78	-0.71	-0.55	7.13
Fixed γ	I	0.42	0.43	17.99	0.46	-0.65	-0.63	-0.60	-0.56	-0.48	-0.40	-0.24	8.46
	II	0.41	0.42	0.43	0.45	-0.66	-0.63	-0.60	-0.56	-0.49	-0.41	-0.25	8.33
Individual m	I	-0.84	-0.84	17.80	-0.90	-0.90	-0.86	-0.87	-0.85	-0.81	-0.76	-0.68	-0.51
	II	-0.84	-0.84	-0.88	-0.91	-0.90	-0.86	-0.87	-0.85	-0.82	-0.77	-0.69	-0.52

Table D.3: Derived c^{WTP} in each scenario (negative premiums per risk units with $c^{WTP} < 0$ are marked in red)

Essay IV

Do Numeracy and Slow Thinking Revise Decision Biases?

Hsiaoyin Chang

Myopic loss aversion and loss probability miscalculation are two decision biases which occur as the long-term risk is perceived as a combination of multiple short-term risks. Whether these short-term risks are considered independently or jointly depends on decision makers' risk editing rule. Numerical literacy helps decision makers to aggregate risks. Taking time analyzing risk prospects may also reduce the biases. This paper uses experiment data to analyze the impacts of numerical literacy and of decision-making time on these decision biases. The result shows that numerate decision makers may resist certain biases. However, when monetary incentives are involved, anxiety, aroused by the risk prospect in the loss domain, deters decision makers from rational thinking. This emotion impact aggravates when they hesitate to make decisions. Therefore, making slow decisions over risk, where a real loss is involved, does not always increase decision rationality.

Keywords: Risk attitudes · Numerical literacy · Decision making theory · Insurance demand anomaly · Behavioral economics

4.1 Introduction/Motivation

Prospect theory (PT) in Kahneman and Tversky (1979) divides the decision-making process into two phases: the initial editing phase and the subsequent evaluation phase. In the editing phase, the prospects are preliminarily analyzed or “edited". The edited prospects are then evaluated in the evaluation phase, where the one with the highest value is chosen. Editing can generally be divided into two rules: segregation and aggregation, or narrowly and broadly bracketing as discussed in Read et al. (1999). Individuals bracket scenarios narrow when each scenario is considered independently. In contrast, broadly bracketing occurs when all the scenarios are bundled and assessed as one. Due to myopic loss aversion discussed in Thaler et al. (1997), risk aversion is aroused when one risk is segregated into several narrow-scope risks, with each appraised separately.

Epper and Fehr-Duda (2018) explained this excessive risk aversion with probability distortion, i.e., overweighting low probabilities while underweighting high probabilities (cf. Quiggin (1982)). Defined as a risk scope in a temporal dimension, a risk period determines how risk is considered within temporal bracketing (cf. Read et al. (1999)). If this period shrinks, a long-term event risk is segregated into multiple short-term ones, each with a divided probability. Divided probabilities are expected to be overweighted to a higher degree. Therefore, the decision maker (DM) becomes more risk averse when the risk is narrowly bracketed with a reduced risk period. In reality, a risk period of a financial asset can be determined by the asset's maturity. Van Binsbergen et al. (2012), Andries et al. (2014), and Eisenbach and Schmalz (2016) showed that a short-maturity asset generally possesses higher risk premium in the financial market as holders of this asset are more risk averse. For the insurance sector, discussed in Johnson et al. (1993), Kunreuther and Pauly (2006), and Epper and Fehr-Duda (2018), flight insurance gains popularity when its risk period is reduced to a fleeting flight. Through probability distortion, a risk period influences DM's risk preference. If this period can be controlled artificially by a risk-related product design (e.g., a financial asset's maturity or an insurance contract duration), DM's risk preference can be “manipulated".

Additionally, segregated salient risk tends to be overestimated due to the representativeness, the availability bias, and the base rate fallacy noted in Tversky and Kahneman (1974). The survey carried out by Johnson et al. (1993) has shown that flight insurance against terrorism act is much more popular than the insurance against general causes. Kunreuther and Pauly (2006) and Schwarcz (2010) noted an increase in demand for certain event insurance shortly after the event occurs. As this occurrence makes the event salient, DM overestimates its probability. Thereby, when one risk is segregated, DM cannot fairly assess each segregated risk. Instead, those salient ones are likely to be overrated. Together with myopic loss aversion, the probability of a salience incident is overestimated while this miscalculated probability is overweighted (cf. Barberis (2013)).

As Webb and Shu (2017) argued, DMs generally make better choices (normative choices with higher expected value) when risk is broadly bracketed. However, with limited cognitive capacity, broadly bracketing is not always attainable. When risk is segregated by the duration of a risk-related product, susceptible to the decision biases (i.e., myopic loss aversion and probability miscalculation), those who have difficulty aggregating risks make worse choices or non-normative choices. Cillo and De Giorgi (2017) conducted an experiment, where multiple scenarios were presented collectively and separately. This experiment implies that aggregating risks by DMs themselves is not always feasible and the willingness to pay for the aggregation is hence positive. However, sophisticated DMs generate lower willingness to pay, suggesting that they are better at risk aggregation and therefore are less susceptible to the decision biases.

Abundant literature have examined the relation between individuals' cognitive capability and their susceptibility to various decision biases. With survey and experiment data, Peters et al. (2006), Peters (2012) and Petrova et al. (2014) demonstrated that highly numerate individuals use appropriate numerate principles and tend to make decisions based on relevant numerate information. On the contrary, low-numerate individuals consider irrelevant information, such as affection, and are more vulnerable to the framing effects. Regarding the base rate fallacy, Stanovich and West (1998b) and Kokis et al. (2002) detected a positive correlation between cognitive capability and the tendency of normative decisions. However, Stanovich and West (1998a) and Stanovich and West (1999) failed to correlate this bias with the cognitive capability. Stanovich and West (2008) conducted several experiments, showing that depending on the experiment tasks, rational decisions may or may not be associated with cognitive capability. Intelligent participants perform better only if the potential biased processing is warned or recognized in advance. Without the warning, intelligent participants make heuristic decisions and are subject to the biases as low-intelligent participants are. Dual-process theory in Kahneman and Frederick (2002) suggests, information is processed with two different modes of thoughts: System I, a heuristic system and System II, an analytic system. Intelligent participants make better choices if System II is employed. However, they are not better at recognizing the potential conflict of the two systems and therefore not better at deciding which system to use.
Time spent on the decision-making process may determine which system is engaged: System I is generally intuitive, effortless, and hence fast, while System II is deliberate and slow. Similar to the dual-process theory, Loewenstein et al. (2001) and Quartz (2009) suggested two distinct thinking models: emotional/heuristic process and cognitive computation. On the one hand,

according to Kahneman and Frederick (2002), those who make fast decisions are assumed to use System I and thus are susceptible to biases. Segregated risks can then be better aggregated when information is processed slowly within System II. On the other hand, anxiety surfaces when facing potential loss. This emotion lingers and grows with prolonged time spent on the decision. As Petrova et al. (2014) asserts, emotional appraisal leads to a higher degree of the loss probability overweighting. Therefore, slow decision-making triggers either cognition (if System II or cognitive computation is acquired) or emotion (if System I or emotional process is in use). The former improves the decision rationality by aggregating risks, while the latter segregates risk, leading to irrational or non-normative choices.

Chang and Schmeiser (2021) conducted an experiment and confirm myopic loss aversion and probability miscalculation when salient risk is segregated, while the experiment participants are considered homogeneous DMs. As DM's individual cognitive capability influences the decision qualities, this paper further examines if the cognitive capability, measured by numerical literacy, relates to these biases. With respect to probability miscalculation, numerate subjects are expected to assess salience risk more properly if System II is in use. If numerate participants can aggregate risk better, they are less subject to myopic loss aversion over risk which is segregated by short-term insurance. However, if the risk period is reduced by the experiment design, the probability distortion, considered as risk preference instead of irrationality, may or may not be associated with numerical literacy. Additionally, this paper explores the relationship between the time spent on the decision process and the decision quality (the tendency of normative decision). It is aimed to study if slow decisions, made by cognitive computation, improve the decision quality. Otherwise, emotion, provoked by prospects, hinders DMs to think rationally and make normative decisions.

The remainder of this paper is set out as follows: Section 2 briefly describes the experiment design. The preliminary results and the testable hypotheses are laid out in the section 3. Section 4 discusses the hypothesis results and the final section concludes the paper.

4.2 Experiment Design

This section briefly describes the experiment design.[44] 178 students from the University of St. Gallen were recruited to participate in this experiment. The experiment lasted around 45 minutes, consisting of two sessions. The first session was done through an on-line survey, and then

[44] A detailed description of the experiment can be found in Chang and Schmeiser (2021).

the subjects played a computer-based game in the second session. The rewards of the experiment are mainly determined by the result of the second session.

Session I includes three scenarios, where the subjects were asked to specify their willingness to pay (WTP) for a full coverage mobile phone insurance within a certain period. A hypothetical mobile phone costs CHF 1,000 with a probability of total loss in one year of 5%. After one year, this phone can be sold for CHF 1,000. In contrast with Session II, in this session, the subjects' decisions did not influence their final rewards:

WTP^A in Scenario A: WTP for a one-year insurance coverage, where WTP^A represents WTP per unit risk (WTP divided by its annual expected loss).

WTP^M in Scenario M: WTP for an insurance contract valid for one single month in which month the loss probability is double the probability of the rest of each month. WTP^M represents WTP per unit risk (WTP divided by its monthly expected loss).[45]

WTP^{Mp} in Scenario Mp: WTP for an insurance contract valid for one single month in which month the loss probability is 0.79%.[46] WTP^{Mp} represents WTP per unit risk (WTP divided by its monthly expected loss). This monthly risk is actually the same as the risk in Scenario M, while in this scenario, the probability is presented with an absolute number.

After these three scenarios, the subjects answered further questions concerning their characteristics: numerical literacies (Numeracy), risk attitudes, and insurance purchase habits in reality (Insurance Habit):

- Numeracy: Numerical literacy is assessed with the eight-item Racsch-based numeracy scale used in Weller et al. (2013).[47]

45 In addition to WTP, subjects are asked to calculate the loss probability in this scenario. Randomly assigned into two groups, the subjects in the treatment group are asked to calculate its monthly loss probability before the WTP choices and those in the control group are asked the probability after the choices. Further details are discussed in the next section.

46 Based on a Poisson distribution, the probability that no loss occurs in one year is $P(0) = 95\%$. Hence, with $\lambda = -\log(0.95)$, the loss probability in Scenario M is calculated as $1 - \exp(-\lambda \cdot 2/13) = 0.786\%$.

47 Note that low numeracy does not necessarily imply that the subjects are incapable of making numerical calculations. It could also be caused by the non-diligent heuristic thinking.

- Risk Attitude: Risk attitude is extracted as the lottery choices in Holt and Laury (2002) but in the loss domain. The subjects were asked to choose between two prospects: A. conservative prospect and B. risky prospect.

A. Conservative Prospect: losing CHF 2 with probability p and CHF 1.6 with probability $1-p$.

B. Risky Prospect: losing CHF 3.85 with probability p and CHF 0.10 with probability $1-p$.

p, $p = 10\%, 20\%, ..., 100\%$, are randomly inserted in a series of A/B choices. When the subjects choose A (B) for p^*, they are assumed to prefer A (B) for all p, $p \geq p^*$ (p, $p \leq p^*$). By the end of the experiment, one of the choices made is randomly selected for each subject, and their rewards earned in Session II would be accordingly deducted.

Risk Attitude, ranging from 0 to 10, is assigned according to the number of chosen B. For example, if the subjects prefer A with $p = 10\%$, they are expected to choose A for the rest of the nine combinations. With Risk Attitude equals 0, they behave as extreme risk-averse agents. Where Risk Attitude equals 10, the subjects prefer to lose CHF 3.85 to CHF 2. Assuming rational DMs prefer more than less, Risk Attitude 10 denotes an irrational decision result.

Table 4.1: Risk Attitude

A: losing CHF 2 with probability p and CHF 1.6 with probability $1-p$.

B: losing CHF 3.85 with probability p and CHF 0.10 with probability $1-p$.

Choices	Expected Loss		Risk Attitude										
			Risk Averse										Risk Seeking
p	A	B	0	1	2	3	4	5	6	7	8	9	10†
10%	CHF -1.64	CHF -0.475	A	B	B	B	B	B	B	B	B	B	B
20%	CHF -1.68	CHF -0.850	A	A	B	B	B	B	B	B	B	B	B
30%	CHF -1.72	CHF -1.225	A	A	A	B	B	B	B	B	B	B	B
40%	CHF -1.76	CHF -1.600	A	A	A	A	B	B	B	B	B	B	B
50%	CHF -1.80	CHF -1.975	A	A	A	A	A	B	B	B	B	B	B
60%	CHF -1.84	CHF -2.350	A	A	A	A	A	A	B	B	B	B	B
70%	CHF -1.88	CHF -2.725	A	A	A	A	A	A	A	B	B	B	B
80%	CHF -1.92	CHF -3.100	A	A	A	A	A	A	A	A	B	B	B
90%	CHF -1.96	CHF -3.475	A	A	A	A	A	A	A	A	A	B	B
100%	CHF -2.00	CHF -3.850	A	A	A	A	A	A	A	A	A	A	B

Note: †Risk Attitude 10: prefer B when $p = 100\%$*, or prefer a loss of CHF 3.85 to a loss of CHF 2, signifying an irrational decision result*

- Insurance Habit, the purchase frequency in smart phone warranty, laptop warranty, and flight accident insurance measures the subjects' risk preference with respect to small-loss risk in the real life. For each insurance product, a score is assigned as zero for “Never", one for “Seldom", two for “Sometimes", three for “Usually", and four for “Always". Summing up the scores for each product, Insurance Habit ranges from zero to twelve. Table (4.2) summarizes the variables.

Table 4.2: Summary of the variables in Session I

Variables	
WTP^A	WTP per unit risk in Scenario A
WTP^M	WTP per unit risk in Scenario M
WTP^{Mp}	WTP per unit risk in Scenario Mp
Numeracy	Numerical literacy score 0-8 (8: the highest numerical literacy)
Risk Score	Risk attitudes 0-10 (0: extreme risk averse; 9: extreme risk seeking; 10: irrational decision result)
Insurance Habit	Insurance purchase habit in the reality for flight accident insurance, smart phone warranty, and laptop warranty 0-12 (0: I never purchase any of these three insurance products; 12: I always purchase all of these three insurance products.)

In Session II, the subjects played a crossing-bridge game, which contains a series of scenarios. Results of this game determined their final rewards. The following describes the game and the related scenarios.

Tommy (a comic figure in the game) delivers the subjects' rewards. The subjects receive their rewards, CHF 32, when Tommy crosses the bridge successfully. This bridge consists of twelve blocks as a year contains twelve months. The probability that the whole bridge breaks down is 5%. Tommy falls if the bridge breaks, and their earned rewards are gone.

The subjects are separated into three groups: Experimental Group: Group I contains 93 students; Control Group: Group II and III contain 50 and 33 students respectively. A whole-bridge insurance is offered for all three groups with a premium of CHF 22. Hence, the subjects can choose to receive either a sure gain of CHF 10 or CHF 32 with 95% (and zero with 5%). If the subjects in Group I and II choose not to take the whole-bridge insurance, at each block, they are offered a block insurance at the cost of CHF 3. In Group I, the third block of the bridge is a risky block, whose breaking probability is twice as high as the probability of other single blocks. In Group II, all blocks are equally likely to break. At each block, the subjects decide whether to insure themselves for the next one. Therefore, in Session II, there are thirteen scenarios in Group I and II and one scenario in Group III. All these scenarios are summarized in Table (4.3).

Table 4.3: Summary of Session II

Group	Scenario	Premium	Expected Loss[†]	Rate per Unit Risk[‡]
I, II, III	W: Whole-bridge insurance	CHF 22	CHF 1.60	CHF 13.75
I	B_i, $i = 1...12$, $i \neq 3$: *i*th block insurance	CHF 3	CHF 0.13	CHF 23.08
	BR_3: Risky block	CHF 3	CHF 0.25	CHF 12
II	$B_i : i = 1...12$: the *i*th block insurance	CHF 3	CHF 0.14	CHF 21.43

Note: [†]Expected loss for each block is computed with the assumption that the subjects have not purchased any block insurance for the previous blocks, i.e. total loss of CHF 32. [†]Rate per Unit Risk is computed as Premium divided by Expected Loss.

Within this experiment setup, each scenarios' risk periods can be defined objectively. In Session I, as the hypothetical phone can be sold without discount in one year, the objective risk period is assumed as one year. In Session II, the subjects receive their rewards once Tommy crosses the bridge successfully. Therefore, the objective risk period is the remaining blocks Tommy has yet to crossed. This period shrinks as Tommy proceeds.

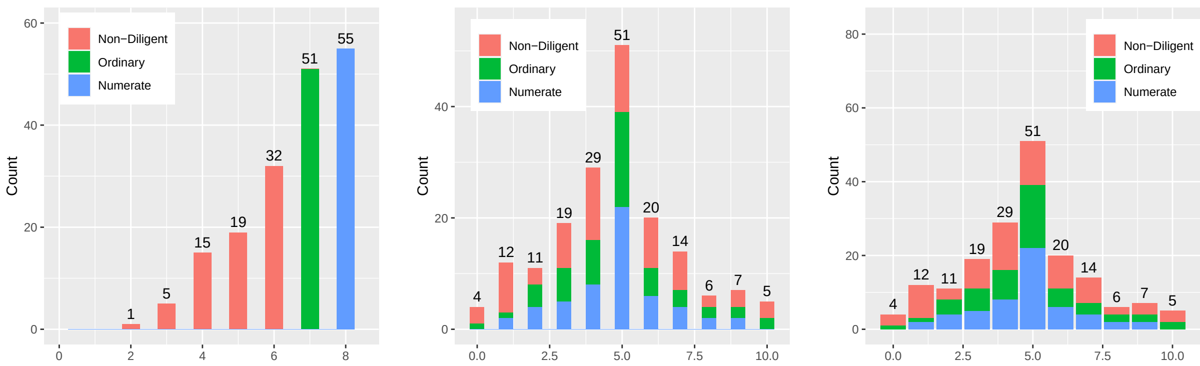
An objective risk period, however, can be different from an evaluation period, within which risk is assessed. As the risk covered by the insurance is emphasized and thus focused, the uncovered risk is neglected. Thereby, insurance products may alter the way that the DM edits the risk. As short-term insurance reduces the evaluation period, it segregates the risk in the temporal dimension. Myopic loss aversion suggests that DMs become more risk averse when risk is segregated. Additionally, when salient risk (e.g. risk in Scenario M and in Scenario BR_3) is isolated, this risk tends to be overestimated due to loss probability miscalculation. DM's excessive risk aversion is therefore provoked by short-term insurance.
Individuals are subject to this insurance's influence on risk editing with different degrees. Those

with high numerical capability are expected to analyze the risk appropriately and better aggregate the risks (which are segregated by the short-term insurance). Therefore, their decisions tend to be consistent and independent from the various insurance durations. Additionally, the excessive risk aversion may be remedied if slow decisions are made with attentive computation. However, when each prospect is analyzed slowly, DMs are exposed to anxiety and can therefore turn into extreme risk averters. The following section demonstrates the experiment results and introduces the hypotheses. These hypotheses examines the relationship between the subjects' numerical literacy and their risk preference and between the decision-making time and the preference.

4.3 Preliminary Results and Hypothesis Test

Figure (4.1) demonstrates the subjects' characteristic distribution. In Figure (4.1(a)), more than 30% of the subjects achieved the highest numerical score (i.e., Numeracy equals eight) and around 28% of them finished with the second highest (i.e., Numeracy equals seven). Those with Numeracy lower than seven are merged into one group as "non-diligent" subjects. Their low numeracy scores reflect that they are either less numerate or conduct this experiment carelessly with heuristic. Thereby, all the subjects are separated into three numeracy groups: "Non-Diligent" with numerate degree 1 for those with Numeracy bigger than 7, "Ordinary" with numerate degree 2 for those with Numeracy equal to 7, and "Numerate" with numerate degree 3 for those with Numeracy equal to 8. A bell-shaped distribution of Risk Attitude is demonstrated in Figure (4.1(b)) with its median equal to five. With Risk Attitude equal to ten, five subjects preferred losing CHF 2 to CHF 3.85. The responses of these five subjects, none of whom is Numerate, are deemed unreliable and are excluded from the data for the further analysis. Figure (4.1(c)) suggests that the majority of the subjects never purchase any of the small-scale insurance.

Figure 4.1: Distribution of subjects' personal characteristics



(a) Numeracy, (mediate 7.00 and mean 6.52)

(b) Risk Attitude, (mediate 5.00 and mean 4.73)

(c) Insurance Habit, (mediate 11.00 and mean 9.92)

Excluding those with Risk Attitude equal to 10 (irrational decisions), the characteristics of the remaining 173 subjects are summarized per numerate group in Table (4.4). As the most risk-averse agents, the non-diligent subjects are shown with the lowest Risk Attitude and acquire small-scale insurance more often in their daily life. Additionally, the heterogeneity among them is the highest with the largest standard deviation (SD). However, as demonstrated in Table (4.5), the Spearman statistic test shows no significant correlation among these characteristics. Both Risk Attitude and Insurance Habit are to measure the subjects' risk attitude. While Insurance Habit is significantly correlated with WTP in Session I, (i.e., WTP^A, WTP^M and WTP^{Mp}), none of WTP is significantly correlated with Risk Attitude. These weak correlations between Risk Attitude and Insurance Habit, and between Risk Attitude and WTP in Session I may be caused by the relatively small loss amount in the experiment design. Therefore, Risk Attitude fails to reflect the subjects' real risk preferences.

Table 4.4: Summary of the subjects' characteristics

		All Subjects	Non-Diligent	Ordinary	Numerate
Numerate Degree			1	2	3
Number of Obs		173	69	49	55
Numeracy		6.55 (1.43)	< 7	$= 7$	$= 8$
Risk Attitude	Mean (SD)	4.57 (2.03)	4.30 (2.30)	4.67 (1.89)	4.82 (1.75)
Insurance Habit	Mean (SD)	2.04 (2.63)	2.14 (2.77)	2.06 (2.63)	1.89 (2.48)

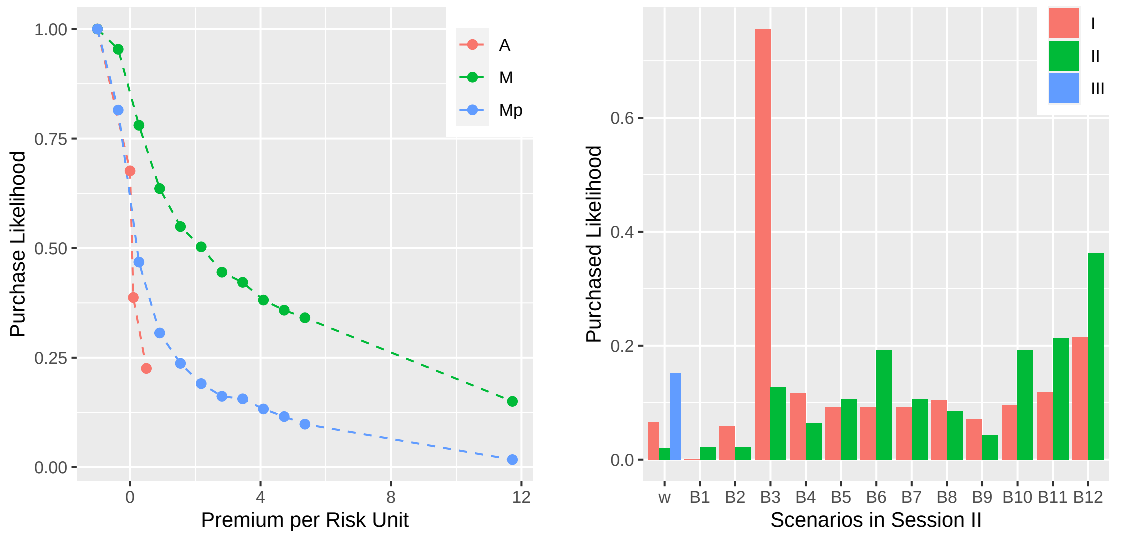
Table 4.5: The correlation matrix among the variables in Session I

	WTP^A	WTP^M	WTP^{Mp}	Numerate Degree	Risk Attitude
WTP^A					
WTP^M	0.34**				
WTP^{Mp}	0.37**	0.58**			
Numerate Degree	0.08	-0.15*	-0.03		
Risk Attitude	0.01	-0.09	-0.14	0.10	
Insurance Habit	0.24**	0.23**	0.30**	-0.04	-0.11

Note: Spearman test for the null hypothesis that no association exists between the two variables
($p < 0.05$; ** $p < 0.01$)*

With the experiment decision results shown in Figure (4.2), Chang and Schmeiser (2021) confirmed two decision anomalies as myopic loss aversion and probability miscalculation. For myopic loss aversion, the loss probability is overweighted in a higher degree when the evaluation period shrinks. Thereby, in Session I, the subjects are more likely to pay a higher premium in Scenario Mp compared to that in Scenario A. In Session II, the subjects tend to take insurance for the last block of the bridge (B_{12}) even though the risks for all blocks except for BR_3 are the same. Regarding probability miscalculation, the DM overestimates the loss probability of the salient risk. Therefore, when facing segregated salient risk, i.e., risks in Scenario M and in Scenario BR_3, the subjects become extremely risk averse as the loss probability is “overweighted and overestimated".

Figure 4.2: Insurance decision results in Session I and II



(a) Insurance decisions for Scenario A, M. Mp in Session I

(b) Insurance decisions for Scenario W, B_i, and BR_3 in Session II for Group I, II, and III

Following, I introduce the hypotheses to test the influences of numerical literacy and decision-making time on the decision anomalies.

H1-1: Numerate subjects tend to make consistent decisions in Scenario M and in Scenario Mp. (Correlation between numeracy and probability miscalculation in Session I)

H1-2: Numerate subjects tend to make consistent decisions in Scenario Mp and in Scenario A. (Correlation between numeracy and myopic loss aversion in Session I)

Susceptible to representative and availability biases discussed in Tversky and Kahneman (1974), a DM tends to overestimate a salient risk's loss probability as the base rate is ignored. Therefore, WTP can be much higher for insurance in Scenario M than in Scenario Mp. From Figure (4.2(a)), participants are more likely to purchase expensive insurance in Scenario M than in Scenario Mp even though the loss probabilities and hence the covered risks in these two scenarios are identical. Numerical literacy may reduce this decisions bias. If numerate subjects can estimate the loss probability fairly, their insurance purchase decisions in Scenario M and in Scenario Mp will be closer.

Additionally, as myopic-risk-averse agents, the subjects' WTP^{Mp} is higher than WTP^{A} if the monthly insurance contract reduces the evaluation period in Scenario Mp into one month. Hypothesis H1-2 tests if less numerate subjects are more prone to the contract influence on the risk editing.

H2: The subjects, first asked to calculate the loss probability in Scenario M before making the insurance decision, tend to make consistent decisions in Scenario M and in Scenario Mp . (Correlation between numeracy and probability miscalculation in Session I)

A rational DM is assumed to make insurance-purchase decisions based on the loss probability as well as the claim size. However, Kunreuther and Pauly (2006) asserts that the coverage-premium ratio is often focused while the loss probability is neglected. Additionally, DMs cannot differentiate the salience risk in its loss probability or in its claim size. Unlike the risk described in Scenario Mp, the risk in Scenario M is predominant, while its loss probability is not explicitly specified. A decision anomaly due to loss probability miscalculation is therefore expected if the subjects make the decision with heuristic. As Stanovich and West (2008) suggests, if a potential pitfall in the decision-making process is warned, numerate or intelligent subjects make better decisions by using the analysis system. Otherwise, even though they are capable of the loss-probability calculation in Scenario M, they do not bother with this calculation and simply employ the heuristic system. This system is prone to bias. Therefore, bringing the loss probability into focus as a warning may reduce the bias caused by the miscalculation.

In Scenario M, the subjects were randomly separated into two groups. In the experiment group, the subjects were asked to calculate the loss probability before specifying their WTP^M. On the contrary, the subjects in the control group first determined their WTP^M and then calculated the loss probability. This treatment is to test if nudging a DM to focus on the probability reduces the decision bias. Thereby, if an appropriate computation is feasible and encouraged, WTP^M would be much closer to WTP^{Mp}.

H3: Numerate subjects spend less on the block insurance. (Correlation between numeracy and the possible decision biases in Session II)

In Session II, the subjects are more likely to purchase insurance for the final block (Scenario B_{12}) even though the risks in scenario B_i for Group I (except for BR_3) and for Group II are the same (cf. Figure (4.2(b)). Chang and Schmeiser (2021) explains this high insurance demand in Scenario B_{12} with myopic loss aversion. As the risk period is reduced to one block, the loss probability is overweighted to a higher degree. Comparing the decisions in Scenario B_{12} made by the subjects with different numerate degrees, I test if numerical literacy correlates with the probability distortion degree. The probability distortion is generally considered as risk preference rather than bias (Tversky and Kahneman (1992) and Quiggin (1982)). Therefore, numerical literacy is expected to be independent from the distortion.

For those who have rejected to buy the whole-bridge insurance, a normative decision is to reject all of the block insurances.[48] However, due to the decision biases, block insurance may appear attractive. As suggested in Epper and Fehr-Duda (2018), risk can be segregated when it is resolved more frequently. With block insurance offered, risk is resolved gradually in a block sequence. Thereby, one-bridge risk is segregated into twelve block risks, each with the evaluation period equal to one block. When one risk is segregated into multiple risks, can the subjects aggregate these segregated ones back by themselves? Cillo and De Giorgi (2017) shows that sophisticated DMs are better at aggregation and make more consistent decisions when risks are presented segregated and aggregated. Risk segregation stimulates myopic loss aversion in Session II. If the risk aggregation requires numerical literacy, the numerate subjects are expected to be more risk tolerant. Numerical literacy is also vital in the loss probability calculation. Therefore, if the numerate subjects are more likely to assess salience risk appropriately, they are less likely to purchase insurance in BR_3.

H4-1: Those who spend more time making decisions tend to make consistent decisions in Scenario M and in Scenario Mp. (Correlation between decision-making time and probability miscalculation in Session I)

H4-2: Those who spend more time making decisions tend to make consistent decisions in Scenario A and in Scenario Mp. (Correlation between decision-making time and myopic loss aversion in Session I)

These two hypotheses further test if the time spent on the decision-making process influences the decision quality, i.e. tendency of normative decisions. The dual-process theory in Kahneman and Frederick (2002) suggests that system I is used to make fast decisions. Without making much effort, this system is usually based on heuristic and therefore is susceptible to biases. On the contrary, System II requires effort and time to process the information before taking a decision. In Session I, those who make quick decisions are expected to utilize System I. Assessing risk with heuristic, they are prone to decision biases. Due to loss probability miscalculation,

[48] The normative decision is based on the expected-loss information. The block insurance's unit rate is generally much higher than the whole-bridge insurance's rate except for Scenario BR_3, where its unit rate is close to the whole-bridge insurance as shown in Table(4.3).

they overestimate the loss probability and therefore are willing to pay a higher premium in Scenario M compared to that in Scenario Mp. Additionally, the monthly insurance segregates the annual risk, while the risk aggregation demands mental effort. Without making such an effort, those who make decisions with heuristic are more risk averse in Scenario Mp than in Scenario A because of myopic loss aversion. In contrast, a slow DM takes time to process the relevant information and makes rational responses with System II. Thereby, the differences between WTP^M and WTP^{Mp} and between WTP^{Mp} and WTP^A are smaller when slow decisions are made.

H5: Those who spend less time making decisions on block insurance spend less on the block insurance. (Correlation between decision-making time and possible decision biases in Session II)

Unlike those in Session I, the prospect resolutions in Session II have monetary impact. This monetary impact together with vivid animation arouse emotions, such as fear and anxiety. On the one hand, slow DMs may spend time on the loss probability calculation. On the other hand, the hesitation evoked by the anxiety could lead to slow decisions. This hesitation reduces the crossing-bridge speed and each block risk is more likely to be perceived individually. The evaluation period is therefore shortened to block units, provoking myopic loss aversion. Additionally, anxiety and fear, inducing and induced by the hesitation, hinder a DM's cognitive evaluation. Therefore, in Session II, the monetary rewards may interfere with the subjects' risk assessment process and prevent them from behaving rationally. This monetary impact enlarges when they spend more time on each block, making them become more risk averse. On the contrary, segregated block risks can be aggregated if a DM deliberately spends little time on each block and, therefore, the focused risk resolves only once (either Tommy falls on a certain block or reaches the end of the bridge). Thereby, the DM becomes more risk tolerant when making fast decisions to aggregate each block risk.

The following table summarizes all the hypotheses.

Table 4.6: Summary of the hypotheses

H1-1	Numerate subjects tend to make consistent decisions in Scenario M and in Scenario Mp.
H1-2	Numerate subjects tend to make consistent decisions in Scenario Mp and in Scenario A.
H2	The subjects, first asked to calculate the loss probability in Scenario M before making the insurance decision, tend to make consistent decisions in Scenario M and in Scenario Mp.
H3	Numerate subjects spend less on the block insurance.
H4-1	Those who spend more time making decisions tend to make consistent decisions in Scenario M and in Scenario Mp.
H4-2	Those who spend more time making decisions tend to make consistent decisions in Scenario A and in Scenario Mp.
H5	Those who spend less time making decisions on the block insurance spend less on the block insurance.

4.4 Result

H1-1: Numerate subjects tend to make consistent decisions in Scenario M and in Scenario Mp.

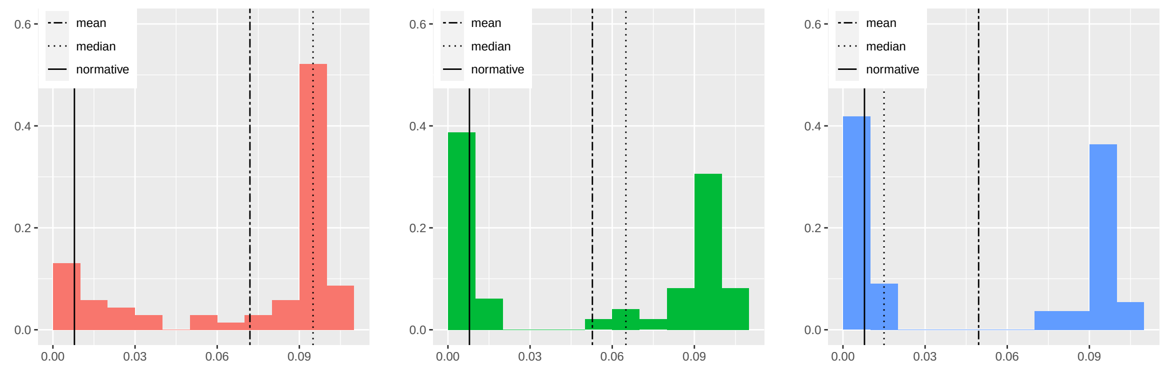
A rational DM calculates the loss probability equal to 0.79% (with the choice “less than 1%"). Figure (4.3) shows the distributions of the estimated loss probability in Scenario M per each numerate group. These distributions concentrate on two ranges, $0-1\%$ and $9\%-10\%$. For the latter, the subjects calculate the loss probability without taking time difference into account (base rate fallacy).[49]

Comparing WTP^M with WTP^{Mp}, log(WTP^M/WTP^{Mp}) measures the bias degree of loss probability miscalculation. As the risks in Scenario M and Mp are identical, log(WTP^M/WTP^{Mp})=0

[49] In one year, the loss probability is 5%. Heuristic mistakenly estimates the loss probability of the salient risk in one month as $2\times5\%$, the double of the annual probability.

for the normative decision. Presented in Figure (4.4), the distribution of $\log(WTP^M/WTP^{Mp})$ are right skewed for all the three numerate groups. Thereby, in each numerate group, there exist subjects who overestimate the loss probability substantially and are willing to pay a much higher premium in Scenario M compared that in Scenario Mp. The distribution for the non-diligent group possesses the heaviest tail. Therefore, non-diligent subjects are generally more likely to overestimate the loss probability and are willing to pay a higher insurance premium in Scenario M.

Figure 4.3: Estimated loss probability in Scenario M per each numerate group: Non-Diligent, Ordinary, and Numerate

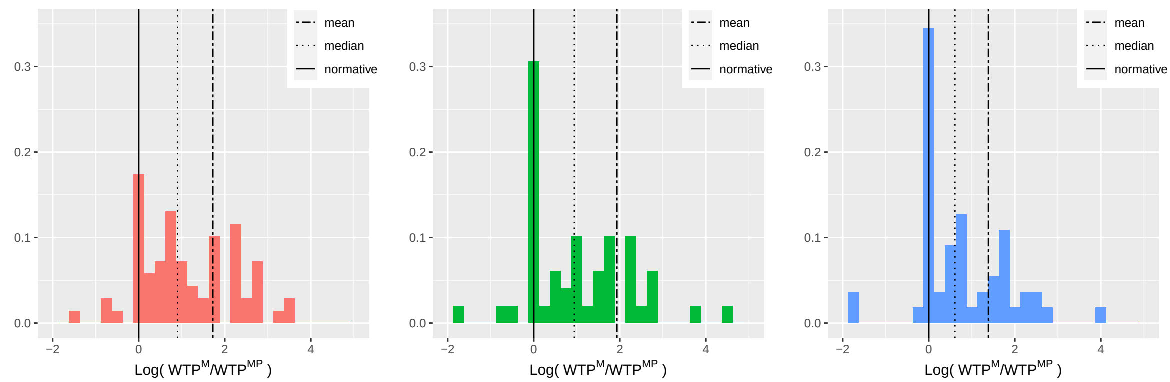


(a) Non-diligent subjects: (mean 7.20% and median 9.50%)

(b) Ordinary subjects: (mean 5.28% and median 6.50%)

(c) Numerate subjects: (mean 4.95% and median 1.50%)

Figure 4.4: $\log(WTP^M/WTP^{Mp})$ per each numerate group: Non-Diligent, Ordinary, and Numerate



(a) Non-diligent subjects: (mean: log(5.59); median log(2.46)

(b) Ordinary subjects: (mean: log(6.90); median: log(2.56))

(c) Numerate subjects: (mean: log(3.99); median log(1.83))

As demonstrated in Table (4.7), by the Spearman test, the correlation between Numerate Degree and the estimated loss probability is significantly negative. It suggests that the subjects with lower numerate degree are more likely to overestimate the loss probability. With a positive correlation between WTP^M/WTP^{Mp} and the calculated loss probability, the subjects generally make the decision based on their calculated loss probability. However, the correlation per numerate group in Table (4.8) shows that non-diligent subjects consider little about the loss probability. It is confirmed that the numerate subjects make more consistent risk choices in Scenario M and in Scenario Mp as they are more likely to calculate the loss probability appropriately and incorporate this information during the decision-making process.

Table 4.7: Correlation between numerate degree, estimated loss probability in Scenario M, and the decision consistencies (i.e. WTP^M/WTP^{Mp} and WTP^{Mp}/WTP^A)

Correlation Matrix	Numerate Degree	Calculated Loss Probability
Calculated Loss Probability	-0.23**	
WTP^M/WTP^{Mp}	-0.15*	0.25**
WTP^{Mp}/WTP^A	-0.13	

Note: Spearman test for the null hypothesis that no association exists between the variables ($*$ $p < 0.05$; $**$ $p < 0.01$)

Table 4.8: Correlation between WTP^M and the estimated loss probability per each numerate group: Non-Diligent, Ordinary, and Numerate

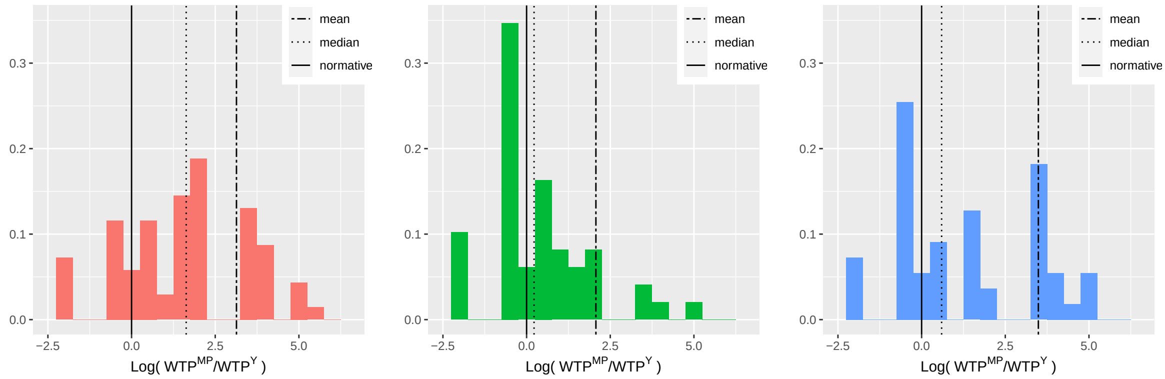
	Cor	p-value		Cor	p-value		Cor	p-value
Non-Diligent	0.01	0.93	Ordinary	0.29*	0.04	Numerate	0.42**	$< 10^{-2}$

Note: Spearman test for the null hypothesis that the correlation between WTP^M and the estimated loss probability is zero ($*$ $p < 0.05$; $**$ $p < 0.01$)

H1-2: Numerate subjects tend to make consistent decisions in Scenario Mp and in Scenario A.

Regarding the correlation between Numerate Degree and myopic loss aversion tested in H1-2, Figure (4.5) exhibits the distribution of $\log(WTP^{Mp}/WTP^A)$ per numerate group. From Table 4.7, WTP^{Mp}/WTP^A is negatively correlated with Numerate Degree as well. However, the significance is minor with p value equals 0.08. In Session I, though the objective risk period is set in one year, without clear monetary structure, it can be influenced by the insurance duration. Therefore, even if the numerate subjects can aggregate risks better, when facing monthly insurance, they behave as myopic loss averters as low numerate subjects do.

Figure 4.5: log (WTP^{Mp}/WTP^{A}) per each numerate group: Non-Diligent, Ordinary, and Numerate



(a) Non-diligent subjects: (mean: log(23.00); median log(5.11))

(b) Ordinary subjects: (mean: log(7.93); median log(1.25))

(c) Numerate subjects: (mean: log(32.7); median log(1.82))

H2: The subjects, first asked to calculate the loss probability in Scenario M before making the insurance decision, tend to make consistent decisions in Scenario M and in Scenario Mp.

Table (4.9) shows that with the Wilcoxon Rank-Sum Test, the Numerate Degree in the experiment group (first loss probability then WTP^{M}) are not significantly different from the degree in the control group (first WTP^{M} then loss probability). The Wilcoxon Rank-Sum Test can neither conclude that WTP^{M}/WTP^{Mp} and calculated probabilities in these two groups are significantly different. Those who are able to do proper calculation make their decisions based on their calculated probability even if they are not asked to perform the computation first. However, for those who cannot or do not quantify the risk fairly, bringing their attention to the loss probability does not reduce their miscalculation bias.

Table 4.9: Mean of Numerate Degree, calculated loss probability, and WTP^M/WTP^{Mp} for the experiment group and for the control group in Scenario M

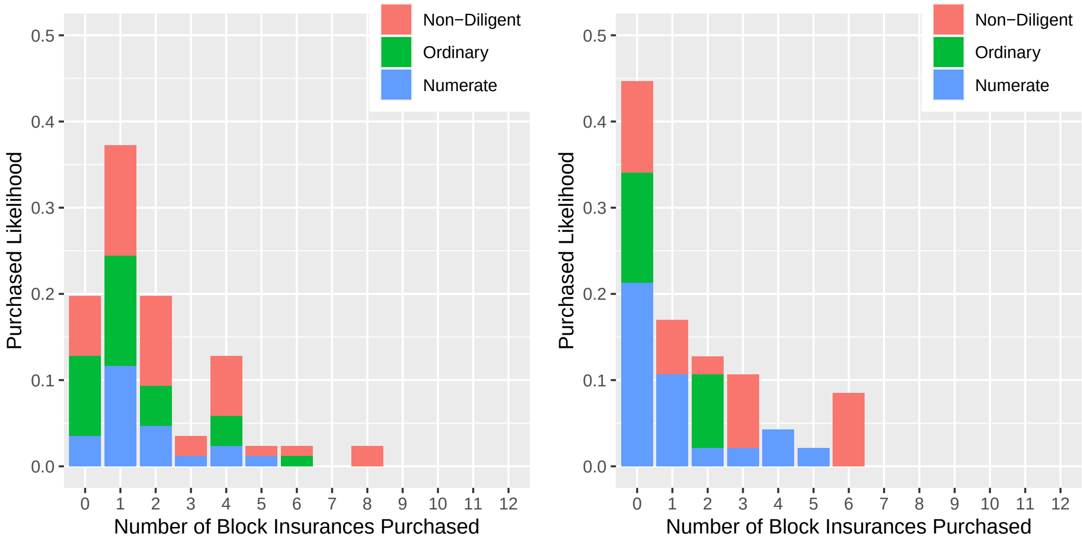
		All Subjects	Non-Diligent	Ordinary	Numerate
Number of Obs	Experiment	84	33	26	25
	Control	89	36	23	30
Numerate Degree	Experiment	1.90			
	Control	1.93	NA	NA	NA
	p-Value†	0.85			
Calculated Loss Probability	Experiment	0.06	0.06	0.06	0.05
	Control	0.06	0.08	0.05	0.05
	p-Value†	0.46	0.06	0.32	0.79
WTP^M/WTP^{Mp}	Experiment	6.13	5.04	8.21	5.41
	Control	6.13	6.10	5.42	2.81
	p-Value†	4.82	0.82	0.59	0.20

Note: † Wilcoxon rank-sum test is performed with the null hypothesis: the distributions of the experiment group and of the control group are equal.

H3: Numerate subjects spend less on the block insurance.

12 out of the 173 subjects preferred CHF 10 with 100% to CHF 32 with 95% and purchased the whole-bridge insurance in Scenario W. The rest of the subjects in Group I and II took the risk, while crossing the bridge per block. Figure (4.6) exhibits the number of the block insurances purchased in each group. The bridge in Group I includes one risky block (Scenario BR_3), whose risk is double the risk in each other block. More than 35% of the subjects in Group I purchased one block insurance on the bridge, and almost 95% of them took that insurance in Scenario BR_3. In Group II, where all the blocks have equal breaking probabilities, almost 40% of the subjects did not acquire any of the block insurances.

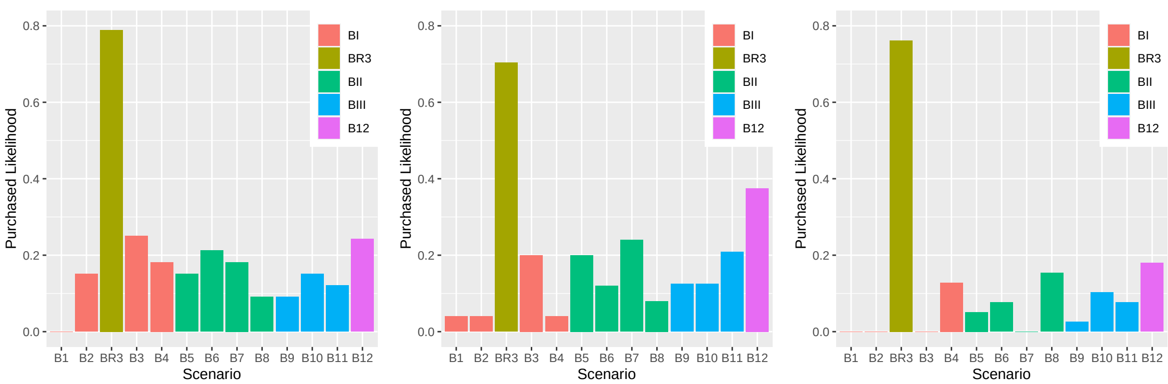
Figure 4.6: Distribution of the number of block insurances purchased in Session II



(a) Group I (b) Group II

The bridge's twelve blocks are separated into five categories: BI, the first category includes the first part of the normal-risk blocks (B_i, $i = 1,2,4$ for Group I and $i = 1\ldots4$ for Group II). Scenario B_i, $i = 5\ldots8$ and B_i, $i = 9\ldots11$ are grouped into the second and the third category as BII and BIII respectively. The subjects make quite distinct decisions in Scenario BR_3 and B_{12}. Thereby, BR_3 and B_{12} are two separate categories as BR3 and B12. Figure (4.7) exhibits the insurance purchase likelihood per block per numerate group and Table (4.10) summarizes this likelihood per category and per numerate group. It shows that the low-numerate subjects are more likely to acquire block insurances especially in BI and BII. Additionally, regardless of their numerical literacy, the subjects are more likely to purchase insurance in BR3 and B12.

Figure 4.7: Insurance purchase decision per each block in Session II



(a) Non-diligent subjects (b) Ordinary subjects (c) Numerate subjects

Table 4.10: Insurance decisions in Session II per each numerate group: Non-Diligent, Ordinary, and Numerate

		Non-Diligent	Ordinary	Numerate
Average of the Purchased Likelihood per Block in each Category (%)				
BI	B_i, $i = 1,2,4$ for Group I and $i = 1...4$ for Group II	10.00	3.30	2.79
BII	B_i, $i = 5...8$	16.82	6.08	6.09
BIII	B_i, $i = 9...11$	14.20	6.31	12.50
BR3	BR_3 for Group I	78.94	70.37	76.19
B12	B_{12}	31.48	16.21	30.00
		Non-Diligent	Ordinary	Numerate
Number of insured blocks		2.15	1.18	1.33
Earned rewards in Session II (CHF)		22.97	27.87	25.65

Spearman test is performed in Table (4.11) to examine the correlation between the numerical literacy and the insurance decisions as well as the final monetary rewards. As the block insurance premium is much higher than the expected loss, those who spend more on the block insurance are expected to receive less rewards. Except for the monetary rewards, insurance decisions are negatively correlated with the Numerate Degree. Thereby, the subjects with higher Numerate Degree are less likely to purchase block insurance and hence receive higher rewards. The negative correlation between Numerate Degree and the insurance decision is especially significant in BI and BII, where the objective risk periods are much longer than the insurance period (i.e. block unit). While these long risk periods are segregated by each block insurance, the numerate subjects aggregate the segregated risks and consider those risk not covered by the block insurance. On the contrary, with the evaluation period reduced to a block unit, the non-diligent subjects become more risk averse and find expensive block insurance attractive. As the game proceeds and the objective risk period shrinks, the magnitude and the significance of the correlation decreases. Though the numerate subjects are better at aggregating risks, numerical literacy is independent from the probability distortion. For the latter, it is generally considered as risk preference instead of decision bias. Thereby, due to myopic loss aversion, the reduced objective risk period evokes the subjects' risk aversion regardless of their numerical capabilities.

Wilcoxon rank-sum test in Table (4.12) examines if Numerate Degrees of those who purchased insurance and of those who did not are discrete. Consistent with what the Spearman test suggests, a significant difference exists in the initial part of the bridge (BI and BII). However, it cannot reject the null hypothesis that the two numerate distributions are equal for BIII and B12. Though, it is confirmed in Hypothesis H1-1 that the numerate subjects are more likely to assess the salient risk fairly, numerical literacy does not remedy the probability miscalculation bias when the decision is monetarily incentivized. Thereby, Numerate Degrees of those who purchased insurance in BR_3 are not significantly different from those who did not.

The result shows that the numerate subjects are less susceptible to the insurance's risk-editing influence. In contrast, non-diligent subjects' risk preference can be easily manipulated by offering insurance with various durations. As they are more likely to purchase expensive block insurances in BI and BII, they generally acquired more over-priced block insurances and end up with lower rewards.

Table 4.11: Correlation between numerate degree and block insurance decisions in Session II

Correlation between Numerate Degree and variable $v^\dagger$		
v	Cor	p-Value
Number of insured blocks per Category in BI	-0.25 **	$< 10^{-2}$
Number of insured blocks per Category in BII	-0.21 *	0.02
Number of insured blocks per Category in BIII	-0.05	0.55
Number of insured blocks for the whole bridge	-0.18 *	0.03
Earned rewards in Session II (CHF)	0.21 *	0.01

Note: †*Spearman test for the null hypothesis that no association exists between Numerate Degree and v*
(* $p < 0.05$; ** $p < 0.01$)

Table 4.12: Numerate degree difference between those who purchased insurance and those who did not per each block in category in Session II

	Numerical Degree (Mean)		
Category	Purchased	Not Purchased	p-Value†
BI	1.46	1.94	0.005
BR3	1.86	1.78	0.682
BII	1.52	1.94	$< 10^{-3}$
BIII	1.82	1.90	0.492
B12	1.86	1.91	0.714

Note: †Wilcoxon rank-sum test is performed with the null hypothesis: the Numerate-Degree distributions for those who purchased insurance and for those who did not are equal.

H4-1: Those who spend more time making decisions tend to make consistent decisions in Scenario M and in Scenario Mp.

H4-2: Those who spend more time making decisions tend to make consistent decisions in Scenario A and in Scenario Mp.

In Session I, each scenario is described in one web page followed by another page where the subjects can choose whether they are willing to purchase the insurance with a specific premium. Time spent on each web page is recorded. The decision-making time for a certain scenario is determined by the time spent on the web page where the first premium choice appears. As the subjects differ over the speeds of general reaction and of general reading habit, the relative time is considered as the absolute time divided by the general time (average time spent on web-pages, where the descriptions for Scenario A, M and Mp are given). t denotes this relative decision making time with t^A, t^M, and t^{Mp} for each scenario A, M, and Mp. The median of the relative time per each numerate group and the correlation between the time and the numerate degree is presented in Table (4.13). The correlation between the decision making time and the numerate degree is significantly negative for Scenario A and Mp while the magnitude and the significance of the correlation is minor in Scenario M. Thereby, generally, non-diligent subjects spend more time on decision making. In Scenario M, numerate subjects recognize that heuristic decision may cause possible bias, they therefore employ their cognitive system to assess the risk.

Table 4.13: Mean of relative decision making time in Session I and Cor: correlation between decision-making time and numerate degree

	Median of Time(Minutes)				
Scenario	All Subjects	Non-Diligent	Ordinary	Numerate	Cor (p-Value)†
Scenario A	0.70	0.84	0.66	0.60	-0.18 (0.01)
Scenario M	0.50	0.54	0.53	0.38	-0.09 (0.25)
Scenario Mp	0.25	0.35	0.20	0.23	-0.21 ($< 10^{-2}$)

Note: †Spearman test for the null hypothesis that no association exists between numerate degree and decision time

Table (4.14) shows the Spearman test result for the correlation between insurance decision and the decision-making time. The positive correlation especially in scenario A and Mp suggests that risk averters hesitate to make decisions. Thereby, slow decision makers tend to have higher willingness to pay for the risk coverage. However, slow decisions can be caused by the usage of the cognitive system, with which decision biases such as loss probability miscalculation can be reduced. Thereby, on the one hand, a slow decision made by a hesitant risk averter generate high WTP^{M}. On the other hand, if this slow decision is made with the cognitive system, WTP^{M} decreases as the risk is more likely to be assessed fairly. The non-diligent subjects make slow decisions mainly due to the hesitation as the correlation in Scenario M is significantly positive. However, for the ordinary and numerate subjects, this correlation coefficient is small and insignificantly different from zero.

If the numerate subjects spend more time on the decision in Scenario M compared to that in Scenario Mp, they are more likely to make rational decisions without overestimating the loss probability. Thereby, Hypothesis H4-1 is confirmed only for the numerate subjects with negative correlation between WTP^{M}/WTP^{Mp} and t^{M}/t^{Mp}. It indicates that slow decisions made by the numerate subjects tend to be generated by cognitive system. t^{Mp}/t^{A} is not significantly correlated with WTP^{Mp}/WTP^{A}, which denotes myopic loss aversion degree. Thereby, Hypothesis H4-2 cannot be confirmed for all the numerate groups.

Table 4.14: Correlation between v1, the insurance purchase decisions and v2, the time spent on the decision, for all the subjects and per each numerate group

v1	v2	All Subjects	Non-Diligent	Ordinary	Numerate
WTP^A	t^A	0.25**	0.07	0.38**	0.40**
WTP^M	t^M	0.11	0.24*	-0.05	0.034
WTP^{Mp}	t^{Mp}	0.16*	0.11	0.05	0.28*
H4-1 WTP^M/WTP^{Mp}	t^M/t^{Mp}	0.00	0.17	0.05	-0.27*
H4-2 WTP^{Mp}/WTP^A	t^{Mp}/t^A	0.10	0.03	0.02	0.20

Note: Spearman test for the null hypothesis that no association exists between v1 and v2 ($p < 0.05$; ** $p < 10^{-2}$; *** $p < 10^{-3}$; *** $p < 10^{-4}$)*

H5: Those who spend less time making decisions on the block insurance spend less on the block insurance.

In Session II, on each block, the subjects make decisions whether to purchase insurance for the next block. The time staying on one block is recorded as the decision-making time for the next block. Twelve blocks are grouped into five categories as in Hypothesis H3 (cf. Table(4.10)). Figure (4.8) exhibits the average time spent per block (i.e., logarithm of time in seconds) per each category. The subjects are shown to spend more time in the initial part of the bridge. Group I also made slow decisions in BR3, where the risk is salient.

Figure 4.8: Average time spent per block (logarithm of time in seconds) per each category

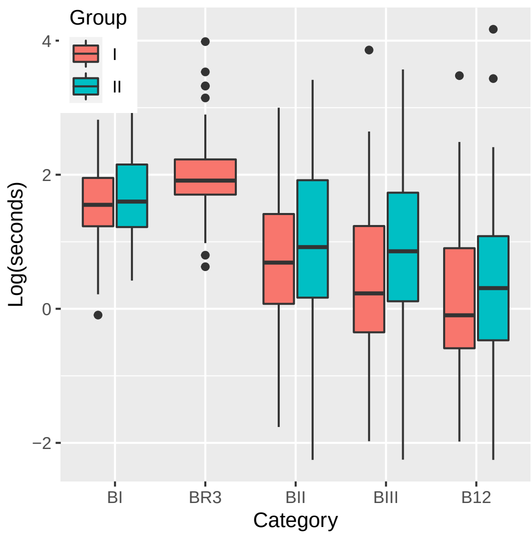


Table (4.15) demonstrates the correlation between the average decision-making time per block and the number of the insured block per each category. Most of the correlation coefficients are positive, suggesting that instead of the cognitive computation, the slow decisions are generally caused by the hesitation and this hesitation leads to risk aversion. Table (4.16) examines if those who purchased insurance and those who did not spent time differently on making these decisions. In the initial part of the bridge (BI), the subjects spent time familiarizing themselves with the game while designing certain strategies. Therefore, the time spent on each block is not significantly correlated with the tendency of block-insurance purchase. When facing salient risk in BR3, the subjects in Group I may spend time calculating the loss probability rationally or hesitate to make decisions as they are overwhelmed by the anxiety. Therefore, a slow decision may lead to either rational (fairly assess the salient risk and not purchase the block insurance) or irrational (overestimate the salient risk and purchase the insurance) behavior. Therefore, no significant correlation is observed in BR3.

The correlations between the time and the insured block number are significantly positive as Tommy approaches the end of the bridge. Wilcoxon rank-sum test in Table (4.16) rejects the null-hypothesis that the decision-making time distributions for those who purchased and those who did not are the same. Thereby, in BII, BIII, and B12, the slow decision makers behave more risk averse. As the subjects who made decisions on block insurance purchase had rejected the whole-bridge insurance, which is much cheaper than the block insurance in terms of rate per unit risk, purchase in any block insurance can be seen as irrational behavior. As irrational behavior, the excessive risk aversion is evoked when the risk is segregated by a series of short-term insurance products. Spending time contemplating does not improve the decision rationality. Especially in the later part of the bridge, with the risk evaluation period decreases gradually, the subjects become increasingly anxious. This anxiety caused hesitation and prevented the subjects from analyzing the risk appropriately for all numerate groups. Therefore contrary to the dual-process theory, where the slow decision is assumed cognitive and analytic, when the DM is overwhelmed by the anxiety and fear, taking time does not enhance the decision rationality. Instead, being exposed to the anxious emotion for a longer time, the DM is more likely to behave irrationally risk averse.

Table 4.15: Correlation between average time spent per block and the number of insured block per category in Session II

	All Subjects		Non-Diligent		Ordinary		Numerate	
Category	Cor	p-Value	Cor	p-Value	Cor	p-Value	Cor	p-Value
BI	0.12	0.18	0.03	0.84	0.28	0.09	0.12	0.45
BII	0.47 ****	$< 10^{-4}$	0.37 **	0.01	0.67 ****	$< 10^{-4}$	0.38 *	0.01
BIII	0.51 ****	$< 10^{-4}$	0.56 ****	$< 10^{-4}$	0.47 **	$< 10^{-2}$	0.45 **	$< 10^{-2}$

Note: Spearman test for the null hypothesis that no association exists between average time spent per block and the number of insured block ($p < 0.05$; ** $p < 10^{-2}$; *** $p < 10^{-3}$; **** $p < 10^{-4}$)*

Table 4.16: Decision-making time for those who purchased insurance and those who did not per each block in category in Session II: T1: median time in seconds for the purchase decision; T2: median time in seconds for the non-purchase decision; p-Value with Wilcoxon test.†

	All Subjects		Non-Diligent		Ordinary		Numerate	
	(T1, T2)	p-Value	(T1, T2)	p-Value	(T1, T2)	p-Value	(T1, T2)	p-Value
BI	(6.02, 4.25)	0.03	(6.02, 4.53)	0.07	(13.66, 4.00)	0.23	(4.61, 3.67)	0.61
BR3	(6.77, 6.67)	0.71	(6.66, 6.82)	0.86	(7.18, 8.13)	0.94	(6.70, 4.18)	0.24
BII	(4.14, 1.66)	$< 10^{-4}$	(3.58, 1.69)	$< 10^{-4}$	(5.70, 1.66)	0.002	(4.84, 1.52)	0.009
BIII	(4.85, 1.15)	$< 10^{-4}$	(4.85, 1.12)	$< 10^{-4}$	(1.12, 3.06)	0.007	(8.69, 1.10)	$< 10^{-3}$
B12	(2.75, 0.69)	$< 10^{-4}$	(2.59, 0.63)	$< 10^{-4}$	(2.50, 0.84)	0.009	(3.08, 0.73)	$< 10^{-3}$

Note: Wilcoxon rank-sum test is performed with the null hypothesis: the decision-making time distributions for the purchase decision and for the no-insurance purchase decision are equal.

4.5 Conclusion

Decision anomalies are noted as DMs' risk preference variations against the same risk. These anomalies can be triggered when the risk is presented differently. Thereby, the DM's risk preference can be "manipulated" by changing the way they edit the risk. This paper examines if the numerical literacy reduces the tendency of the decision anomalies and the numerate DM is less subject to the manipulation of insurance products with the experiment data from Chang and Schmeiser (2021). In this experiment, the subjects are asked to make both hypothetical and incentivized insurance decisions in various scenarios, where the risk is presented (framed) within different periods.

When a long-term risk is segregated by several short-term insurance contracts, the results show that the numerate subjects are more likely to estimate the loss probabilities of segregated salient risk appropriately. In contrast, the low-numerate subjects tend to overestimate salient risk due to the base rate fallacy. Additionally, bringing the probability information into focus does not enhance the rationality. Therefore, those who can calculate the loss probability properly make decisions based on the fairly assessed risk even if the probability information is not emphasized. As suggested in Peters et al. (2006), Peters (2012) and Petrova et al. (2014), without monetary incentives, the numerate subjects incorporate the estimated probability while making decisions, whereas the low-numerate subjects fail to do so. Due to the bias of representativeness, the low-numerate subjects behave as extreme-risk averse agents when facing segregated salient risk. However, when monetary incentives are involved, all the subjects are exposed to emotions such as anxiety. Therefore, they tend to act extremely risk averse when the risk is presented salient.

Myopic loss aversion occurs when the risk evaluation period is reduced and the divided loss probability is overweighted. As discussed in Barberis (2013), probability distortion is considered as preference rather than bias. In the experiment, the subjects decided whether to buy a block insurance while crossing the bridge. Even though most of the blocks have the same breaking probability, regardless of their numerical literacy, all the subjects tended to be more risk averse when facing the last block of the bridge (Scenario B_{12}), where the evaluation period is reduced to one block. Insurance products can further influence the way DM edits the risk prospects by altering their contract periods. As insurance is offered with short-term period, the subjects consider risk in a reduced time period. As this evaluation period is shortened, WTP is higher for a monthly insurance contract compared to that for an annual contract in terms of the risk unit rate. This evaluation period can, however, be controlled by other external factors. In this experiment, as the subjects received their rewards when Tommy reached the end of the bridge, the risk period can be objectively determined as the remaining block Tommy has yet to cross. Block insurance reduces this period into block unit and segregates the whole-bridge risk. While the numerate subjects are able to aggregate these segregated risks, block insurance reduces the low-numerate subjects' evaluation period and triggered their risk aversion. As a result, the numerate subjects received higher rewards than did the low-numerate subjects did. In a real world environment, Benartzi and Thaler (1995) employed myopic loss aversion to explain the equity premium puzzle: the equity returns appear to be much more volatile if the evaluation period is shortened. As numerate individuals are better at aggregating risks, Almenberg and Widmark (2011) confirmed with empirical data in Sweden that those with high numeracy are more likely to participate in a stock market.

Generally, the results also show that taking time to make decisions does not improve the decision rationality. It contradicts with what Kahneman and Frederick (2002) documents: a rational decision is expected when the DM takes time to analyze its risk with analytic/cognitive system. Otherwise, people make fast decisions based on a hunch and subject to biases. However, in the loss domain, negative emotions such as anxiety can be provoked while analysing risk prospects. This negative emotion prevents DMs from cognitive computation. As discussed in Loewenstein et al. (2001), emotion takes a central role in the decision-making process. Instead of cognitive evaluations, emotion appraisals are prone to decision biases, such as loss probability overestimation of salient risks. In Session I, DM does not behave more rationally when more time is spent on making decisions. When the monetary incentive is involved, slow decisions are caused by the hesitation in the emotional system. The slower the decision are made, the longer the DMs are exposed to the anxiety emotion. The more anxious DMs are, the longer the time they spend on making decisions and more risk-averse (especially myopic loss averse) they become. Thereby, slow decision makers are more susceptible to myopic loss aversion and behave as extreme risk-averse agents.

This paper examines if numerical literacy reduces the two decision biases: probability miscalculation and myopic loss aversion. The numerate DMs are shown to estimate the salient risk more properly. However, when monetary incentive is involved, they are susceptible to these biases and become as extreme risk averse as non-numerate are. Myopic loss aversion is a risk preference rather than a decision bias, and is therefore independent from the numerical literacy. However, numerate DMs aggregate the segregated risks better and can resist the evaluation-period manipulation by risk-related product (i.e. insurance contract or financial contract). Moreover, risk stimulates emotions. Therefore, taking time to make decision does not always improve the decision rationality.

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